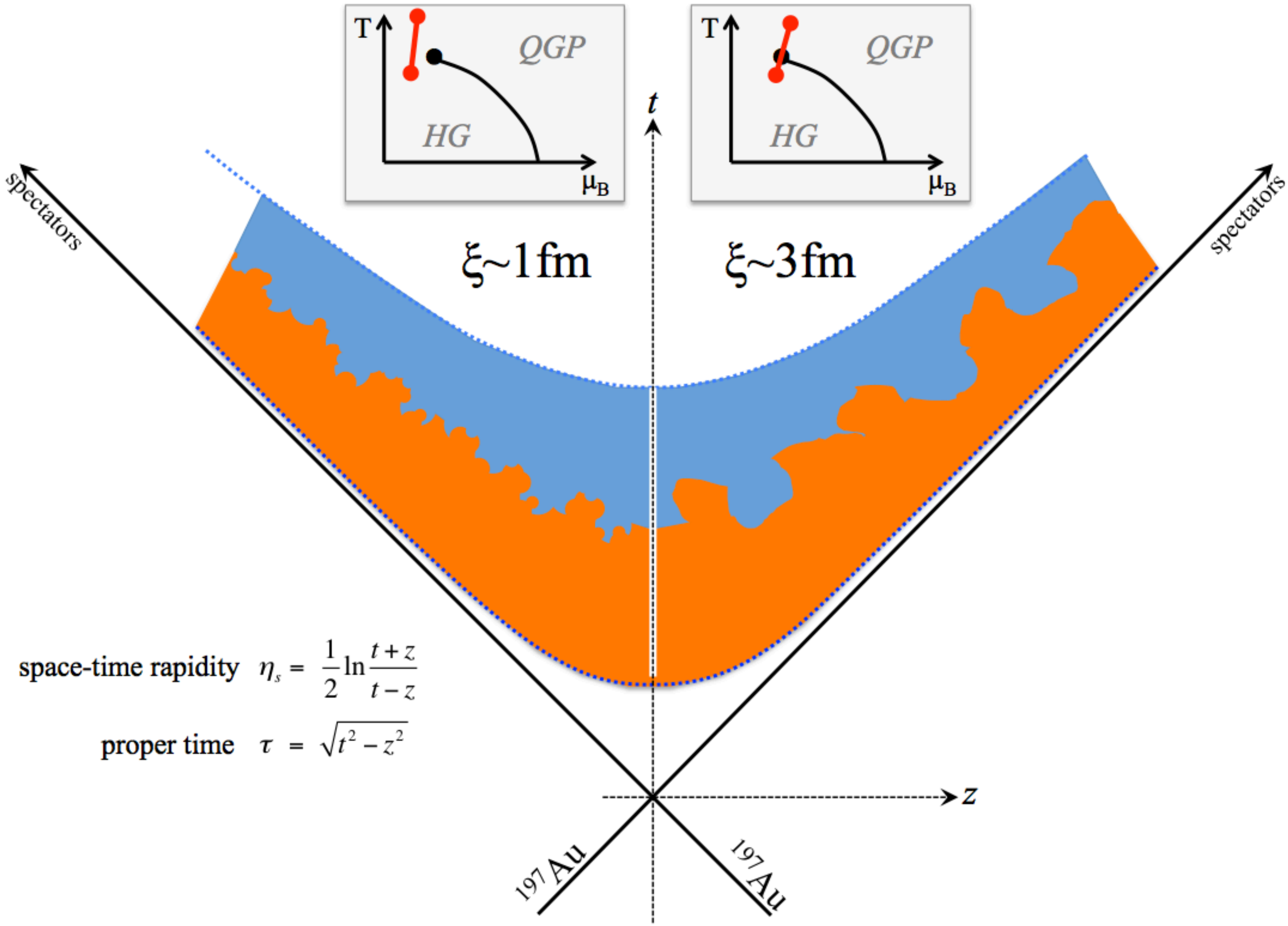
Rapidity Correlations

W.J. Llope for the STAR Collaboration
Wayne State University



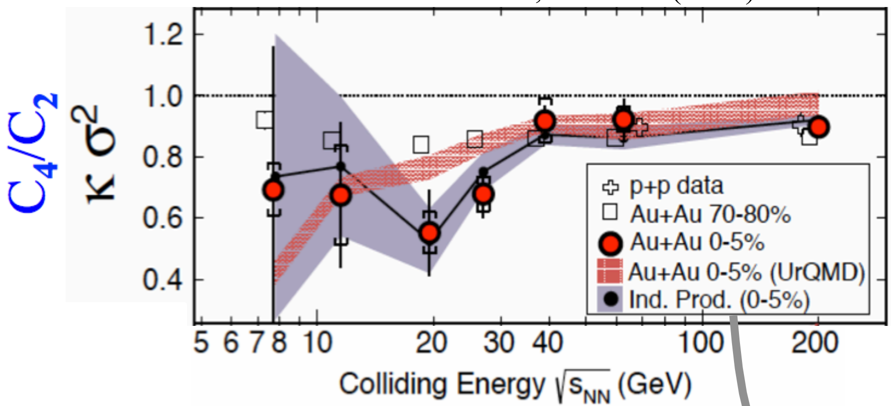
space-time rapidity $\eta_s = \frac{1}{2}\ln\frac{t+z}{t-z}$

proper time $\tau = \sqrt{t^2 - z^2}$

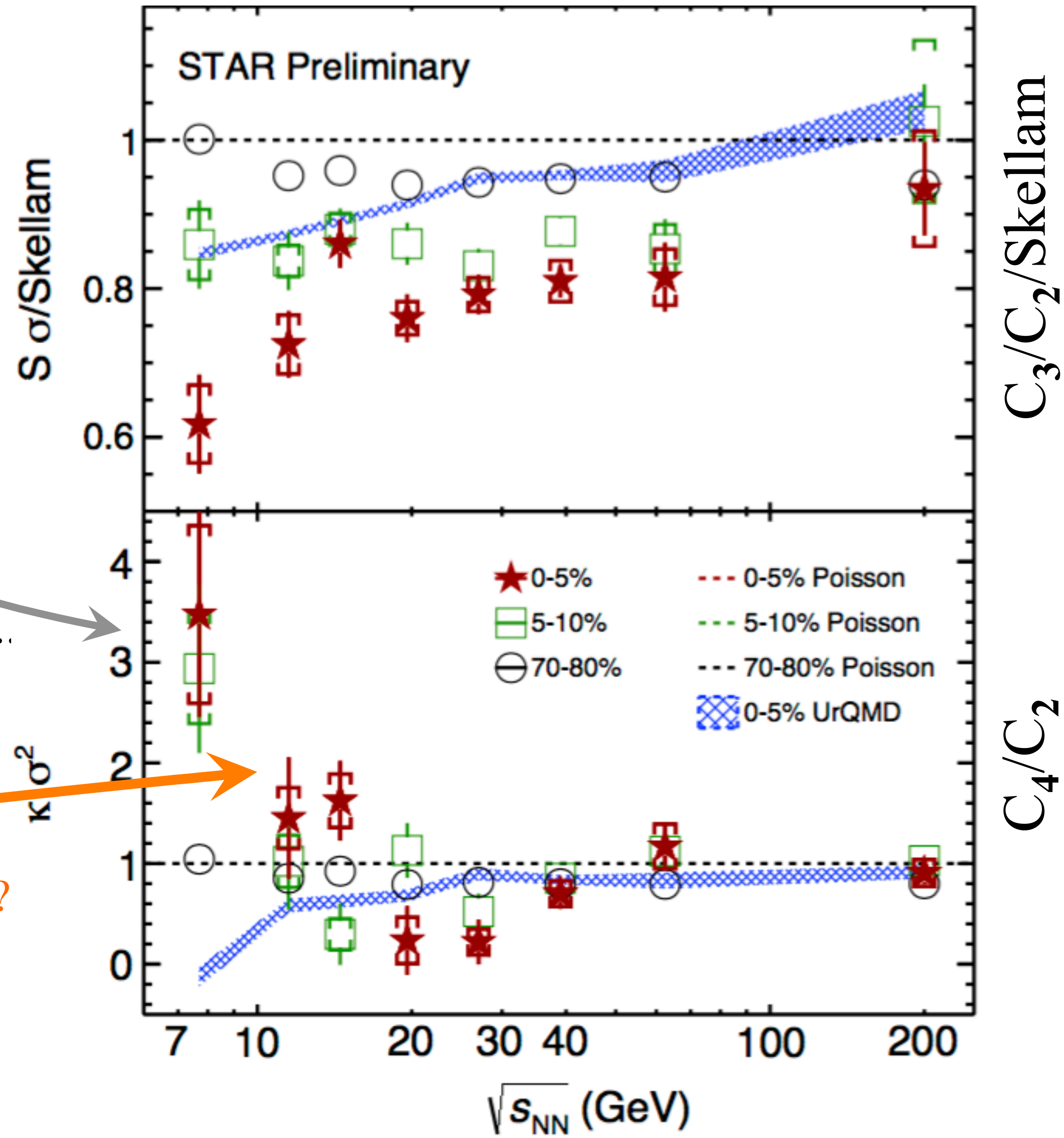
Net-proton multiplicity cumulant ratios...

STAR net-p multiplicity cumulant ratios

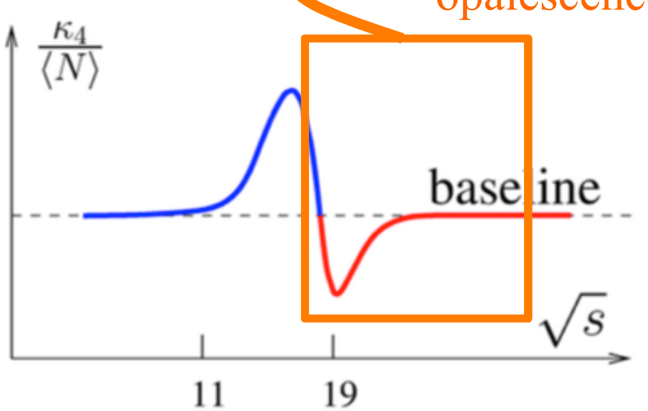
STAR, PRL 112 (2014) 032302



Widen the acceptance
from $0.4<P_T<0.8$ to $0.4<P_T<2.0$...



critical
opalescence?



A wider P_T acceptance made the deviations from Poisson much larger!

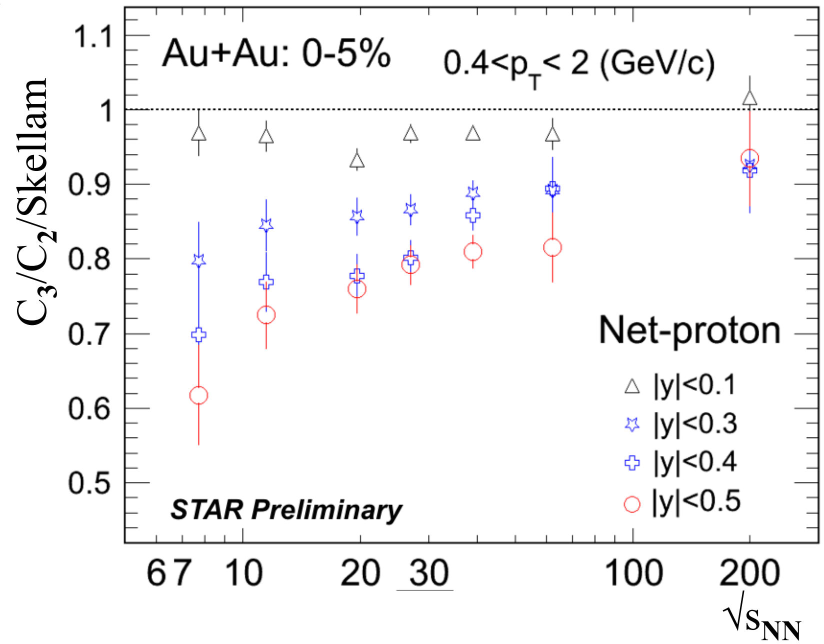
What (was learned in) BES-I? Net-proton multiplicity cumulant ratios...

In a small acceptance, you will see Poissonian cumulant ratios, CP or not....

V. Koch, RIKEN BNL Research Center Workshop on Fluctuations, Correlations and RHIC Low Energy Runs, October 3-5, 2011
http://quark.phy.bnl.gov/~htding/fcrworkshop/Koch.pdf

decreasing rapidity acceptance in the analysis also drives the C_4/C_2 values to Poisson:

see also D. Mahapatra *et al.*, arXiv nucl-ex/0108011v2



Net-baryon Acceptance:

0% — 100%

Poisson fluctuations — Zero fluctuations (baryon # conservation)

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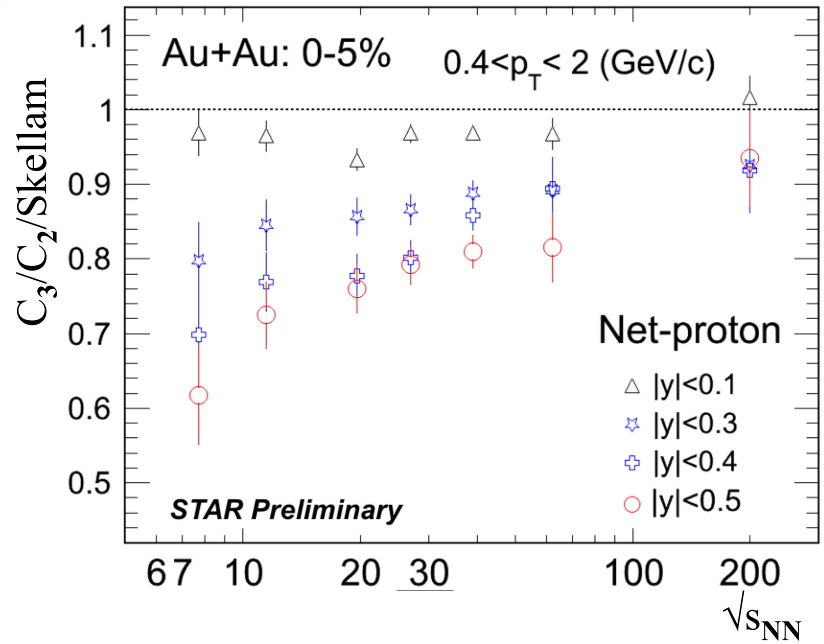

Net-baryon Acceptance:

0% — 100%

Poisson fluctuations

No signal!

Maximum signal?

B. Ling & M. Stephanov, arXiv 1512.09125

Zero fluctuations (baryon # conservation)

No signal!

What (was learned in) BES-I? Net-proton multiplicity cumulant ratios...

In a small acceptance, you will see Poissonian cumulant ratios, CP or not....

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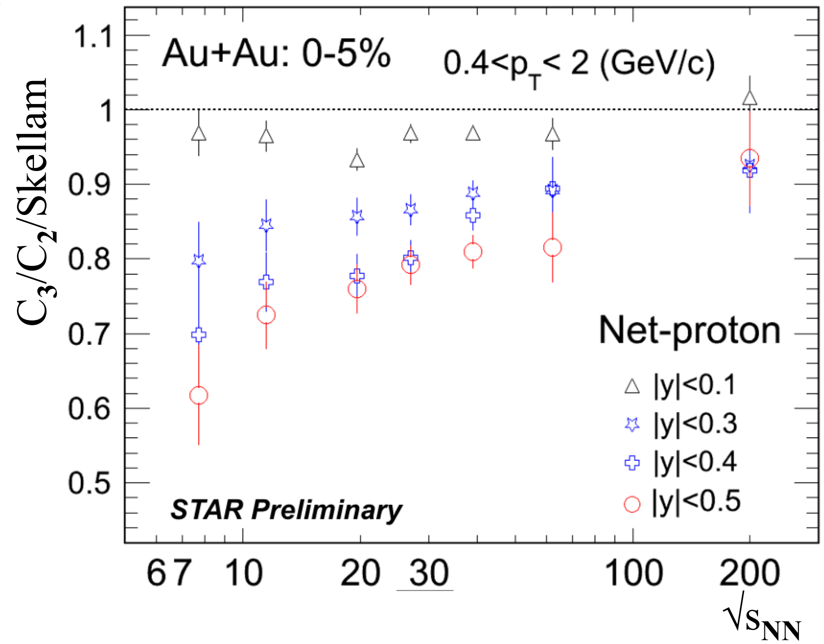


Net-baryon Acceptance:

0% 5% 14% 100%

Poisson fluctuations

Zero fluctuations (baryon # conservation)

net-protons
$0.4<P_T<0.8$, $\|y\|<0.5$
$C_1 \sim 8$ @ 19.6GeV

net-protons
$0.4<P_T<2.0$, $\|y\|<0.5$
$C_1 \sim 22$ @ 19.6GeV

Let's look differentially in rapidity.

