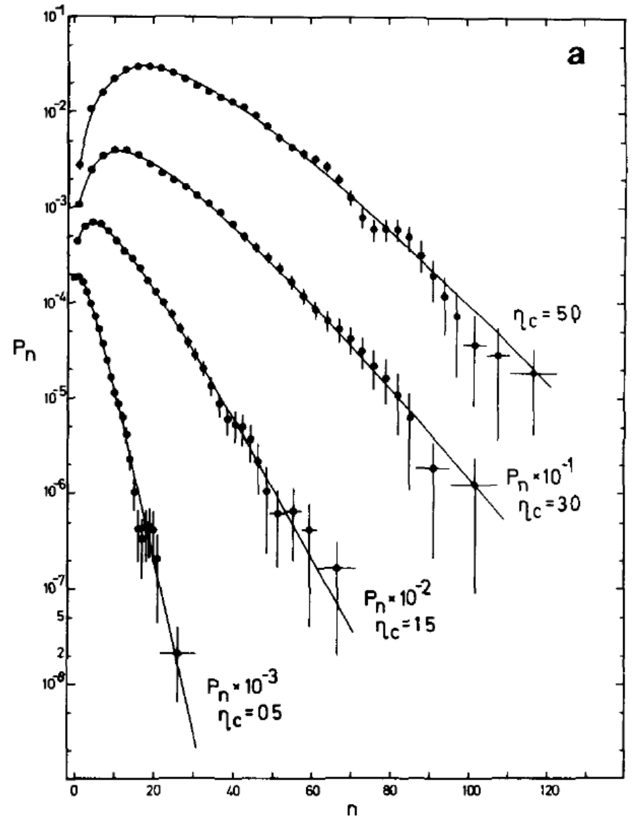
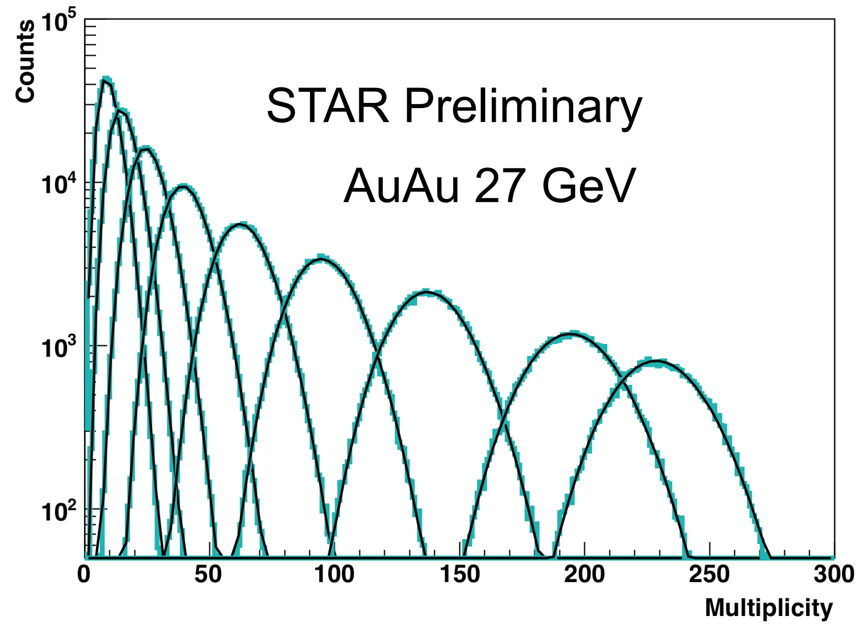
Beam Energy Dependence of Clan Multiplicity

Aihong Tang for STAR

Introduction : The Success of Neg. Binomial Distrib.





UA5 Collab. Phys. Lett. 160B 193 (1985)
(and many others ……)

Negative Binomial has been widely used to describe particle productions in high energy collisions, and it can also describe particle production at RHIC very well.

Introduction : Transformation to Clan Parameters

Recall the negative binomial pdf , $f(n) = C_{k+n-1}^{n}(1-p)^{n} p^{k}$

Its mean is given by $\mu = \frac{(1-p)k}{p}$

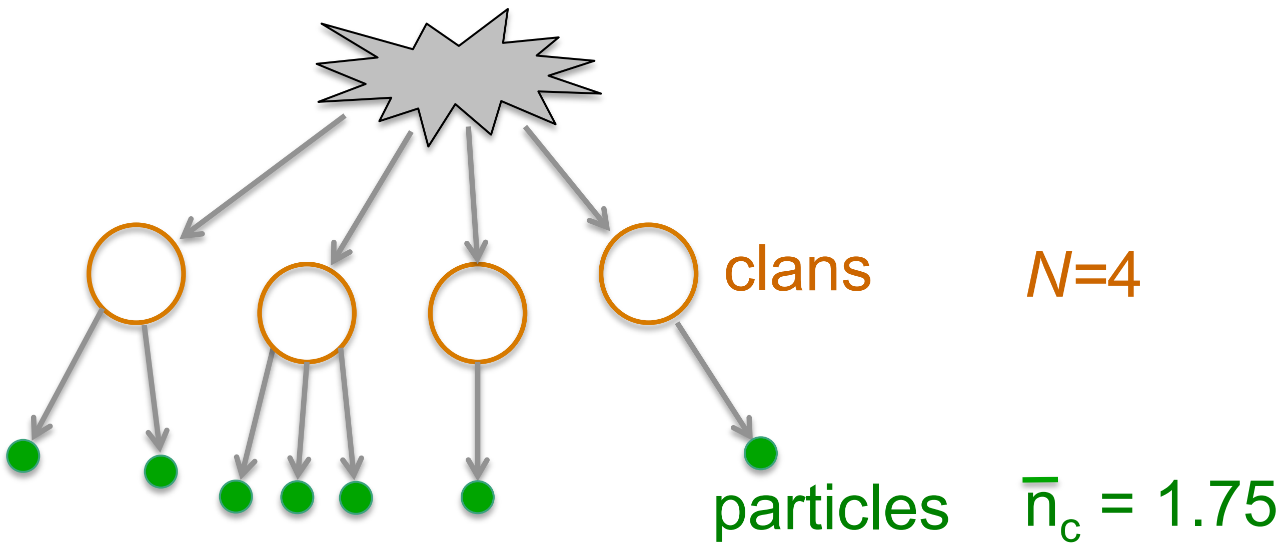
and variance $\sigma^{2} = \frac{(1-p)k}{p^{2}}$

In order to interpret the wide occurrence of the negative binomial charged particle multiplicity distribution, a parameterization alternative to the standard NB parameters has been proposed. They are, namely the average number of groups of particles of common ancestor (average number of clans, $\overline{N} = k \ln\left(1 + \frac{\mu}{k}\right)$) and the average number of particles per clan, $\overline{n}_{c} = \mu / \overline{N}$

A. Giovanni, L. Van Hove, Z. Phys. C30 391 (1986)

Introduction : Clan Concept

With this transformation of parameters, it turns out that clans in various classes of events in a collision are independently produced whereas particles within each clan are distributed according to a logarithmic multiplicity distribution. See next slide for its mathematical justification.



