



Two- and Multi-particle Cumulant Measurements of v_n and Isolation of Flow and Nonflow in $\sqrt{s_{NN}} = 200$ GeV Au+Au Collisions by STAR

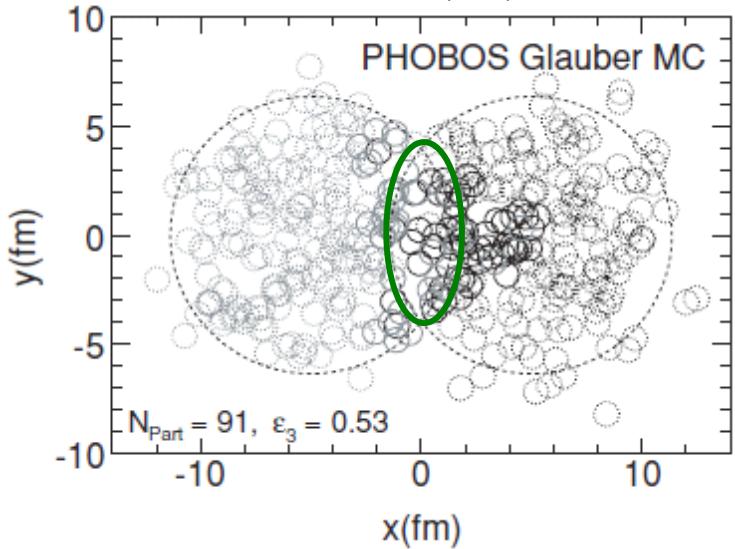
Li Yi (for the STAR collaboration)
Purdue University

Outline

- Physics motivation
- Results
 - 2- and multi-particle anisotropy
 - 2- and 4-particle η - η cumulants
→ Isolation of flow and nonflow
- Summary

Azimuthal Anisotropy Flow and Nonflow

Alver, Roland, PRC 81, 054905 (2010)

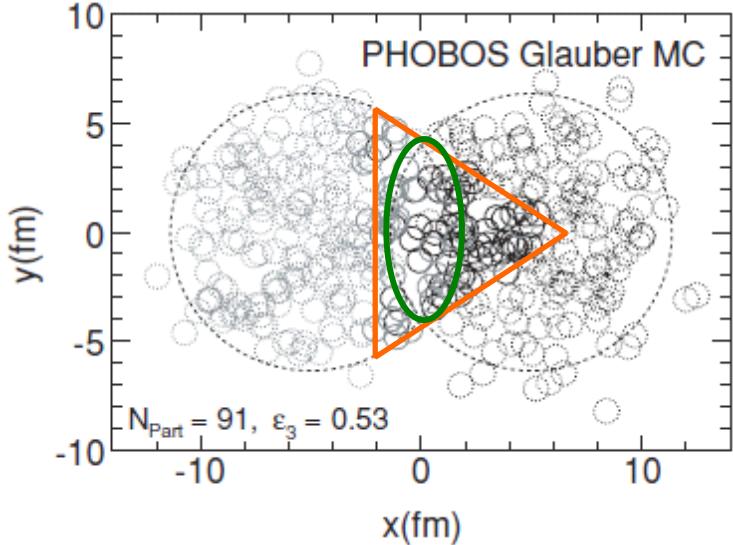


- Hydrodynamic expansion
→ anisotropic flow;
- Flow is sensitive to early stage of heavy ions collisions

$$dN/d\phi \propto 1 + \sum_{n=1}^{\infty} 2\nu_n \cos(n(\phi - \Psi_{nR}))$$

Azimuthal Anisotropy Flow and Nonflow

Alver, Roland, PRC 81, 054905 (2010)



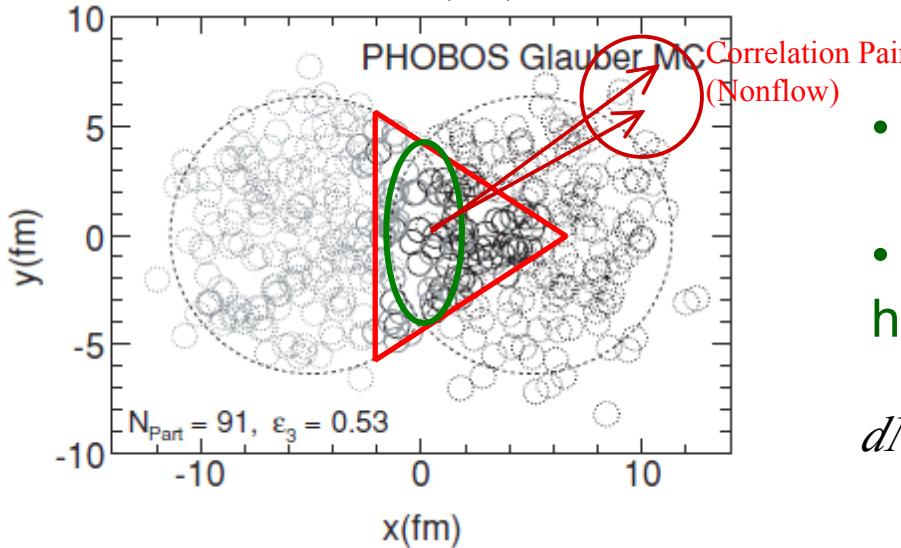
- Event-by-event initial geometry fluctuation
→ odd harmonics

- Hydrodynamic expansion
→ anisotropic flow;
- Flow is sensitive to early stage of heavy ions collisions

$$dN/d\phi \propto 1 + \sum_{n=1}^{\infty} 2\nu_n \cos(n(\phi - \Psi_{nR}))$$

Azimuthal Anisotropy Flow and Nonflow

Alver, Roland, PRC 81, 054905 (2010)



- Hydrodynamic expansion
→ anisotropic flow;
- Flow is sensitive to early stage of heavy ions collisions

$$dN/d\varphi \propto 1 + \sum_{n=1}^{\infty} 2v_n \cos(n(\varphi - \Psi_{nR}))$$

- Event-by-event initial geometry fluctuation
→ odd harmonics
- The reaction plane azimuthal angle is unknown
→ the measured anisotropies = flow(v) + flow fluctuation (σ) + nonflow (δ)

particle correlation unrelated to the reaction plane

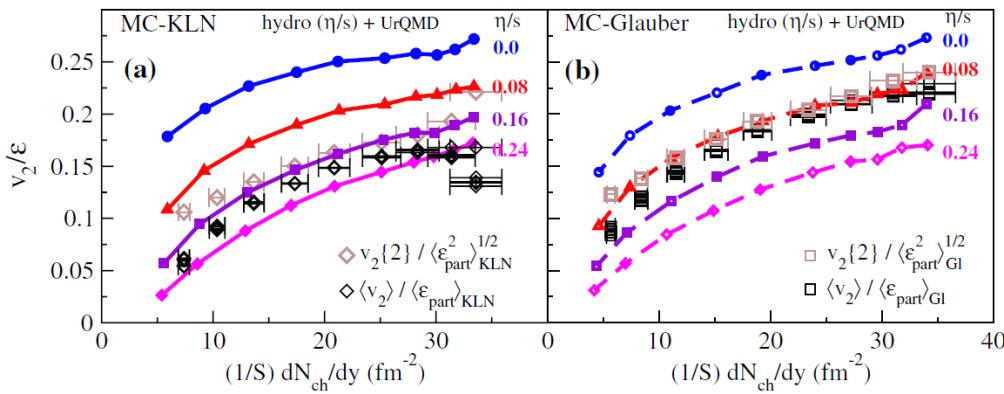
Constraint on η/s

20% uncertainty in $v_2/\varepsilon \rightarrow 100\%$ uncertainty in η/s

The question is how to reduce uncertainty in v_2/ε :

1. ε from theoretical part
2. v_2 from experimental part

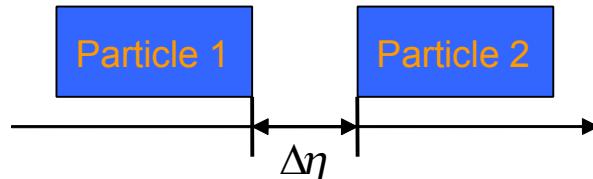
Song, Bass, Heinz, et al. PRL 106, 192301 (2011)



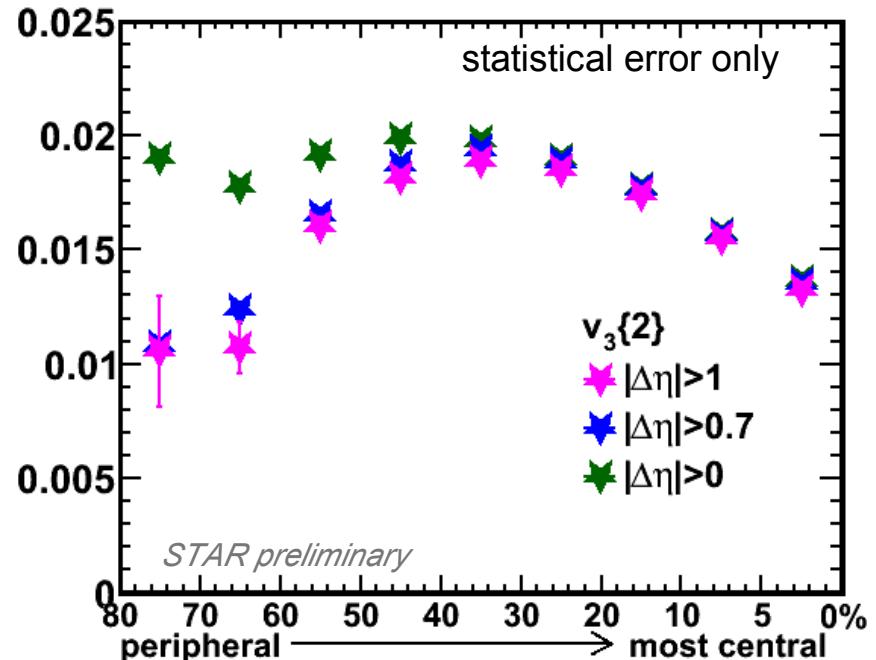
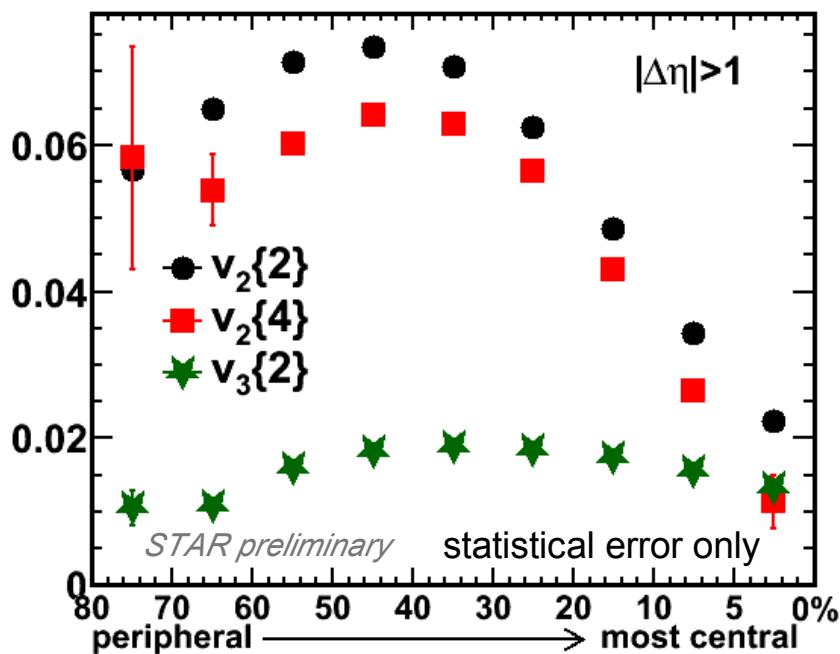
"The extraction of η/s from a comparison with hydrodynamics thus requires careful treatment of both fluctuation and nonflow effects"