Rapidity Correlations

$R_2(y_1,y_2)$ – developed at ISR & FNAL in 1970s to describe two particle correlations in (psuedo)rapidity
$R_2>0$ correlations, $R_2<0$ anticorrelations, $R_2=0$ uncorrelated.

$$R_2 = \frac{C_2(y_1,y_2)}{\rho_1(y_1)\rho_1(y_2)} = \frac{\rho_2(y_1,y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1$$

same event
mixed events or tensor product of 1D

lead to "cluster" picture...
- clusters decay to FS particles
- clusters uncorrelated w/ each other
- isotropic decay of clusters in their rest frames
- Lorentz-invariant translation of clusters in pseudorapidity

Exposes short and long-range correlations:
E & p conservation
minijets
HBT, Bose-Einstein, *etc.*

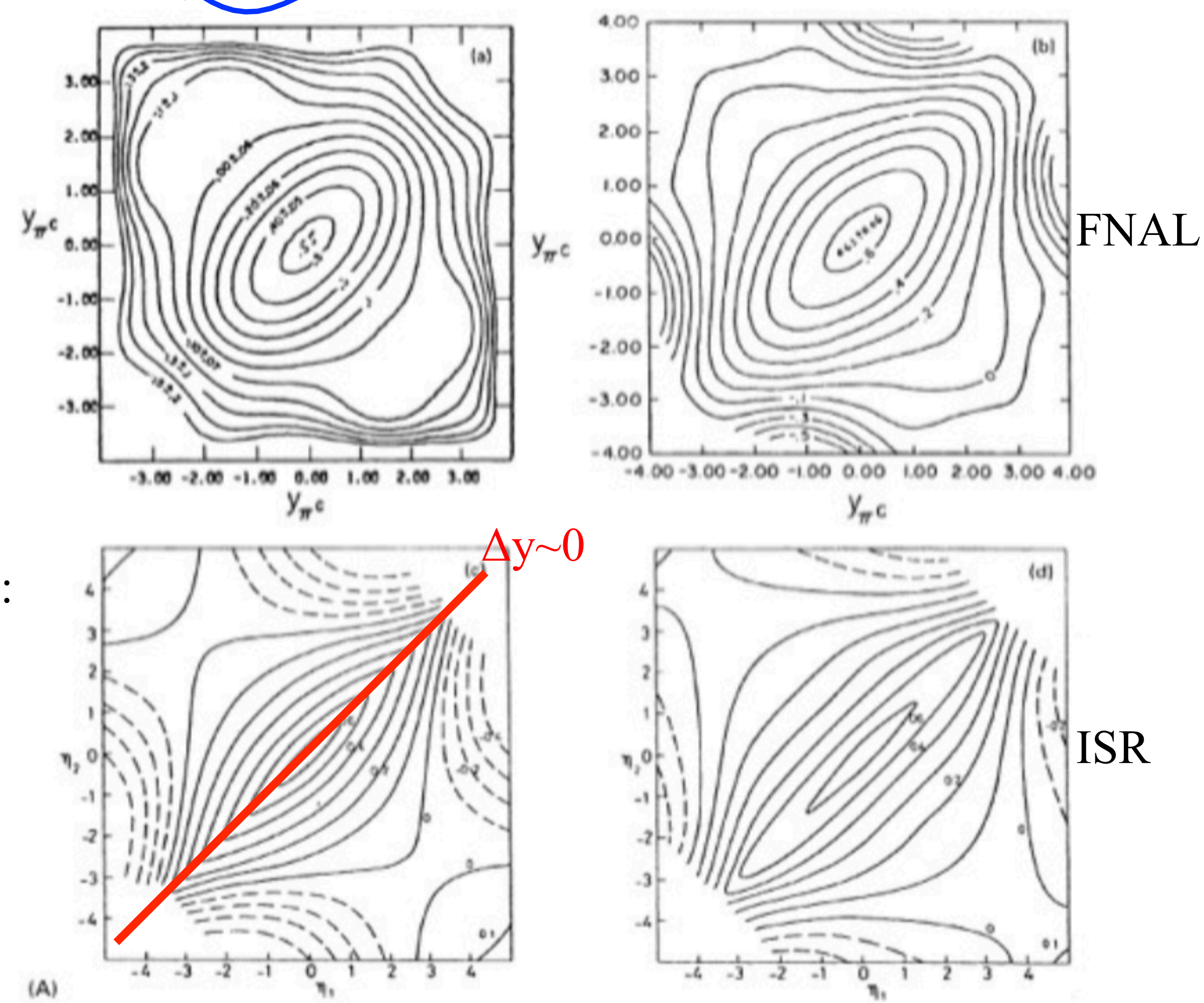


Figure 3.5: R_2^{cc} for $p+p$ collisions at FNAL (a-b)and CERN ISR (c-d): $\sqrt{s} = 13.7$, 27, 23, 63 GeV.

L. Foà, Phys. Lett. **C22**, 1 (1975)
H. Bøggild, Ann. Rev. Nucl. Sci. **24**, 451 (1974)
M. Jacob, Phys. Rep. **315**, 7 (1999)

Recent interest from theory side for such information

Recall how fourier decomposition of azimuthal angle distrubutions leads to all sorts of interesting information on elliptic flow, flow fluctuations, triangularity....

A similar approach can be applied to study the shape of the fireball in the longitudinal direction!

Long-range rapidity correlations as fluctuating rapidity density of the fireball:

A. Bialas, A. Bzdak, and K. Zalewski, Phys. Lett. B **710**, 332 (2012).
A. Bialas and K. Zalewski, Acta Phys. Pol. B **43**, 1357 (2012).

...possibly with a significant asymmetric component in fireball's rapidity shape:

B. I. Abelev *et al.* (STAR Collaboration), Phys. Rev. Lett. **103**, 172301 (2009).

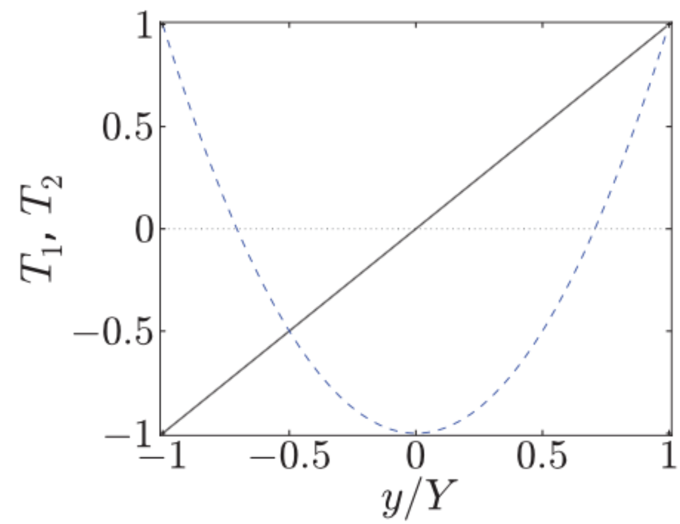
...Generalize!

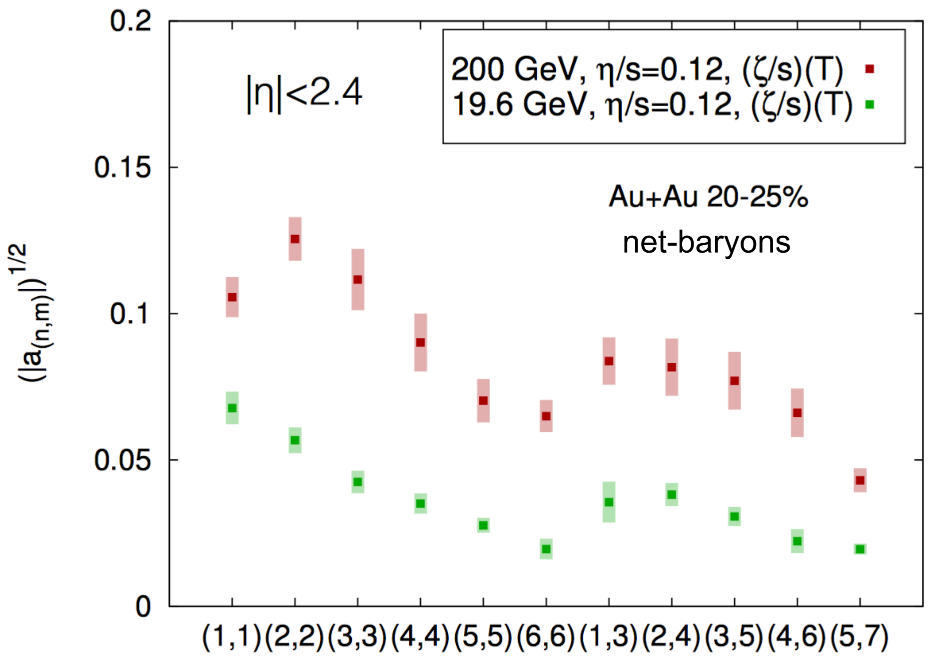
A. Bzdak and D. Teaney, Phys. Rev. C **87**, 024906 (2013)

$$C(y_1, y_2) \equiv \rho_2(y_1, y_2) - \rho(y_1)\rho(y_2)$$

...decompose rapidity cumulant into Chebyshev polynomials...

$$\frac{C_2(y_1, y_2)}{\langle\rho(y_1)\rangle\langle\rho(y_2)\rangle} = \sum_{i,k} \langle a_i a_k \rangle T_i(y_1/Y) T_k(y_2/Y)$$





information on the number of sources, baryon stopping mechanisms, viscosity, ...

See also:
A. Bzdak, Phys. Rev. C **85**, 051901(R) (2012)
T. Lappi & L. McLerran, Nucl. Phys. A **832**, 330 (2010)
A. Monnai, B. Schenke, arXiv:1509.04103
A. Bzdak (QM2015) 29/9/2015 16:00-16:20
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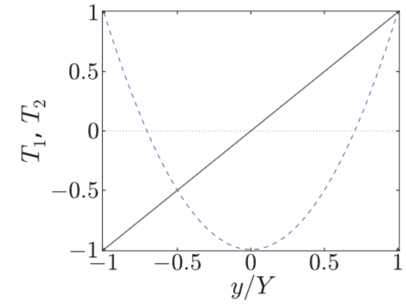
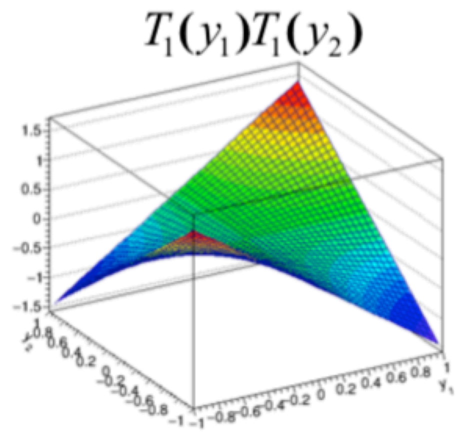
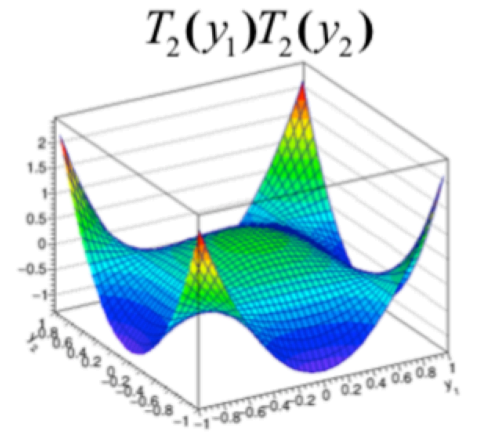
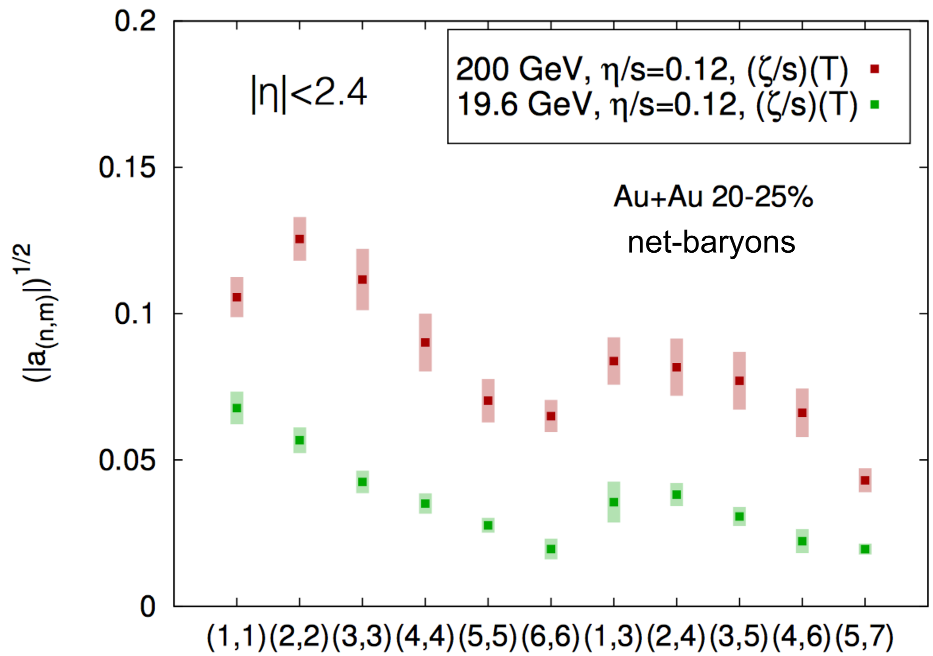


...decompose rapidity cumulant into Legendre polynomials...

$$\frac{C_2(y_1,y_2)}{\langle\rho(y_1)\rangle\langle\rho(y_2)\rangle} = \sum_{i,k} \langle a_i a_k\rangle T_i(y_1/Y)T_k(y_2/Y)$$







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$R_2(y_1,y_2)$ – developed at ISR & FNAL in 1970s to describe two particle correlations in (psuedo)rapidity
$R_2>0$ correlations, $R_2<0$ anticorrelations, $R_2=0$ uncorrelated.

Recently, this variable has reappeared with a new name: $C(y_1,y_2)$… $C(y_1,y_2) = R_2(y_1,y_2) + 1$

$$R_2 = \frac{C_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} = \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1$$

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$$C(y_1,y_2) = 1 + \frac{1}{2} < a_0 a_0 > + \frac{1}{\sqrt{2}} \sum_{n=1}^{\infty} < a_0 a_n > (T_n(y_1) + T_n(y_2)) + \sum_{n,m=1}^{\infty} < a_n a_m > \frac{T_n(y_1)T_m(y_2) + T_n(y_2)T_m(y_1)}{2}$$

J. Jia, S. Radhakrishnan, and M. Zhou, Phys. Rev. C93, 044905 (2016), arXiv:1506.03496

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reflects the multiplicity fluctuations in the event

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reflects the multiplicity fluctuations in the event

represents residual centrality dependence in the shape of <N(y)>

With a special normalization, the residual centrality dependence is largely eliminated.

$$C_N(y_1,y_2) = \frac{C(y_1,y_2)}{C_p(y_1)C_p(y_2)}$$

$$C_p(y_1) = \frac{\int_{-Y}^{Y} C(y_1,y_2)\,dy_2}{2Y}, C_p(y_2) = \frac{\int_{-Y}^{Y} C(y_1,y_2)\,dy_1}{2Y}$$

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reflects the multiplicity fluctuations in the event

represents residual centrality dependence in the shape of <N(y)>

encodes the dynamical shape fluctuations for events with the same centrality

With a special normalization, the residual centrality dependence is largely eliminated.

$$C_N(y_1,y_2) = \frac{C(y_1,y_2)}{C_p(y_1)C_p(y_2)}$$

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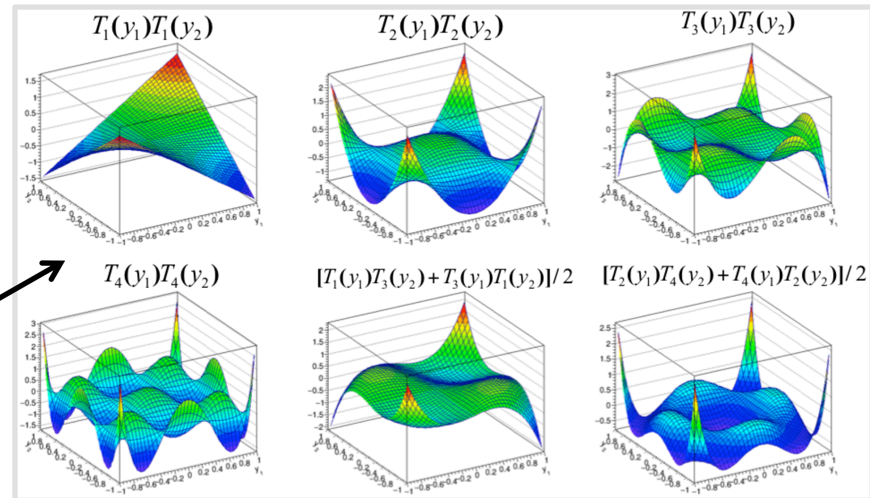


Dynamical shape fluctuations (and correlations) can be quantified by decomposing the measured distributions onto a basis set of Legendre polynomials, with "strength" coefficients $<a_m a_n>$

Rapidity analog of decomposition of azimuthal anistropies onto cos(nφ…) bases with strengths v_n

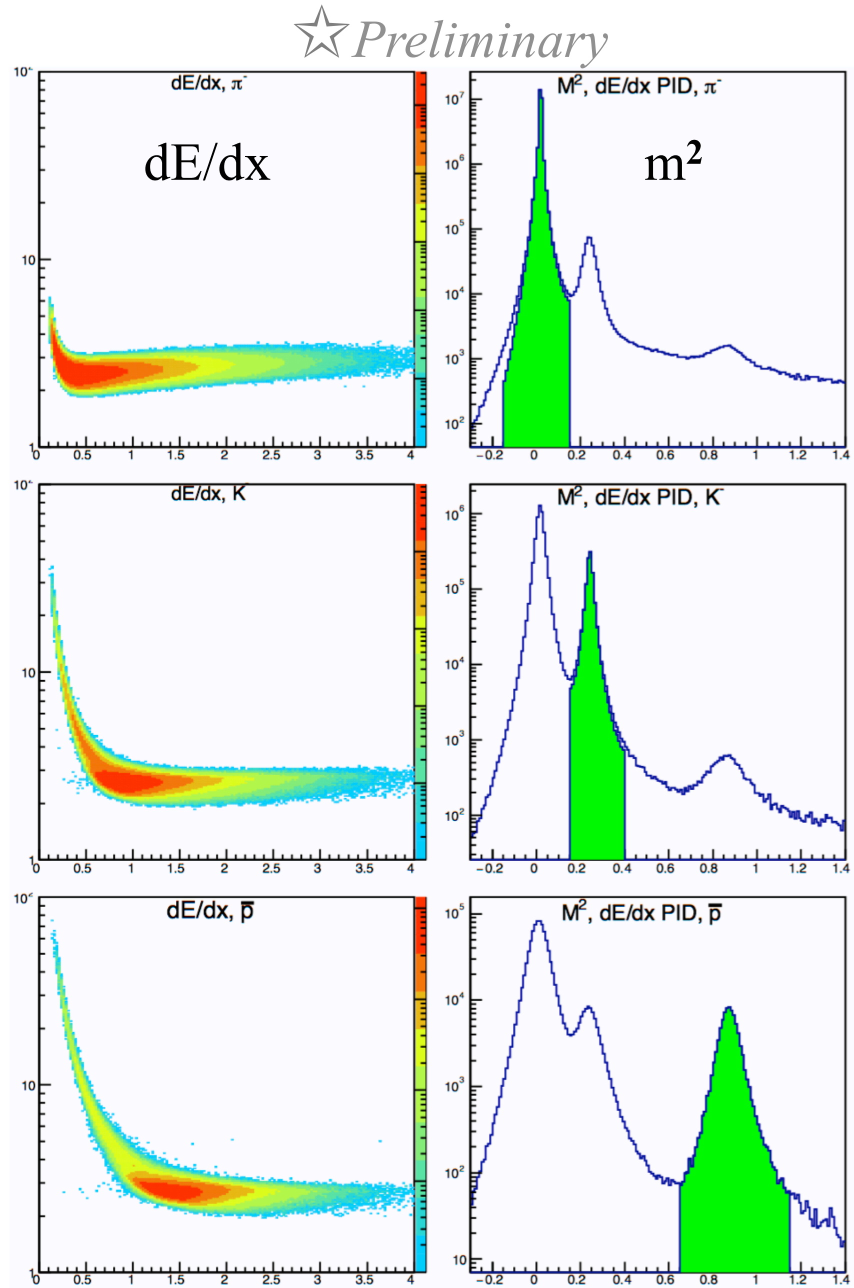
Datasets, cuts, centrality

Datasets: All 8 BES energies

POI: h±, K±, & p±
2σ on dE/dx, then require good TOF m^2

General cuts: $|Zvtx|<30$cm at all $\sqrt{s_{NN}}$
Nhitsfit>15
gDCA<2cm
p_T^{min} : 0.2 for h± & K±, 0.4 for p±
p_T^{max} : 2.0
p^{max} : 1.6 for h± & K±, 3.0 for p±