A. Giovanni, L. Van Hove, Z. Phys. C30 391 (1986)

Introduction : Mathematical Justification

The generating function of logarithmic distribution is :

$$G_{\ln}(x) = \frac{\ln(1-zx)}{\ln(1-z)}$$ with z is set to be $$z = \frac{\mu}{\mu+k}$$

Because clans are independent of each other, they can be described by Poisson distribution, for which the generating function is :

$$G_{poisson}(x) = e^{\overline{N}(x-1)}$$

The generating function of the compound distribution is then given by :

$$G_{poisson}(G_{\ln}(x)) = e^{\overline{N}(G_{\ln}(x)-1)} = \bullet\bullet\bullet = \left(\frac{\frac{k}{\mu}}{1+\frac{k}{\mu}-x} \right)^k$$

by which we recover the generating function of negative binomial distribution.

A. Giovanni, L. Van Hove, Z. Phys. C30 391 (1986)

Introduction : Examine the Clan Parameters

With the introduction of the clan concept, the negative binomial distribution, if rearranged properly, yields a similarity to the grand-canonical partition function. Such analogy leads to the "clan thermodynamics" ([1]). Here, adopting the concept of clan does not necessary implies that we endorse the "clan thermodynamics".

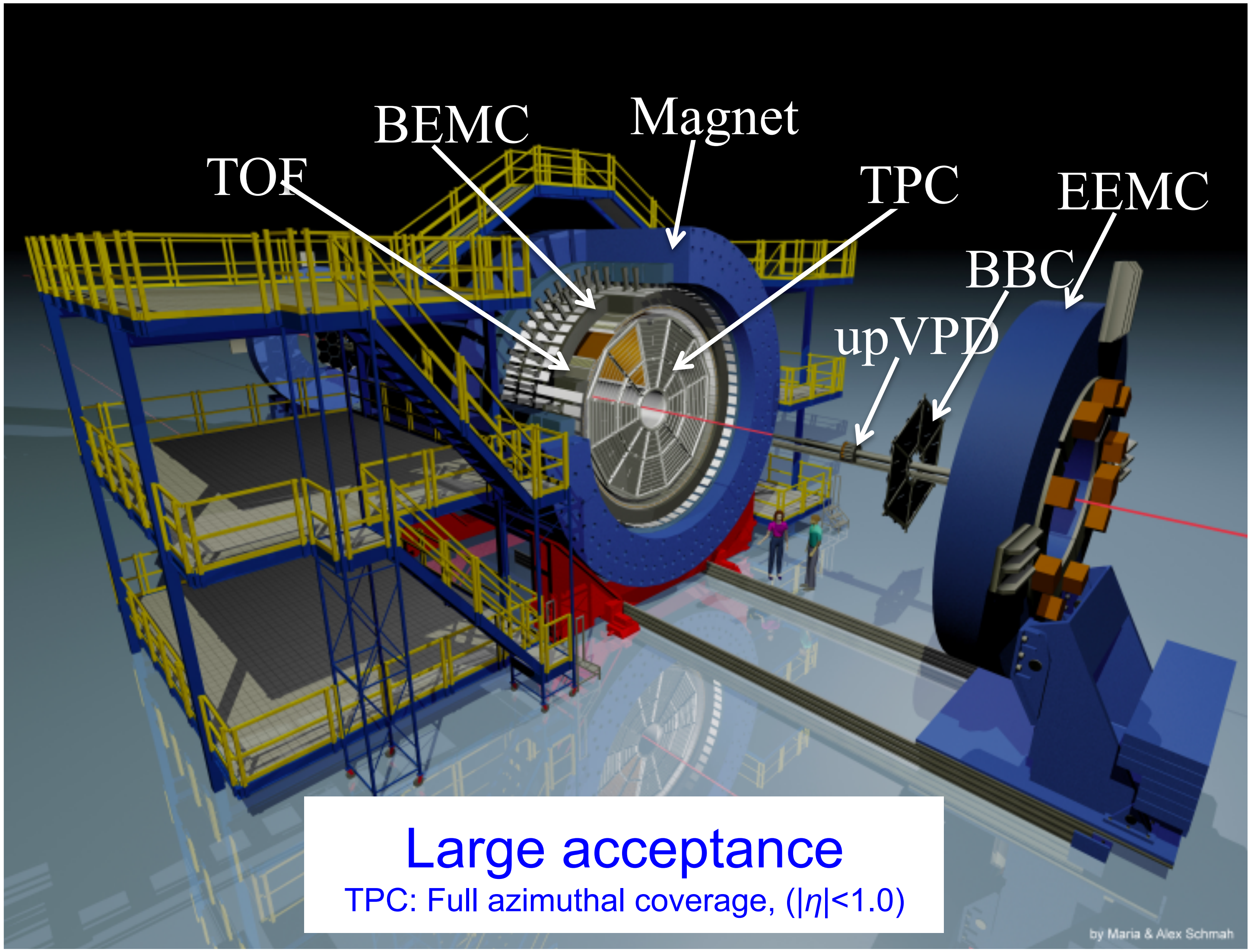
Clan concept describes clustering characteristics.

The clan parameters have been used to identify abnormalities due to phase transition([2-7]).

In this study, we examine the average charged particles per clan as a function of collision energy.

(1) Giovannini, Lupia and Ugoccioni, PRD 65 094028 (2002)
(2) UA5 : Phys. Rep.154, 247(1987).
(3) EMC: Z. Phys. C 35, 335 (1987) [Erratum-ibid.36, 512 (1987)
(4) NA22 : Z. Phys. C37, 215 (1988)in heavy ion collisions :
(5) E802 : Phys. Rev. C 52, 2663 (1995)
(6) NA35 : Z. Phys. C 57 541 (1993)
(7) PHENIX: Phys. Rev. C 78 044902, (2008)].