v_3 vs Centrality

AuAu@200GeV
 $-1 < \eta < 1$
 $p_T < 2 \text{ GeV}/c$



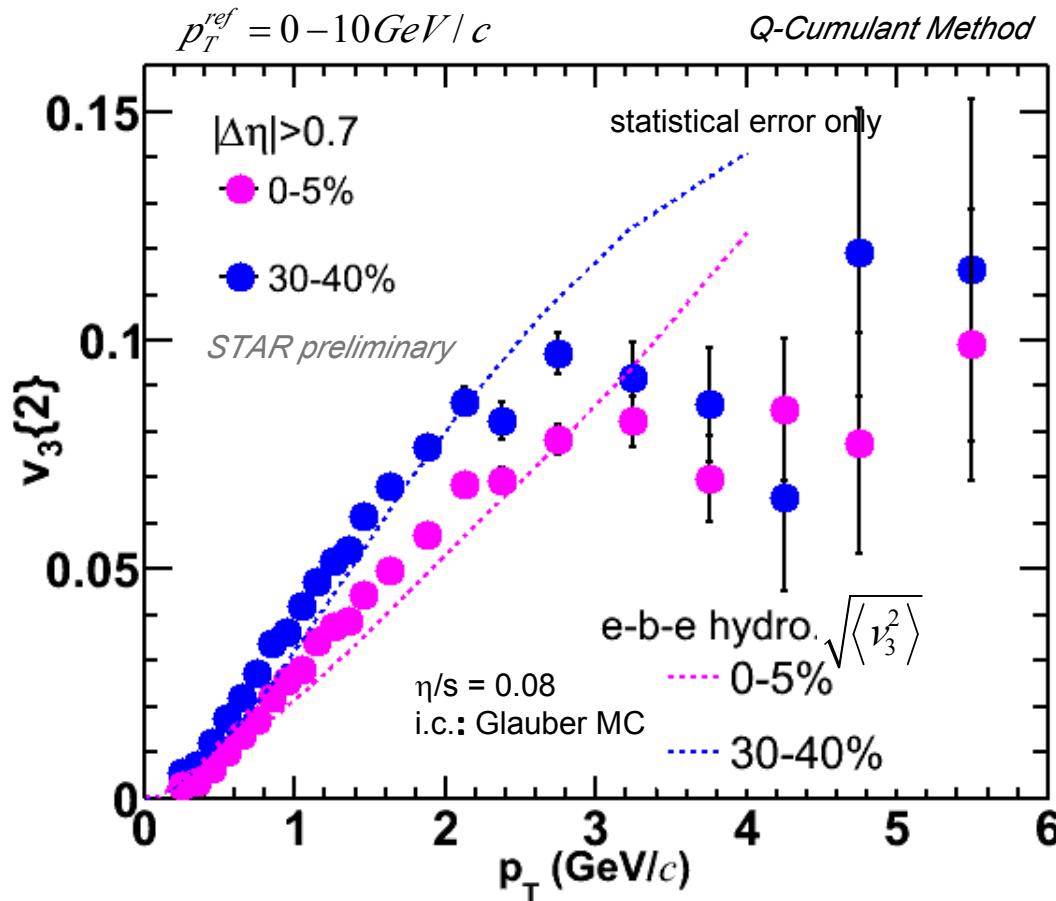
*Q-Cumulant Method
with η -gap*



- v_3 shows modest centrality dependence (Also see: Pandit, 1A, Tue.)
- v_3 is $3\times$ smaller than v_2 in peripheral to mid-central collisions

v_3 VS p_T

- v_3 : more sensitive to η/s than v_2



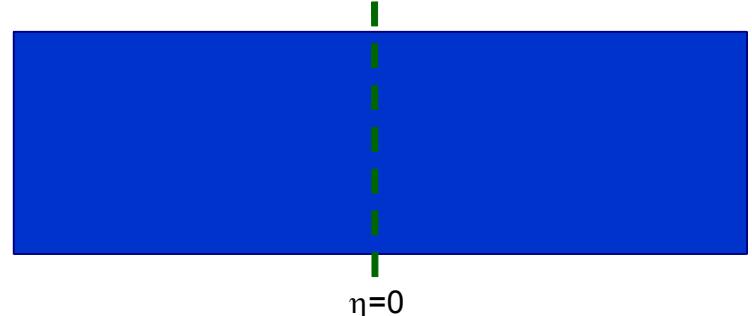
- hydro describes data trend well at $p_T < 2 \text{ GeV}/c$
- Data may contain nonflow

Isolation of Flow and Nonflow

using 2-, and 4-Particle η - η Cumulants

Xu, LY, arXiv:1204.2815

- 2-particle cumulant:



$$\mathcal{V}\{\eta_\alpha, \eta_\beta\} = \nu(\eta_\alpha)\nu(\eta_\beta) + \sigma(\eta_\alpha)\sigma(\eta_\beta) + \sigma'(\Delta\eta) + \delta(\Delta\eta)$$

‘flow’ flow fluct. $\Delta\eta$ -dep fluct. $\Delta\eta$ -dep nonflow

$$\Delta\mathcal{V}\{2\} = \mathcal{V}\{-\eta_\alpha, -\eta_\beta\} - \mathcal{V}\{-\eta_\alpha, \eta_\beta\}$$

$$\nu(-\eta_\beta) = \nu(\eta_\beta) \quad \sigma(-\eta_\beta) = \sigma(\eta_\beta)$$

- 4-particle cumulant:

$$\mathcal{V}^{1/2}\{\eta_\alpha, \eta_\alpha, \eta_\beta, \eta_\beta\} = \nu(\eta_\alpha)\nu(\eta_\beta) - \sigma(\eta_\alpha)\sigma(\eta_\beta) - \underline{\sigma'(\Delta\eta)}$$

$$\Delta\mathcal{V}\{4\}^{1/2} = -\Delta\sigma'$$

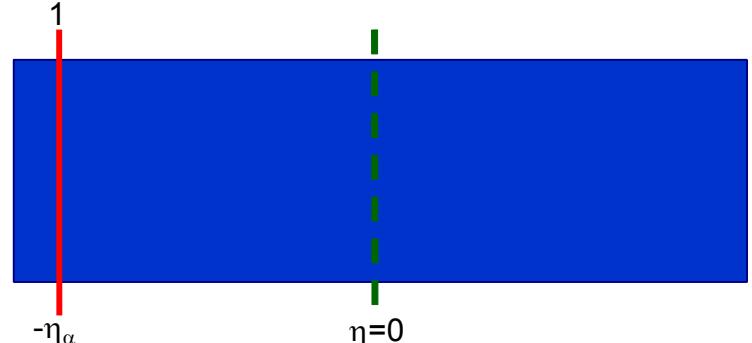
~ 0 (next slide)

Isolation of Flow and Nonflow

using 2-, and 4-Particle η - η Cumulants

Xu, LY, arXiv:1204.2815

- 2-particle cumulant:



$$\mathcal{V}\{\eta_\alpha, \eta_\beta\} = \nu(\eta_\alpha)\nu(\eta_\beta) + \sigma(\eta_\alpha)\sigma(\eta_\beta) + \sigma'(\Delta\eta) + \delta(\Delta\eta)$$

‘flow’ flow fluct. $\Delta\eta$ -dep fluct. $\Delta\eta$ -dep nonflow

$$\Delta\mathcal{V}\{2\} = \mathcal{V}\{-\eta_\alpha, -\eta_\beta\} - \mathcal{V}\{-\eta_\alpha, \eta_\beta\} = \Delta\sigma' + \Delta\delta$$

$$\nu(-\eta_\beta) = \nu(\eta_\beta) \quad \sigma(-\eta_\beta) = \sigma(\eta_\beta)$$

- 4-particle cumulant:

$$\mathcal{V}^{1/2}\{\eta_\alpha, \eta_\alpha, \eta_\beta, \eta_\beta\} = \nu(\eta_\alpha)\nu(\eta_\beta) - \sigma(\eta_\alpha)\sigma(\eta_\beta) - \underline{\sigma'(\Delta\eta)}$$
$$\Delta\mathcal{V}\{4\}^{1/2} = -\Delta\sigma'$$

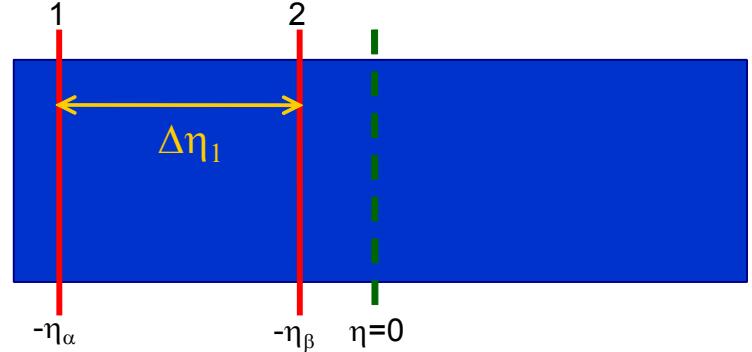
~ 0 (next slide)

Isolation of Flow and Nonflow

using 2-, and 4-Particle η - η Cumulants

Xu, LY, arXiv:1204.2815

- 2-particle cumulant:



$$\mathcal{V}\{\eta_\alpha, \eta_\beta\} = \nu(\eta_\alpha)\nu(\eta_\beta) + \sigma(\eta_\alpha)\sigma(\eta_\beta) + \sigma'(\Delta\eta) + \delta(\Delta\eta)$$

‘flow’ flow fluct. $\Delta\eta$ -dep fluct. $\Delta\eta$ -dep nonflow

$$\Delta\mathcal{V}\{2\} = \mathcal{V}\{-\eta_\alpha, -\eta_\beta\} - \mathcal{V}\{-\eta_\alpha, \eta_\beta\} = \Delta\sigma' + \Delta\delta$$

$$\nu(-\eta_\beta) = \nu(\eta_\beta) \quad \sigma(-\eta_\beta) = \sigma(\eta_\beta)$$

- 4-particle cumulant:

$$\mathcal{V}^{1/2}\{\eta_\alpha, \eta_\alpha, \eta_\beta, \eta_\beta\} = \nu(\eta_\alpha)\nu(\eta_\beta) - \sigma(\eta_\alpha)\sigma(\eta_\beta) - \underline{\sigma'(\Delta\eta)}$$

$$\Delta\mathcal{V}\{4\}^{1/2} = -\Delta\sigma'$$

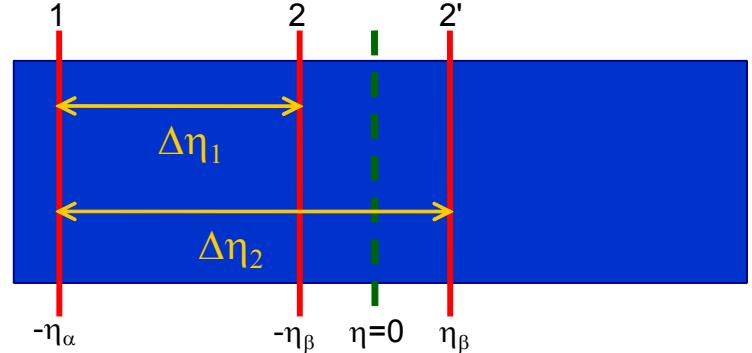
~ 0 (next slide)

Isolation of Flow and Nonflow

using 2-, and 4-Particle η - η Cumulants

Xu, LY, arXiv:1204.2815

- 2-particle cumulant:



$$\mathcal{V}\{\eta_\alpha, \eta_\beta\} = \nu(\eta_\alpha)\nu(\eta_\beta) + \sigma(\eta_\alpha)\sigma(\eta_\beta) + \sigma'(\Delta\eta) + \delta(\Delta\eta)$$

'flow' flow fluct. $\Delta\eta$ -dep fluct. $\Delta\eta$ -dep nonflow

$$\Delta\mathcal{V}\{2\} = \mathcal{V}\{-\eta_\alpha, -\eta_\beta\} - \mathcal{V}\{-\eta_\alpha, \eta_\beta\} = \Delta\sigma' + \Delta\delta$$

$$\nu(-\eta_\beta) = \nu(\eta_\beta) \quad \sigma(-\eta_\beta) = \sigma(\eta_\beta)$$

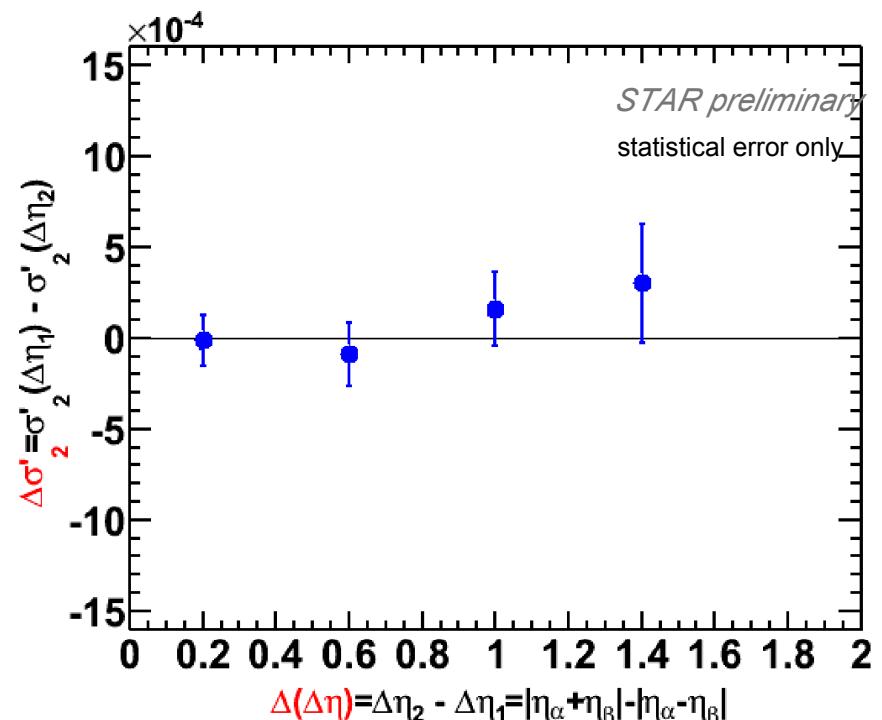
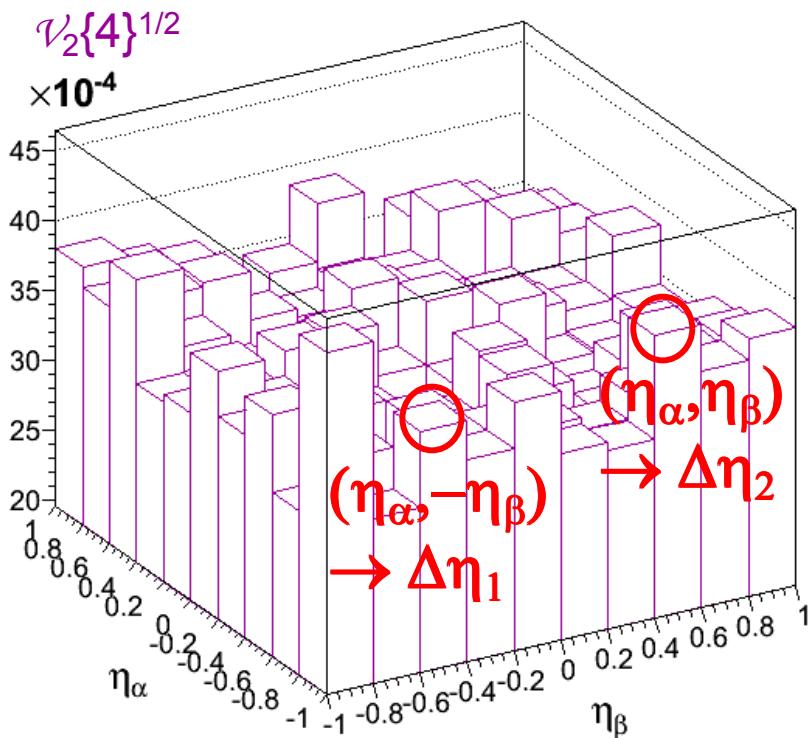
- 4-particle cumulant:

$$\mathcal{V}^{1/2}\{\eta_\alpha, \eta_\alpha, \eta_\beta, \eta_\beta\} = \nu(\eta_\alpha)\nu(\eta_\beta) - \sigma(\eta_\alpha)\sigma(\eta_\beta) - \underline{\sigma'(\Delta\eta)}$$

$$\Delta\mathcal{V}\{4\}^{1/2} = -\Delta\sigma' \qquad \sim 0 \text{ (next slide)}$$

$\Delta\eta$ -dependence $\sigma'(\Delta\eta)$

AuAu@200GeV 20-30%

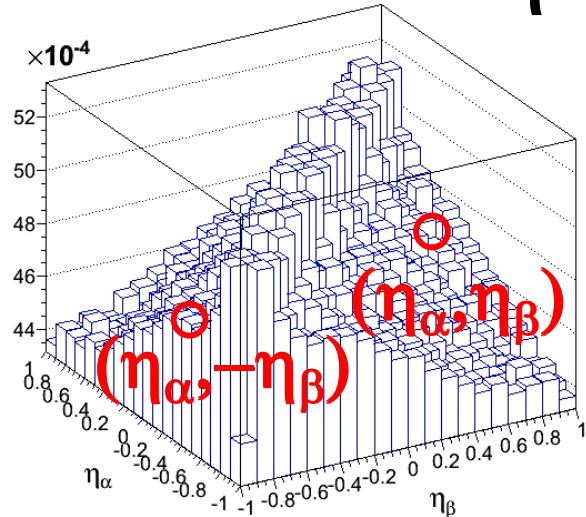


Flow fluctuation appears independent of $\Delta\eta$.

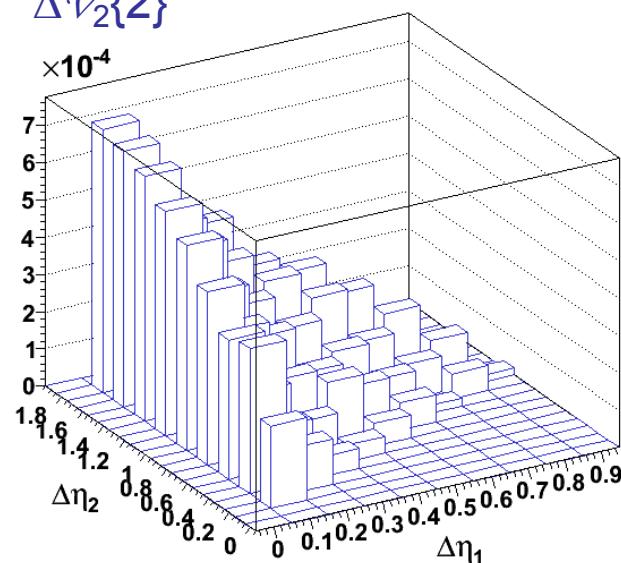
$\Delta\eta$ -dependent $\delta(\Delta\eta)$

AuAu@200GeV 20-30%

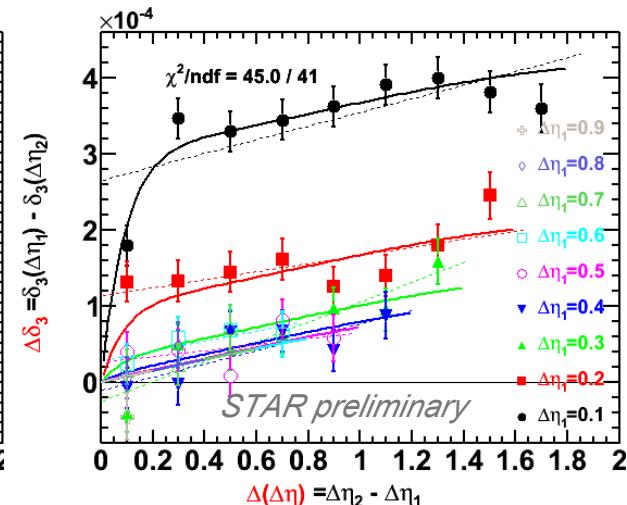
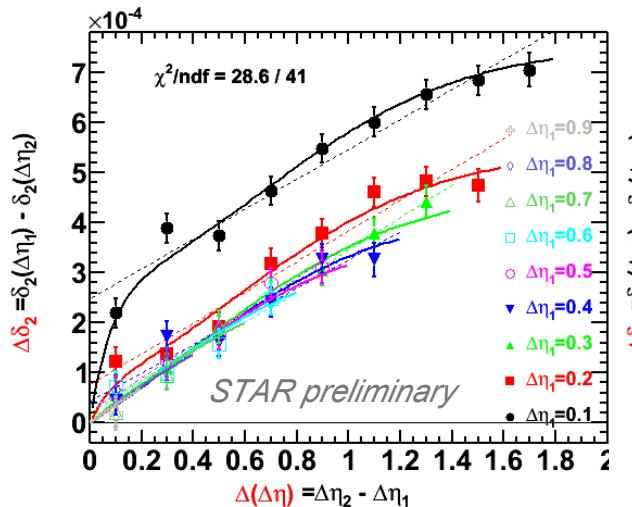
$\mathcal{V}_2\{2\}$



$\Delta \mathcal{V}_2\{2\}$



- $\delta(\Delta\eta_2)-\delta(\Delta\eta_1)$ linear in $\Delta\eta_2-\Delta\eta_1$ at a given $\Delta\eta_1$ with similar slopes
- Intercept changes with $\Delta\eta_1$ exponentially



statistics error only

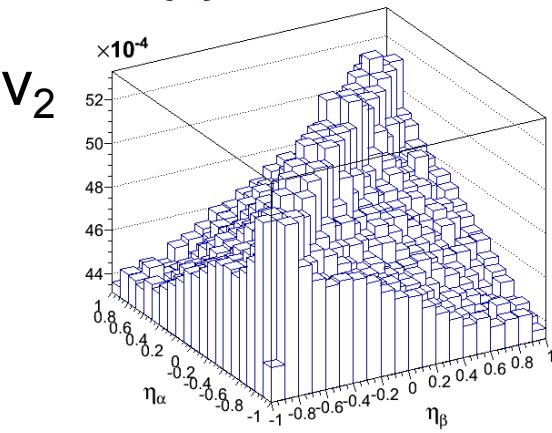
$$\Delta\delta(\Delta\eta_1, \Delta\eta_2) = a(e^{-\Delta\eta_1/b} - e^{-\Delta\eta_2/b}) + A(e^{-\Delta\eta_1^2/2\sigma^2} - e^{-\Delta\eta_2^2/2\sigma^2})$$

$$\delta(\Delta\eta) = ae^{-\Delta\eta/b} + Ae^{-\Delta\eta^2/2\sigma^2}$$

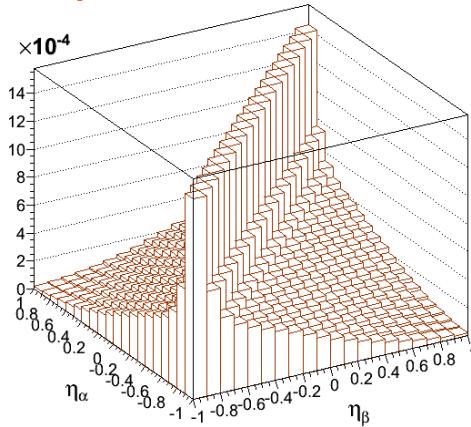
Cumulant $\mathcal{V}\{2\}$, Nonflow δ , 'Flow' v_n^2

AuAu@200GeV 20-30%

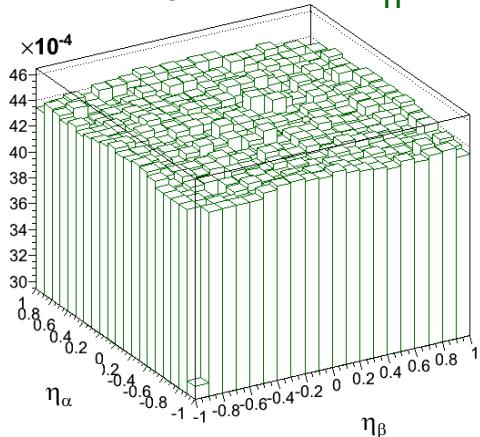
$\mathcal{V}_n\{2\}$



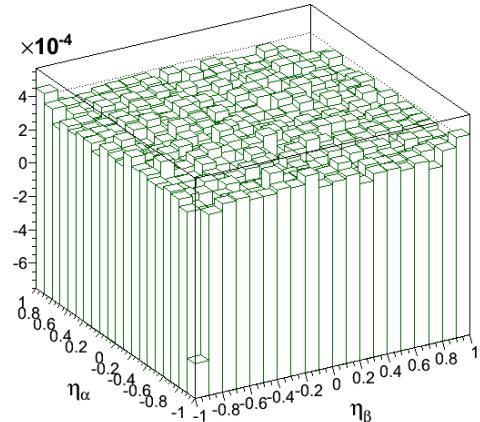
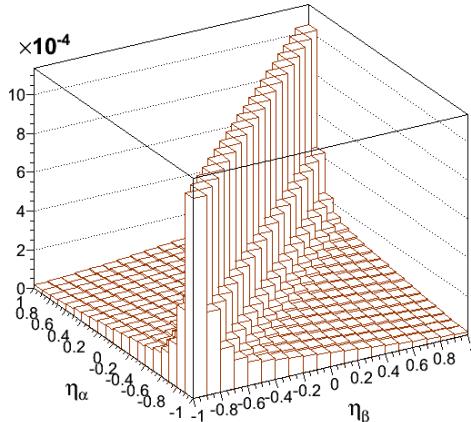
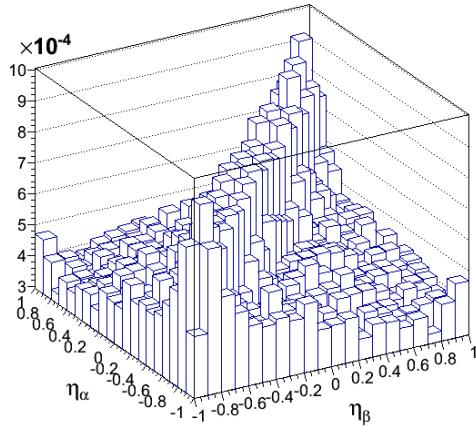
δ_n parameterized