Centrality: N_{tracks} with $0.5<\eta<1$ for h± & K±
$N_{\pi,K}$ with $0.5<\eta<1$ for p±



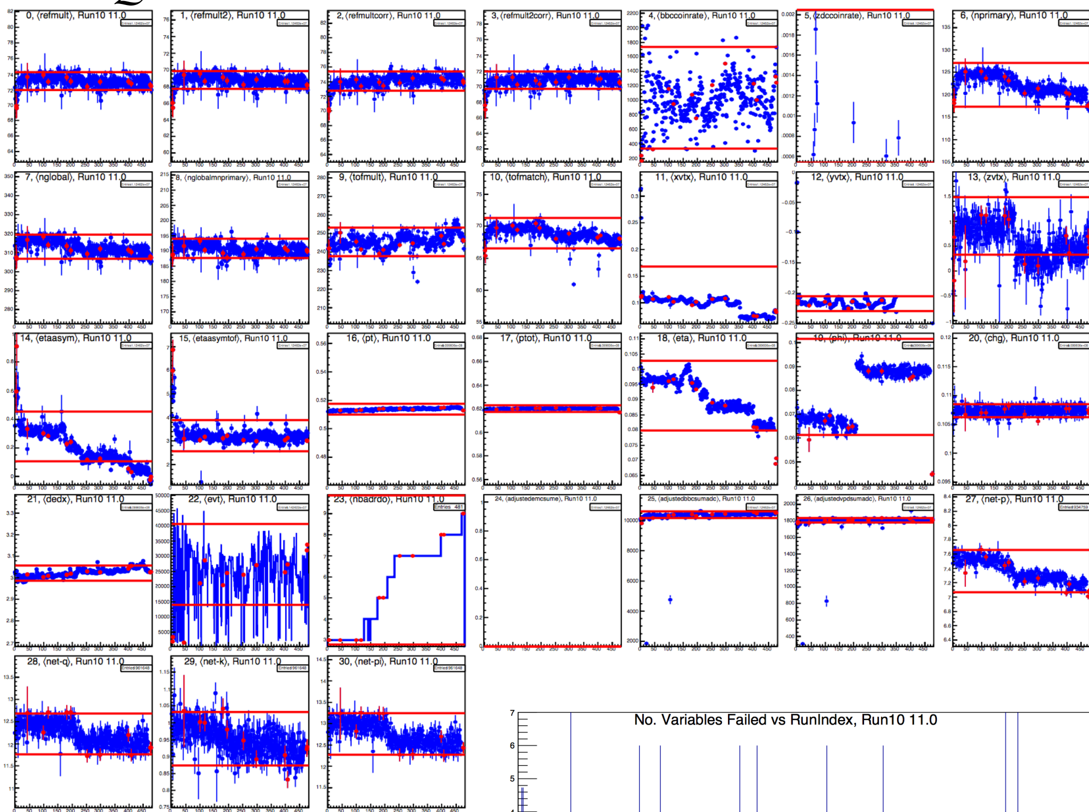
Cuts & centrality intentionally very close to those used in recent ☆ multiplicity cumulant analyses.

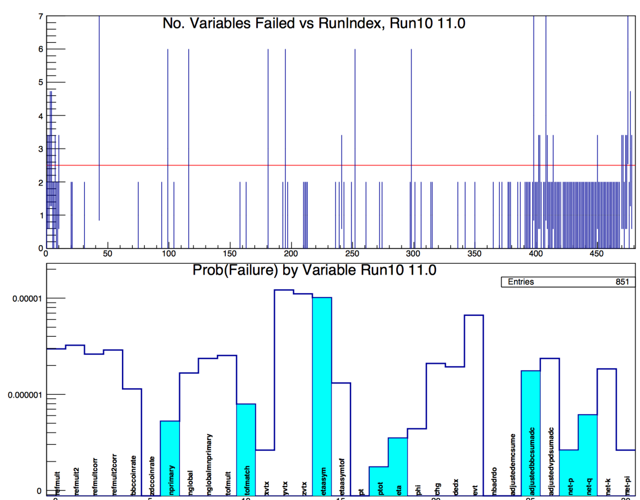
Same analysis code used for UrQMD events
~20M min. bias events available at each $\sqrt{s_{NN}}$…
Parameterized exp. efficiency vs (PID,pt,y,cent)…
Centrality via cuts on b when integrated w/ RM>1

Extremely careful run and event QA

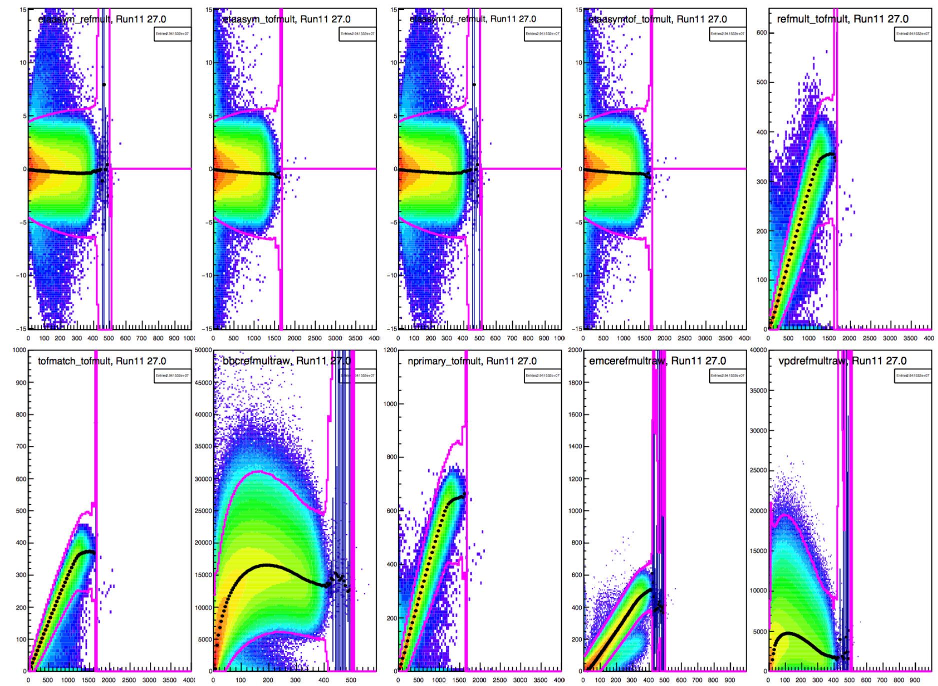
Careful good run & good event QA performed...

Run QA





Event QA (in good runs)



Turn now to some experimental effects:

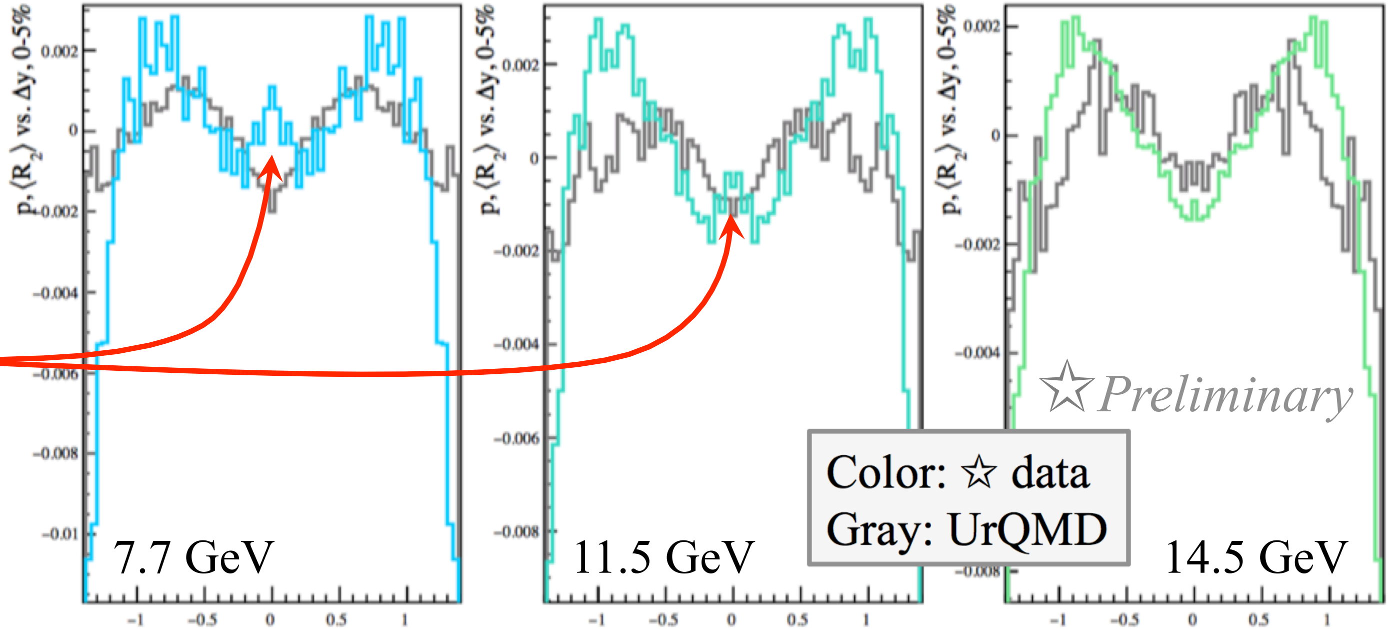
- Zvtx smearing
- Track merging
- rapidity binning

Zvtx averaging

Pseudocorrelations
$\langle R_2 \rangle$ vs Δy

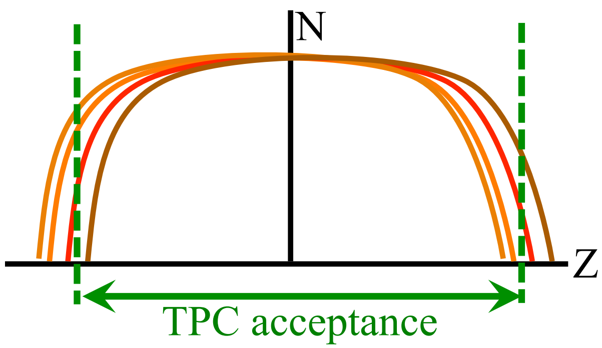
low Δy enhancement…

not seen in UrQMD evts…

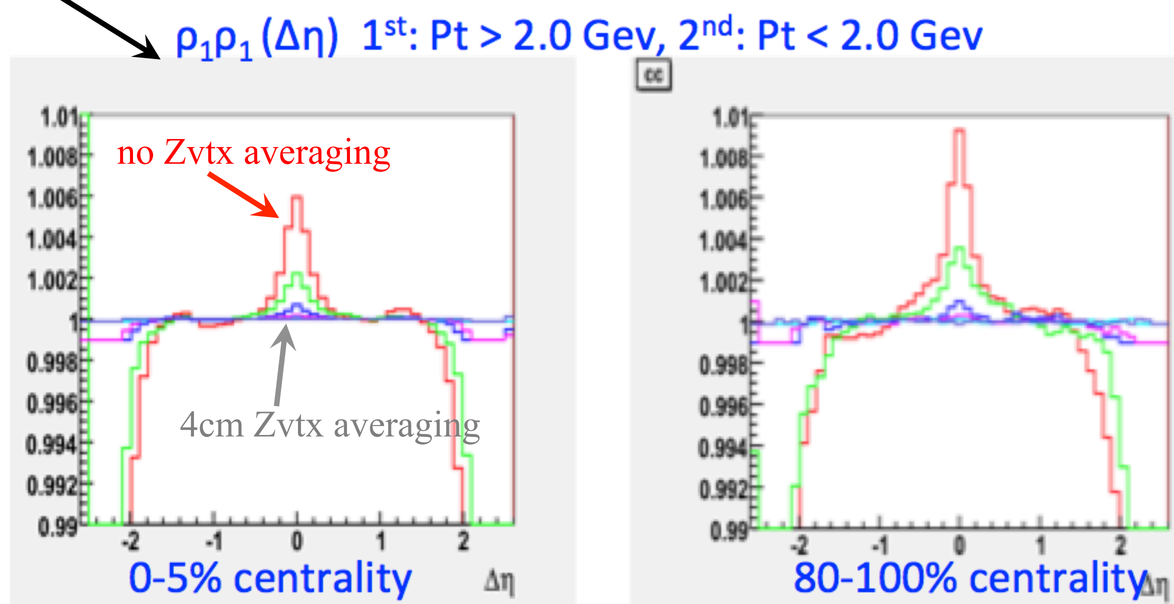


Caused by rapidity dependence of experimental efficiency coupled with Zvtx smearing…

See L. Tarini, Ph.D. Thesis, and his talk at the STAR Analysis Meeting, MIT, 7/10/2009



✔ Now I analyze in 2cm-wide Zvtx bins then weight-average the results…



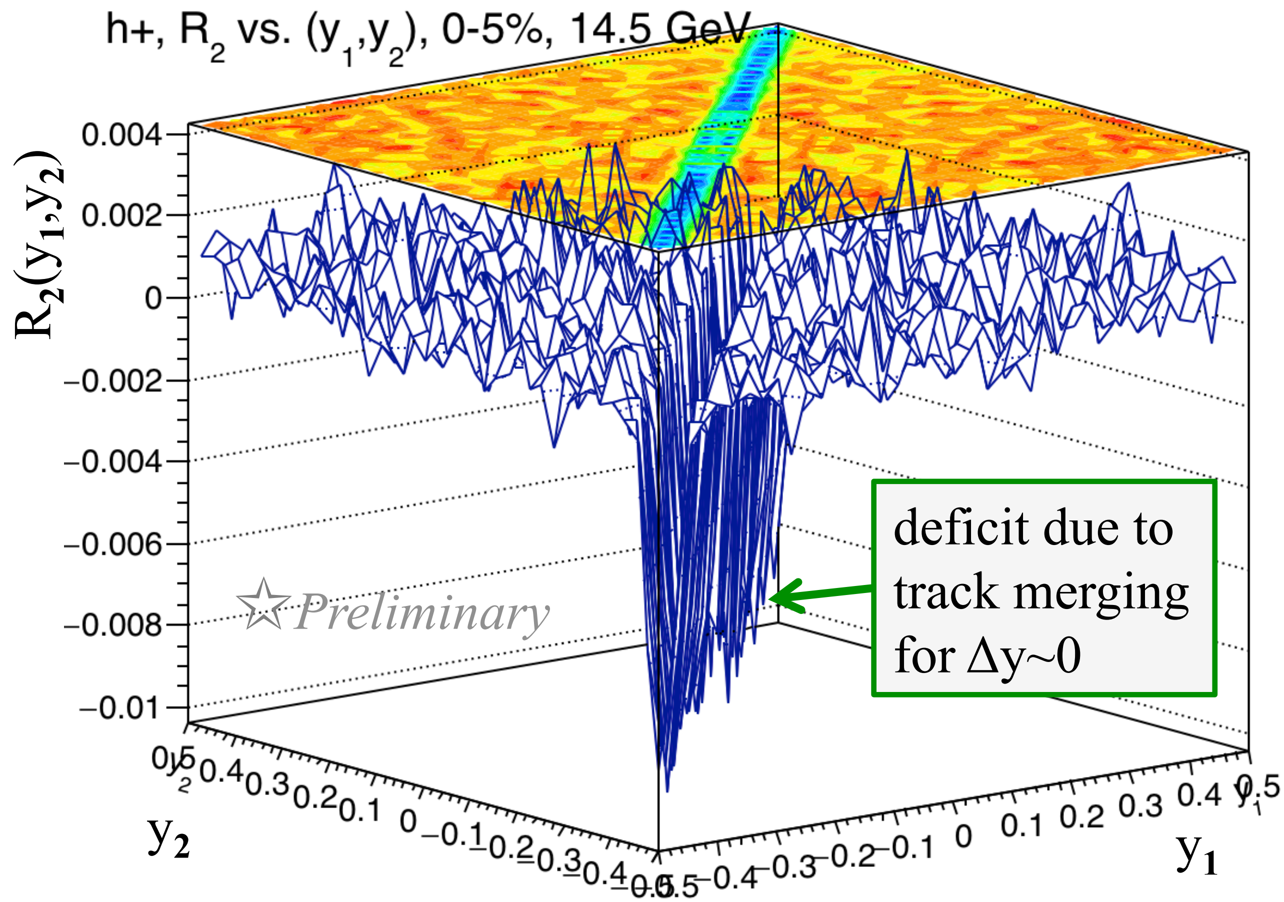
Slightly reduced efficiency for nearby tracks…