STAR Detector Setup



Large acceptance

TPC: Full azimuthal coverage, ($|\eta|<1.0$)

Datasets and Cuts

Energy (GeV)	Year	Total # of events (M)
7.7	2010	4.4
11.5	2010	7.2
19.6	2011	44
27	2011	80
39	2010	37
62.4	2010	25
200	2010	25

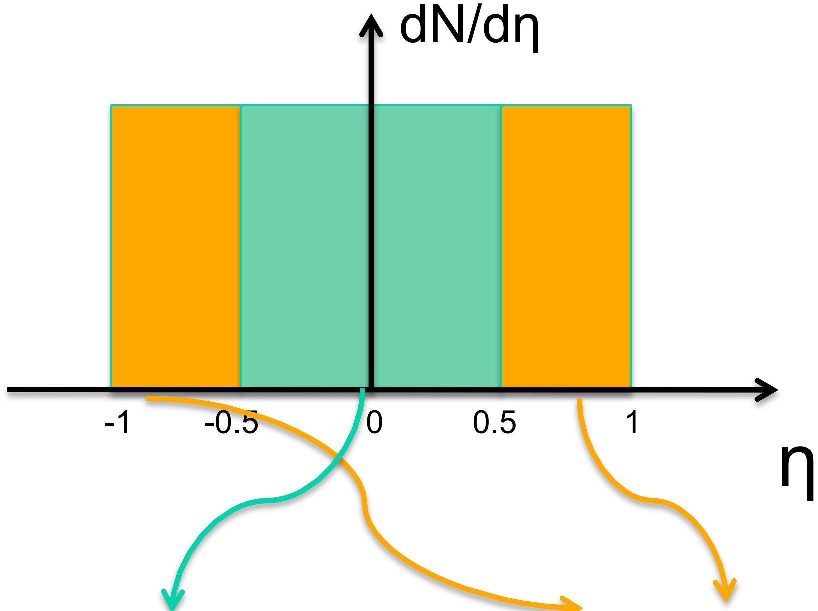
Event-wise cuts
$|Vz|<30$ cm
$Vr < 2$ cm
Should have at least one TOF matched track

Track-wise cuts
numHitsFit>=15
1.02 > ratio = (nHitsFit/nHitsPoss) > 0.52
$|eta|<0.5$ for particle of interest

For 62 and 200 GeV, use low luminosity data (BBCX<25k)

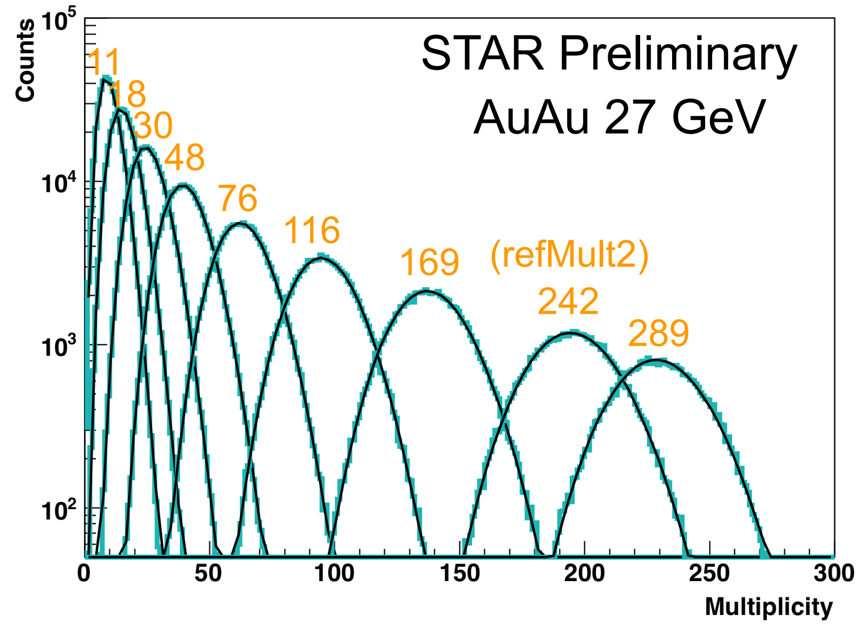
PID cut is based on 2 sigma cut on TPC dE/dx.

Centrality Definition and Multiplicity Distribution



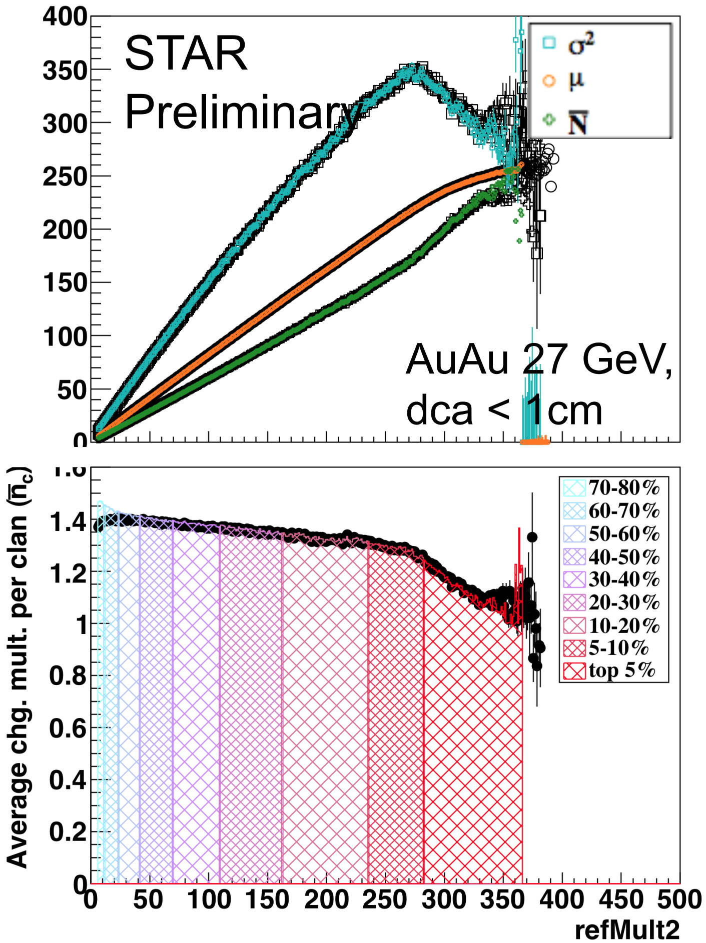
For the study of the multiplicity distribution

For the definition of centrality and reference multiplicity (refMult2)



multiplicity distribution in |η|<0.5 fitted with NB.

Average Charged Multiplicity per Clan



Black : σ^2 μ from direct calculation

Colored : σ^2 μ from fitting NB

We are interested in produced particles only, thus protons and heavier positive particles are excluded.

To avoid the additional fluctuation from a wide multiplicity bin, which can significantly bias the variance, the analysis is performed with the finest refMult2 bin-width, and the final result for a centrality is presented by taking the average over fine bins.