decomposed $\langle v_n^2 \rangle$



v_3



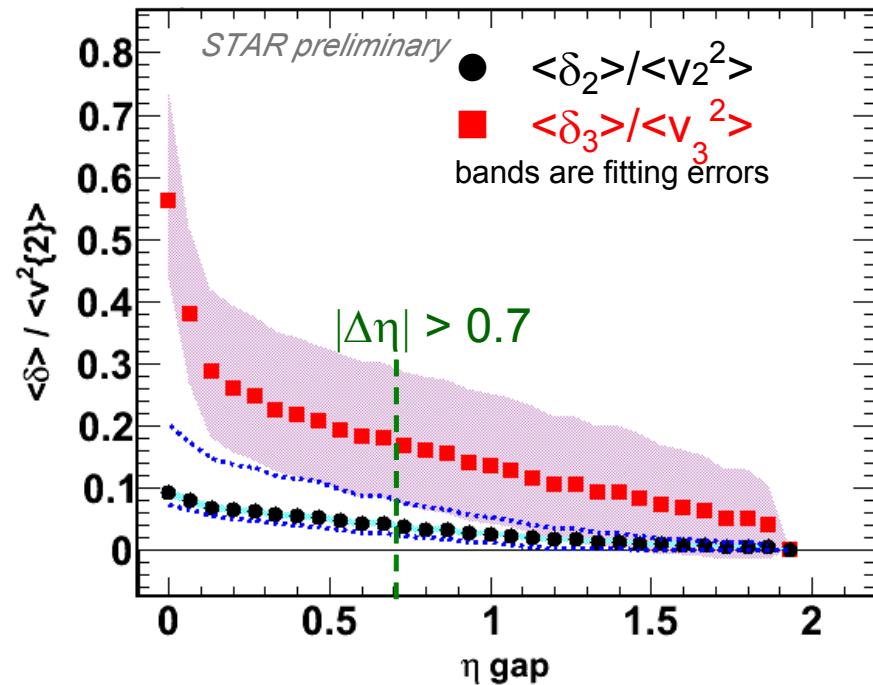
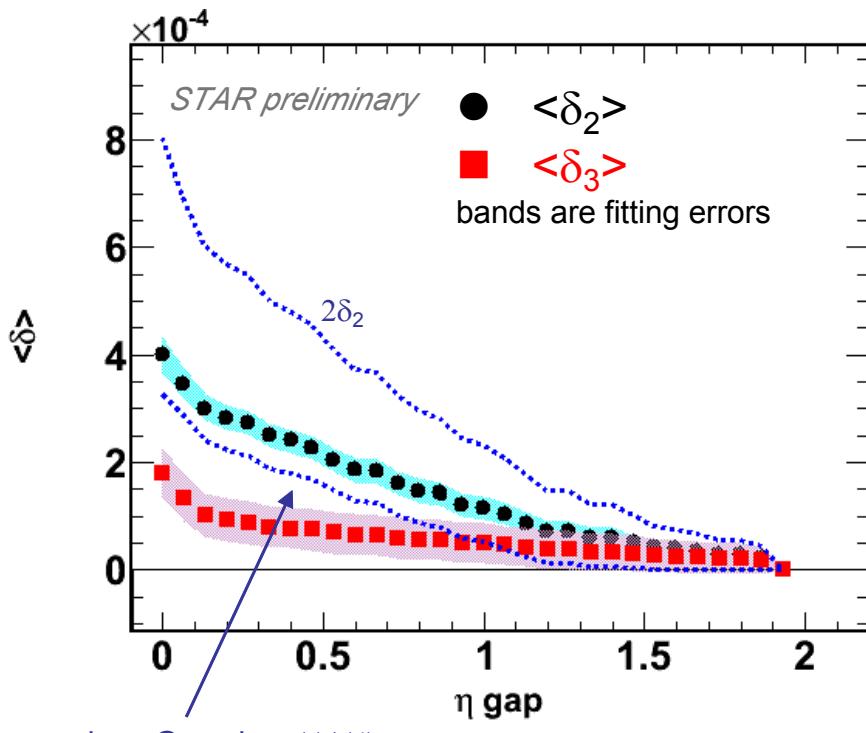
No Assumption about flow η dependence in our analysis

The decomposed 'flow' appears to be independent of η .

$\Delta\eta$ -dep Near-side Nonflow

AuAu@200GeV 20-30%

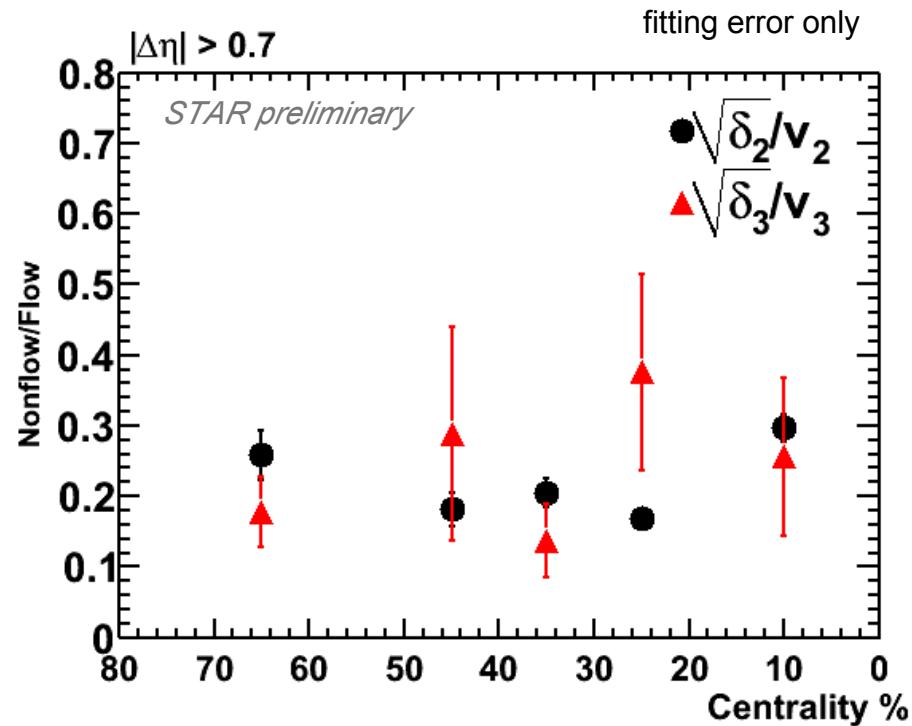
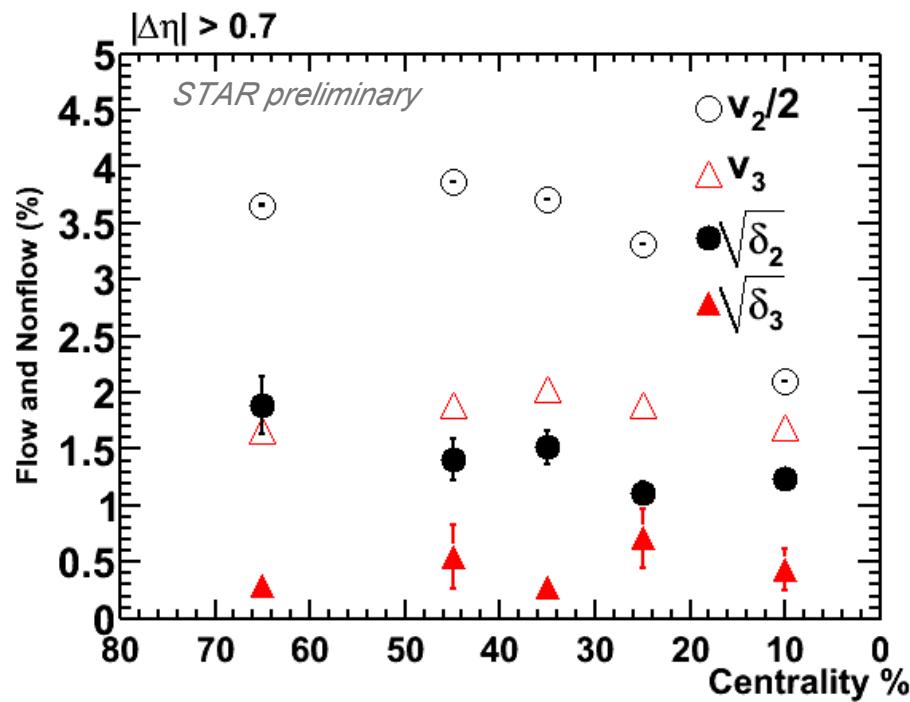
- Calculate $\langle \text{nonflow} \rangle$ of all $(\eta_\alpha, \eta_\beta)$ bins with $x < \eta\text{-gap} < 2$. (x = horizontal axis).



- With $|\Delta\eta| > 0.7$, significant nonflow still exists.

'Flow' and Nonflow vs Centrality

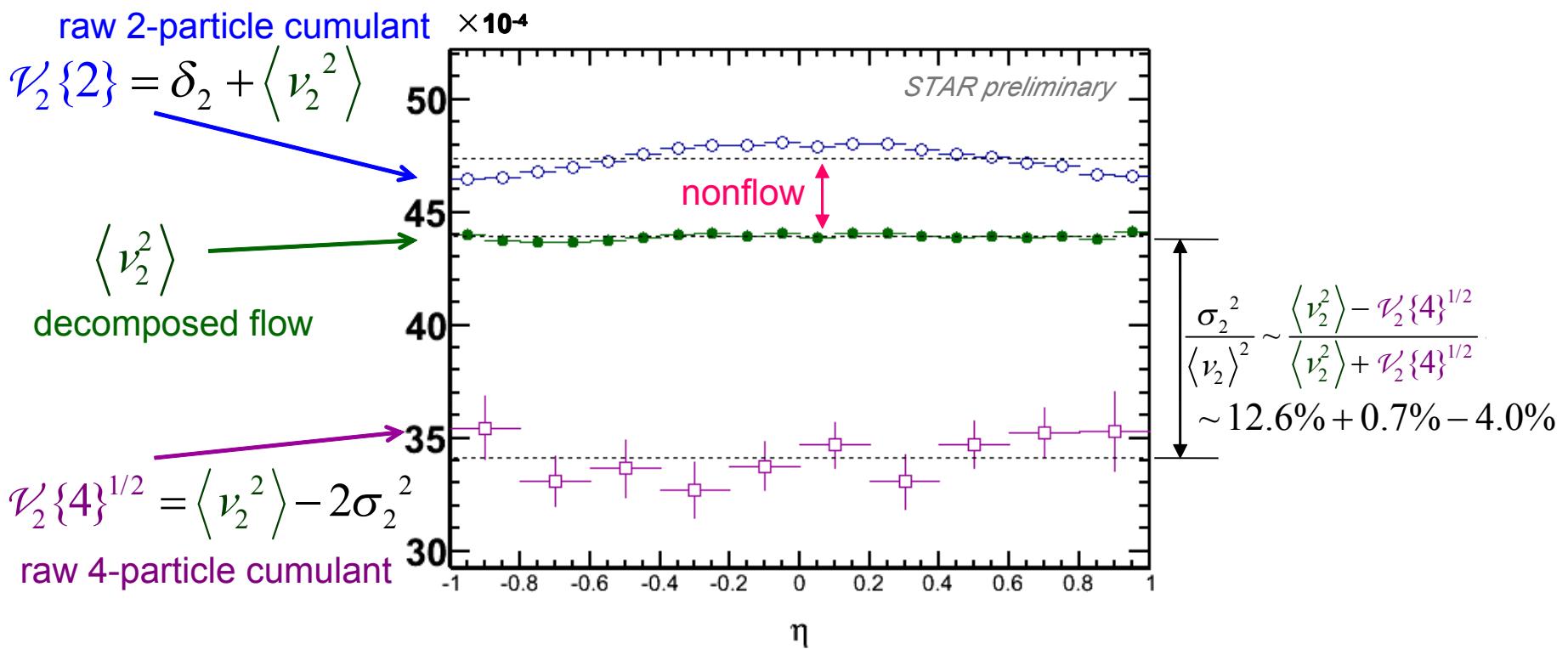
Au+Au@200GeV



- $\sqrt{\delta_2} / v_2 \sim 20\%$ for $|\Delta\eta| > 0.7$
 $\delta_2 / v_2^2 \sim 4\%$

'Flow' vs η

AuAu@200GeV 20-30%



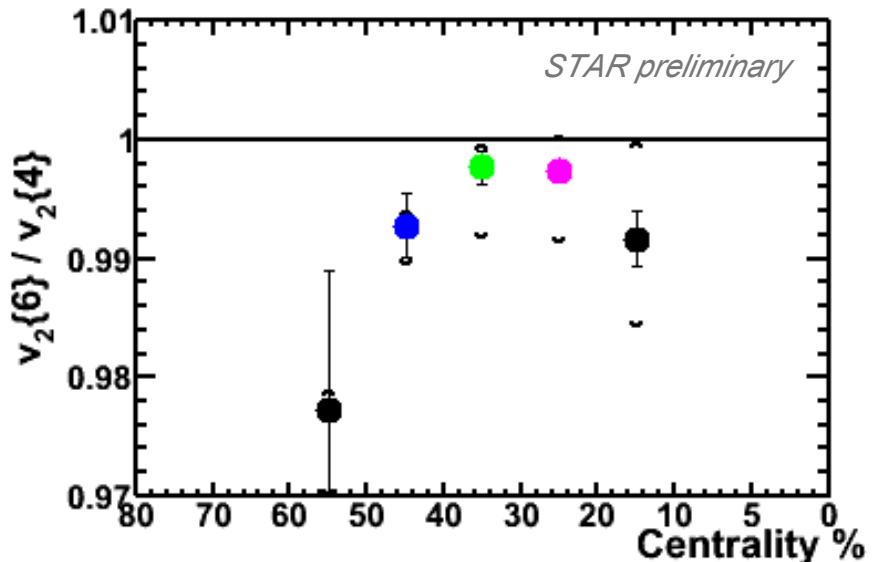
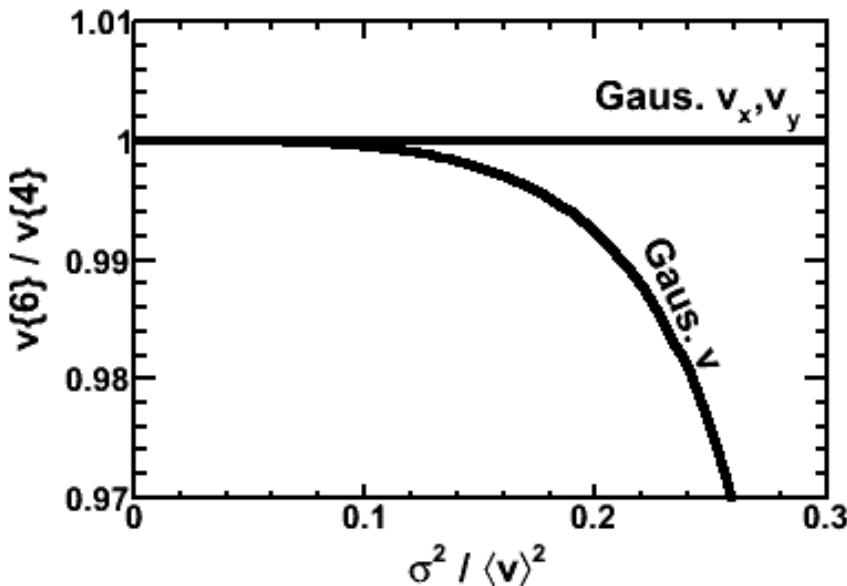
- Flow seems independent of η . Note no assumption of η dependence in our approach.
- $(\text{Fluctuation / flow})^2 \sim 13\%$

