STAR Analysis Note:

Measurement of transverse polarization of Λ in unpolarized pp collision at 200GeV

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140 Dataset

The data set (summarized in Table 1.1) used in this analysis includes pp200long_2015, pp200long2_2015 and pp200trans_2015 at present, which were taken in RHIC-STAR at $\sqrt{s} = 200$ GeV in pp collision with 689, 557 and 686 good physics runs respectively. The sum of the integrated luminosity of the three samples is about 133 pb^{-1} . Jet-Patch triggers(JP1, JP2), as shown in Table.1.2, are used in the analysis.

System and energy	pp collisions at $\sqrt{s}=200$ GeV						
Data	pp200long_2015	pp200trans_2015	pp200long2_2015				
Number of run	Number of run 689		557				
Total events	436 M	862 M	728 M				
Luminosity(pb ⁻¹)	29	52	52				
Production	P16id						
Trigger	•)404, 480404, 480414,)401, 480401, 480411,	,				

Table 1.1: Dataset in this analysis.

Trigger	ID	Threshold (ADC channels)	Equivalent E _T (GeV)
JP1	490404	28	5.4
JP2	490401	36	7.3

Table 1.2: Triggers used in the analysis

Some sub-detectors of STAR such as the TPC, BEMC, and EEMC are used in this analysis. The Events with primary vertex z within ± 90 cm from the center of TPC along the beam direction are selected. The primary vertex rank must be larger than 10e6, with about 5.93×10^8 events after z cuts. Fig. 1.1 showed the primary vertex z distribution before the selection of primary vertex z.

$\Lambda/\overline{\Lambda}$ reconstruction

The Λ hyperon characterized by self-analyzing weak decay has played a special role in the field of spin physics [1]. The $\Lambda(\overline{\Lambda})$ candidates are reconstructed via the weak decay channel: $\Lambda \to p + \pi^ (\overline{\Lambda} \to \overline{p} + \pi^+)$, following a similar procedure as in Ref. [2] except that the Time of Flight (TOF) hit matching is not required for the pion track. Firstly, good-quality tracks are obtained by following criteria:

• Track flag: $0 \sim 1000$

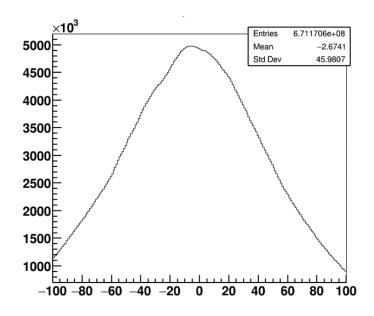


Figure 1.1: distribution of the primary vertex z.

- $p_T :> 0.15 \,\mathrm{GeV}$
- NHits > 15

- NHits/NHitsPoss > 0.52
 - DCA < 30cm

The TPC detector provides charge tracking and particle identification, which is used to select protons and π from a bunch of particles by ionization energy loss dE/dx. Because of the limited resolution of TPC detector, the capability of particle identification is reduced for charge particles with large momentum that are shown in Fig.2.1 (a) [3] that present ionization energy loss of four type particles, e^{\pm} , $p(\bar{p})$, π^{\pm} and K^{\pm} . The $n\sigma$ cut of proton candidate, for example, was required to be within $\pm 3\sigma$ to the theoretical values of dE/dx for proton. This cut is a reasonable value to balance the statistics and particle identification quality. Two daughter tracks with opposite charges are paired and hyperon p_T -dependent topological selection criteria, summarized in Tab. 2.1 and 2.2, are applied to suppress the background with an acceptable percentage of about 10%. Figure 2.2 shows the invariant mass distribution of Λ .

V_0 jet reconstruction

In order to implement the measurement of Λ polarization contribution from the fragmentation process, we need to reconstruct jet. The momentum direction of jet will be regarded as the direction of the fragmenting parton. This is also critical to determine the polarization direction of Λ . In this analysis, the jet was reconstructed with anti- k_T algorithm with following parameter sets.

- Reconstruction: anti- k_T with R = 0.6
- Tracks: primary track with $p_T > 0.2$ GeV and DCA < 3 cm
 - Towers are required to have $E_T > 0.2 \text{ GeV}$

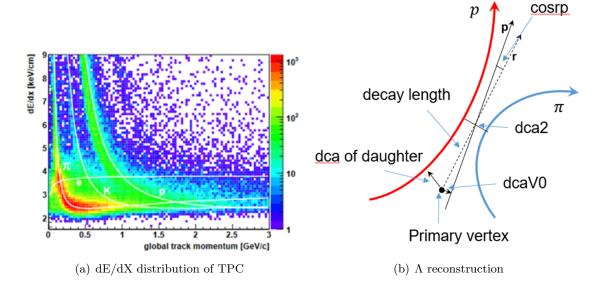


Figure 2.1: (Left) particle identification of TPC by dE/dx, (Right) the schematic of Λ reconstruction.

	$\Lambda(\overline{\Lambda})$ topological cuts									
$p_T [\mathrm{GeV}/c]$	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	> 6			
$ n\sigma <$	1.5	1.5	1	1	1	1	1			
DCA2(cm) <	0.65	0.65	0.60	0.55	0.50	0.45	0.40			
$DCA_p(cm) >$	0.45	0.35	0.30	0.15	0.005	0.005	0.005			
$DCA_{\pi}(cm) >$	0.65	0.65	0.60	0.55	0.50	0.50	0.50			
DCAV0(cm) <	0.55	0.65	0.75	1.0	1.0	1.0	1.0			
DecayLength(cm) >	3.0	3.0	3.5	3.5	4.0	4.5	4.5			
cosrp >	0.995	0.995	0.995	0.995	0.995	0.995	0.995			

Table 2.1: The table of $\Lambda(\overline{\Lambda})$ topological cuts at different p_T ranges

K_s^0 topological cuts									
$p_T [\mathrm{GeV}/c]$	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	> 6		
$ n\sigma <$	1.35	1.35	1.35	1.40	1.45	1.50	1.70		
DCA2(cm) <	0.65	0.65	0.65	0.55	0.55	0.50	0.35		
$DCA_p(cm) >$	0.60	0.55	0.50	0.35	0.30	0.25	0.20		
$DCA_{\pi}(cm) >$	0.60	0.55	0.50	0.35	0.30	0.25	0.20		
DCAV0(cm) <	0.65	0.70	0.80	0.90	0.90	0.90	0.90		
DecayLength(cm) >	3.55	3.60	3.70	3.75	3.80	4.0	5.5		
cosrp >	0.995	0.995	0.995	0.995	0.995	0.995	0.995		

Table 2.2: The table of K_s^0 topological cuts at different p_T ranges

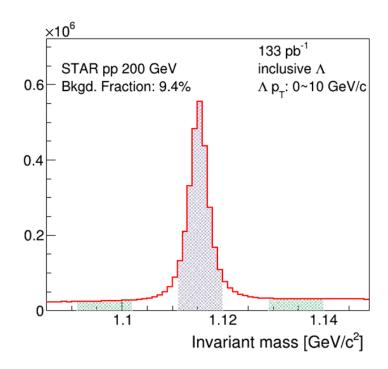


Figure 2.2: The invariant mass distribution of reconstructed Λ .