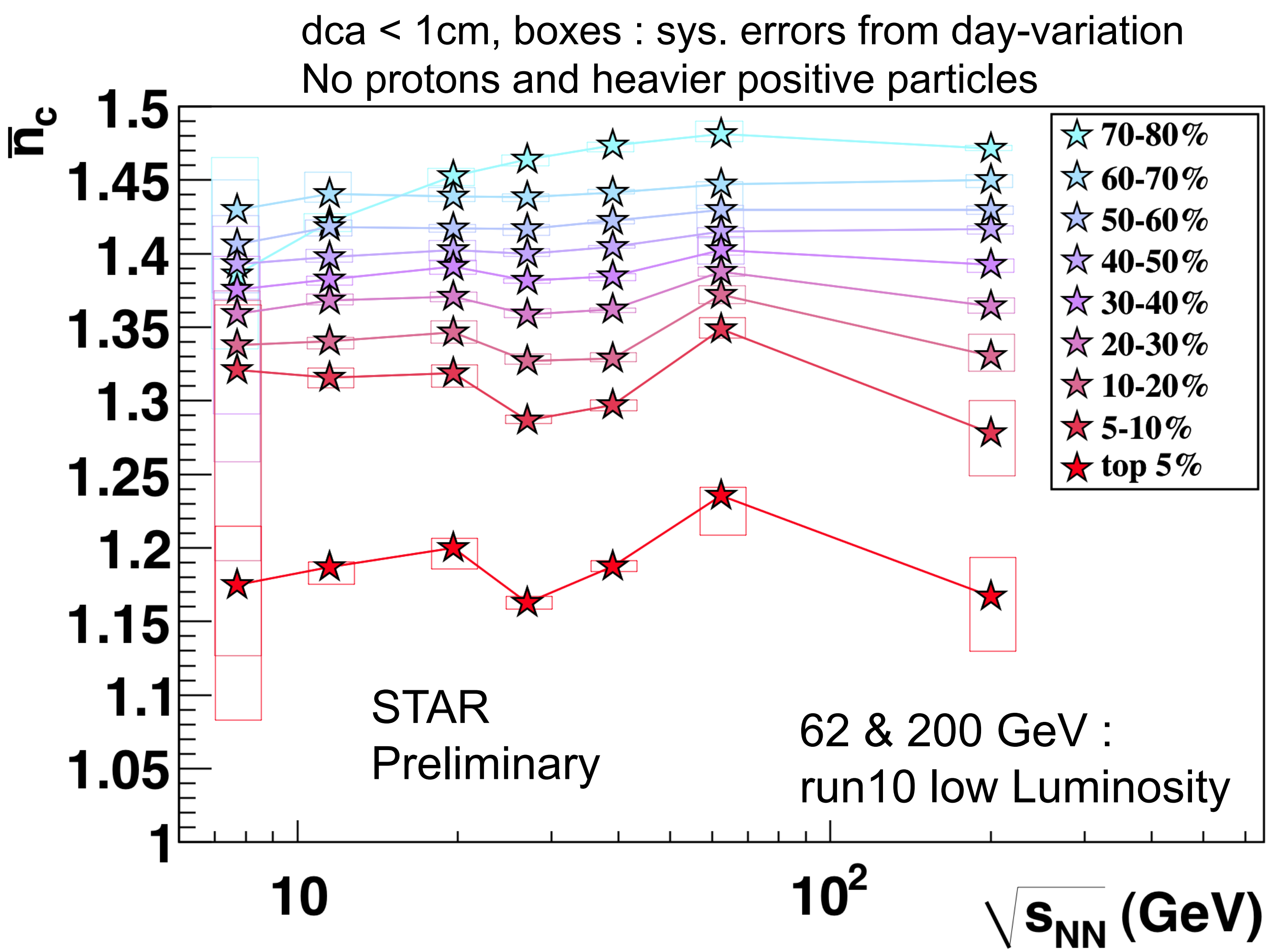
$$f(n) = C^{n}_{k+n-1}(1-p)^{n} p^{k}$$

$$p = \frac{\mu}{\sigma^2} \qquad k = \frac{\mu p}{1-p}$$

$$\overline{N} = -k \ln p$$

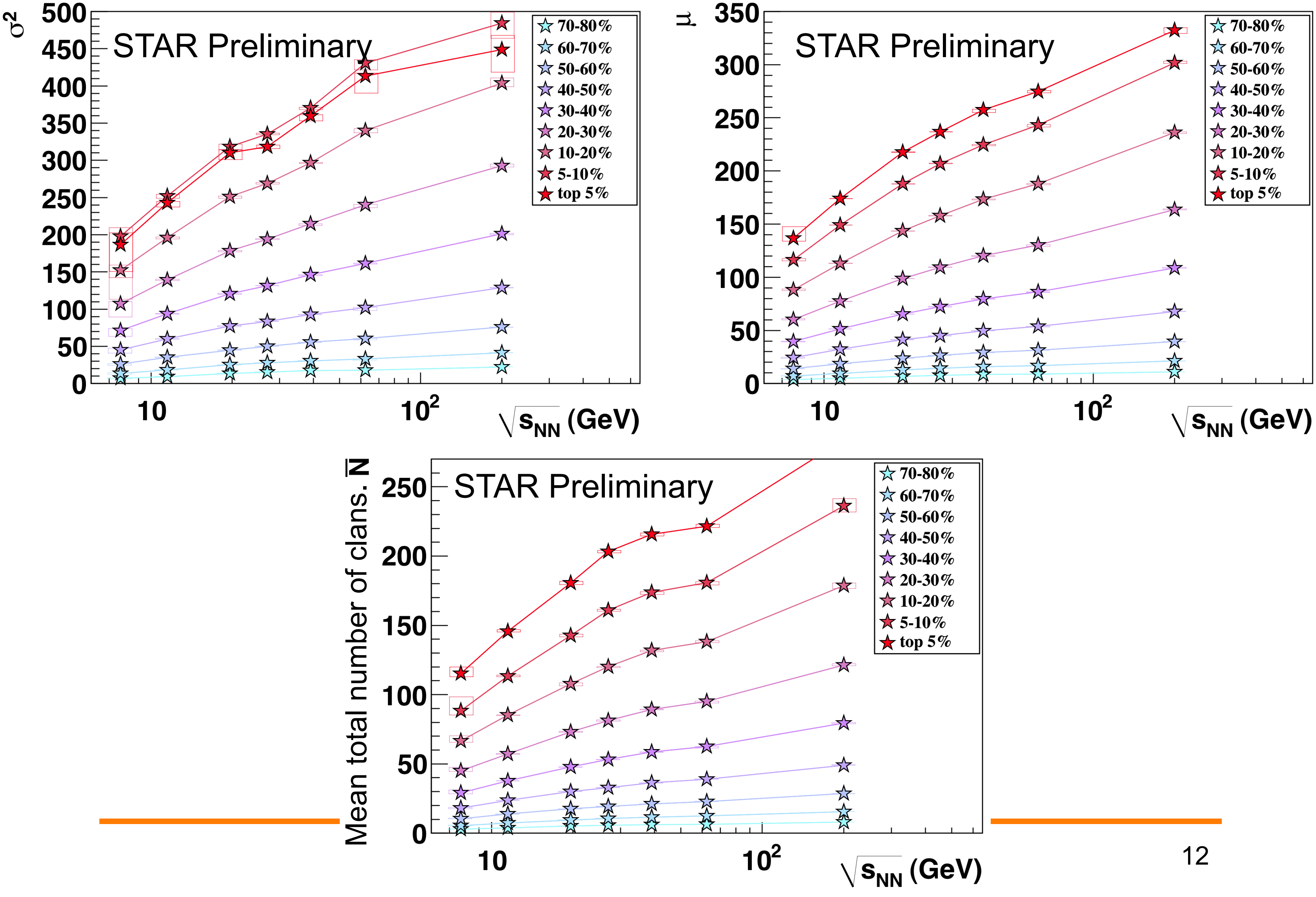
$$\overline{n}_c = \frac{\mu}{\overline{N}} = \frac{1-p}{-p\ln p}$$

Average Chgd Mult. per Clan, Energy Dependence



Interesting structure in energy dependence.
Most prominent in central collisions.