4- and 6-Particle Cumulant

Assuming the flow fluctuations are Gaussian, we have two options:

1. v_x, v_y are Gaussian: $v\{6\} = v\{4\}$ Voloshin, Poskanzer, Tang, Wang, PLB
2. v is Gaussian: LY, Wang, Tang, arXiv: 1101.4646

$$v_n\{6\}/v_n\{4\} \approx 1 - \sigma^6 / 3\langle v_n \rangle^6 , \text{ if } \sigma^2 / \langle v_n \rangle^2 \ll 1$$



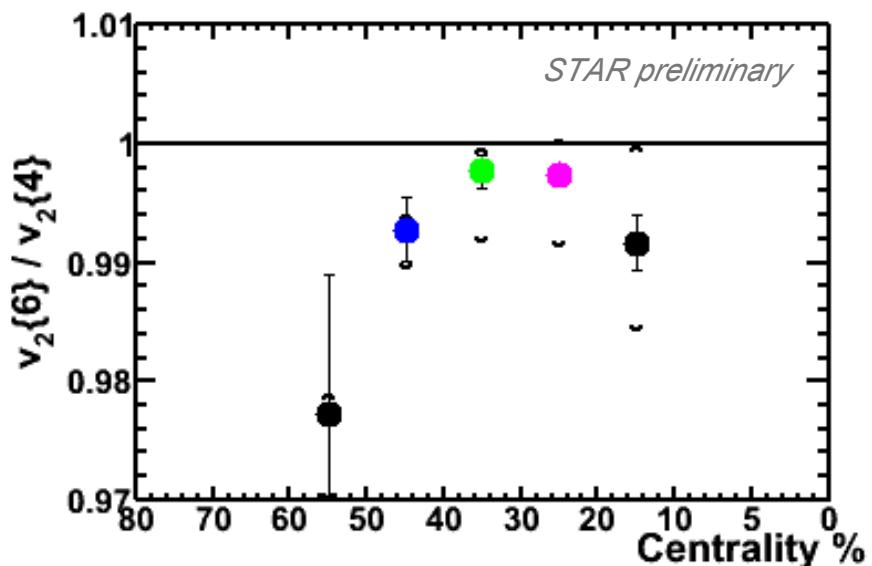
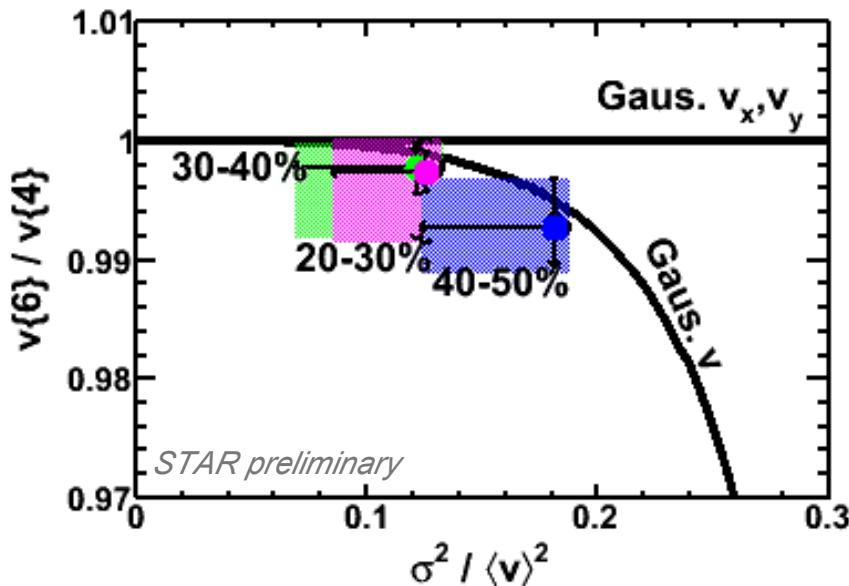
* No-weight applied, non-uniform acceptance corrected. Systematic errors estimated by applying weight and no acceptance correction

4- and 6-Particle Cumulant

Assuming the flow fluctuations are Gaussian, we have two options:

1. v_x, v_y are Gaussian: $v\{6\} = v\{4\}$ Voloshin, Poskanzer, Tang, Wang, PLB
2. v is Gaussian: LY, Wang, Tang, arXiv: 1101.4646

$$v_n\{6\}/v_n\{4\} \approx 1 - \sigma^6 / 3\langle v_n \rangle^6 , \text{ if } \sigma^2 / \langle v_n \rangle^2 \ll 1$$



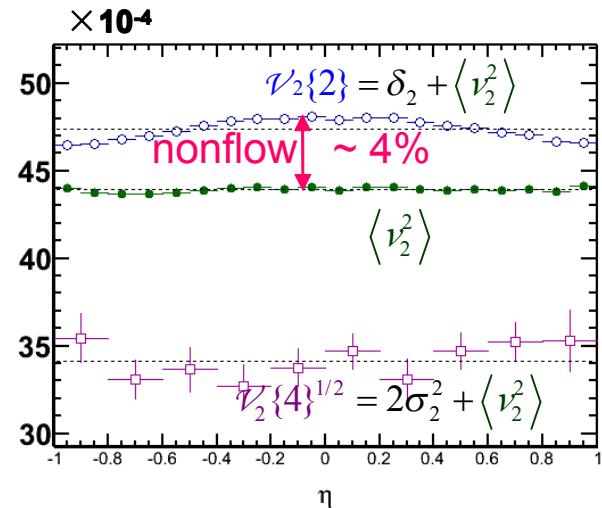
* No-weight applied, non-uniform acceptance corrected. Systematic errors estimated by applying weight and no acceptance correction

Summary

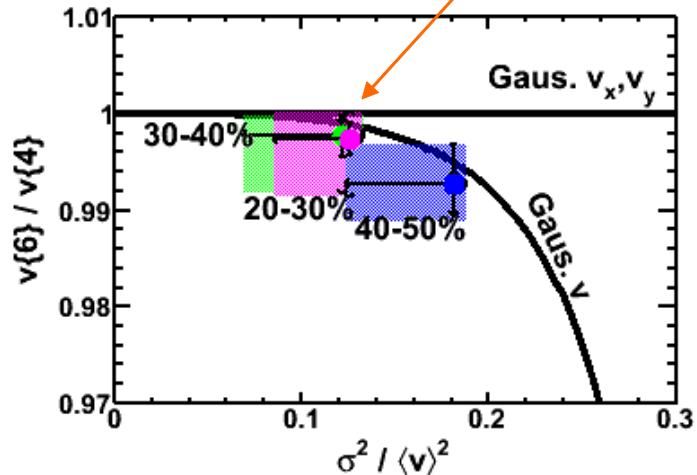
- 2-, 4- and 6-particle cumulants vs p_T , centrality are presented.
- Isolation of $\Delta\eta$ -dependent (near-side nonflow) and $\Delta\eta$ -independent (flow-dominant + small away-side nonflow) correlations, using 2- and 4-particle cumulants between η bins
 - the decomposed 'flow' appears to be independent of η within ± 1 unit
 - nonflow estimation $\sim 4\%$ in v_2^2
 - flow fluctuation estimation $\sim 13\%$ in v_2^2

AuAu@200GeV 20-30%
 $-1 < \eta < 1$ $p_T^{< 2\text{GeV}/c}$

Aug. 15th, 2012



$$\frac{\sigma_2^2}{\langle v_2^2 \rangle^2} \sim \frac{\langle v_2^2 \rangle - \langle v_2^4 \rangle^{1/2}}{\langle v_2^2 \rangle + \langle v_2^4 \rangle^{1/2}} \sim 12.6\% + 0.7\% - 4.0\%$$



Abstract

Azimuthal anisotropic flows v_n , arising from the anisotropic collision geometry, reflect the hydrodynamic properties of the quark gluon plasma created in relativistic heavy-ion collisions. A long standing issue in v_n measurements is the contamination of nonflow, caused by intrinsic particle correlations unrelated to the collision geometry. Nonflow limits, in part, the precise extraction of the viscosity to entropy density ratio η/s from data-model comparisons. Isolation of flow and nonflow is critical to the interpretation of the Fourier decomposition of dihadron correlations.

In this talk we report measurements of v_n azimuthal anisotropies using the two- and multi-particle Q-cumulants method from STAR in Au+Au collisions at 200 GeV. The centrality and pT dependence of v_n will be presented. We compare the four- and six-particle cumulant measurements to gain insights on the nature of flow fluctuations [1,2]. We further analyze two- and four-particle cumulants between pseudo-rapidity (η) bins. Exploiting the collision symmetry about mid-rapidity, we isolate the $\Delta\eta$ -dependent and $\Delta\eta$ -independent correlations in the data with a data-driven method [3]. The $\Delta\eta$ -dependent part arises from near-side nonflow correlations, such as HBT interferometry, resonance decays, and jet-correlations. The $\Delta\eta$ -independent part is dominated by flow and flow fluctuations with relatively small contribution from away-side jet-correlations. The method does not make assumptions about the η dependence of flow. Our isolated $\Delta\eta$ -independent part from data, dominated by flow, however, is found to be also η -independent within the STAR TPC of ± 1 unit of pseudo-rapidity. The $\Delta\eta$ drop in the measured two-particle cumulant appears to entirely come from nonflow. We assess the effect of the nonflow on η/s extraction. We reexamine the high-pT triggered dihadron correlations with background subtraction of our decomposed flows.

- [1] S.A. Voloshin, A.M. Poskanzer, A. Tang, and G. Wang, Phys. Lett. B659, 537 (2008).
- [2] L. Yi, F. Wang, and A. Tang, arXiv:1101.4646 [nucl-ex].
- [3] L. Xu, L. Yi, D. Kikola, J. Konzer, F. Wang, and W. Xie, arXiv:1204.2815 [nucl-ex].