Very strong trench in R_2 when particle multiplicities/event of POI get large:
h± for all centralities and $\sqrt{s_{NN}}$, and only most central for K±

Numerator and denominator of R_2 & C_N uses only measured tracks…
but there is a slight 2-particle efficiency loss when two tracks are nearby ($\Delta y \sim 0$)

The STAR track-finder `"sti"` does not share spacepoints!

a new one does `"stiCA"` (10%)



Beating the track merging deficits at Δη~0

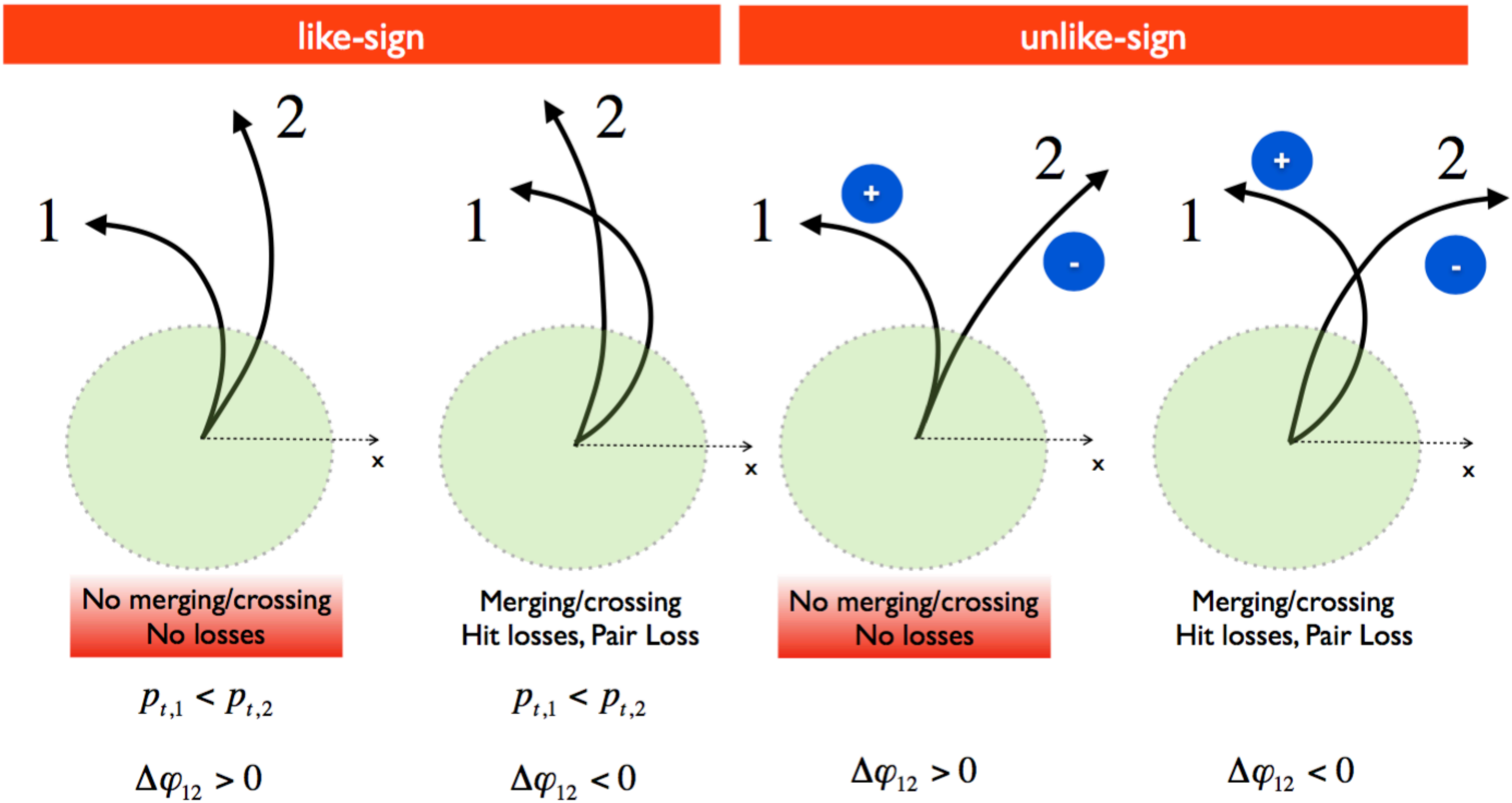


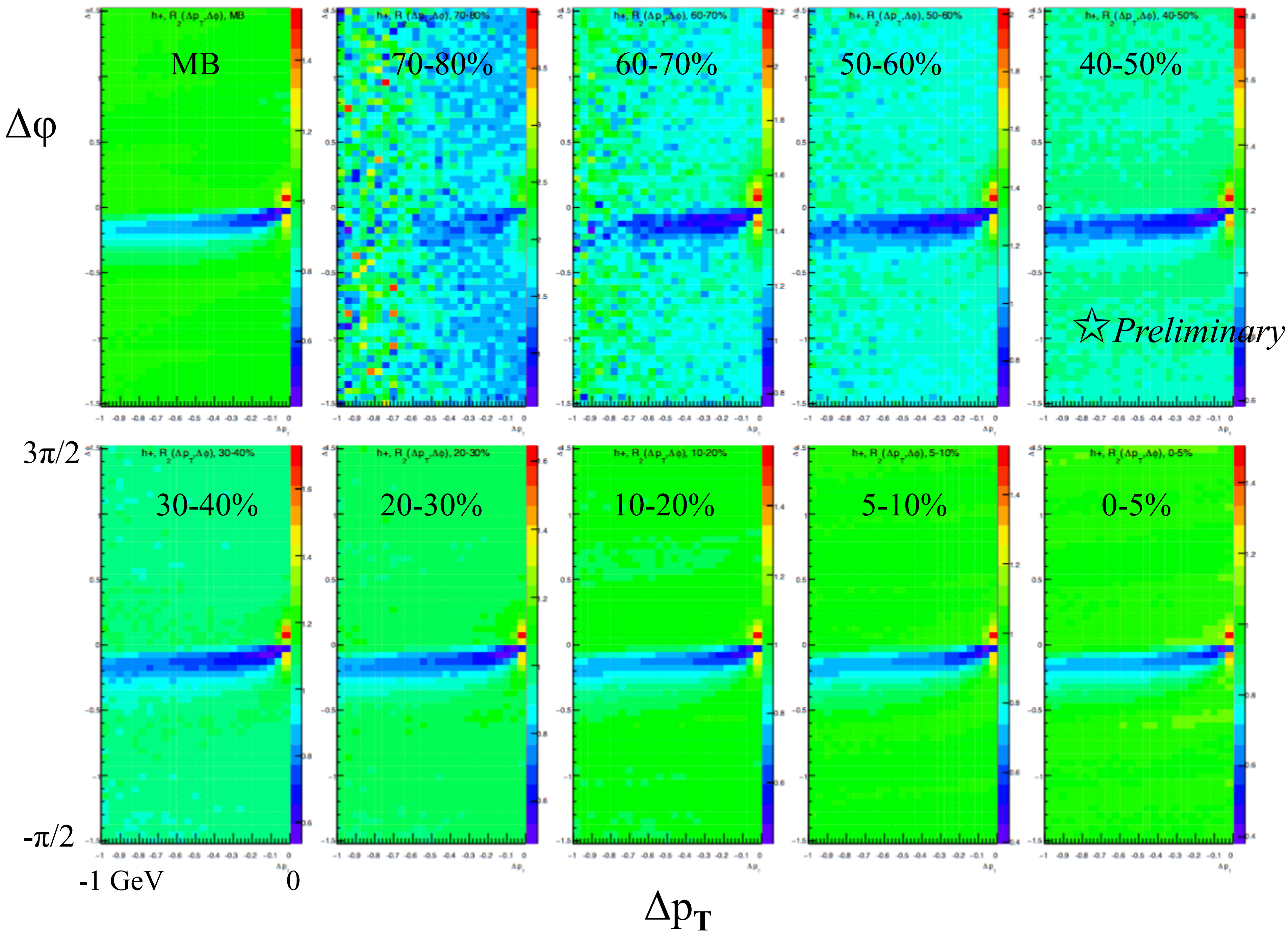
Image from P. Pujahari

LS & US: reflect clean area in Δφ to replace problem area

US: nothing special in fill method

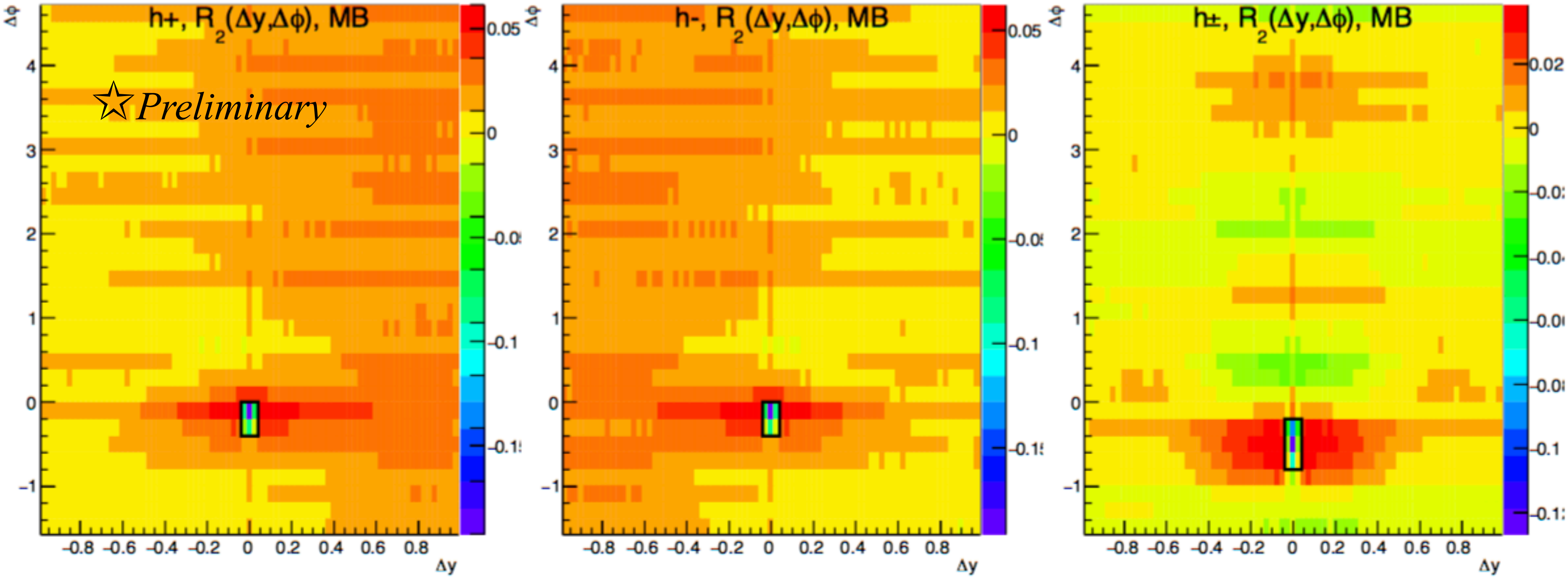
LS: pT order the tracks, fill numerator for upper triangle only, then symmetrize

h+ LS, $R_2(\Delta p_T,\Delta\varphi)$ for $|\Delta y|<0.04$, AuAu 200 GeV, MB and by centrality



h LS & US, $R_2(\Delta y,\Delta\varphi)$, AuAu 200 GeV 0-5%

unsymmetrized $R_2(\Delta y,\Delta\varphi)$



Thus cannot simply start from $\rho_2(y_1,y_2)$ and $\rho_1(y_1)*\rho_1(y_2)$…

…I must calculate each as 3D hists as $(y_1,y_2,\Delta\varphi)$

Several symmetrization approaches tried to "fill" merging hole... All ***tricky***

For now, just cut out the merging hole in both same and mixed event histograms

Binning

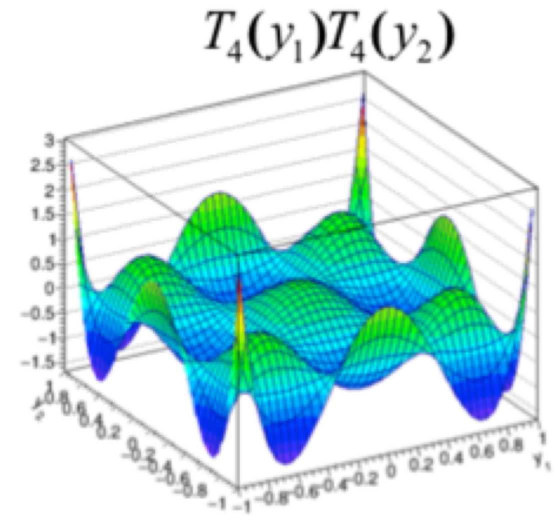
The cut used is $|\Delta y|<0.04$ and $-5\pi/12 \leq \Delta\varphi < 0$

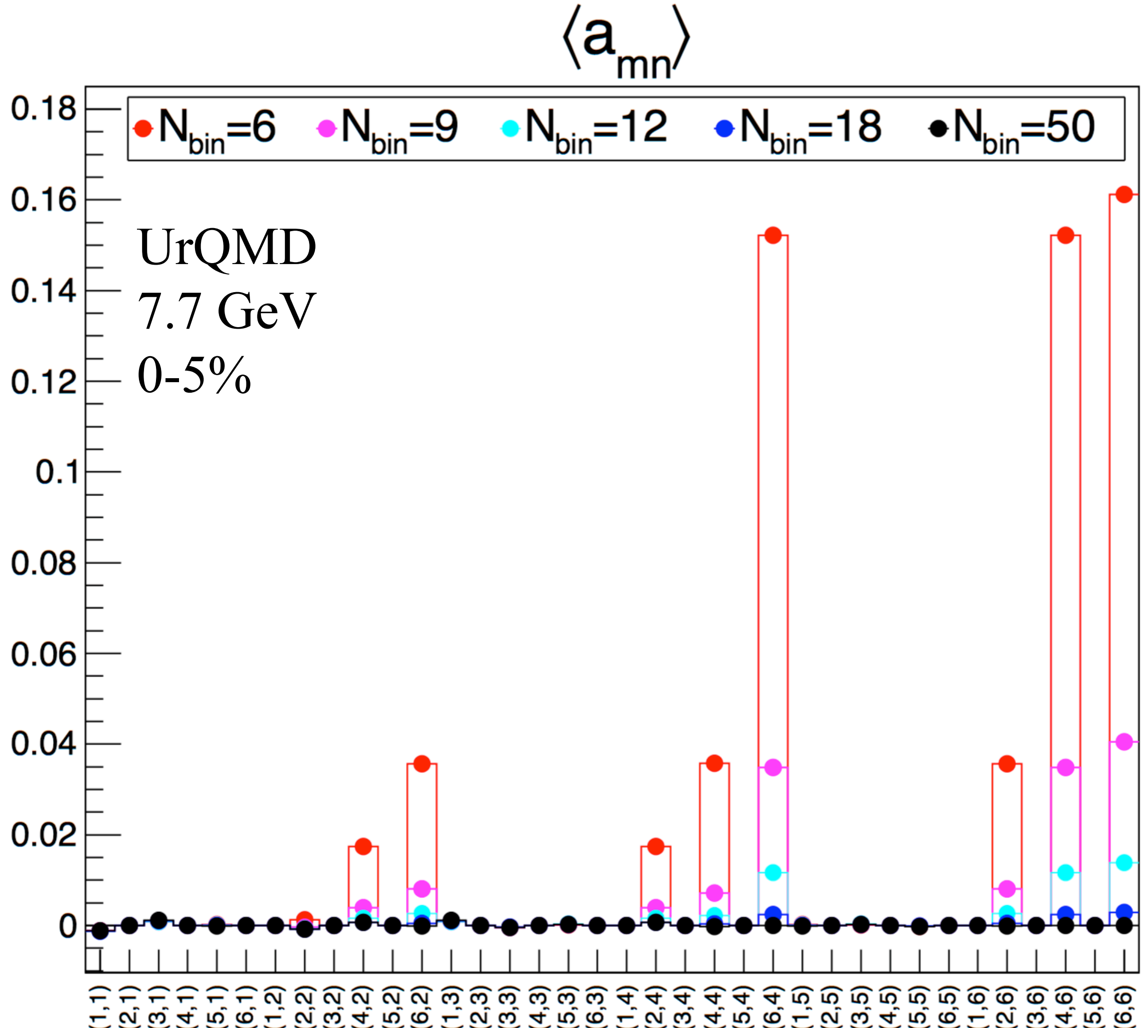
Given this cut, I cannot bin the (y1,y2) parts of the TH3D too finely!

(or there will never be any counts in the $\Delta y=0$ bins)

Rapidity bin width must be near or larger than 2*0.04…

But this can cause non-physical artifacts in the $\langle a_{mn} \rangle$ values!