Other Parameters



Consideration of Inefficiency

Consider the generating function for the negative binomial distribution

$$G(z) = \left(\frac{\frac{k}{\mu}}{1 + \frac{k}{\mu} - z} \right)^k = \left(\frac{p}{1 - (1-p)z} \right)^k$$

where $p = \frac{\mu}{\sigma^2} = \frac{k}{\mu + k}$

Treating being detected and not being detected as two "decay" modes of a particle, by replacing *z* by *g(y) = 1 - ε + εy* (here *ε* contains both detector inefficiency and finite acceptance effect, *ε*=detector inefficiency x Acceptance) the generating function becomes

$$G(y) = \left(\frac{\frac{k}{\mu}}{1 + \frac{k}{\mu} - (1 - \varepsilon + \varepsilon y)} \right)^k = \left(\frac{\frac{k}{\varepsilon\mu}}{1 + \frac{k}{\varepsilon\mu} - y} \right)^k = \left(\frac{p'}{1 - (1-p')y} \right)^k$$

where we recover the generating function for a negative binomial distribution, while with the mean (*μ*) scaled down to *εμ*, $p' = \frac{k}{\mu\varepsilon + k} = \frac{p}{\varepsilon + p - \varepsilon p}$, and *k* not changed.

Tang & Wang, Phys Rev C 88, 024905 (2013)

The detecting inefficiency does not distort a negative binomial distribution.

Consideration of Inefficiency

With the *μ'* and *p'* extracted from the measured negative binomial distribution, we can recover the corresponding parameters of the original distribution by applying the following identities :

$$\mu = \mu'/\varepsilon \qquad\qquad p = \frac{p'\varepsilon}{1-p'(1-\varepsilon)}$$

Thus we obtain the formula with detecting inefficiency as :

$$\overline{N} = -k\ln p = -k\ln[\frac{p'\varepsilon}{1-p'(1-\varepsilon)}]$$

$$\overline{n}_c = \frac{\mu}{\overline{N}} = \frac{\mu'/\varepsilon}{-k\ln[\frac{p'\varepsilon}{1-p'(1-\varepsilon)}]}$$

Note that *k* is unchanged : $k' = \frac{\mu' p'}{1-p'} = \frac{\mu p}{1-p} = k$

Tang & Wang, Phys Rev C 88, 024905 (2013)

The detecting inefficiency can be corrected for (but VERY model dependent).

A Side Note

$$\mu = \mu' / \varepsilon \qquad p = \frac{p'\varepsilon}{1 - p'(1-\varepsilon)}$$

The two identities above are also useful in making corrections to other NB based observables, e.g. expected Kurtosis from a NB.

Products of Moments for Single Distributions

Negative Binomial

$$p = \frac{\mu}{\sigma^2} < 1$$

$$\frac{C_3}{C_2} = S\sigma = \frac{2-p}{p}$$

$$\frac{C_4}{C_2} = \kappa\sigma^2 = \frac{6-6p+p^2}{p^2}$$

$$\frac{C_6}{C_2} = \frac{120-240p+150p^2-30p^3+p^4}{p^4}$$

Binomial

$$p = \frac{\mu}{\sigma^2} > 1$$

$$\frac{C_3}{C_2} = S\sigma = 1-2p$$

$$\frac{C_4}{C_2} = \kappa\sigma^2 = 1-6p+6p^2$$

$$\frac{C_6}{C_2} = 1-30p+150p^2-240p^3+120p^4$$

The ratios of these cumulants depend only on *p*

Simply replace everywhere *p* by

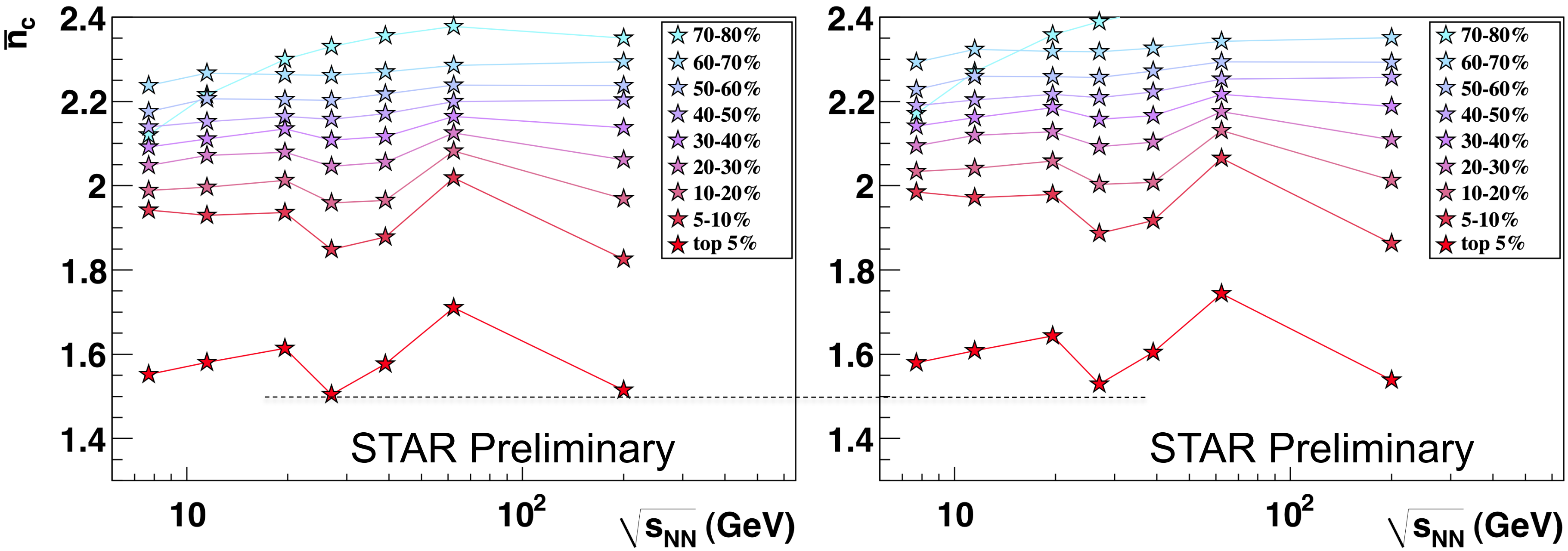
$$p = \frac{p'\varepsilon}{1 - p'(1-\varepsilon)}$$

Tang & Wang, Phys Rev C 88, 024905 (2013)

Can Efficiency Variation Cause the Structure ?