• The jet $p_T > 5 \text{ GeV}$

• Anti-proton energy correction

The final production of the whole fragmentation process consists of a variety of charge particles and neutral particles. We aim to probe the Λ polarization in final states. Therefore, the jets used here is full-jet consisting of both charge tracks from TPC and neutral energy from EEMC and BEMC. Only primary tracks with DCA < 3cm are utilized for jet reconstruction. To reduce noise background, the track p_T and tower energy E_T are required to be larger than 0.2 GeV. In case of the additional energy deposits in detector from possible annihilation effects of \bar{p} with proton from material of BEMC and EEMC, the \bar{p} annihilation correction is necessary(see Section 3.2). Besides, to reduce the other effects from underlying events (UE), we applied off-axis method to do the UE corrections, which helps to reduce the pile-up events. The jet candidates satisfying follow selection cuts are considered in this analysis.

- Jet p_T UE $p_T > 5$ GeV and pass trigger threshold
- Neutral fraction R < 0.95
- Jet η : $-1 < \eta < 1$
- Jet detector η_{det} : $-0.7 < \eta_{det} < 0.9$

The goal of neutral fraction R < 0.95 requirements is to avoid the contribution from charge tracks of TPC is too low. The difference between jet η and detector η_{det} is that η_{det} indicates the pseudorapidity of tower position in EMC relative to the TPC center.

3.1 Modification of jet reconstruction

Unlike traditional jet reconstruction in STAR, in this analysis, the reconstructed $\Lambda/\overline{\Lambda}$ candidates will also be added to the input list for jet reconstruction. Meanwhile, the primary tracks associated with the $\Lambda/\overline{\Lambda}$ daughter tracks will be excluded to avoid double counting. The diagram of this process is presented in Fig. 3.1. In some cases, $\Lambda/\overline{\Lambda}$ and K_S^0 may share the same daughter track due to the misidentification between protons and pions. This effect will introduce potential double counting if $\Lambda/\overline{\Lambda}$ and K_S^0 are both added to the same input list for jet reconstruction. To avoid such double counting, the $\Lambda/\overline{\Lambda}$ -jets and K_S^0 -jets were reconstructed separately.

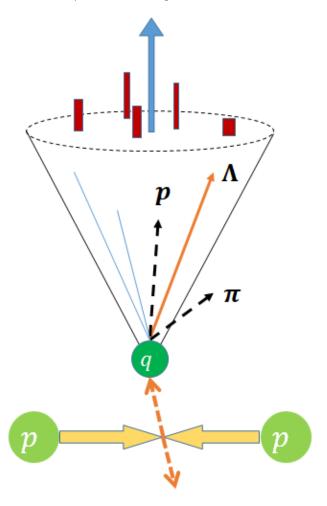


Figure 3.1: The Λ jet reconstruction process, where dashed black lines inside cone denote daughter tracks: p, π that will be excluded from particle list. The red rectangle means tower energy deposited in BEMC or EEMC. The big blue arrow indicates the reconstructed jet direction.

3.2 Anti-proton annihilation correction

The annihilation effects of antiproton produced in the final state with materials of BEMC/EEMC are non-negligible. For example, the \bar{p} decayed from $\bar{\Lambda}$, especially for low momentum, would likely annihilate with protons from BEMC/EEMC materials and deposit additional energy in BEMC/EEMC. This additional energy will also impact the neutral fraction in the process of jet reconstruction and increase the original actual jet energy. Fig.3.2 displays the tower energy

distribution deposited in BEMC and EEMC that match to p and \bar{p} . According to parity conservation, the behaviors of p and \bar{p} should be similar, which are different from the results in the plots. There is an apparent enhancement at large tower energy for \bar{p} . And the mean value of proton tower energy is 0.6 GeV, even only about half of that for \bar{p} .

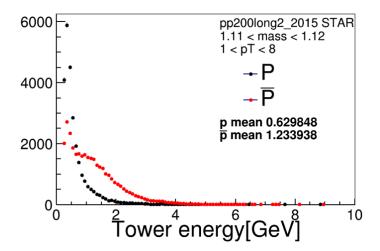


Figure 3.2: Comparison of tower energy of p and \bar{p} matched to BEMC or EEMC.

Nevertheless, the deposited energy of \bar{p} was still less than the theoretical value (twice of proton mass), if annihilated with other detector protons. One of the reasons we suppose might be that the additional energy extended to surrounding towers, which caused the tower energy matched to \bar{p} shift to the low energy range. To include annihilation energy of \bar{p} deposited in calorimeters as much as possible, the tower region matched to charge particle expands from one tower to surrounding 9 towers. As shown in Fig.3.3, the number denotes the tower index in detectors within the phase space constructed by η and ϕ axis.

Significantly, the energy distribution including 9 towers matched to \bar{p} shifts to the large value range with a peak at about 2 GeV. At the same time, No significant changes were observed for p. Such results demonstrate that the annihilation effects of \bar{p} can not be ignored and it is necessary to make corrections. In this analysis, 3×3 towers energy with it central tower matched to \bar{p} are removed from the jet reconstruction.

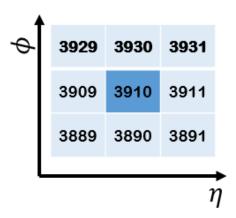


Figure 3.3: Tower map of BEMC that p and \bar{p} matched.

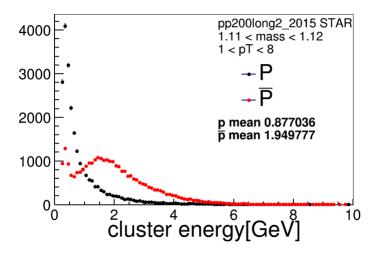


Figure 3.4: Comparison of 3×3 tower energy of p and \bar{p} matched to BEMC or EEMC.