Analysis Cuts

AuAu@200GeV

Year2004 data

19 million min-bias events

|Vertex z| < 30 cm

$|\eta| < 1$

Dca < 2 cm

nfit ≥ 20

nhits / nfit-pos > 0.51

Year2010 data

80 million min-bias events

$|\text{Vertex } z| < 30 \text{ cm}$

$|\text{vpdz-Vz}| < 3$

$|\text{Vr}| < 2$

TiggerId: 260001, 260011, 260021, 260031

$|\eta| < 1$

Dca < 2 cm

nfit ≥ 15

$1.02 > \text{nhits} / \text{nfit-pos} > 0.52$

$\text{flag}() < 1000$

2-, 4-, 6-Particle Q-Cumulant Method

- Particle azimuthal moments:

In a single event δ is nonflow

$$\langle 2 \rangle_n = \left\langle e^{in(\phi_i - \phi_j)} \right\rangle = \nu_n^2 + \delta_n \quad \boxed{\langle 2 \rangle_n = \frac{|\mathcal{Q}_n|^2 - M}{MM-1} \quad \mathcal{Q}_n \equiv \sum_{i=1}^M e^{in\phi_i}}$$

$$\langle 4 \rangle_n = \left\langle e^{in(\phi_i + \phi_j - \phi_k - \phi_l)} \right\rangle \approx \nu_n^4 + 4\nu_n^2\delta_n + 2\delta_n^2$$

$$\langle 6 \rangle_n = \left\langle e^{in(\phi_i + \phi_j + \phi_k - \phi_l - \phi_m - \phi_n)} \right\rangle \approx \nu_n^6 + 9\nu_n^4\delta_n + 18\nu_n^2\delta_n^2 + 6\delta_n^3$$

- 2-, 4-, 6-particle azimuthal anisotropy

Average over all events

$$\nu_n \{2\}^2 \equiv \langle \langle 2 \rangle \rangle_n$$

$$\nu_n \{4\}^4 \equiv 2\langle \langle 2 \rangle \rangle_n^2 - \langle \langle 4 \rangle \rangle_n$$

$$\nu_n \{6\}^6 \equiv (\langle \langle 6 \rangle \rangle_n - 9\langle \langle 2 \rangle \rangle_n \langle \langle 4 \rangle \rangle_n + 12\langle \langle 2 \rangle \rangle_n^3) / 4$$

For exclusive region :

$$\langle 2' \rangle_n = \left\langle e^{in(\phi'_i - \phi'_j)} \right\rangle = \nu'_n \nu_n + \delta'_n$$

$$\nu_n \{2'\} \equiv \langle \langle 2' \rangle \rangle_n / \sqrt{\langle \langle 2 \rangle \rangle_n}$$

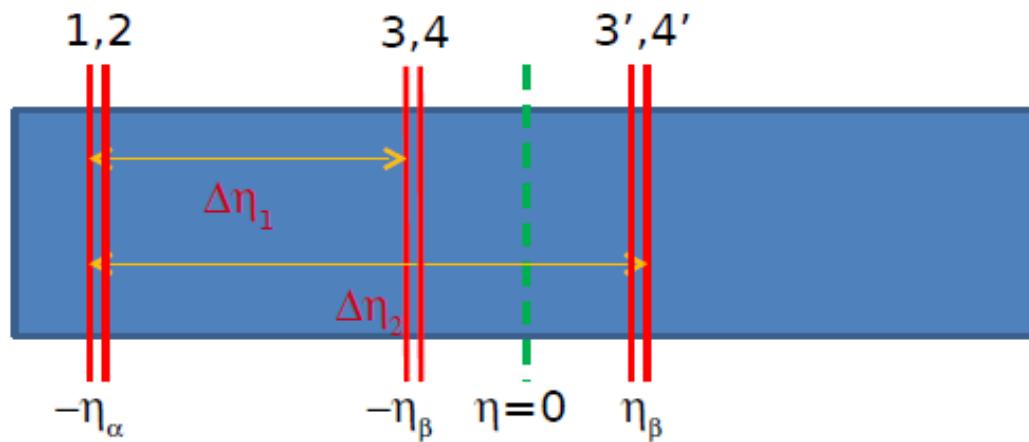
*Flow analysis with cumulants: direct calculations, Ante Bilandzic, Raimond Snellings and Sergei Voloshin, arXiv:1010.0233 [nucl-ex].

*Li Yi, Fuqiang Wang, and Aihong Tang, arXiv: 1101.4646.

4-Particle Cumulant between η Bins

$$V^{1/2}(\eta_\alpha, \eta_\alpha; \eta_\beta, \eta_\beta) = v(\eta_\alpha)v(\eta_\beta) - \sigma(\eta_\alpha)\sigma(\eta_\beta) - \sigma'(\Delta\eta)$$

\longleftrightarrow
fluctuations



$$\begin{aligned} & V^{1/2}\{-\eta_\alpha, -\eta_\alpha; -\eta_\beta, -\eta_\beta\} \\ & \quad \longleftrightarrow \\ & V^{1/2}\{-\eta_\alpha, -\eta_\alpha; \eta_\beta, \eta_\beta\} \end{aligned}$$

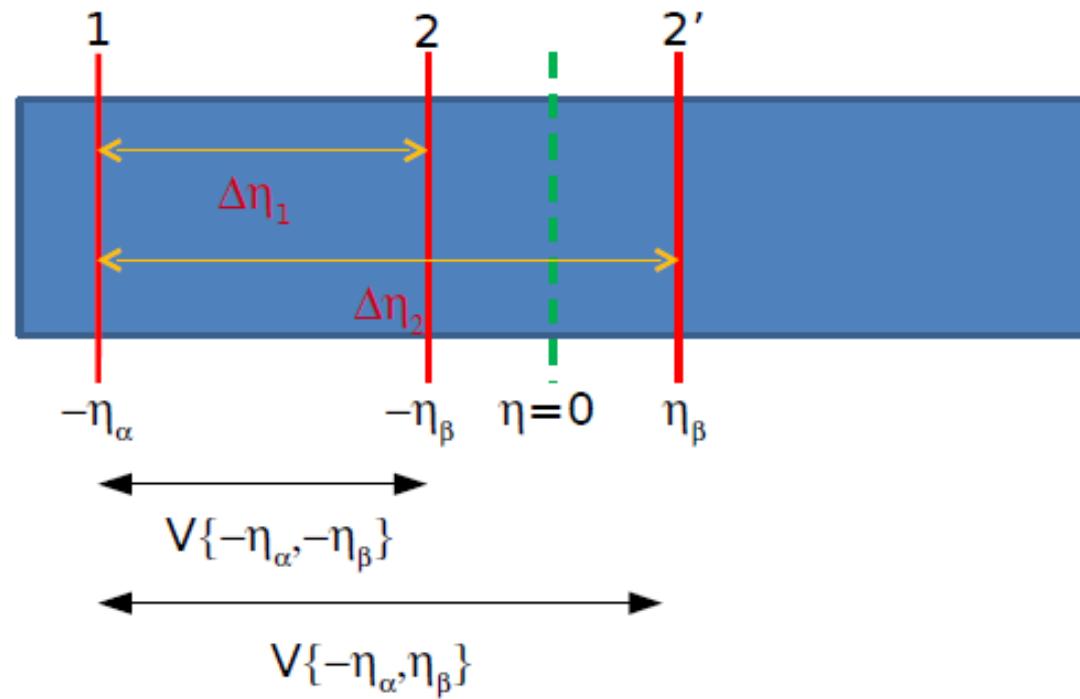
$v(-\eta_\beta) = v(\eta_\beta)$
 $\sigma(-\eta_\beta) = \sigma(\eta_\beta)$

$$V^{1/2}\{-\eta_\alpha, -\eta_\alpha; -\eta_\beta, -\eta_\beta\} - V^{1/2}\{-\eta_\alpha, -\eta_\alpha; \eta_\beta, \eta_\beta\} = -\sigma'(\Delta\eta_1) + \sigma'(\Delta\eta_2)$$

2-Particle Cumulant between η Bins

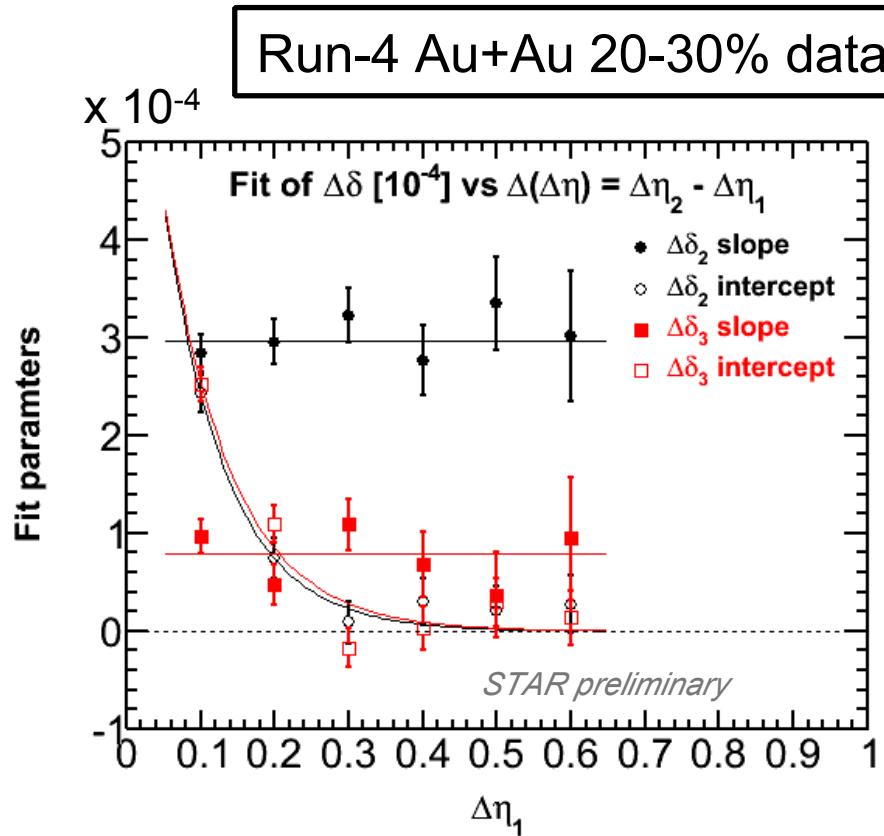
$$V\{\eta_\alpha, \eta_\beta\} = v(\eta_\alpha)v(\eta_\beta) + \sigma(\eta_\alpha)\sigma(\eta_\beta) + \sigma'(\Delta\eta) + \delta(\Delta\eta)$$

fluctuations **nonflow**



$$V(-\eta_\alpha, -\eta_\beta) - V(-\eta_\alpha, \eta_\beta) = \delta(\Delta\eta_1) - \delta(\Delta\eta_2) + \sigma'(\Delta\eta_1) - \sigma'(\Delta\eta_2)$$

Nonflow Parameterization



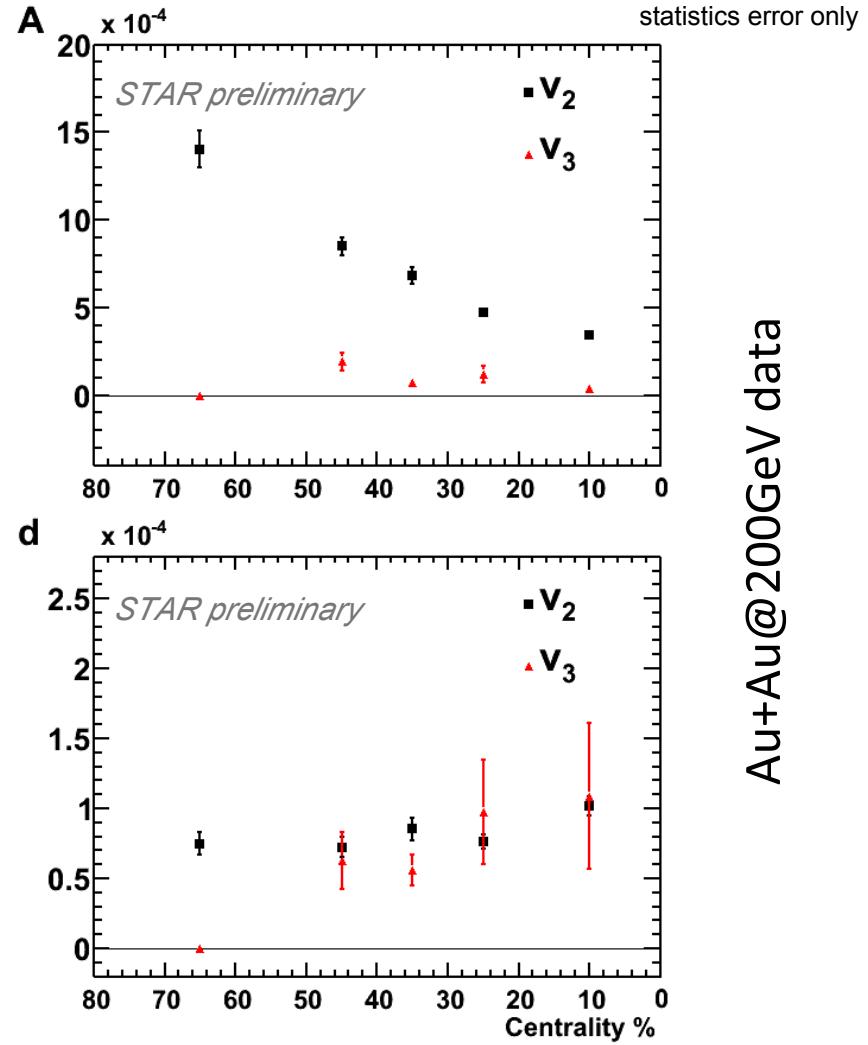
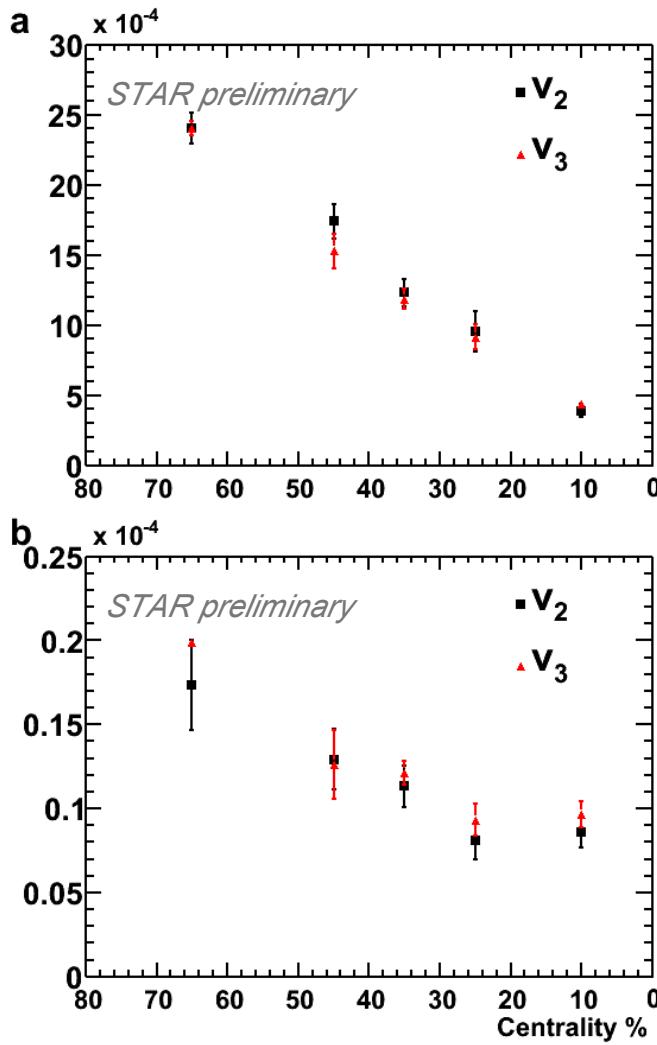
- Intercepts drops quickly at large $\Delta\eta_1$ and then saturates

$$\delta(\Delta\eta) = a * \exp(-\Delta\eta/b) - k(\Delta\eta - \Delta\eta_{max}) + c$$

c set to 0 (arbitrary)