Let's take a blind guess of ε = 0.3, and check how much a 5% change in efficiency can change the result.

For both : no protons and heavier positive particles, dca<1cm



ε = 0.3 ε = 0.3*0.95

The variation of inefficiency cannot make up the structure

Summary

- A proper transformation of the NB parameters gives insights into the clan cluster production.

- Clan parameters have been measured in RHIC BES to study phase transitions.

- For the mean number of particles per clan we observed a reduction between 19 GeV and 62 GeV, with the minimum around 27 GeV. The structure is visible for most centralities, and most prominent for central collisions. So far we haven't identified a non-physical source for the structure.

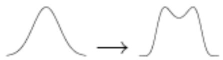
- Looking forward to checking with more BES points.

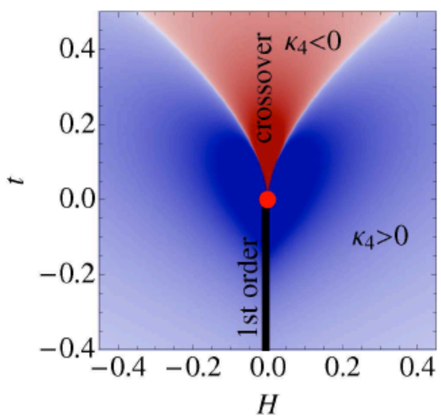
Back Up Slides

Speculations

Negative kurtosis

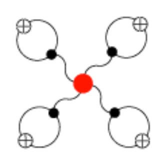
- Not only kurtosis becomes large, but it also changes sign
  (PRL 107:052301,2011)

- $P(\sigma_V)$:    Thus $\langle \sigma_V^4 \rangle_c < 0$ on the crossover line ($\lambda_3 = 0$).
  And around it.



- Universal Ising eq. of state $M(H)$:
  $M = R^\beta \theta, \quad t = R(1-\theta^2), \quad H = R^{\beta\delta} h(\theta)$
  - here κ_4 is $\kappa_4(M) \equiv \langle M^4 \rangle_c$
  - in QCD $M \to \sigma_V$,
    and $(t, H) \to (\mu - \mu_{\rm CP}, T - T_{\rm CP})$

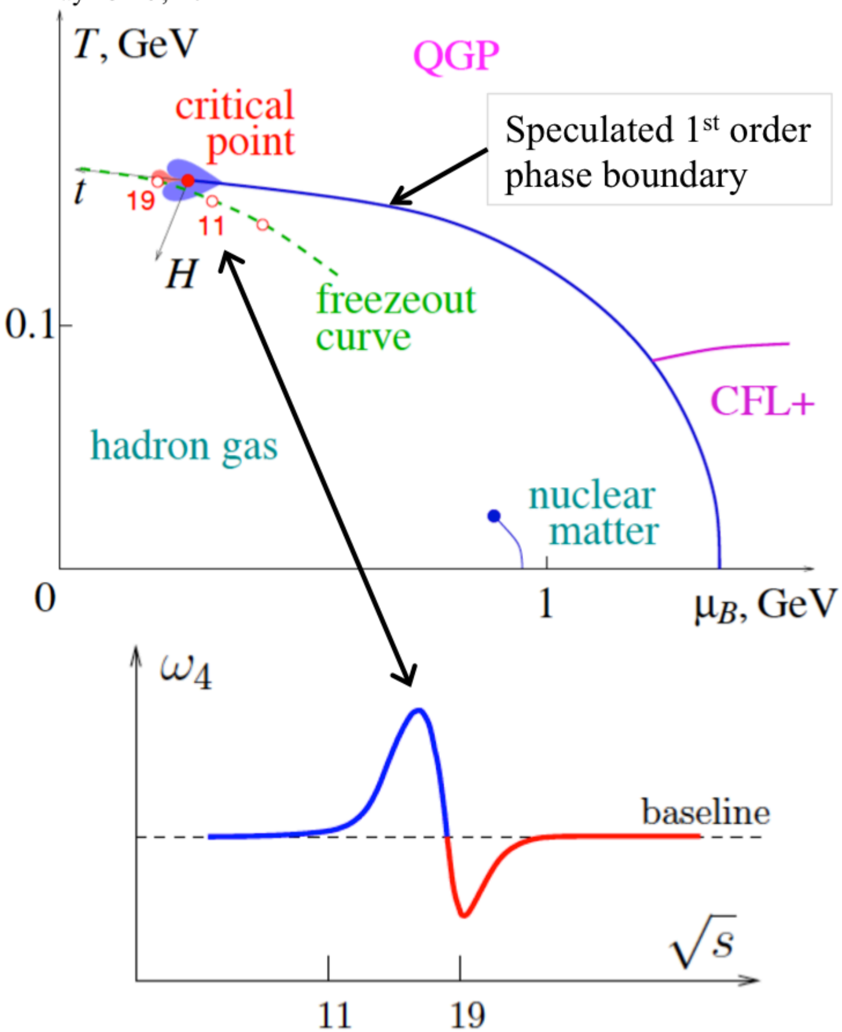
$$\langle (\delta N)^4 \rangle_c = \langle N \rangle + \langle \sigma_V^4 \rangle_c \left(\frac{g}{T} \int_{\boldsymbol{p}} \frac{v_{\boldsymbol{p}}^2}{\gamma_{\boldsymbol{p}}} \right)^4 + \ldots,$$