3.3 Underlying events correction

The typical method, off-axis cone[4], was used in this analysis to subtract contributions from underlying events (UE), which contribute mostly low p_T tracks. They are corresponding to all particles produced directly from pile-up or hard scattering of partons, which are regarded as the contamination of jet. The two cones with the same η as jet, but perpendicular to the jet cone, are adopted to evaluate the UE particle yield. As shown in Fig. 3.5, the UE cones, dashed circular line with the radius equal to the jet resolution parameter (R = 0.6), are offset by an azimuthal angle $\phi = \pi/2$ with respect to the jet axis.

A general strategy for the UE contamination correction is to subtract the UE contribution to the jet p_T jet-by-jet. The p_T spectra of all particles inside these two UE cone are accumulated and divided by cone area, namely $2\pi R^2$, to obtain the UE p_T density ρ . Hence, the average UE p_T could be obtained through $\rho \times A_{jet}$, where A_{jet} is the area of the jets calculated by the Fastjet package[5].

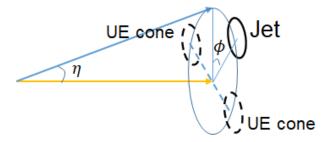


Figure 3.5: Diagram of Off-axis method.

However, in the multi-jet events, two or more jets with the same η but the $\Delta \phi = \pi/2$ probably occurred in the same event. It means the UE contribution to the jet p_T would be significantly overestimated, which will enhance the UE p_T . Figure 3.6 shows the UE p_T spectra with jet number dependence, and the average UE p_T increases with jet numbers. As a result, the jet p_T will be over corrected, if using these raw UE p_T that was enhanced by contribution from a real jet. What we did for this issue is to modify the UE region selection by including a protection

that when a jet was found nearby UE cones ($\Delta R \leq 1.2$), particles in that UE cones will be excluded from the UE p_T calculations.

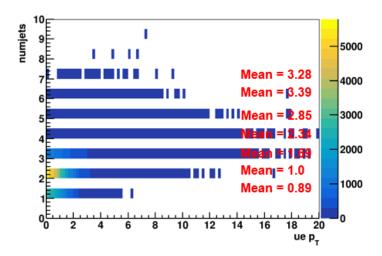


Figure 3.6: Underlying events p_T and average UE p_T versus number of jets.

The threshold of the jet that was regarded as a jet found nearby UE cones is set as 4 GeV. Following plots, Fig. 3.7, show the UE results after applying a protection mechanism in two UE cones. Apparently, this protection mechanism impacts largely on the UE p_T calculations, especially for multi-jets events. On the other hand, the threshold setup of a jet is also a crucial factor. Lower threshold means a jet would be identified as a real jet easier. See for the two plots of Fig. 3.7, the different minimum jet p_T are 4 GeV and 2.5 GeV respectively and resulted in different average UE p_T . In the left plot, the label '3coneUE' denotes another cone at opposite azimuth relative to the jet was regarded as UE cone either, which aimed to compensate the deficiency of UE cone resulted by protection mechanism but was canceled at final analysis. To keep things consistent, all parameters of jet nearby UE cones are the same as jet parameters above.

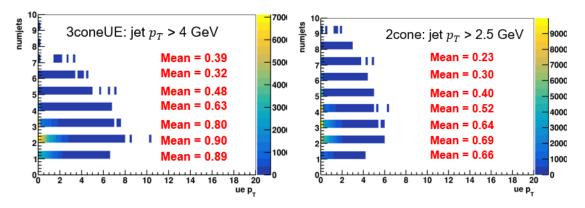


Figure 3.7: Underlying events p_T and average UE p_T versus number of jets.

²⁶² 4 MC Simulation

To correct acceptance effects from limited detector acceptance range and efficiency, we need to obtain acceptance functions corresponding to the STAR detector, which could be available by Monte Carlo (MC) simulation. There are many MC generators for the simulation of the pp collisions. In this analysis, simulation events are generated by PYTHIA6.4.28 [6] and then run through GEANT3 [7] based on STAR detectors.

268 4.1 Parameters set

The simulated events should be embedded into "zero-bias" data which was taken by triggered randomly in the period of run. Because these events with zero-bias trigger could be used to simulate beam background and pile-up events to make the simulation closer to the actual conditions. However, based on our study, we find it does not greatly affect the acceptance function without zero-bias data from simulation. The simulation setup are listed following:

- PYTHIA6.4.28 + GEANT3
- ptHard > 4 GeV

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- Energy 200 GeV
 - Geometry: y2015c
 - $\Lambda/\overline{\Lambda}$ filter: promise every event include at least one $\Lambda/\overline{\Lambda}$ with $p_T > 0.5$ GeV
 - Primary vertex: Gaussian distributions with $\sigma_x = 0.026$ cm, $\sigma_y = 0.015$ cm, $\sigma_z = 41.48$ cm

The reason why ptHard is larger than 4 GeV, rather than the usual several separate regions 280 from 2 to 35, is to increase simulation efficiency with jet-patch trigger as much as possible while 281 suppress edge effects of trigger threshold as low as possible, simultaneously. Figure 4.1 shows 282 the ratio of contributions of different ptHard ranges to jet p_T spectra. The left plot is for the 283 JP1 trigger and the right one is for the JP2 trigger. The percentage of the contribution to jet p_T 284 spectrum from ptHard $2 \sim 3$ GeV is about 5.68% and from ptHard $3 \sim 5$ GeV is about 7.53%. 285 Moreover, the efficiency for a event from ptHard $2 \sim 4$ GeV that passes trigger threshold is 286 too low to obtain sufficient statistics within acceptable time duration. Therefore, 4 GeV is an 287 appropriate value for minimum ptHard. 288

The goal of applying $\Lambda/\bar{\Lambda}$ filter is to increase simulation efficiency and save disk space by selecting events that include at least one Λ or $\bar{\Lambda}$ with $p_T > 0.5$ GeV. For the JP1 and JP2 triggers, we also applied the trigger simulator to simulate the trigger response. The same algorithms as the data are applied in MC simulation to reconstruct $\Lambda/\bar{\Lambda}$ and jet.