Nonflow Fit Parameters

$$\delta(\Delta\eta) = \mathbf{a}^* \exp(-\Delta\eta/\mathbf{b}) + \mathbf{A}^* \exp(-\Delta\eta^2/2\mathbf{d}^2) + \mathbf{c}$$

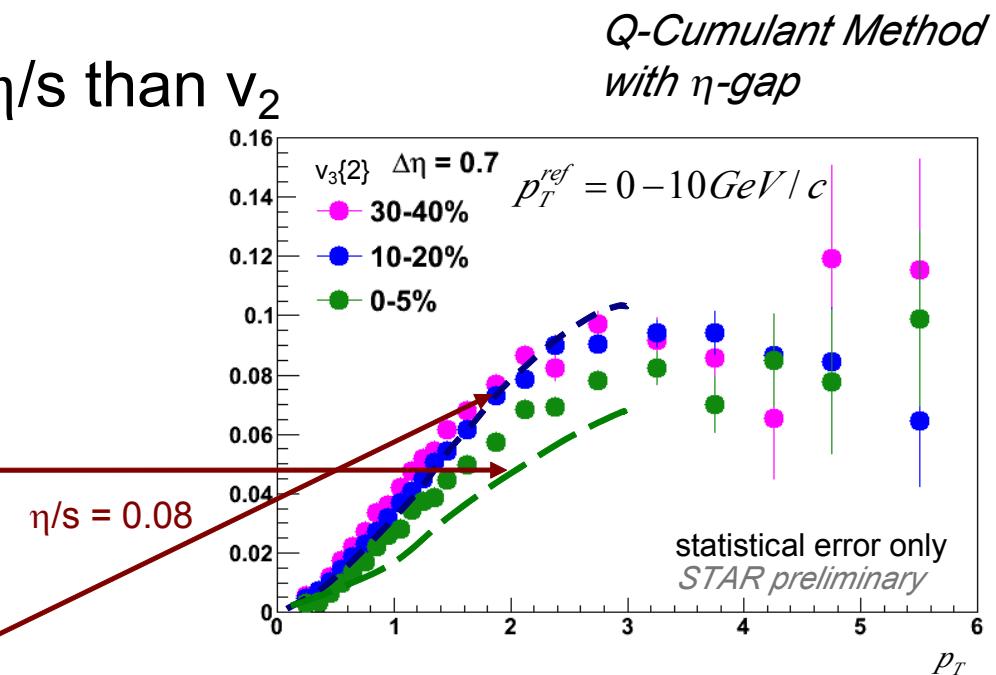
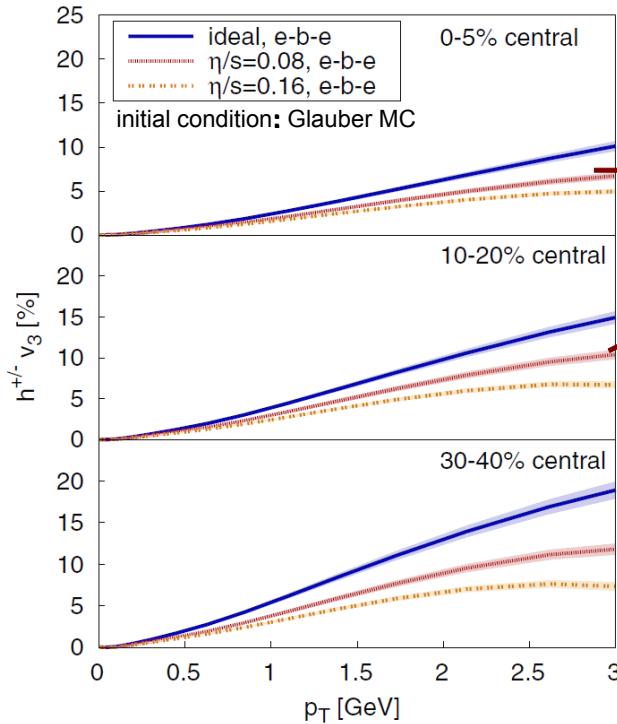


Au+Au@200GeV data

v_3 vs p_T and Model Comparison

- v_3 more sensitive to η/s than v_2

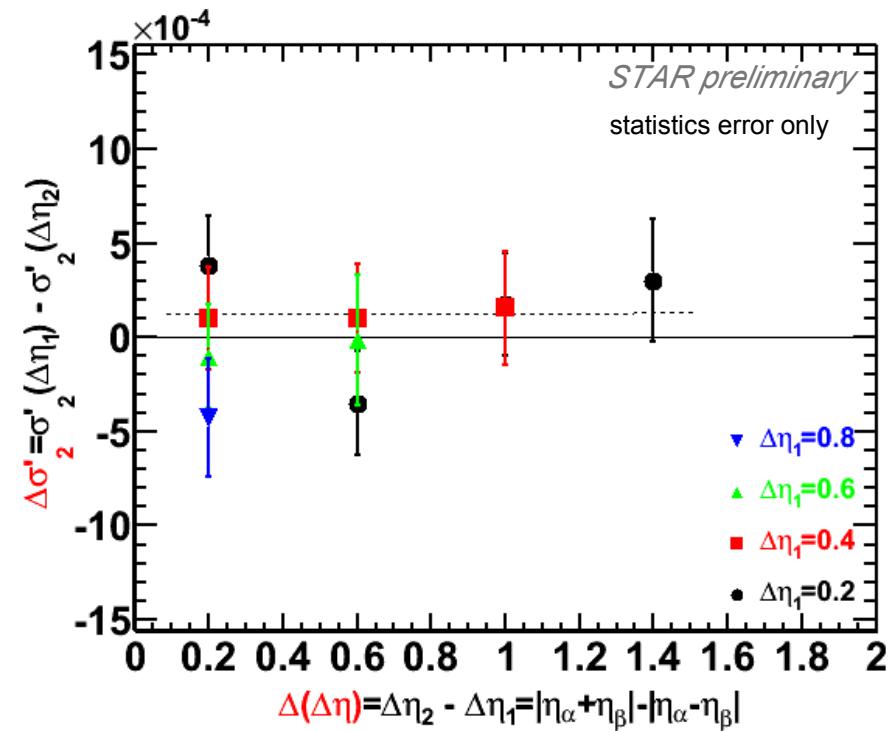
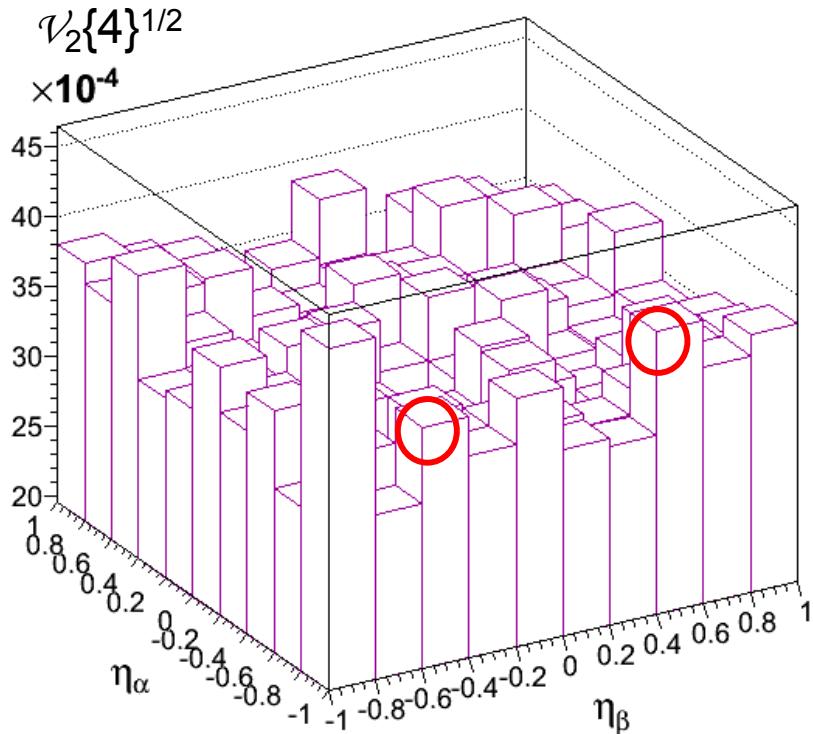
B. Schenke, S. Jeon, and C. Gale
 PRL 106, 042301 (2011)



- Top 5%, hydro under-predicts data
- Non-central hydro describes data well at $p_T < 2$ GeV hydro deviates from data at $p_T > 2$ GeV
- Data may contain nonflow

$\Delta\eta$ -dependence $\sigma'(\Delta\eta)$

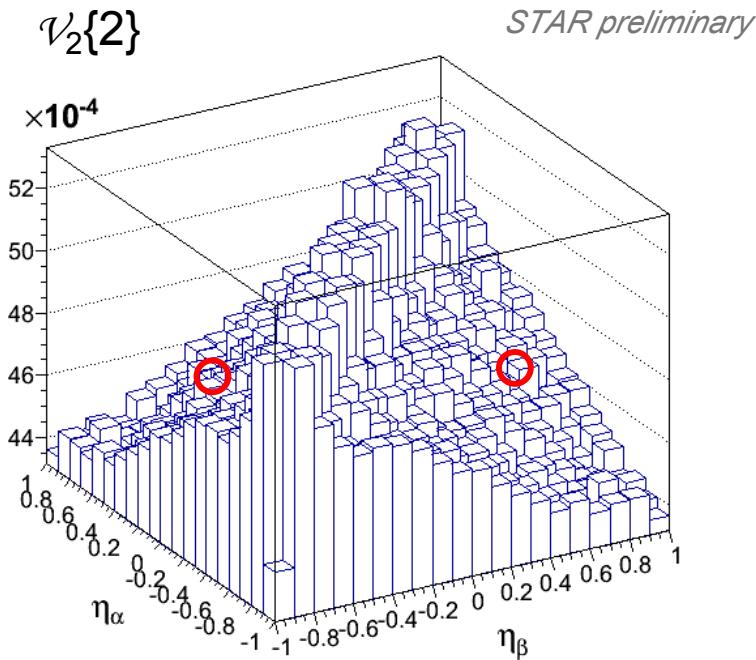
Au+Au@200 GeV 20-30% data



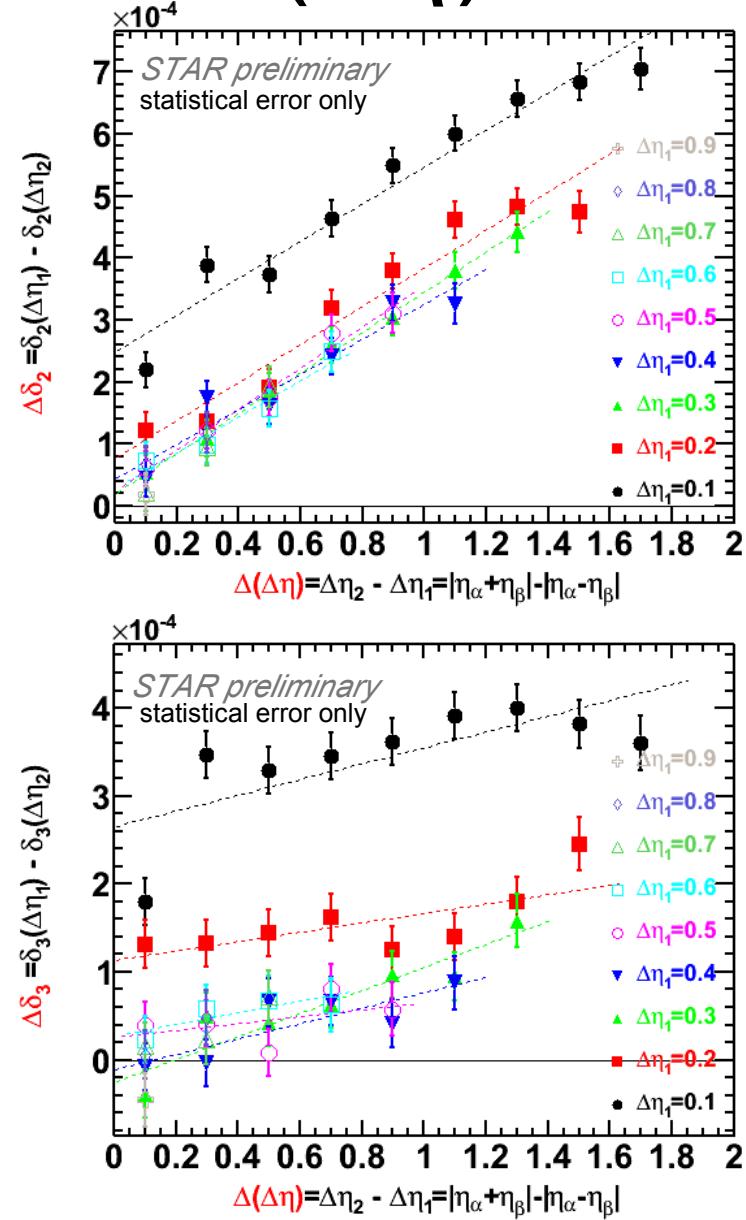
Flow fluctuation appears independent of $\Delta\eta$.