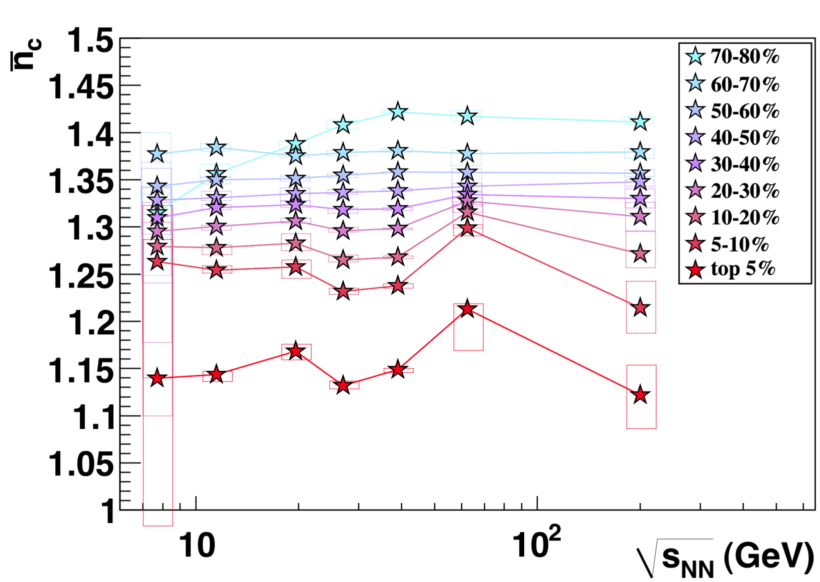
$\langle \sigma_V^4 \rangle_c < 0$ means $\quad \omega_4(N) \equiv \langle (\delta N)^4 \rangle_c / \langle N \rangle < 1$

Speculations

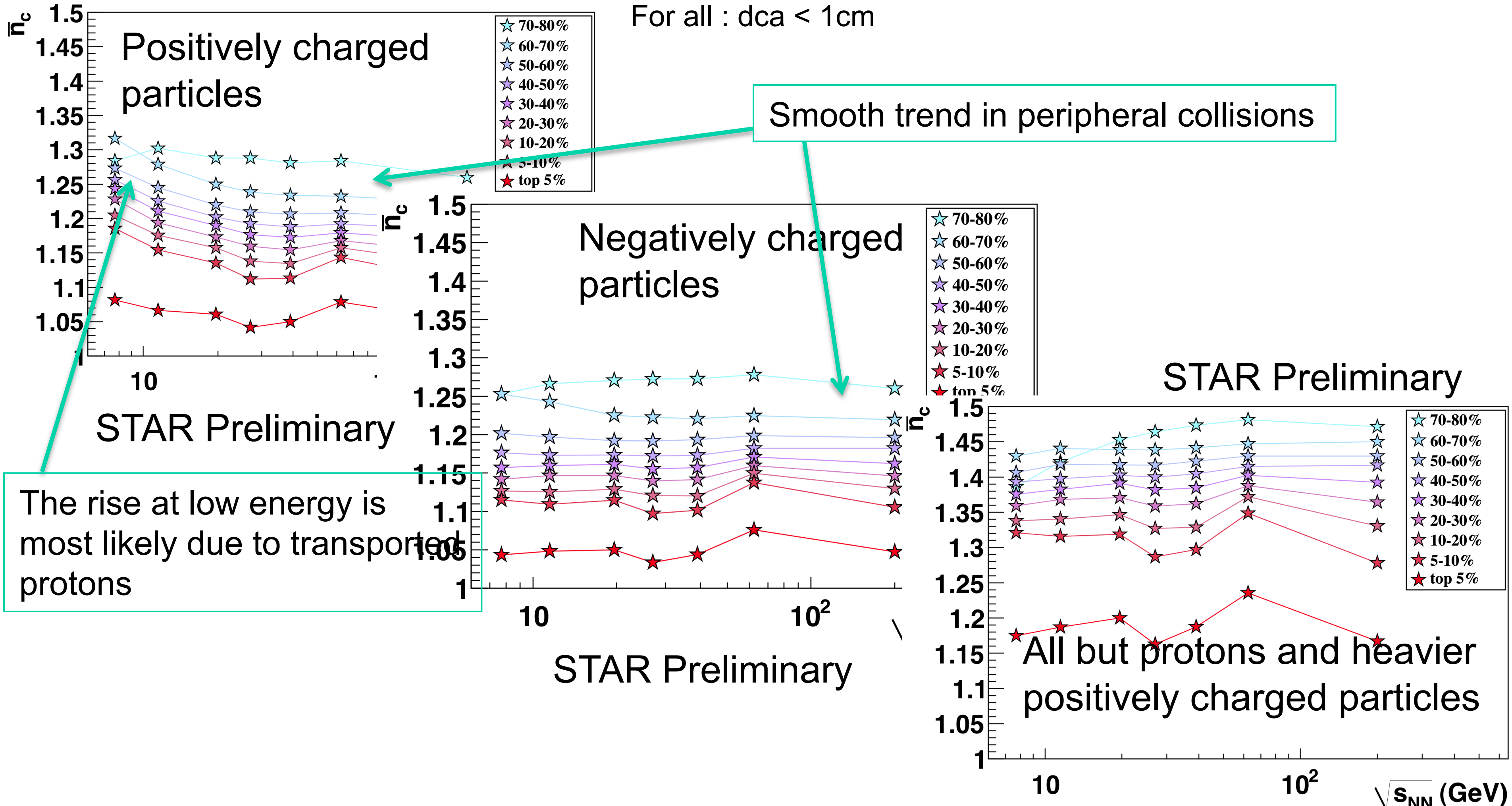
M. Stephanov, Rice Workshop, May 23-25, 2012



In the vicinity of the CP, there is a finite domain one would expect abnormal fluctuations. The energy dependence of observables depends on the trajectory of BES.