4.2 Particle identification correction

In the analysis, we encountered a severe issue with the MC sample: the central value of $n\sigma$ 294 distribution from the MC sample significantly deviated from its theoretical value and also differed 295 from the real data distribution. The distributions of $n\sigma$ for protons in both the MC and real 296 data samples are shown below in Fig.4.2. The center of the proton $n\sigma$ distribution in the MC 297 sample is shifted towards negative values by approximately one sigma. In contrast, the center of 298 the proton $n\sigma$ distribution in the data sample is consistent with zero. This issue will introduce 299 potential biases to the measurements as same $n\sigma$ selection cuts were applied to both read data 300 and MC samples. 301

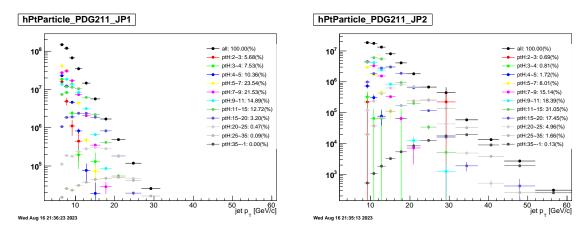


Figure 4.1: Jet contributions from different ptHard ranges.

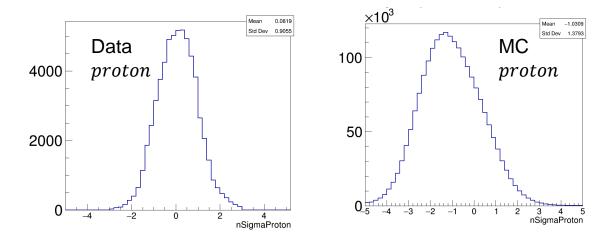


Figure 4.2: $n\sigma$ distributions of proton in data and MC sample

Upon careful examination, we found that the cause of this phenomenon is due to inadequate simulation of particle ionization energy loss in the gas during the generation of the MC sample. The blue and green lines in the Fig. 4.3 below represent the fits to the ionization energy loss as a function of momentum for protons at the detector level and association level in the MC sample, respectively. These do not match the distribution of ionization energy loss versus momentum for protons in the real data sample. Similar issues are observed for other types of particles as well.

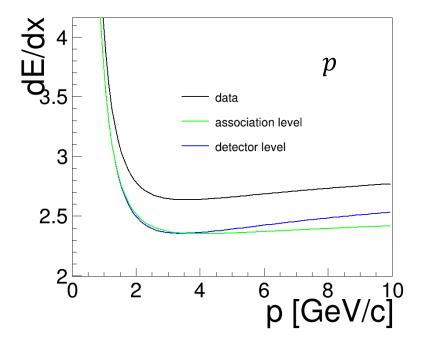


Figure 4.3: dE/dx vs momentum distributions of proton in data and MC sample

To avoid the bias introduced by suboptimal simulation of ionization energy loss, we must apply a correction. The method involves fitting the distribution of the $n\sigma$ mean values as a function of momentum to ascertain the deviation from the theoretical curve. For this step, we require a clean sample of particles, so we extracted particles at the association level, which are associated directly with pure particles produced by PYTHIA. The left plot of Fig.4.4 shows a 2-dimensional distribution of proton $n\sigma$ as a function of momentum. And right plot is the distribution of the mean value of $n\sigma_p$ versus proton momentum, which shows a complex dependence. Then, we subtract the corresponding deviation value from each particle's $n\sigma$, realigning it with the theoretical value.

4.3 Comparison of pure MC and data

The reconstruction of Λ , $\overline{\Lambda}$, and K_s^0 in both MC and data employed identical reconstruction methods, selection criteria, and topological cuts to ensure consistency. Comparisons of the data and MC simulation are shown in the Appendices. We can find a good agreement for p_T between the data and MC simulation.

For pseudo-rapidity η and azimuth angle ϕ , some sectors of TPC issued this year resulted in the nonuniform distributions of azimuth angle ϕ and asymmetrical η distribution relative to zero. However, MC simulation is not consistent with data, which means GEANT3 based on

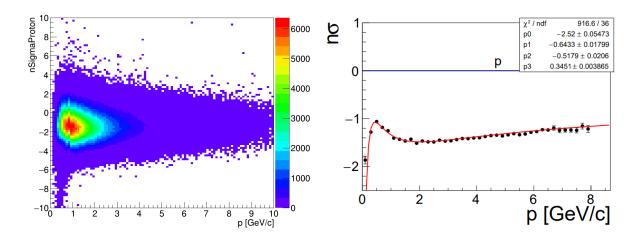


Figure 4.4: Left: 2-dimensional distribution of proton $n\sigma$ as a function of momentum; Right: the mean value of $n\sigma_p$ versus proton momentum

the STAR detector did not simulate perfectly the true status of the STAR detector. These will influence acceptance correction. Simultaneously, the statistic of the MC simulation sample is highly hard to produce due to low efficiency and limited resources. We just utilize it to check the new method of acceptance correction and estimate trigger bias.

330 5 Mixed Events

The biggest disadvantage of MC simulation is its statistics are still not enough for acceptance correction of data, which resulted statistical uncertainty of results are too large to obtain a definite conclusion. Thus, another alternative method, named mixed-event method[8], is proposed for this analysis. This is a popular method utilized widely to estimate combination backgrounds by mixing different tracks from randomly different events, the details can be found in reference [8]. An important reason we want to use the mixed event method is its fast production and smaller storage space, which could save lots of time and computer resources.

In this analysis, the mixed method is a little different but with the same principle. A reconstructed Λ particle will be embedded into a different event to form a mixed event, then using this event to reconstruct Λ jet. The procedures are shown in Fig.5.1. Of course, these two events must be required with the same trigger and their discrepancy of primary vertex z is smaller than 5 centimeters, and mixed events must be applied to the individual run aiming to ensure similar conditions as much as possible. Owing to there being no correlation between Λ and jet from different events, no physic signal of polarization will be obtained theoretically, and the original correlation between Λ and jet at the SE is also broken simultaneously.