$\Delta\eta$ -dependent $\delta(\Delta\eta)$

Au+Au@200GeV 20-30% data

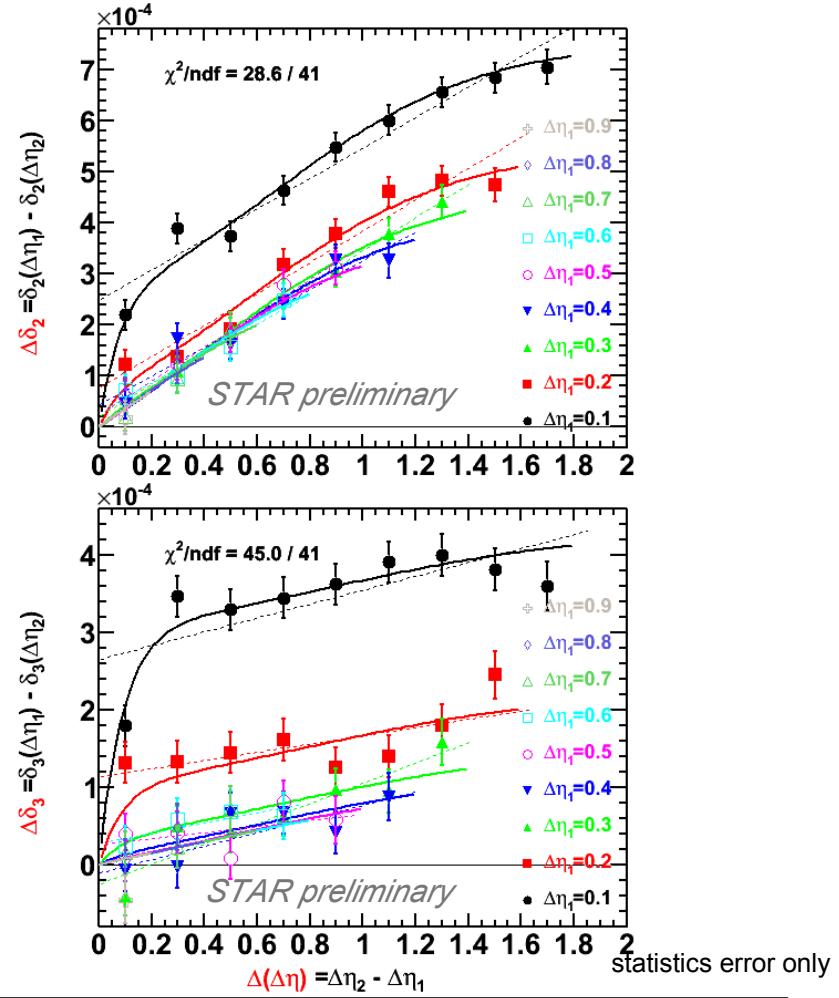
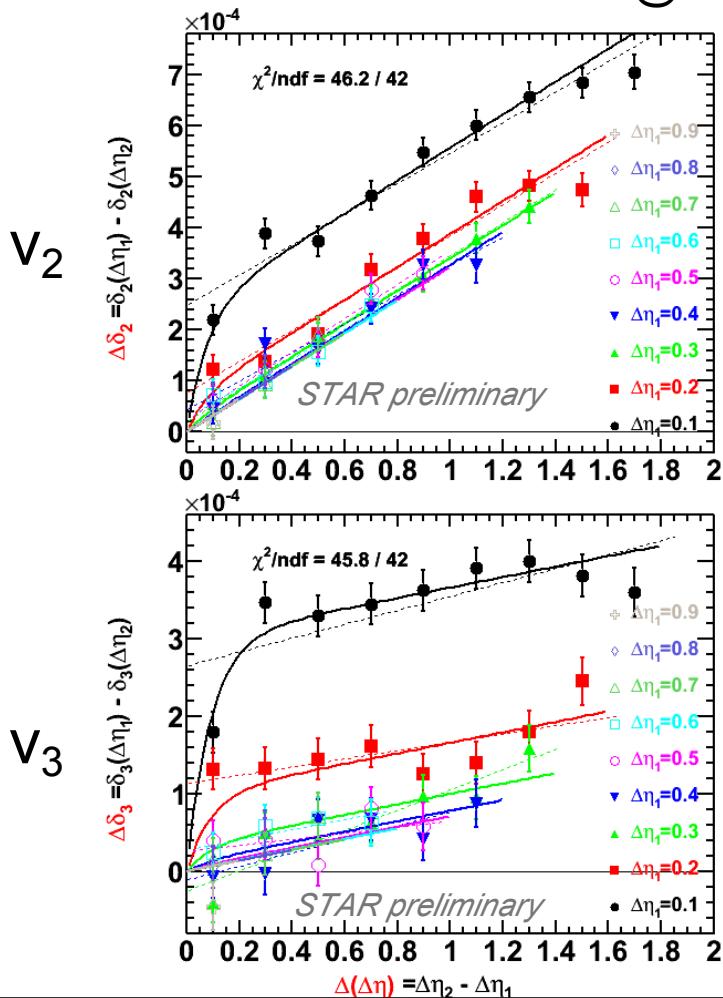


- $\delta(\Delta\eta_2) - \delta(\Delta\eta_1)$ linear in $\Delta\eta_2 - \Delta\eta_1$ at a given $\Delta\eta_1$ with similar slopes
- Intercept changes with $\Delta\eta_1$ exponentially



Improved Nonflow Fit

Au+Au@200GeV 20-30% data



$$\Delta\delta(\Delta\eta_1, \Delta\eta_2) = a * [\exp(-\Delta\eta_1/b) - \exp(-\Delta\eta_2/b)] - k(\Delta\eta_1 - \Delta\eta_2)$$

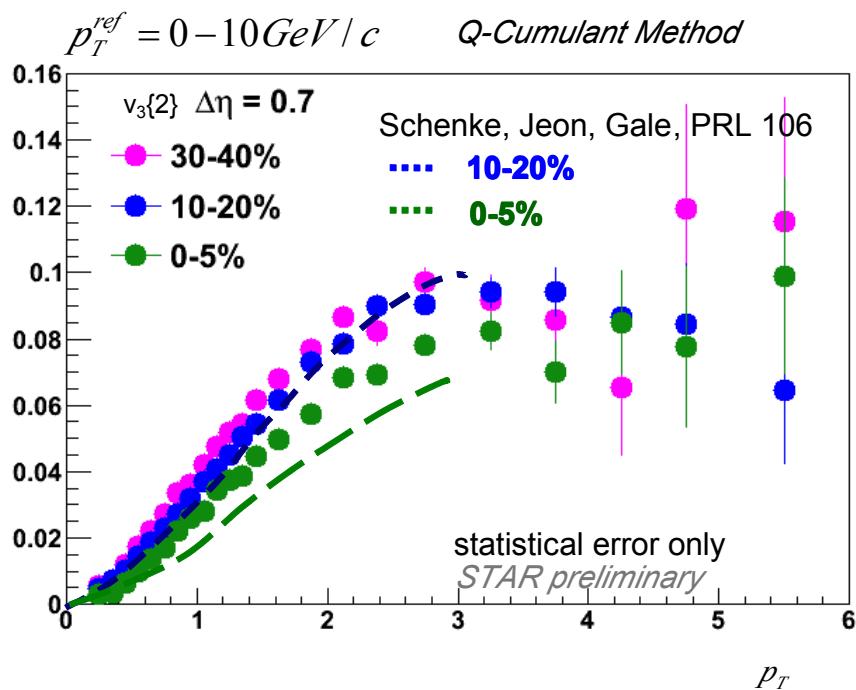
$$\delta(\Delta\eta) = a * \exp(-\Delta\eta/b) - k\Delta\eta + c$$

$$\Delta\delta(\Delta\eta_1, \Delta\eta_2) = a * [\exp(-\Delta\eta_1/b) - \exp(-\Delta\eta_2/b)] + A * [\exp(-\Delta\eta_1^2/2d^2) - \exp(-\Delta\eta_2^2/2d^2)]$$

$$\delta(\Delta\eta) = a * \exp(-\Delta\eta/b) + A * \exp(-\Delta\eta^2/2d^2)$$

v_3 vs p_T and Model Comparison

- v_3 more sensitive to η/s than v_2



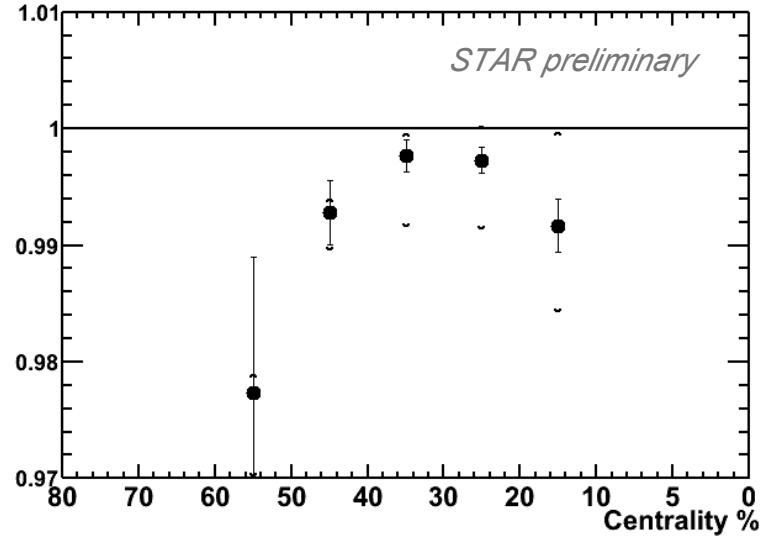
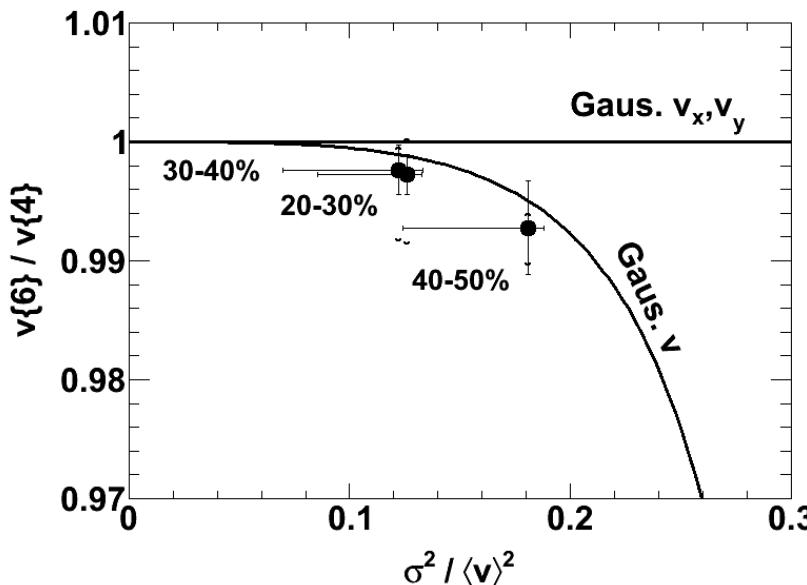
- Top 5%, hydro under-predicts data
- Non-central hydro describes data well at $p_T < 2 \text{ GeV}$
hydro deviates from data at $p_T > 2 \text{ GeV}$
- Data may contain nonflow

4- and 6-Particle Cumulant

Assuming the flow fluctuations are Gaussian, we have two options:

1. v_x, v_y are Gaussian: $v\{6\} = v\{4\}$ Voloshin, Poskanzer, Tang, Wang, PLB
2. v is Gaussian: LY, Wang, Tang, arXiv: 1101.4646

$$v_n\{6\}/v_n\{4\} \approx 1 - \sigma^6 / 3\langle v_n \rangle^6 , \text{ if } \sigma^2 / \langle v_n \rangle^2 \ll 1$$



* No-weight applied, non-uniform acceptance corrected. Systematic errors estimated by applying weight and no acceptance correction

Aug. 15th, 2012