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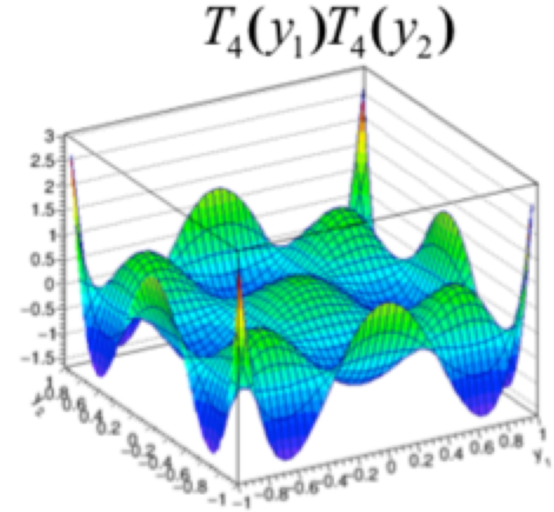

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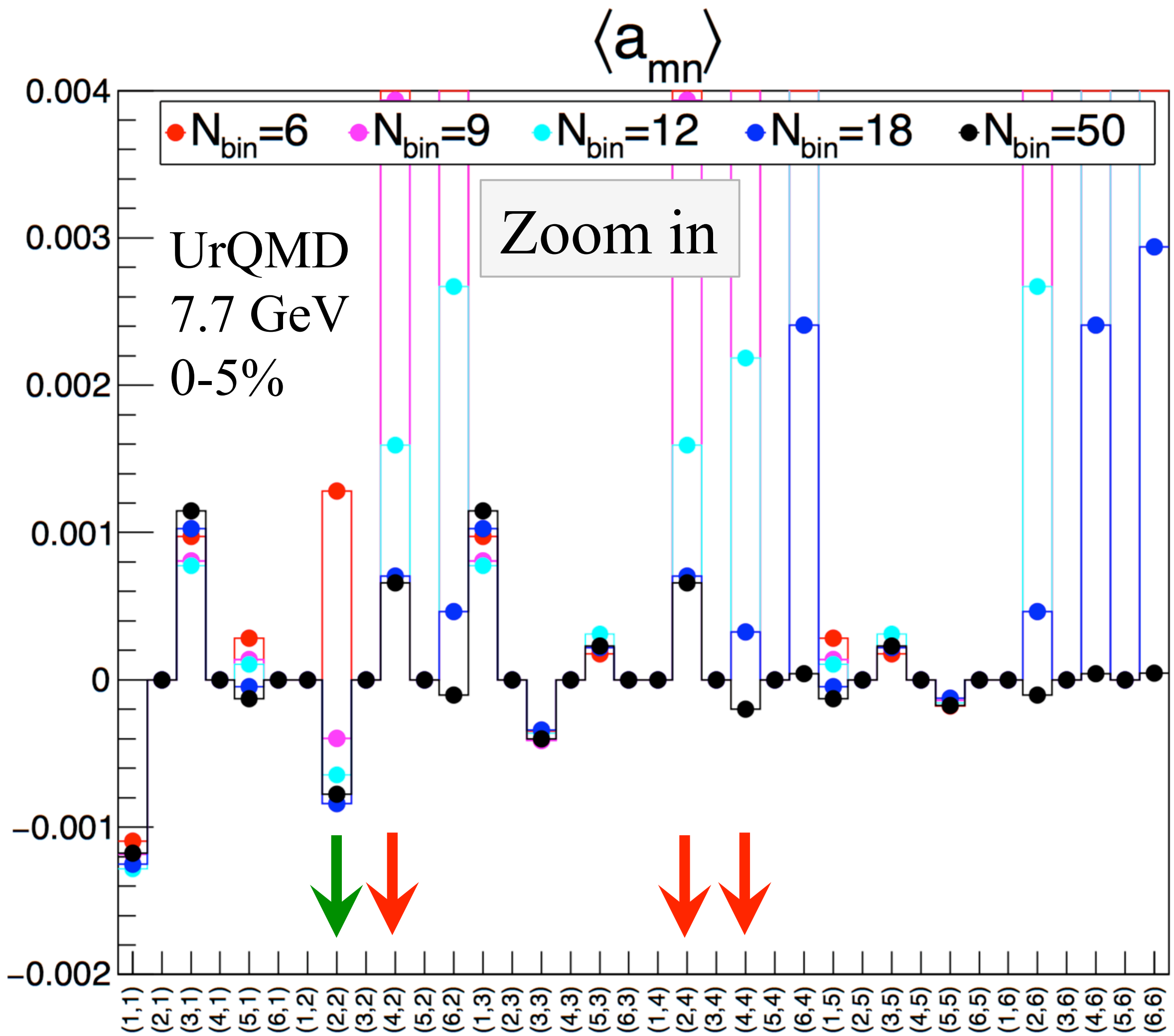
Rapidity bin width must be near or larger than 2*0.04…

But this can cause non-physical artifacts in the $\langle a_{mn}\rangle$ values!



$-0.72<y<0.72$

(2,2) stable for $N_{bin}\geq 12$

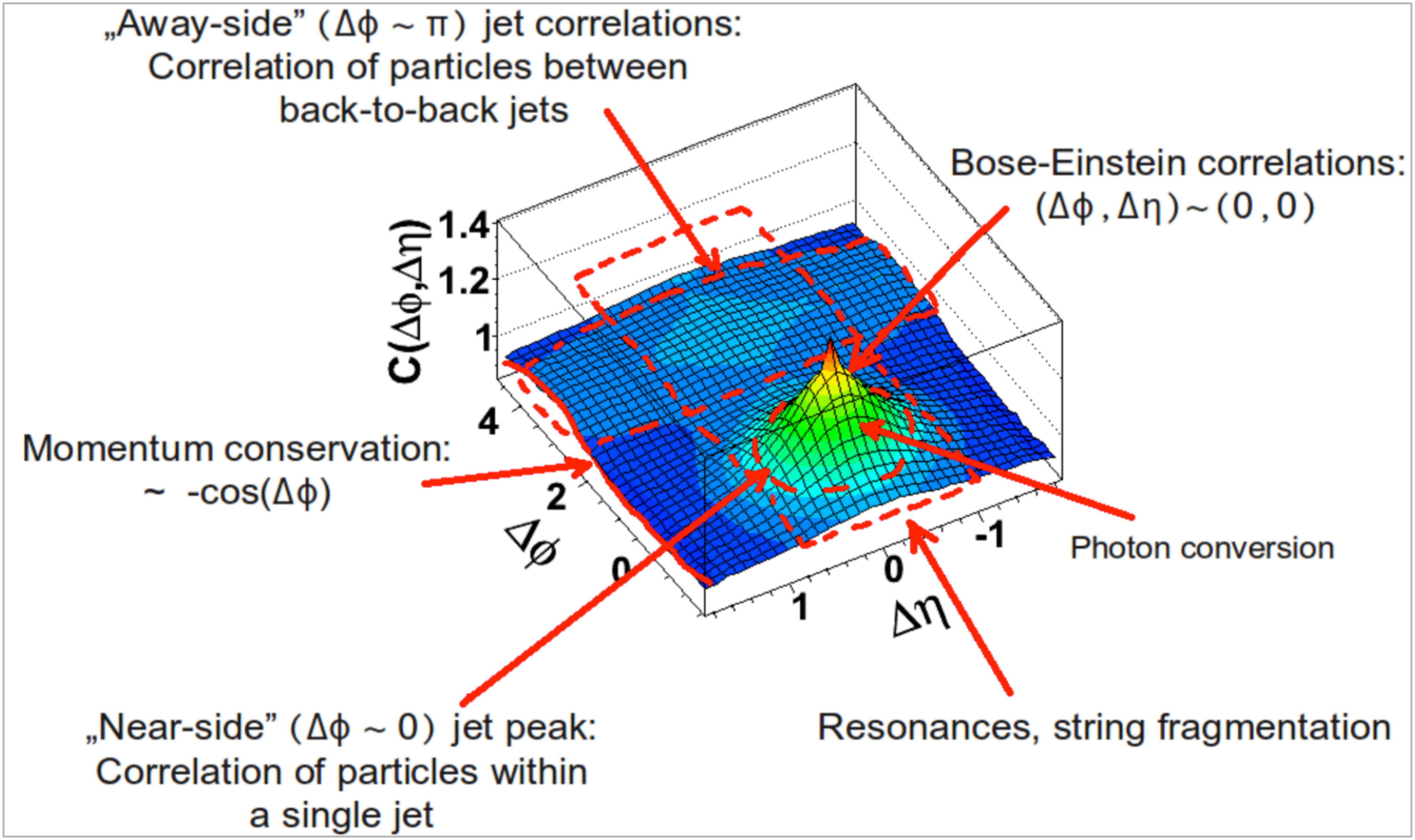
(2,4) stable for $N_{bin}\geq 18$

(4,4) stable for $N_{bin}>>18$

So, use 12 bins
and don't show $\langle a_{mn}\rangle$
for (2,4) or (4,4)

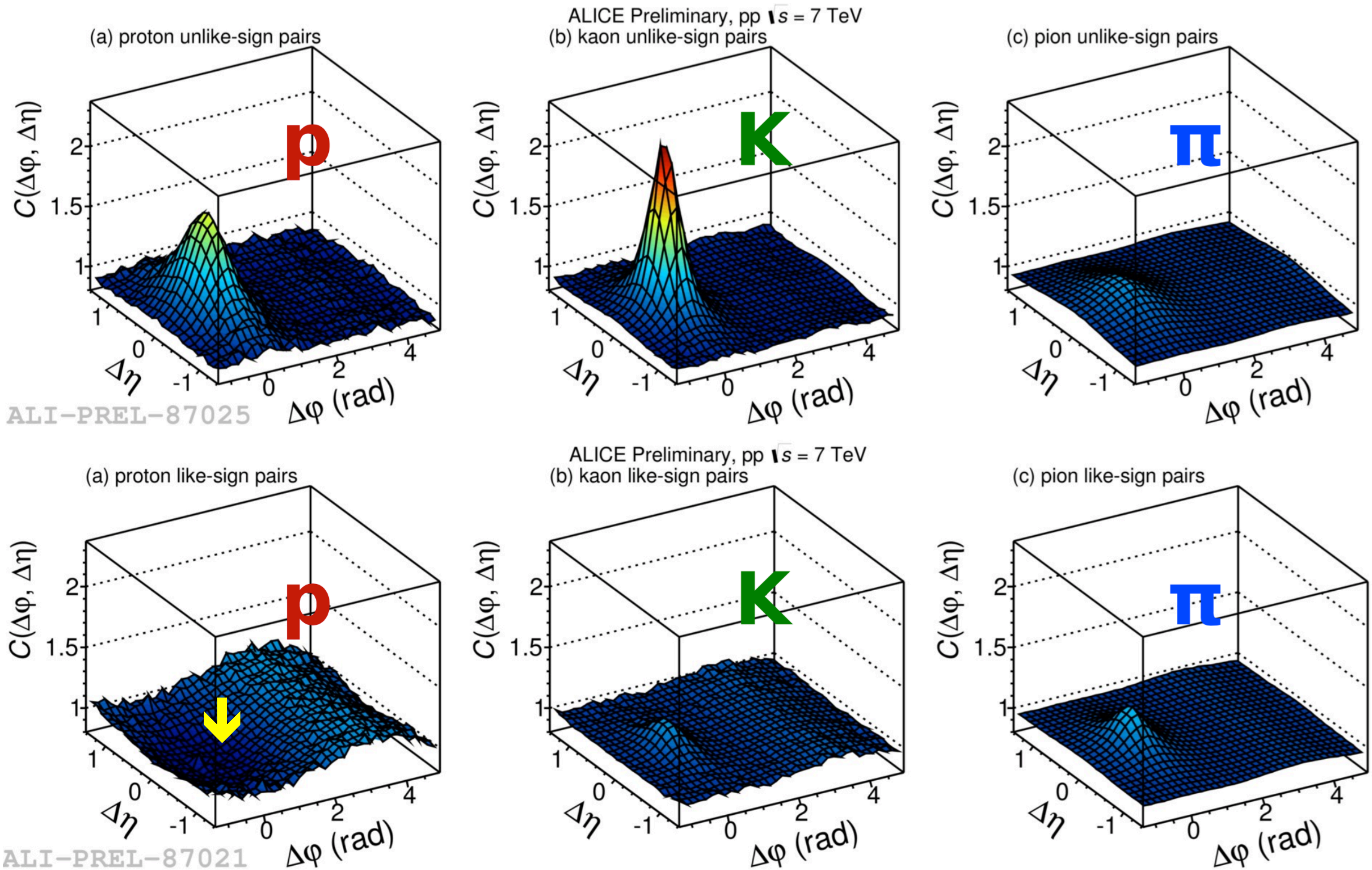
$R_2(\Delta y,\Delta\varphi)$ from $R_2(y_1,y_2,\Delta\varphi)$

With these 3D distributions, I can then make the CF plots *vs.* $(\Delta y,\Delta\varphi)$



Małgorzata Janik, X Workshop on Particle Correlations and Femtoscopy, Gyöngyös, Hungary, Aug 26, 2014

ALICE p+p $\sqrt{s_{NN}}$=7 TeV



Małgorzata Janik, X Workshop on Particle Correlations and Femtoscopy, Gyöngyös, Hungary, Aug 26, 2014

ALICE p+p $\sqrt{s_{NN}}$=7 TeV

Minijet peak at (Δy,$\Delta\varphi$) ~ (0,0) seen for all PID'd pairs *except* LS protons…

$\Delta\varphi$ dependence is $-\cos(\Delta\varphi)$ so consistent with momentum conservation…

But comparison with models with strict mom'n conservation do not have this hole *(?!?)*

ALICE p+p $\sqrt{s_{NN}}$=7 TeV

Minijet peak at $(\Delta y,\Delta\varphi) \sim (0,0)$ seen for all PID'd pairs *except* LS protons…

$\Delta\varphi$ dependence is $-\cos(\Delta\varphi)$ so consistent with momentum conservation…

But comparison with models with strict mom'n conservation do not have this hole *(?!?)*

In RHIC BES data, another possibility is lack of available energy to create nearby 2nd baryon…

This idea used to describe e^+e^- data at 20-30 GeV

Małgorzata Janik, X Workshop on Particle Correlations and Femtoscopy, Gyöngyös, Hungary, Aug 26, 2014

From mechanism of jet production: Two primary hadrons with the same **baryon number** (or **charge** or **strangeness**) **are separated** by at least two steps in rank ("rapidity").

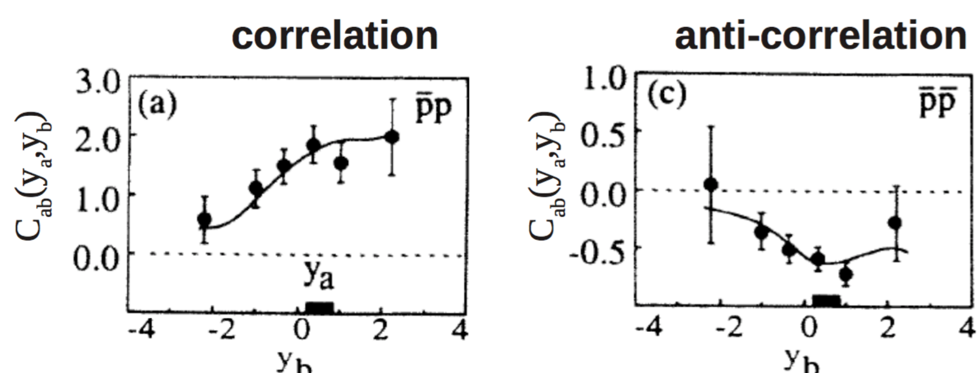
=>

To conservations of the quantum numbers:
- global conservation
- **local conservation**

A Parametrization of the Properties of Quark Jets
R.D. Field, R.P. Feynman (Caltech). Nov 1977. 131 pp.
Published in Nucl.Phys. B136 (1978) 1

We are not likely to find two baryons or two antibaryons at the same rapidity.

- y_a (black bar) rapidity range of first particle
- y_b rapidity of second particle
- C_{ab} correlation function



Study of baryon correlations in e+e− annihilation at 29-GeV
TPC/Two Gamma Collaboration (H. Aihara et al.), Phys.Rev.Lett. 57 (**1986**) 3140

26/08/2014, WPCF '14 Małgorzata Janik – Warsaw University of Technology

Turning now to the ☆ data...

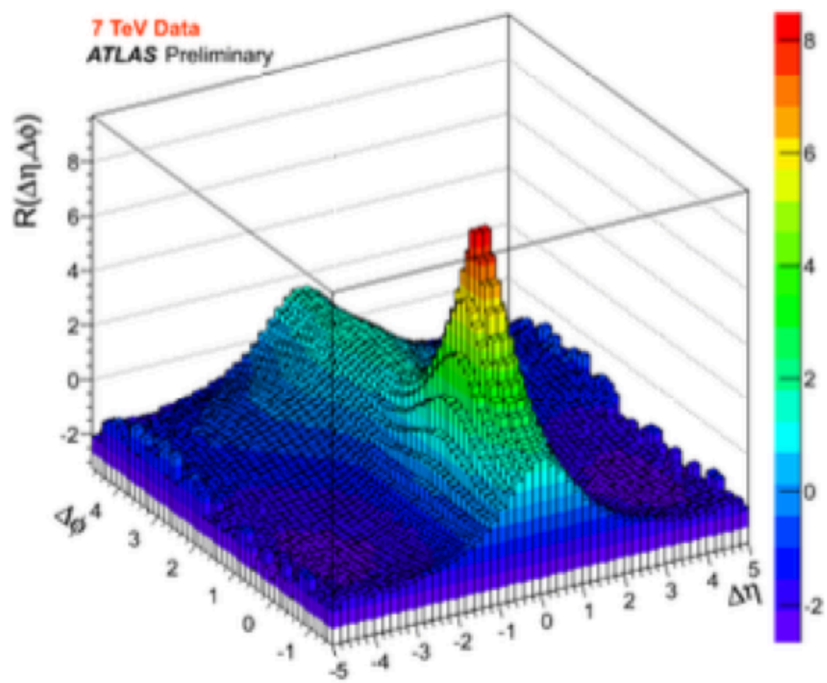
Caveats...