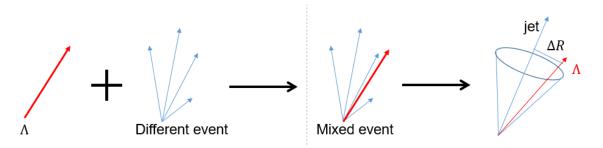


Figure 5.1: Mixed event procedure

5.1 The research of Mixed-event methods

There are two types of mixed events in this analysis based on constraints of Λ and jet in different events. For example, at one event, the azimuth phase space is separated into two sections, the jet areas and off-axis regions, as shown in Fig.5.2. The Fig. 5.3 shows near-jet mixed events and corresponding comparison of ΔR distribution. If there are no constraints between Λ and jet at mixed event, the Λ will located randomly at any region that was described above that named as random mixed events. Therefore, it is possible for Λ to reconstruct a fake jet when it located at off-axis regions where none jet exist. It means this jet was dominated by Λ particle, which was verified in Fig. 5.4. This condition might affect jet p_T distribution and acceptance correction. The Fig. 5.4 shows mixed events when Λ located at off-axis regions, likely to underlying event (UE) cone. The ΔR distribution is inconsistent with the same event.

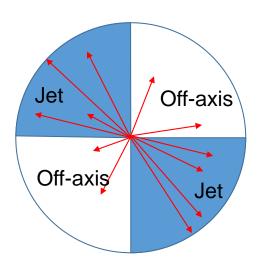


Figure 5.2: Azimuth phase space

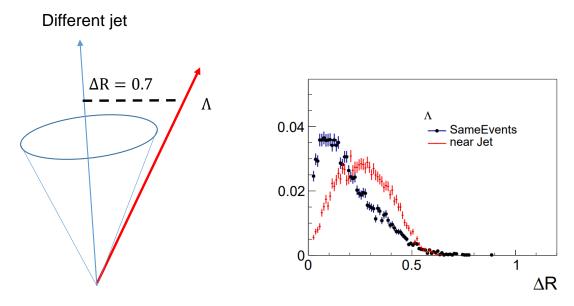


Figure 5.3: The near-jet mixed events

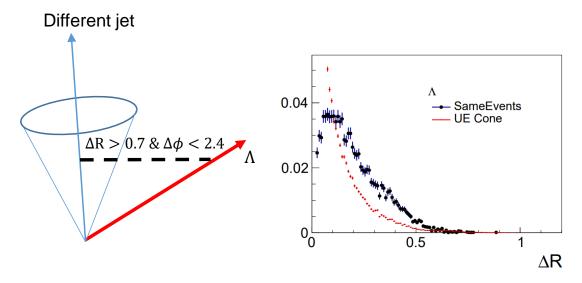


Figure 5.4: The off-jet mixed events

 To assess the magnitude of the influence, Λ was required to be near the jet with $\Delta R < 0.7$ in mixed events prior to jet reconstruction. The quality comparison between random mixed events and near-jet mixed events is illustrated in the following figures. There is no significant difference in the jet p_T distribution, with the exception of the low p_T range. Removing mixed events from the off-axis region would significantly reduce the number of fake jets with low p_T that are predominantly composed of Λ particle. A positive outcome is that near-jet mixed events have improved the consistency of the z distribution with SE. Nevertheless, j_T distribution has not seen substantial improvement, and inconsistency persists.

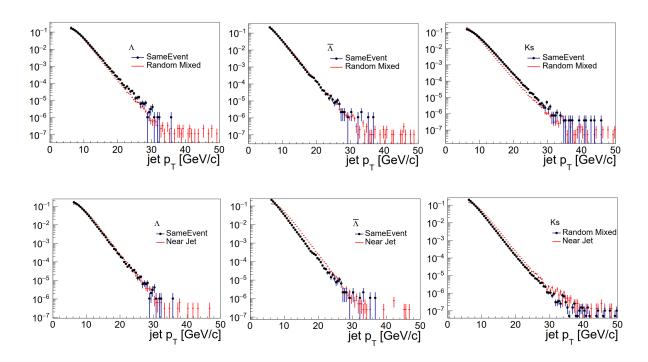


Figure 5.5: Top panel: comparisons of jet p_T between SE and random ME; Bottom panel: comparisons of jet p_T between SE and near-jet ME

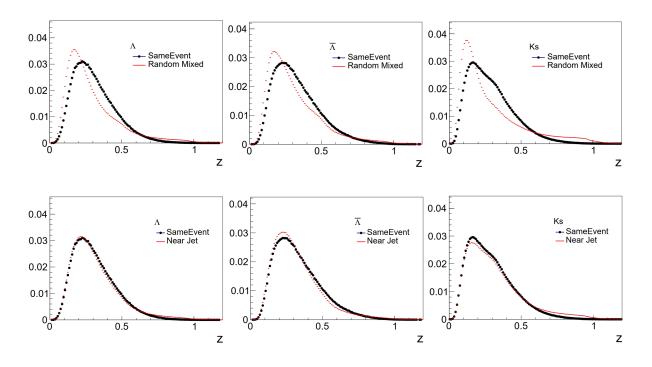


Figure 5.6: Top panel: comparisons of z between SE and random ME; Bottom panel: comparisons of z between SE and near-jet ME

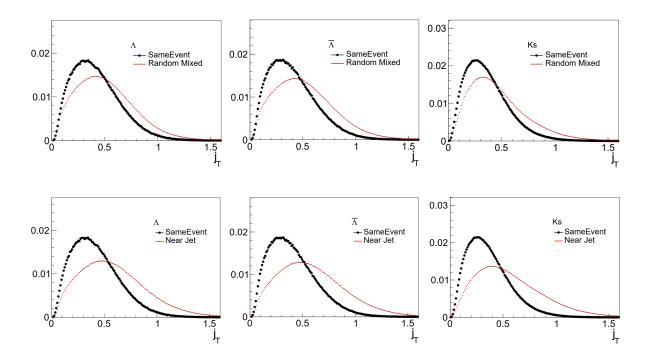


Figure 5.7: Top panel: comparisons of j_T between SE and random ME; Bottom panel: comparisons of j_T between SE and near-jet ME

We also compared Λ and $\overline{\Lambda} \cos \theta^*$ distributions of mixed events generated by the different methods. The consistency of their distributions was very good, indicating that the off-axis region has a minor impact on the correction of the acceptance. However, the near-jet mixed events were closer to the true events, so we still used this method for acceptance correction in this analysis.

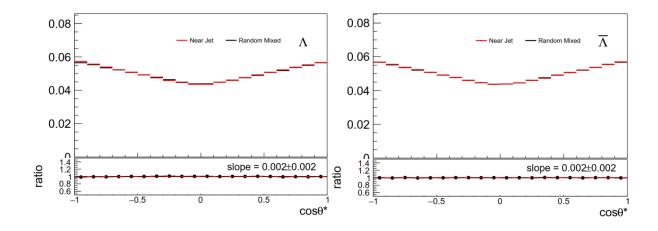


Figure 5.8: Comparison of $\cos \theta^*$ between random and near-jet mixed event

5.2 Closure test in MC

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The closure test for this method is essential to determine whether the mixed-event technique can be effectively applied for acceptance correction in this analysis. The following investigation of mixed events is based on unpolarized Monte Carlo (MC) samples, generated using Pythia 6 and GEANT3, as previously mentioned. First, we must verify whether the mixed-event method introduces any non-physical spurious signals.