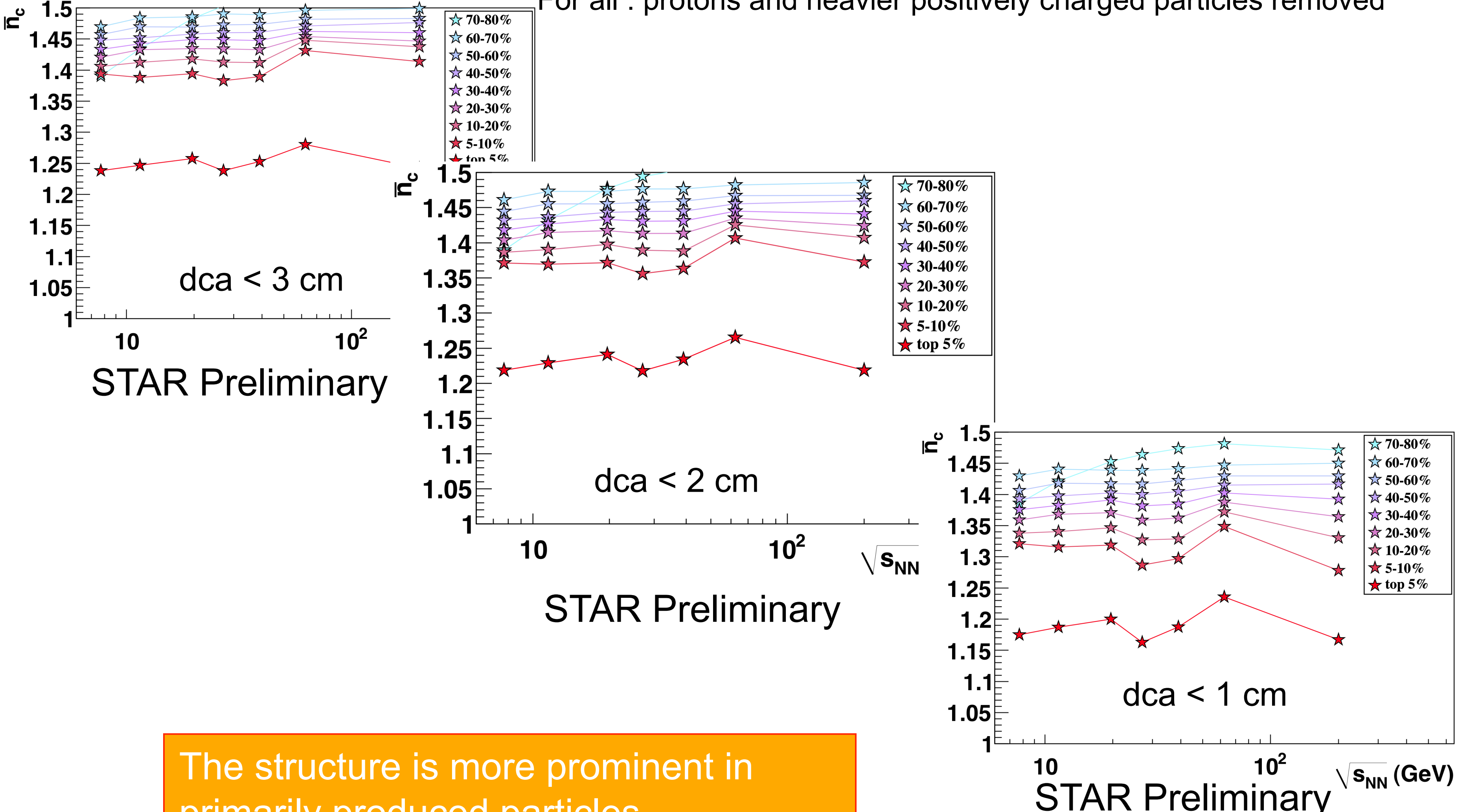
Check Different Particle Compositions



We are interested in “produced” particles only. Protons, which are contaminated with produced ones, can change the feature and are thus removed.

Check Different DCA

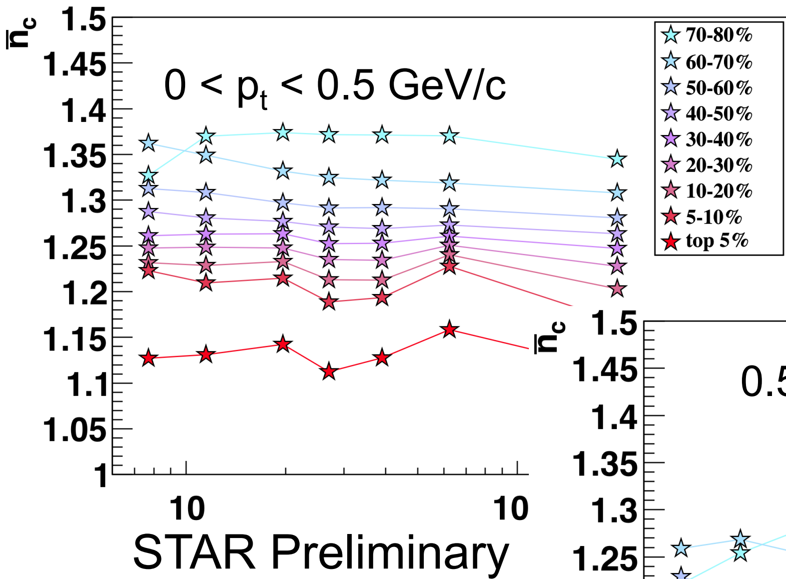
For all : protons and heavier positively charged particles removed

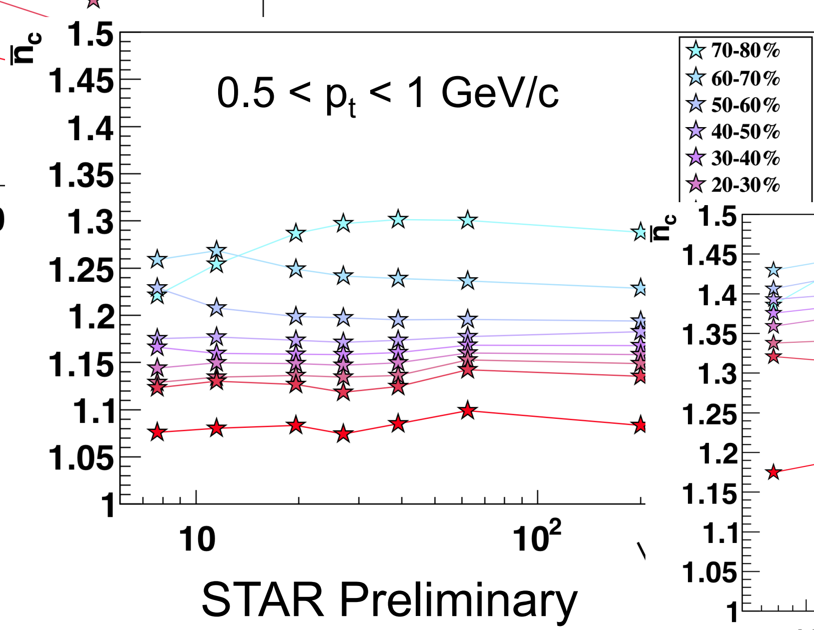


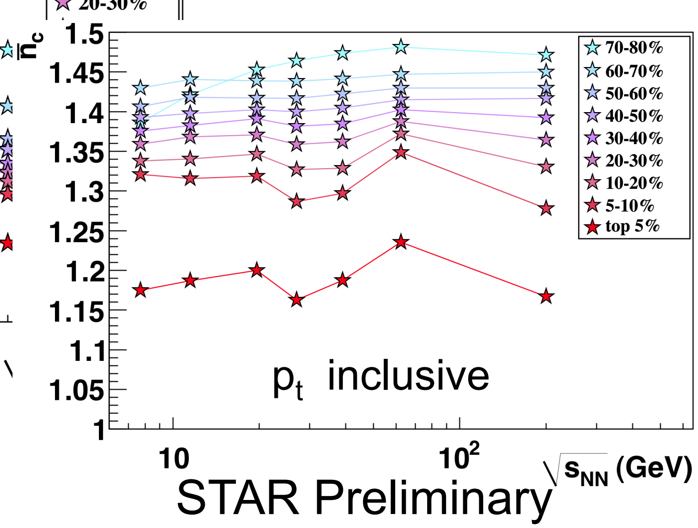
The structure is more prominent in primarily produced particles

Check Different Pt

For all : protons and heavier positively charged particles removed







No obvious dependence on p_t