Hard cut to remove effects from track merging (reflection might be better)

Denominator from mixing (convolution might be better)

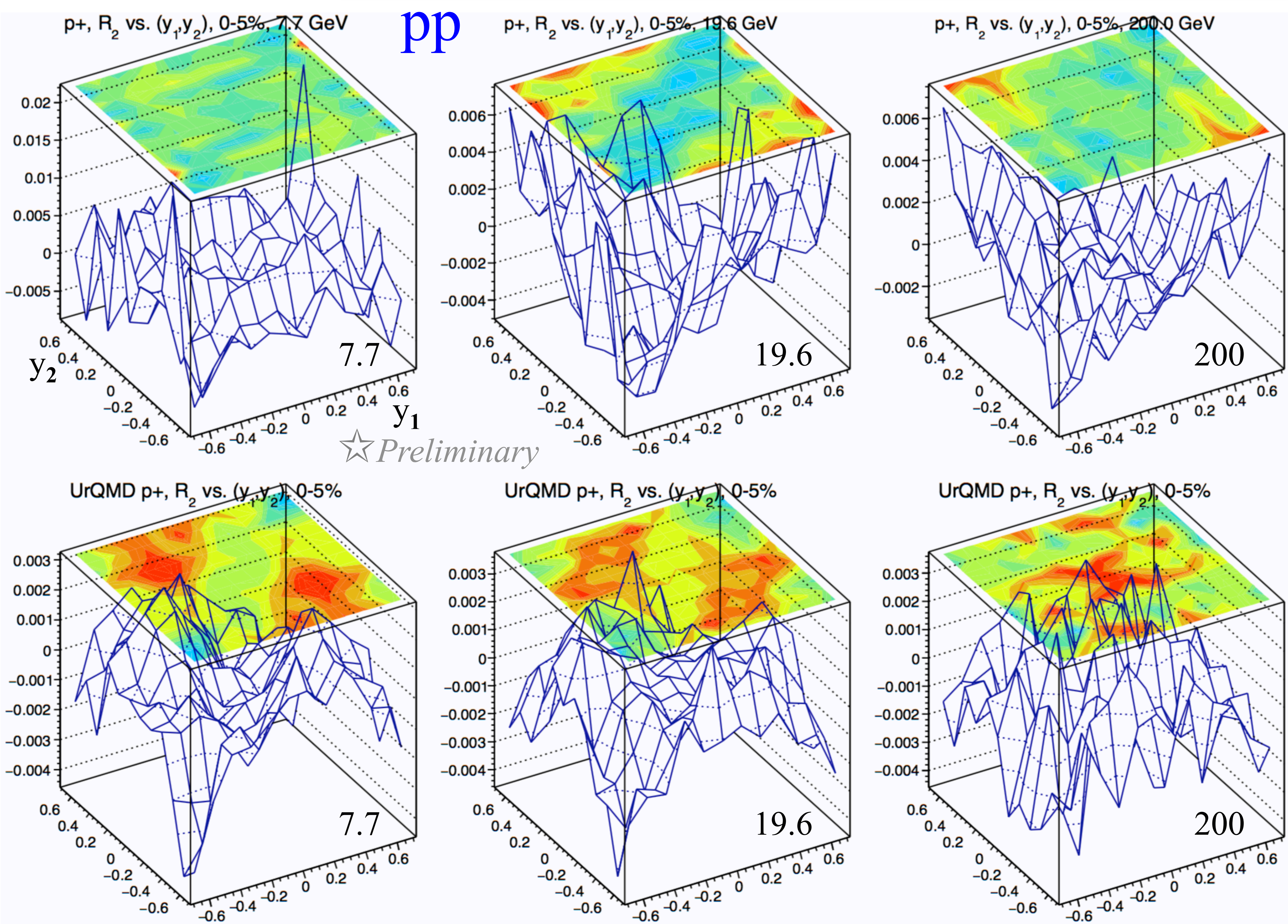
Not yet scaling R_2 by N_{part}

Systematic uncertainities not yet determined.

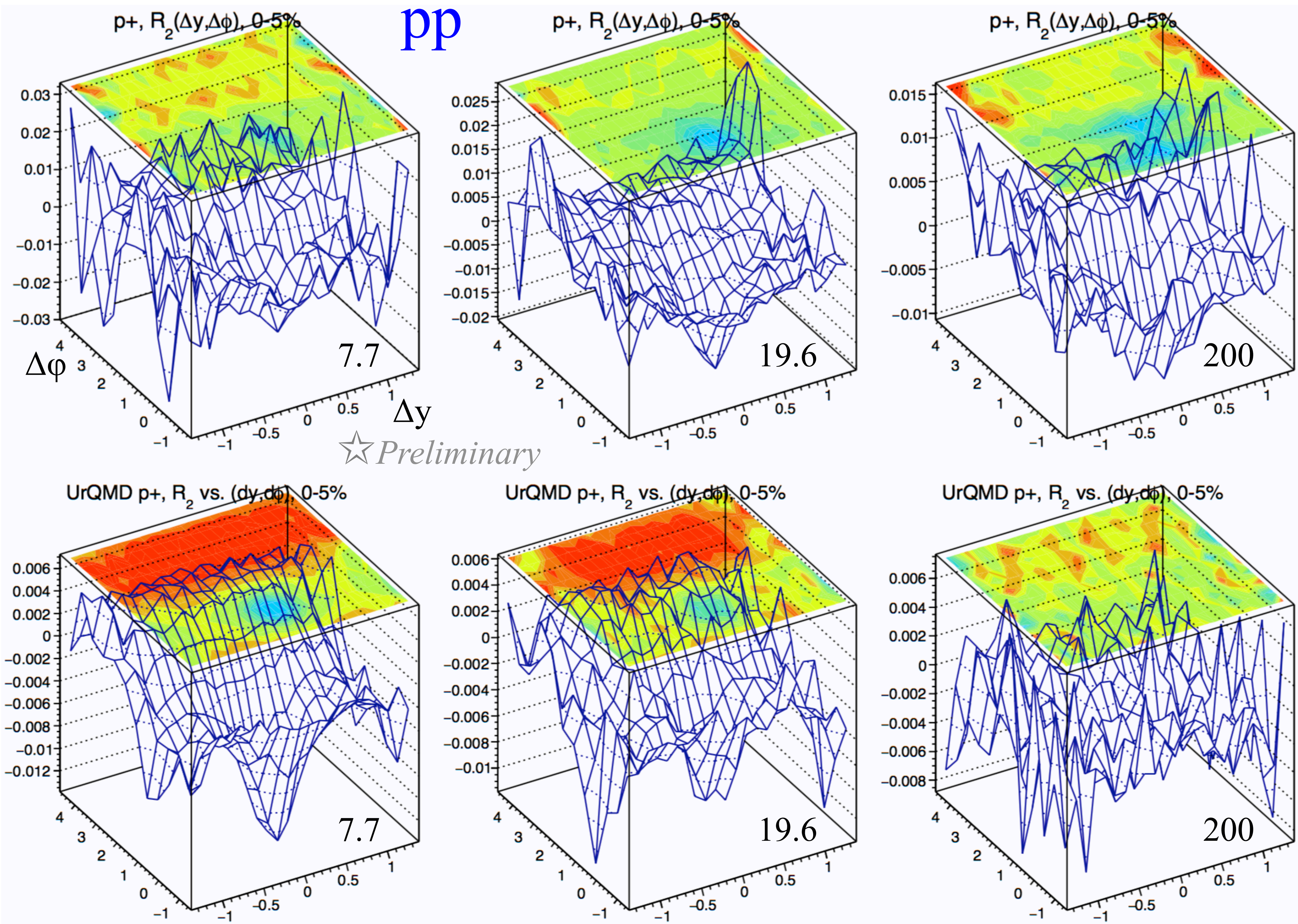


Don’t expect beautifully smooth plots like produced at the LHC.
Event sample sizes are similar, but the LHC has *many* more pairs/event.

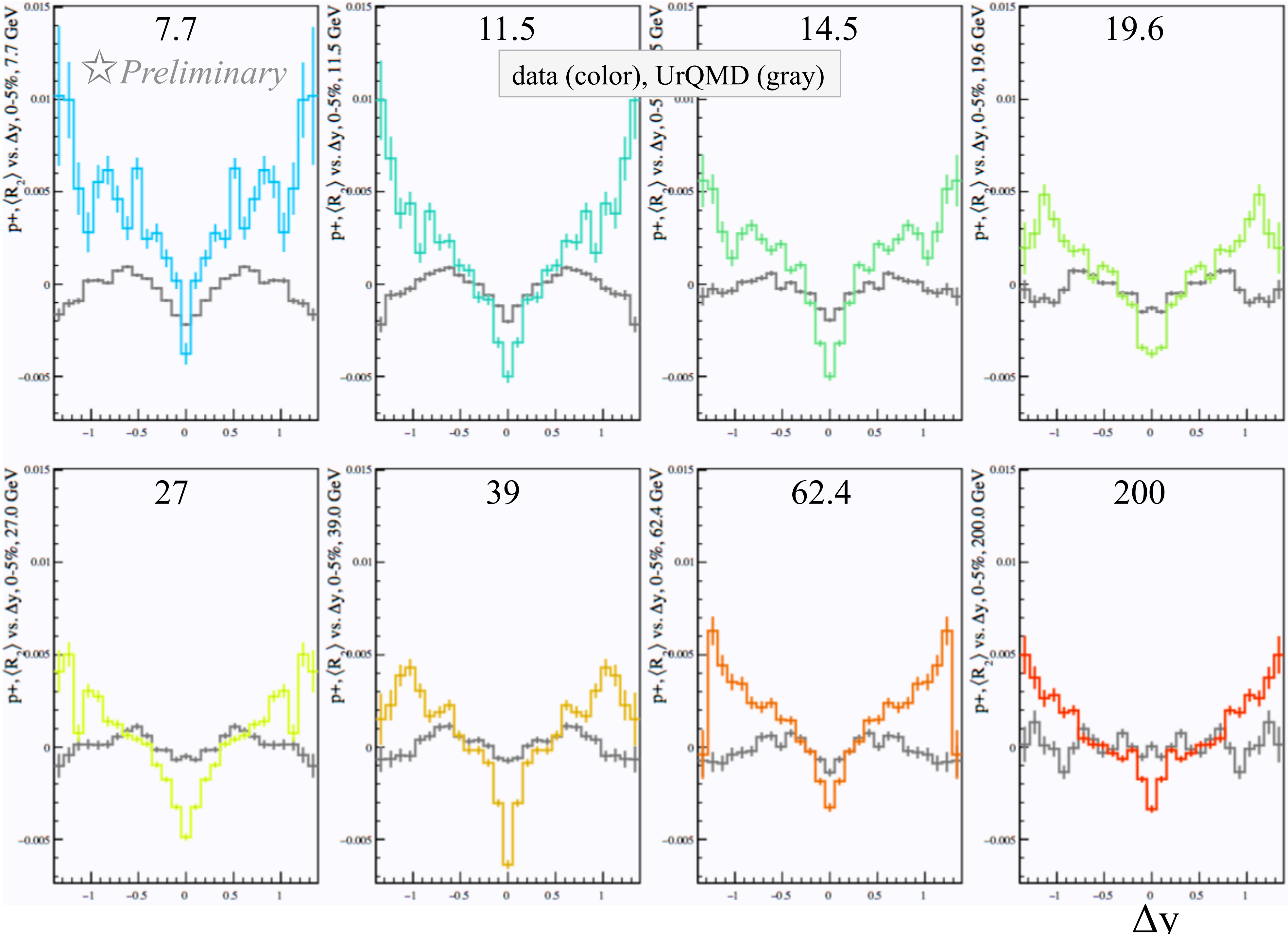
R_2 *vs.* (y_1,y_2) for protons by $\sqrt{s_{NN}}$, 0-5%, STAR BES data



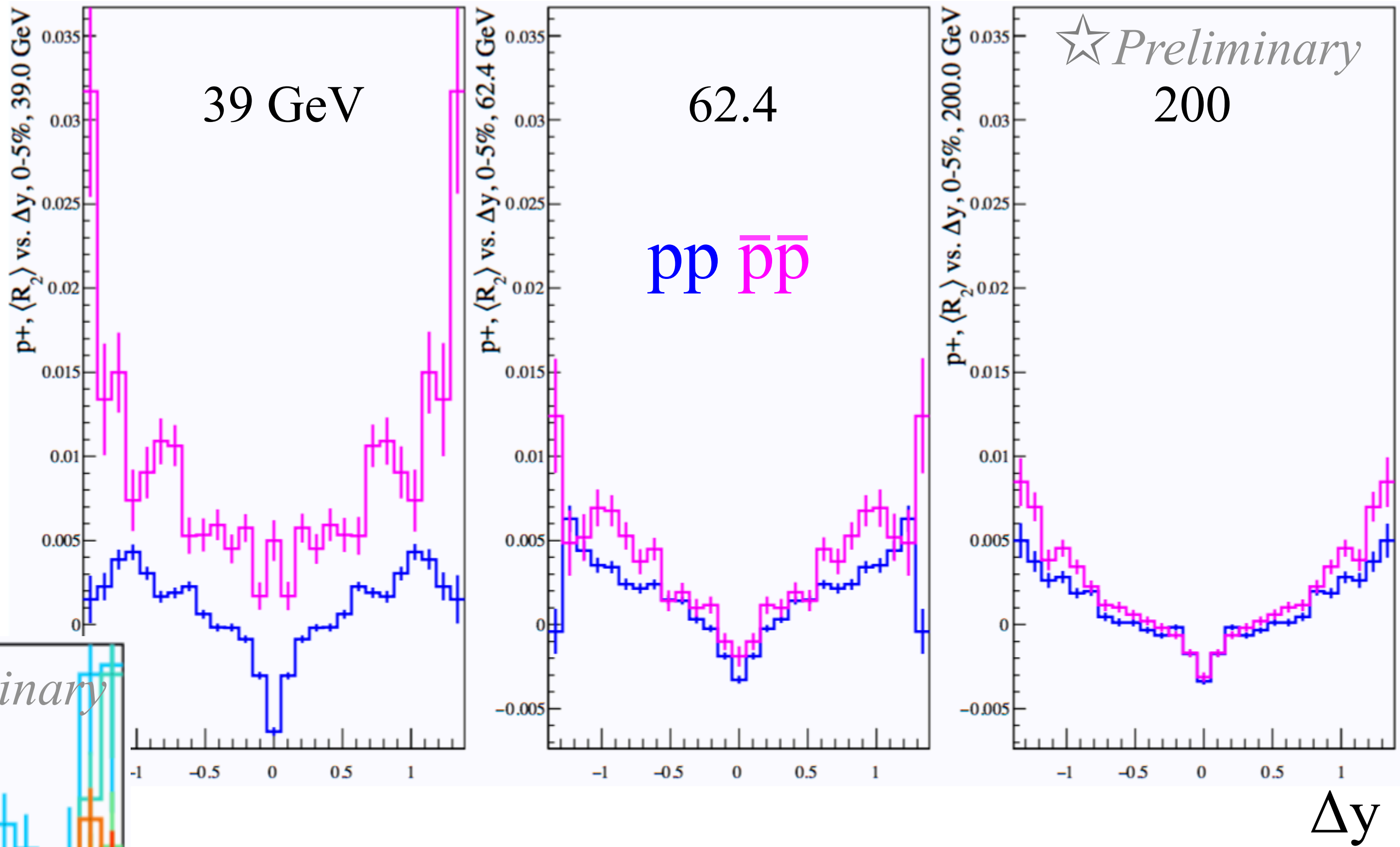
R_2 *vs.* $(\Delta y, \Delta\varphi)$ for protons by $\sqrt{s_{NN}}$, 0-5%, STAR BES data