We have generated a mixed-event sample using MC simulation data, applying the same jet reconstruction algorithm and selection criteria as in the original analysis. Since Λ hyperons in mixed events do not originate from the same hard scattering process as the jets, the Λ -jet correlation in mixed events is expected to differ from that in same events. Figure 5.11 compares the distributions of key observable ΔR , which characterize the Λ -jet correlation between mixed and same events.

• ΔR represents the angular separation between the Λ hyperon and the jet axis in the η - ϕ plane.

$$\Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} \tag{5.1}$$

To improve the consistency of correlations quantities between the mixed and the same events, we implement a reweighting procedure to match the ΔR distribution between data and mixed events. However, a purely one-dimensional ΔR reweighting fails to capture important angular correlations, particularly the distinct η and ϕ -dependence of Λ -jet correlations. This is especially critical for the $\Delta \eta$ distribution, which exhibits significant pseudorapidity dependence due to detector acceptance effects. As illustrated in Figure 5.9 and Figure 5.10, the two-dimensional distributions of $(\Delta \eta, \Delta \phi)$ versus jet η reveal complex correlation patterns that cannot be reproduced by ΔR alone. These distributions show:

- $\Delta \eta$ broadening at high $|\eta|$
- Edge effects near detector acceptance boundaries

To properly account for these effects, we implement a three-dimensional reweighting scheme based on:

$$\left(\Delta\eta, \Delta\phi, \eta^{\text{jet}}\right) \tag{5.2}$$

This comprehensive approach ensures proper descriaption of:

- The full angular correlation structure
- η -dependent detector effects

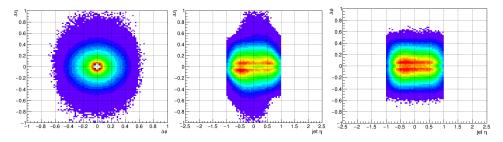


Figure 5.9: Two-dimensional distributions showing the correlations between $\Delta \eta$, $\Delta \phi$ and jet η at mixed events.

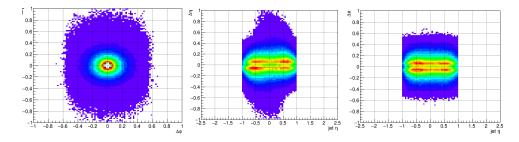


Figure 5.10: Two-dimensional distributions showing the correlations between $\Delta \eta$, $\Delta \phi$ and jet η at the same events.

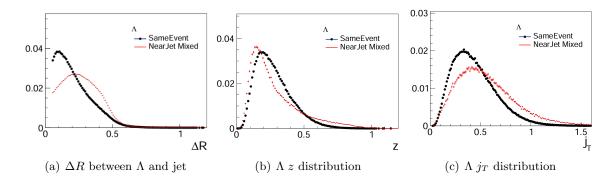


Figure 5.11: Comparison of $\Delta R, z, j_T$ distribution of Λ between SameEvents and MixedEvents before reweighting.

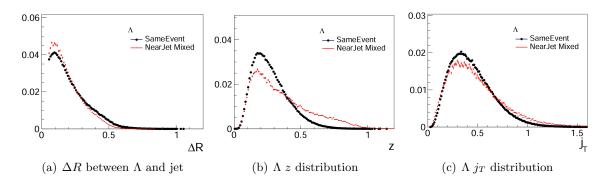


Figure 5.12: Comparison of $\Delta R, z, j_T$ distribution of Λ between same events and mixed events after reweighting.

In addition, the Λ p_T distribution shows a discrepancy between same-event (SE) and mixed-event (ME) samples in Fig. 5.13, particularly in the high- p_T region. The 3D reweighting procedure not only failed to improve this inconsistency but even exacerbated it. However, as demonstrated in Fig. 5.15, the detector acceptance effect decreases with increasing Λ p_T . Combined with the relatively small fraction of high- p_T Λ particles, this discrepancy has no significant impact on the acceptance correction procedure.

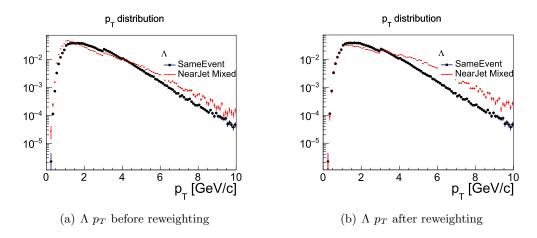


Figure 5.13: Comparison of p_T distribution of Λ between same events and mixed events

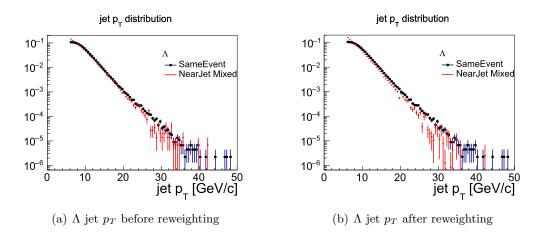


Figure 5.14: Comparison of p_T distribution of Λ jet between same events and mixed events

The key consideration is whether the mixed-event method can adequately account for detector acceptance effects in our analysis. Figure 5.16(a) demonstrates that the mixed events successfully reproduce the $\cos \theta^*$ distribution, including acceptance effects, even *before* reweighting.

After applying the reweighting procedure, we observeed improved consistency in the $\cos \theta^*$ distribution, as evidenced by the ratio plot slope being compatible with zero (Figure 5.16(b)). This indicates that:

- The mixed-event sample introduces no artificial polarization signal
- Acceptance effects are properly modeled by the method

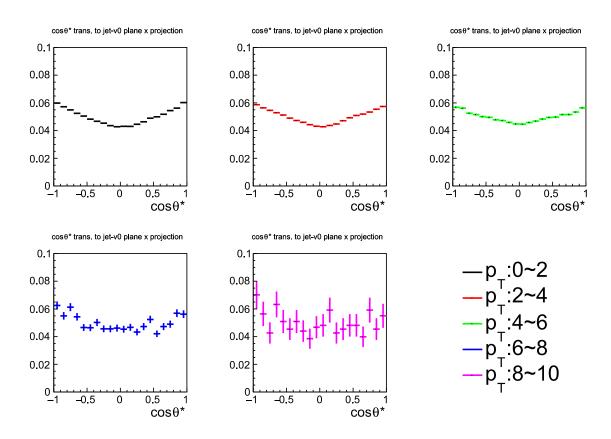


Figure 5.15: $\cos \theta^*$ distribution of in the different Λ p_T bins

However, this validation does not imply that the mixed-event technique can be directly applied to extract Λ polarization from polarized samples. The critical remaining question is how the mixed-event procedure affects the $\cos\theta^*$ distribution for polarized Λ hyperons - a key systematic uncertainty that must be quantified through:

$$\Delta P = P_{\text{true}} - P_{\text{measured}}^{\text{(mixed events)}} \tag{5.3}$$

where P denotes the polarization magnitude. This constitutes the final stage of our closure test and will determine the applicability limits of the method for this analysis.

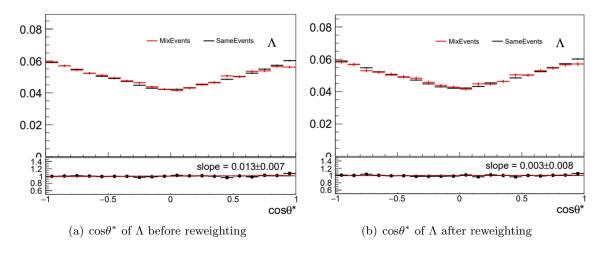


Figure 5.16: Comparison of $\cos \theta^*$ of Λ between same events and mixed events.

To confirm whether the mixed-event method works well in polarization extraction and how large impacts are for the polarized Λ sample, we generate a MC sample with polarized Λ by throwing some Λ randomly by a linear function of $\cos \theta^*$:

$$dN/d(\cos\theta^*) = (1 + \alpha P_{\Lambda} \cos\theta^*) \tag{5.4}$$

where P_{Λ} is the input polarization and α is the weak decay constant of Λ . The blue flat line in the left plot of Fig. 5.17 is $\cos \theta^*$ distribution with $P_{\Lambda} = 0$, and the red line is $\cos \theta^*$ distribution with $P_{\Lambda} = -0.1$. We fit this red line and get the same polarization signal as the input value. So we used the same method at the detector level. Then, we use this polarized lambda sample to make the mixed event.

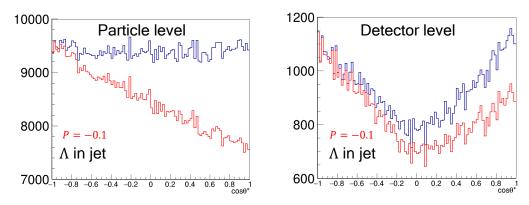


Figure 5.17: Left: $\cos\theta^*$ of Λ at particle level. Right: $\cos\theta^*$ of Λ at detector level

The validation of polarization extraction is presented in Fig. 5.18, showing the correlation between input (x-axis) and extracted (y-axis) Λ polarization values. The points are fitted with a linear function:

$$f(x) = p_0 x + p_1 (5.5)$$

where p_0 and p_1 parameters denote the slope and y-intercept of the fit function, respectively.

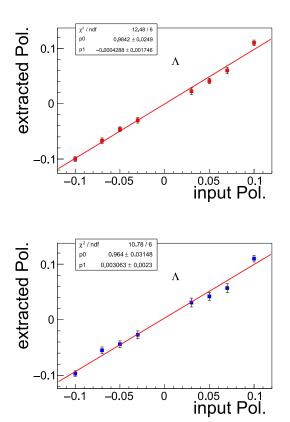


Figure 5.18: Extracted polarization vs input polarization of Λ ; (Red points) before reweighting; (Blue points) after reweighting.

Notably, the extracted polarization values show excellent agreement with the input values both before and after reweighting, as evidenced by the goodness-of-fit ($\chi^2/\text{ndf} = 12.48/6$ 431 $\chi^2/\text{ndf} = 10.78/6$). This consistency confirms the reliability of the mixed-event method for 432 Λ polarization measurement. Furthermore, the observed discrepancy in the Λ p_T distribution 433 does not have a significant impact on the acceptance correction procedure. The linear response 434 with slope $p_1 = 0.96 \pm 0.03$ and intercept $p_0 = 0.003(2)$ % demonstrates mixed events method and 435 our reweighting procedure throughout the closure test without introducing significant system-436 atic bias. The method properly accounts for detector acceptance variations and combinatorial 437 background effects, with statistical uncertainties dominating the measurement precision while 438 keeping systematic uncertainties below 7%, making it fully suitable for precision measurements. 439

440 5.3 Mixed-events sample

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A rigorous quality assessment (QA) of the mixed-event sample is essential prior to applying acceptance corrections. As previously discussed, this requires detailed comparison of kinematic distributions between mixed and single-event samples. Figure 5.19 shows the two-dimensional $\Delta \eta$ versus $\eta_{\rm jet}$ distribution for K_S^0 candidates, revealing significant discrepancies between mixed and same-event data. Most notably, the mixed events exhibit a pronounced $\Delta \eta$ asymmetry in opposite $\eta_{\rm jet}$ regions, which is absent in the genuine correlated events. To mitigate potential biases in the acceptance correction, we implemented the three-dimensional reweighting procedure $(\Delta \eta, \Delta \phi, \eta_{\rm jet})$ described in Section 5.2.

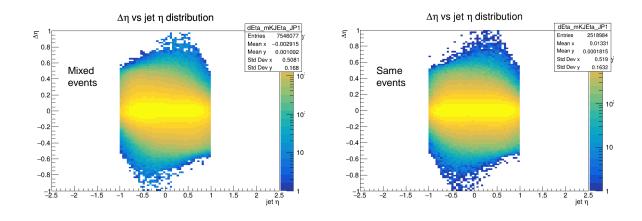


Figure 5.19: Left: $\Delta \eta$ vs η_{jet} in mixed events. Right: $\Delta \eta$ vs η_{jet} in same events.

The kinematic distributions, including pseudorapidity (η) and azimuthal angle (ϕ) , remain well-preserved after reweighting, while inconsistency is observed in the transverse momentum (p_T) spectrum. However, studies confirm that this p_T discrepancy has no significant effect on the polarization extraction.