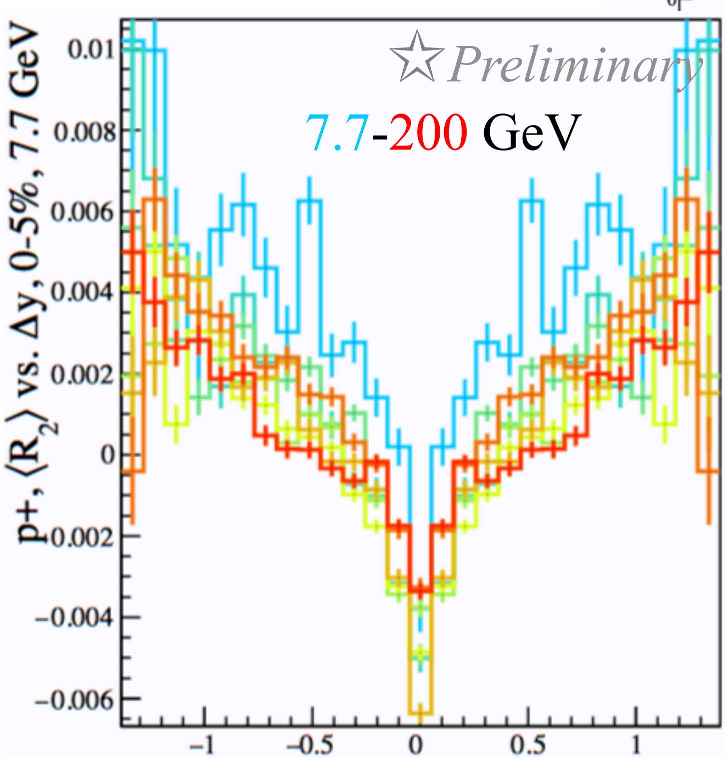


R_2 *vs.* Δy for protons by $\sqrt{s_{NN}}$, 0-5%, STAR BES data



R_2 *vs.* Δy for protons

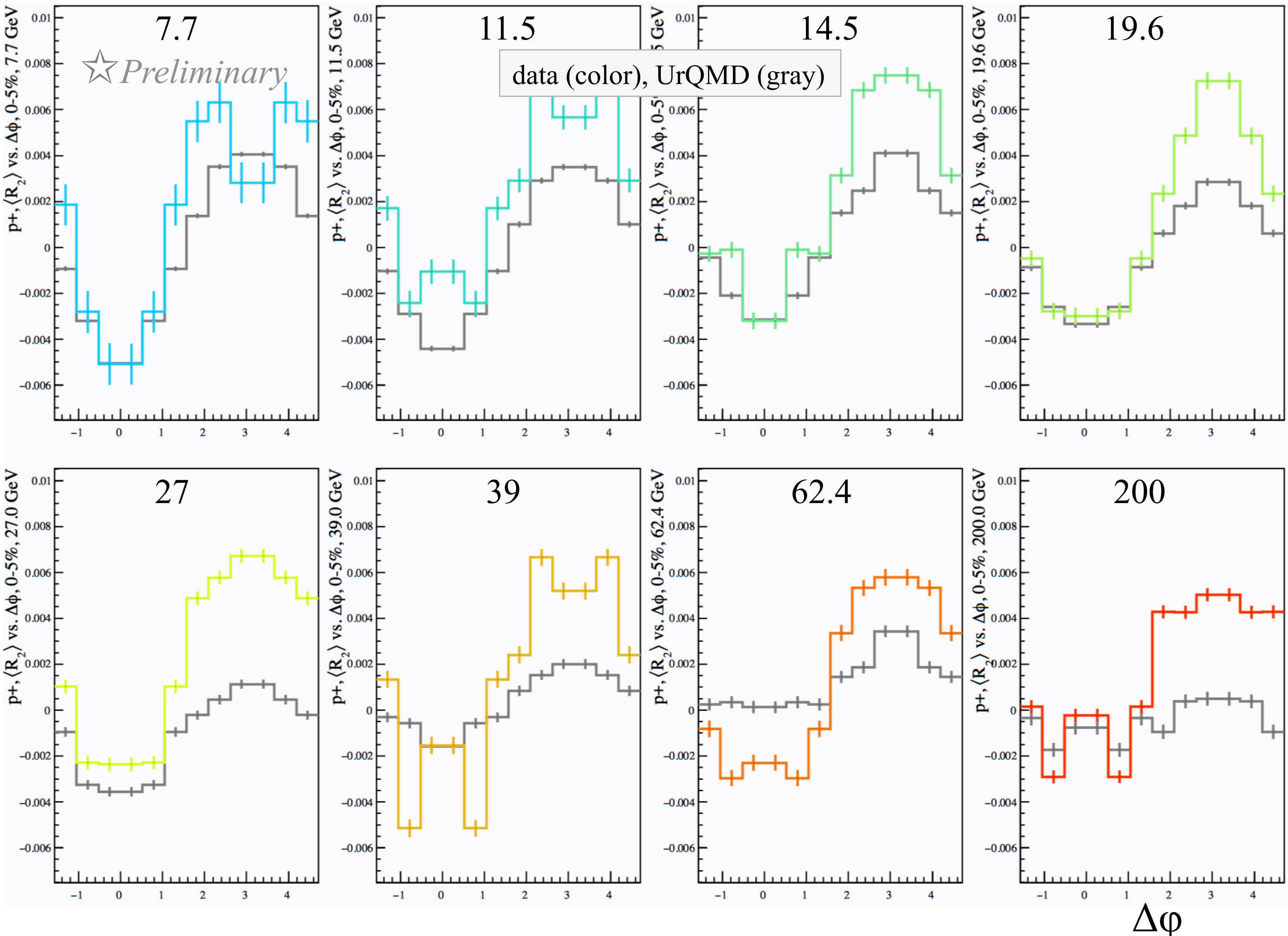




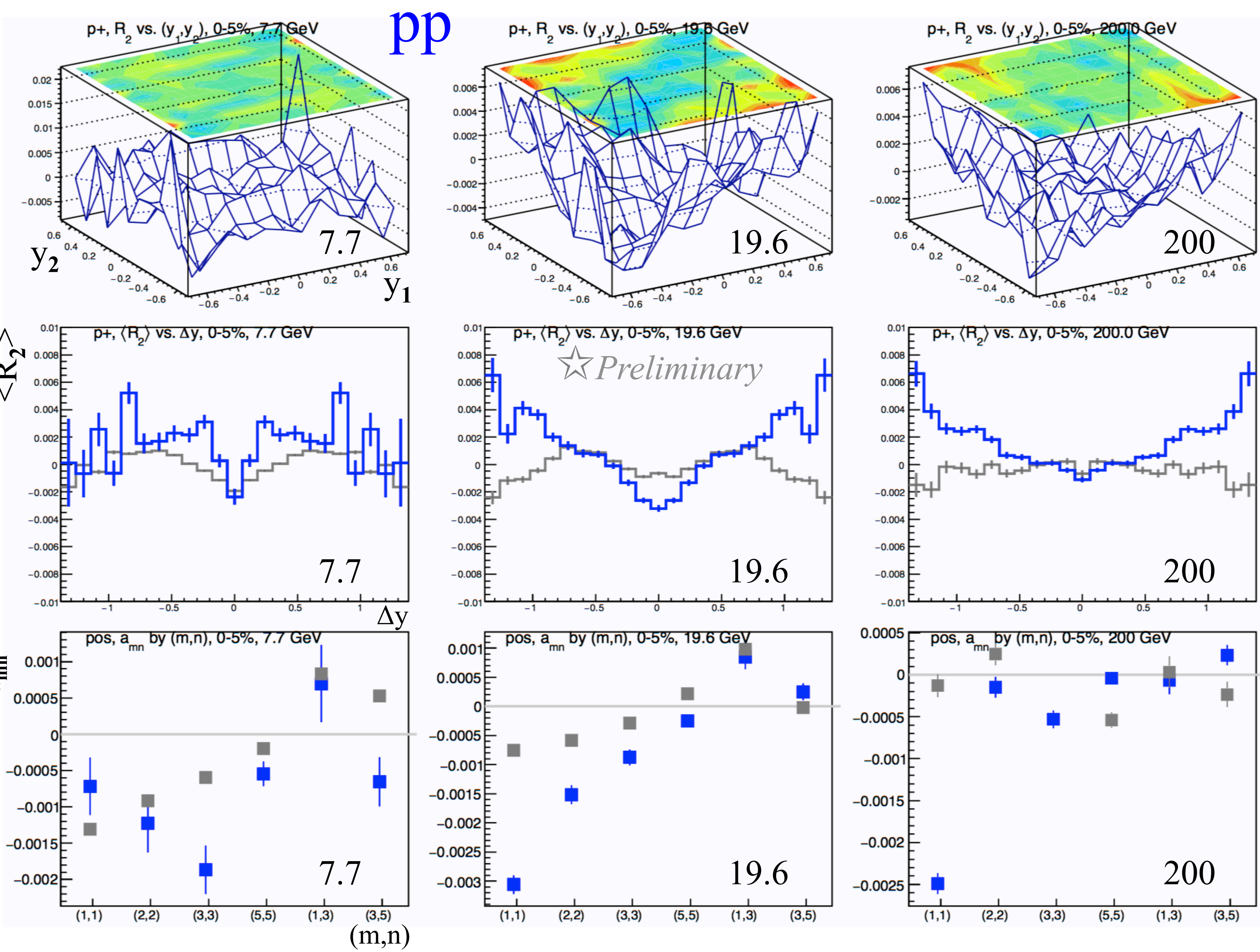
LS p and pbar R_2 the same at 200 GeV

Very little dependence of R_2 on $\sqrt{s_{NN}}$ for LS p

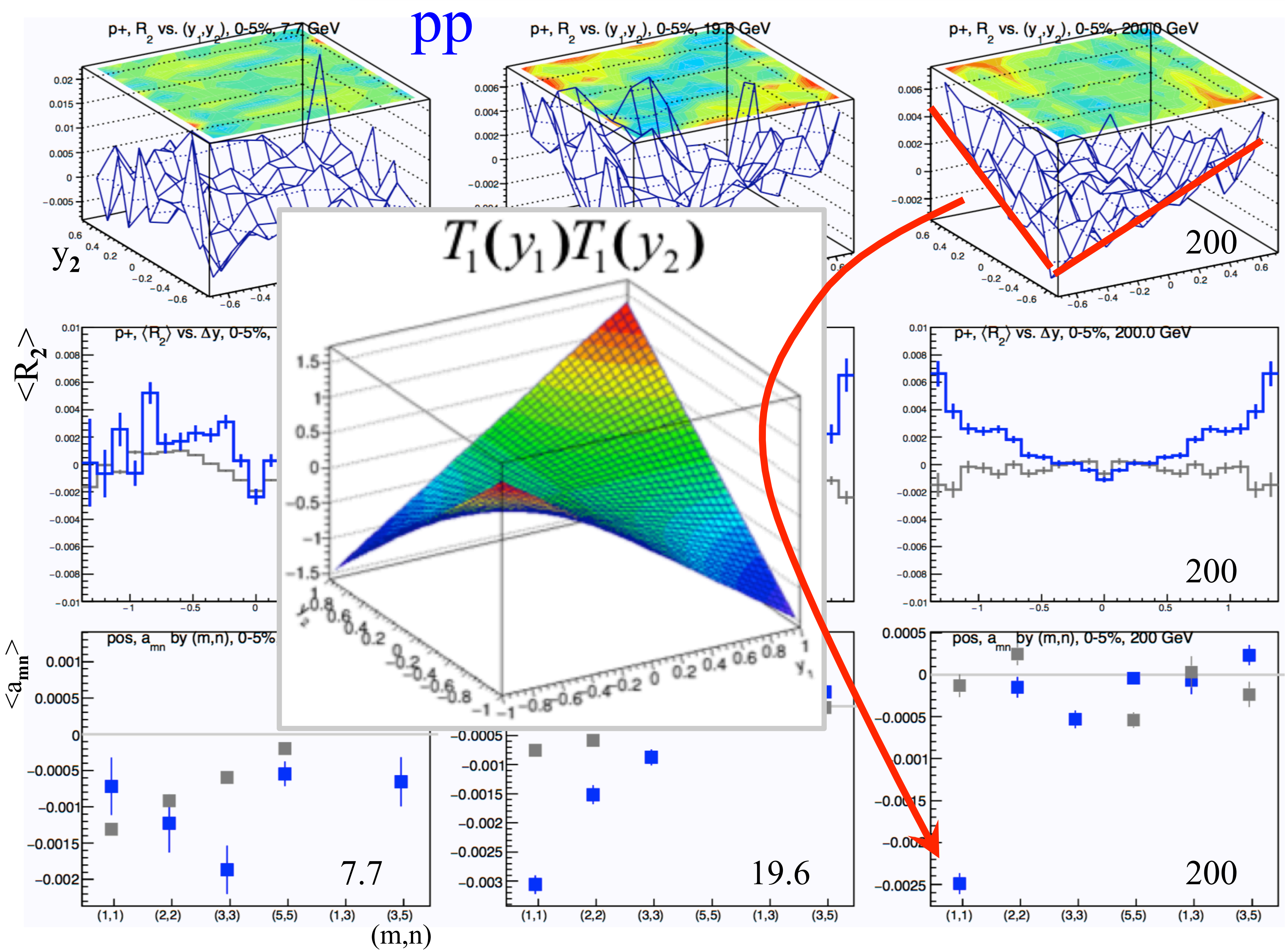
R_2 *vs.* $\Delta\varphi$ for protons by $\sqrt{s_{NN}}$, 0-5%, STAR BES data



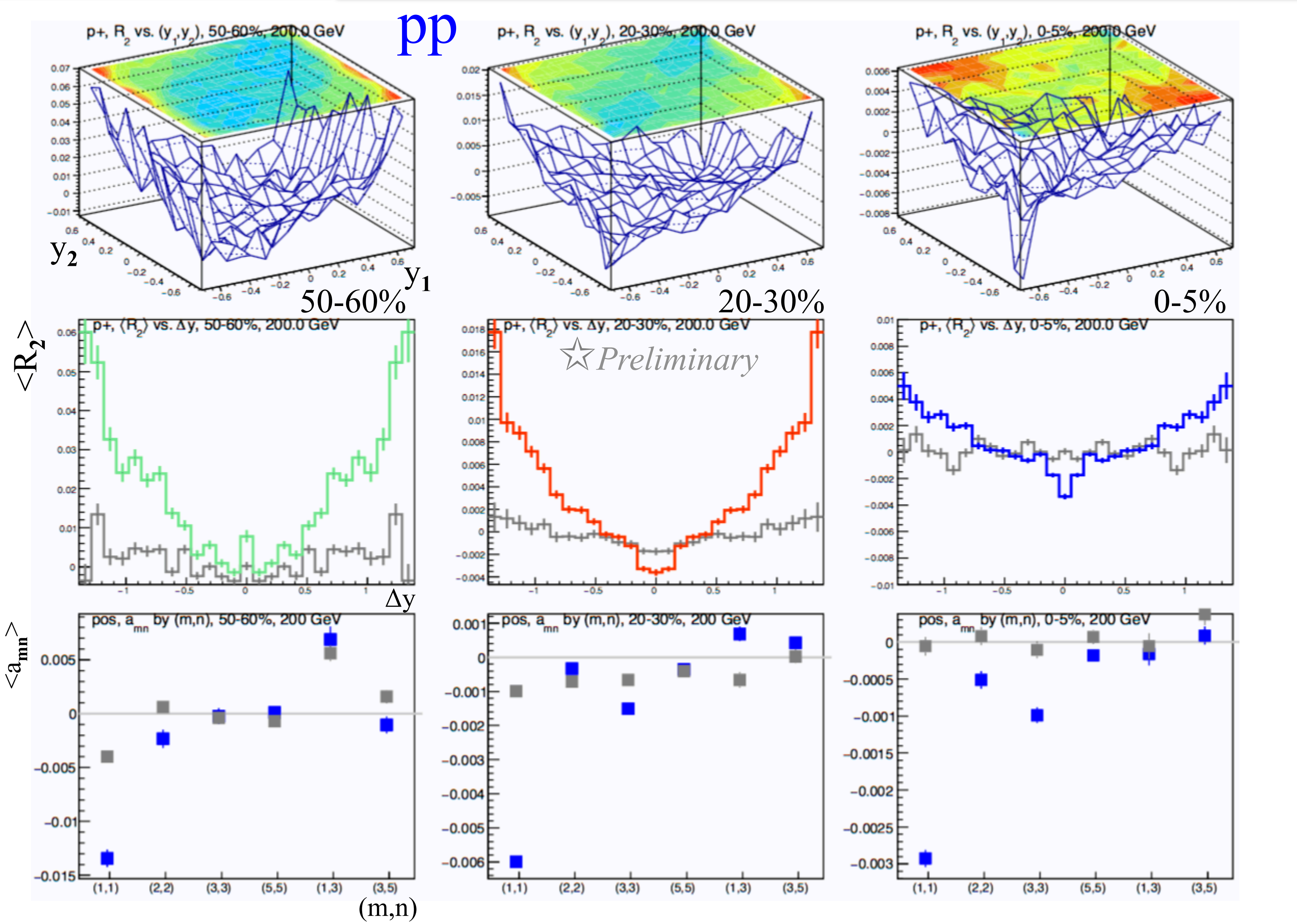
R_2 & a_{mn} *vs.* (y_1,y_2) & Δy for protons by $\sqrt{s_{NN}}$, 0-5%, data & UrQMD



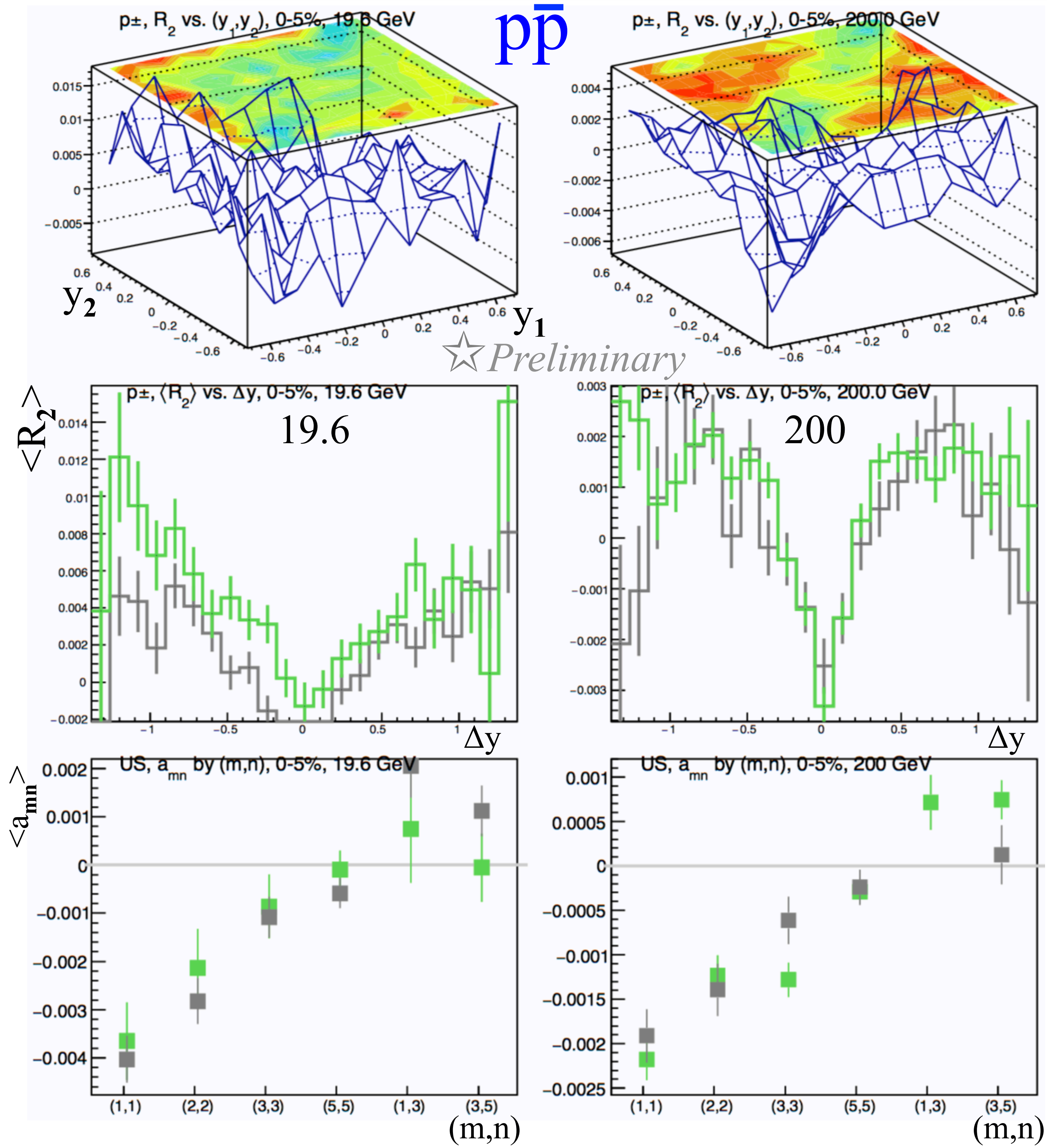
R_2 & a_{mn} *vs.* (y_1,y_2) & Δy for protons by $\sqrt{s_{NN}}$, 0-5%, data & UrQMD



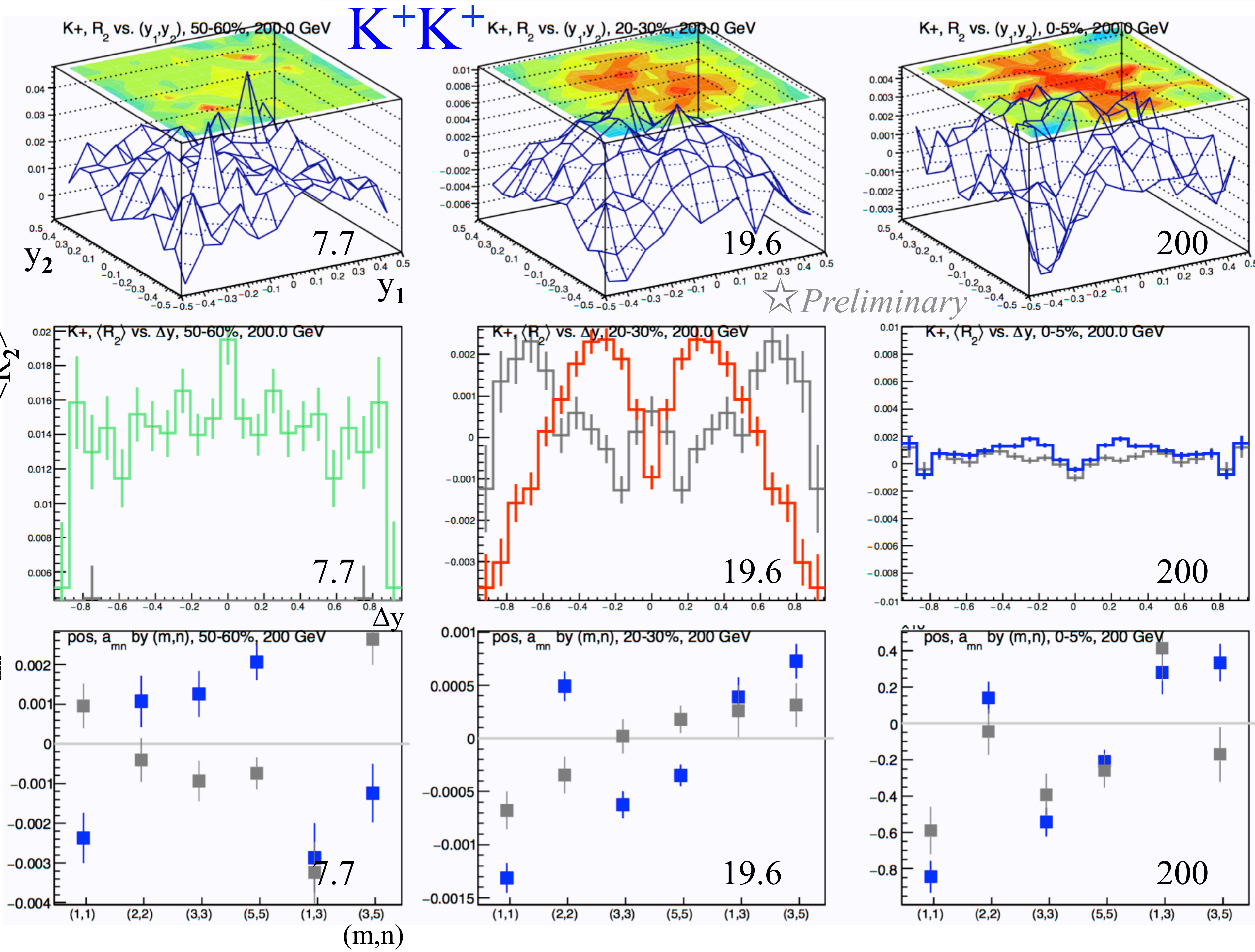
R_2 & a_{mn} *vs.* (y_1,y_2) & Δy for protons by centrality, 200 GeV, data & UrQMD



R_2 & a_{mn} *vs.* (y_1,y_2) & Δy for US protons by centrality



R_2 & a_{mn} *vs.* (y_1,y_2) & Δy for K+ by $\sqrt{s_{NN}}$, 0-5%, data & UrQMD



Summary

Rapidity correlation variables R_2 and C_N studied for LS and US h, K, and p as a function of the centrality and $\sqrt{s_{NN}}$

C_N can be decomposed using basis set of Legendre polynomials to quantify the importance of different shaped (anti)correlations.

This approach is the analog in the rapidity direction of quantifying azimuthal anistropies with v_n observables.

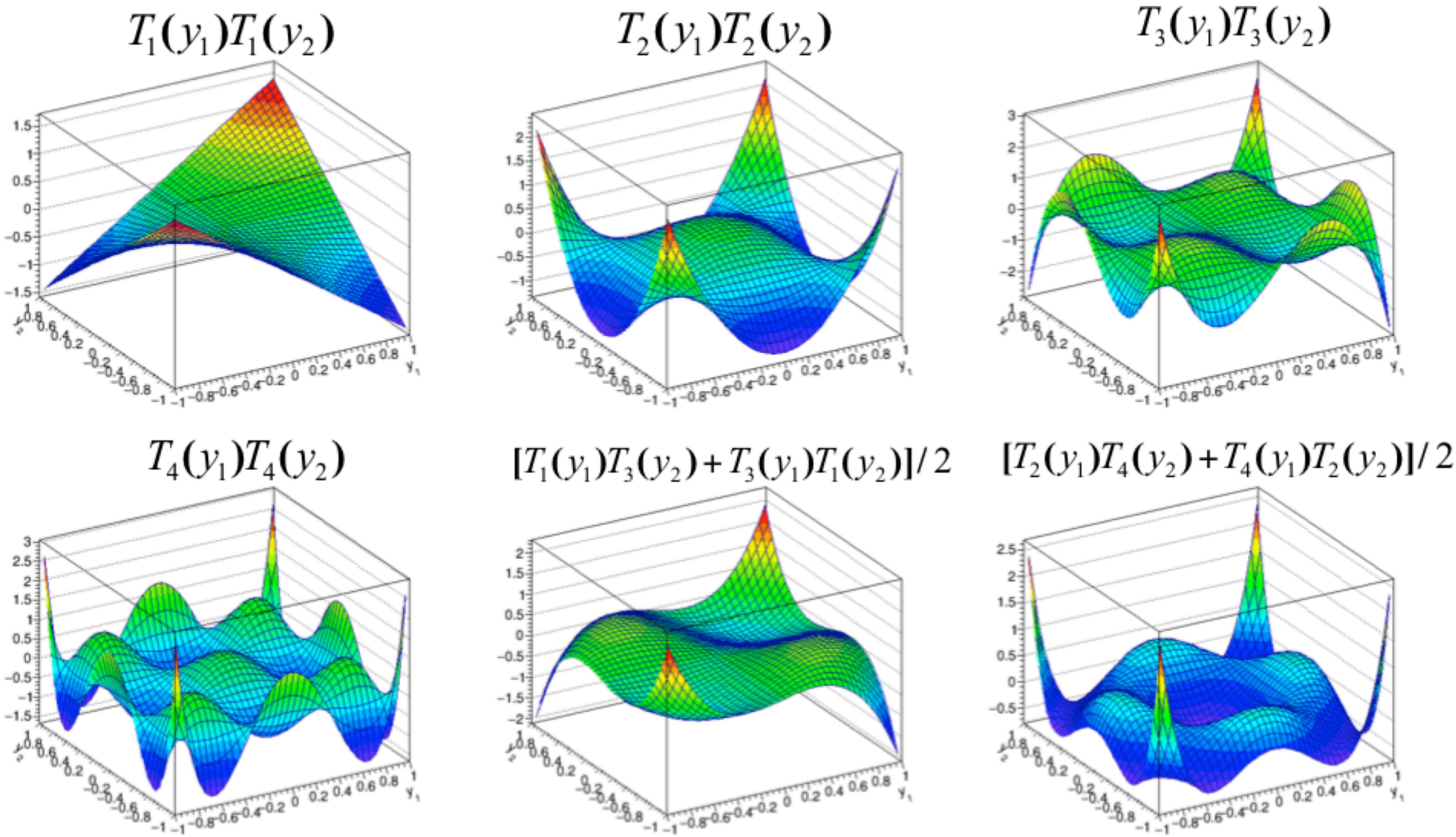
Both STAR BES data and large samples of UrQMD events analyzed.

Careful run and event QA, experiment efficiencies applied to UrQMD events

Signal everywhere.
UrQMD generally does not reproduce the observations.

Would very much appreciate any thoughts!

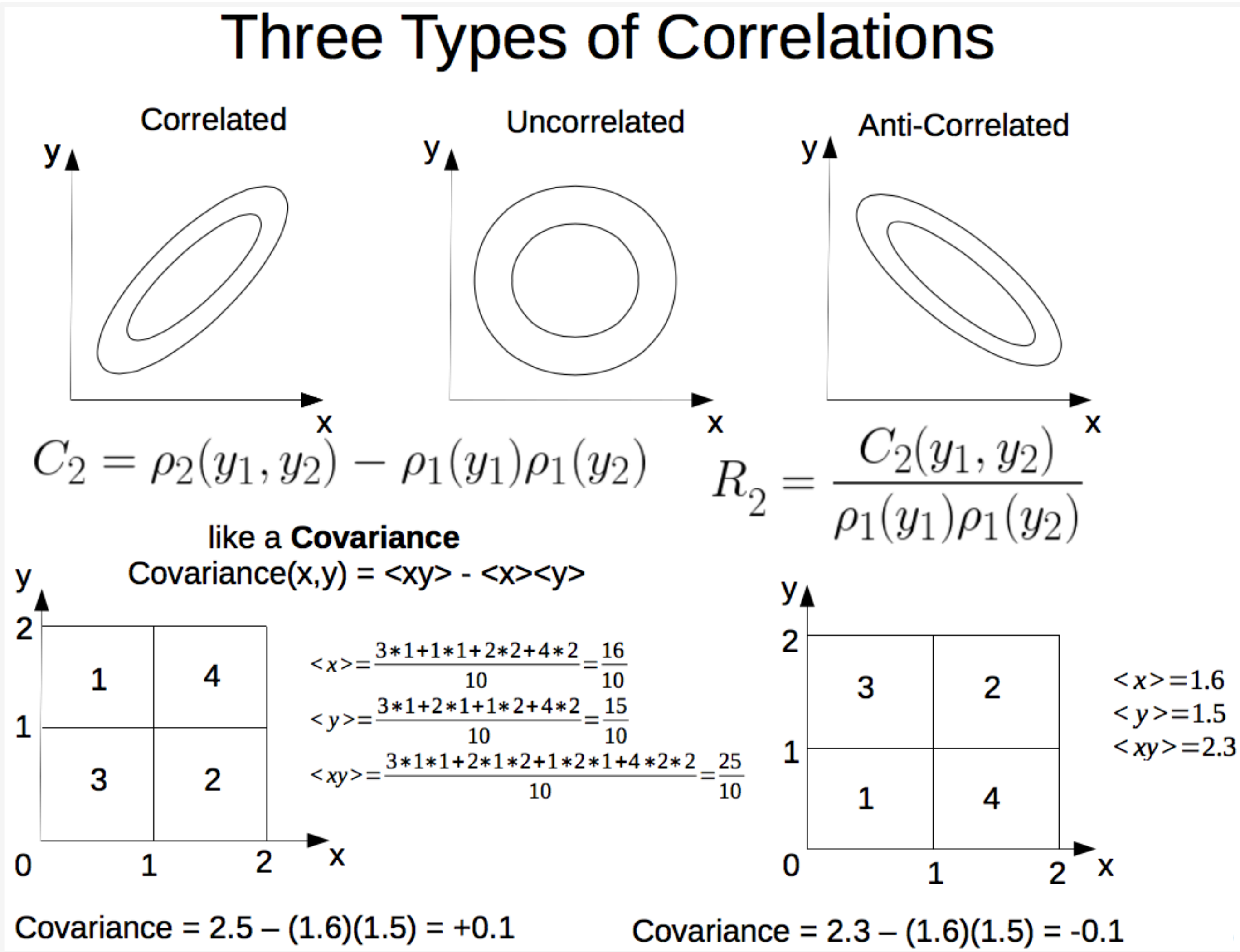
Legendre basis functions



Let's look at the correlations in a different way.

$$C_2 = \rho_2(y_1, y_2) - \rho_1(y_1)\rho_1(y_2)$$

$$R_2 = \frac{C_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} = \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1$$



C_2 is a covariance
R_2 is the covariance per pair

Correlated:	$C_2, R_2 > 0$
Uncorrelated:	$C_2, R_2 \sim 0$
AntiCorrelated	$C_2, R_2 < 0$

Magnitude normalization:

$$R_2^{bs} = \frac{\langle n(n-1)\rangle}{< n >^2} - 1$$

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Structure of correlation functions

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(Received 9 October 1990)

$$C_2(x_1,x_2)=\rho_2(x_1,x_2)-\rho_1(x_1)\rho_1(x_2)\ ,$$

$$C_3(x_1,x_2,x_3)=\rho_3(x_1,x_2,x_3)-\sum_{(3)}\rho_2(x_1,x_2)\rho_1(x_3)+2\rho_1(x_1)\rho_1(x_2)\rho_1(x_3)\ ,$$

$$C_4(x_1,x_2,x_3,x_4)=\rho_4(x_1,x_2,x_3,x_4)-\sum_{(4)}\rho_3(x_1,x_2,x_3)\rho_1(x_4)-\sum_{(3)}\rho_2(x_1,x_2)\rho_2(x_3,x_4)$$
$$+2\sum_{(6)}\rho_2(x_1,x_2)\rho_1(x_3)\rho_1(x_4)-6\rho_1(x_1)\rho_1(x_2)\rho_1(x_3)\rho_1(x_4)\ .$$

See also:

L. Foà, Phys. Lett. **C22**, 1 (1975)
H. Bøggild, Ann. Rev. Nucl. Sci. **24**, 451 (1974)
M. Jacob, Phys. Rep. **315**, 7 (1999)

Lower-order correlations explicitly removed.
R_k is just these rapidity cumulants C_k scaled by the number of pairs, triplets, quadruplets, ...
R_k thus manifestly independent of experimental inefficiencies by definition...

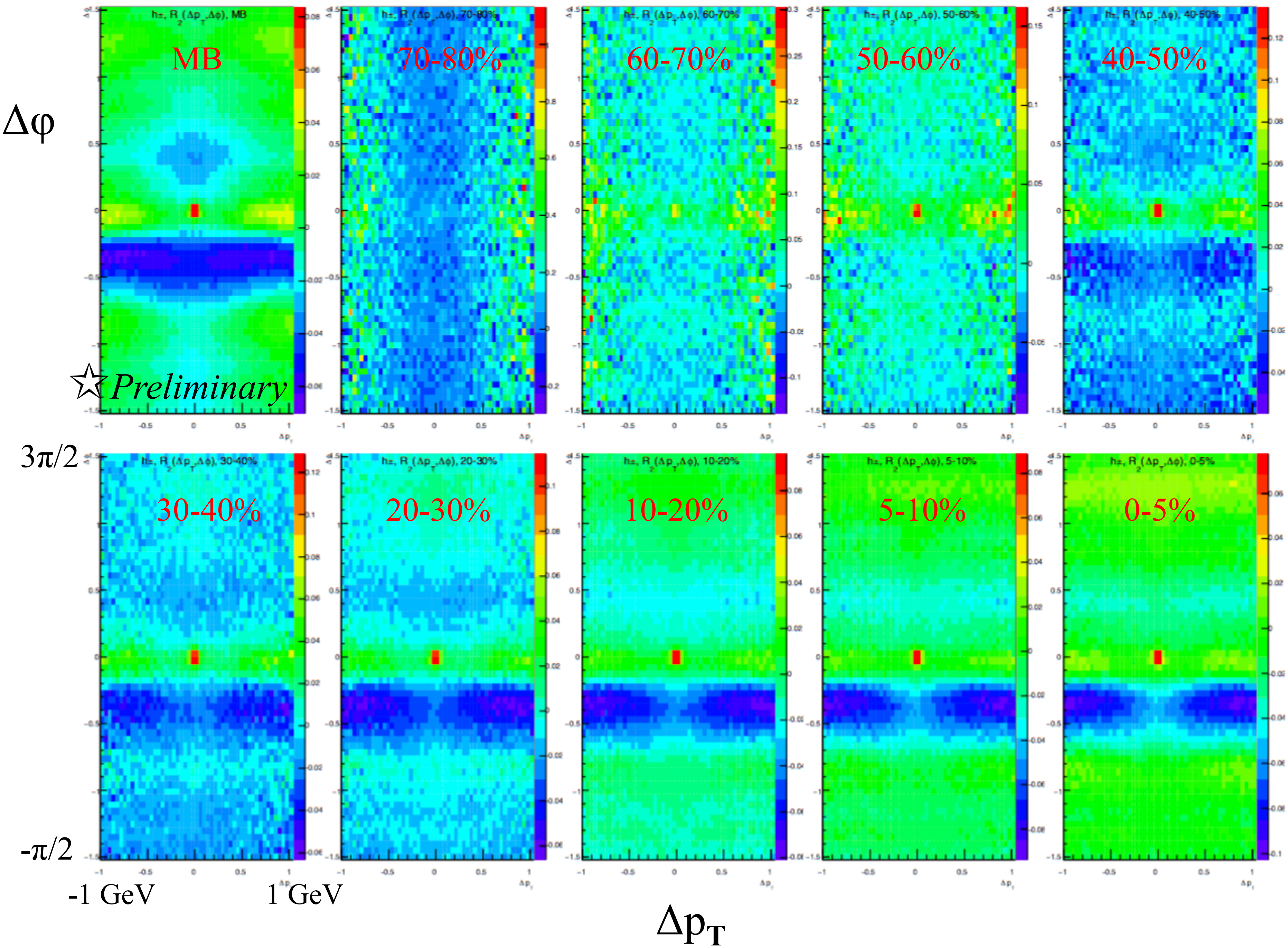
R_2 baseline: $R_3 = \dfrac{\langle n(n-1)\rangle}{\langle n\rangle^2} - 1$

R_3 baseline: $R_3 = \dfrac{\langle n(n-1)(n-2)\rangle}{\langle n\rangle^3} - 3\dfrac{\langle n(n-1)\rangle}{\langle n\rangle^3}\langle n\rangle + 2$

Robust indicator of N-fold (anti)correlations, explicitly as a function of Δy and <y>…

By construction, independent of single-particle inefficiencies…

h± US, $R_2(\Delta p_T,\Delta\varphi)$ for $|\Delta y|<0.04$, AuAu 200 GeV, MB and by centrality



h± US, $R_2(\Delta p_T, \Delta\varphi)$ for $|\Delta y|<0.04$, AuAu 200 GeV, 0-5%