Additionally, we compare correlated observables between hyperons and jets after reweighting. The results show excellent agreement in the angular separation ΔR , while other variables exhibit slight deviations that resulted by the inconsistency of p_T . Nevertheless, these inconsistencies do not affect the acceptance correction, as verified by closure tests.

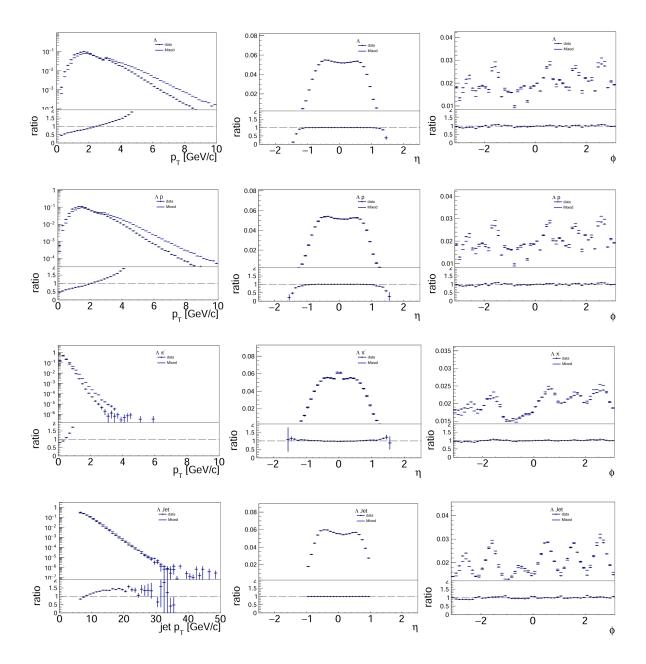


Figure 5.20: Comparisons of three kinematic quantities p_T, η, ϕ of Λ and jet between SE and ME.

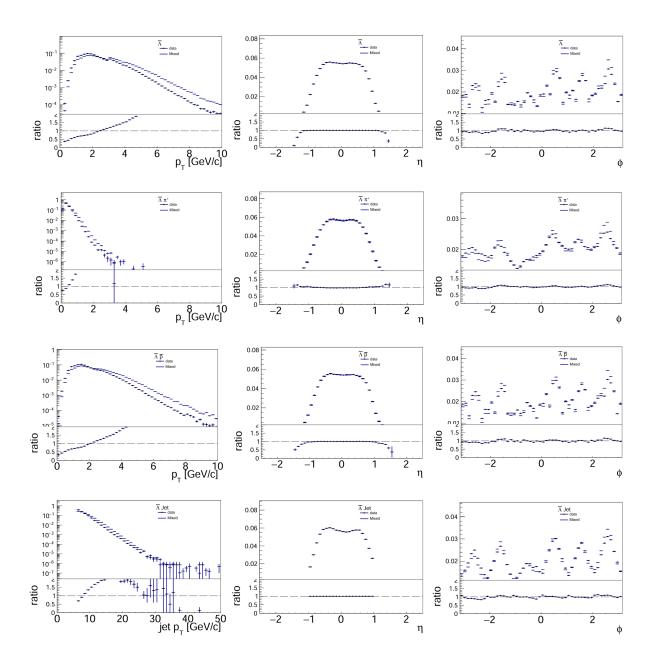


Figure 5.21: Comparisons of three kinematic quantities p_T, η, ϕ of $\overline{\Lambda}$ and jet between SE and ME.

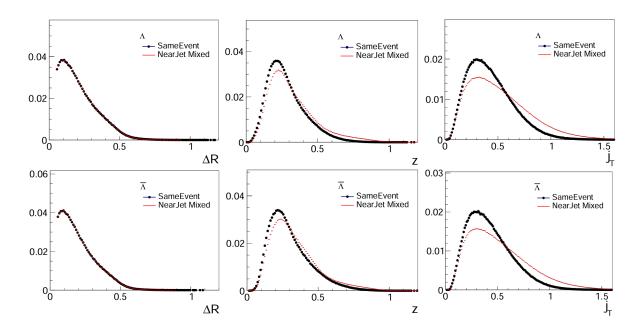


Figure 5.22: Comparisons of $\Delta R, z, j_T$ of Λ between SE and ME.

⁴⁵⁷ 6 Transverse polarization $P_{\Lambda/\overline{\Lambda}}$ extraction of $\Lambda/\overline{\Lambda}$

6.1 Detector acceptance correction

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Here shows the procedure of acceptance correction and lambda polarization extraction. The $\cos\theta^*$ distribution of Λ is not linear, as shown in Figure 6.1, which is attributed to the detector acceptance effects. Here, the mass peak window of the candidates Λ is set at 1.112 \sim 1.120 GeV/c and background contribution had been subtracted from the $\cos\theta^*$ distribution under the mass peak using the sideband method, as shown in Figure 2.2.

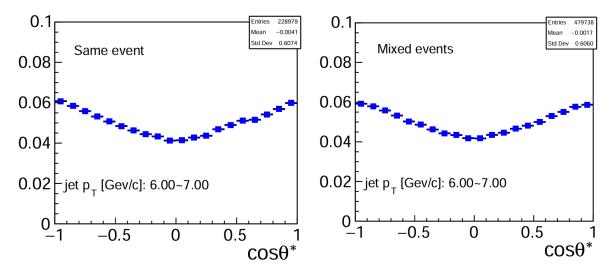


Figure 6.1: $\cos \theta^*$ distribution of Λ for the same event(left) and mixed events(right)

The acceptance correction can be done via mixed events. The $\cos\theta^*$ distribution of Λ that could reflect detector acceptance can be seen in the right panel of Figure 6.1. The same background subtraction procedure was also applied for mixed events. Once the acceptance correction is done, polarization can be extracted by fitting the $\cos\theta^*$ distribution with a linear function:

$$dN/d(\cos\theta^*) = A(\cos\theta^*)(1 + \alpha P_{\Lambda}\cos\theta^*)$$
(6.1)

where A(cos θ^*) denotes acceptance function. The α is the weak decay constant of Λ , which is $\alpha = 0.747 \pm 0.009$ [9]. The magnitude of weak decay constant for $\overline{\Lambda}$ is $\alpha = 0.757 \pm 0.004$.

Figure 6.2, as an example, shows the $\cos\theta^*$ distribution of Λ after acceptance correction, and it was fitted by above function Eq. (6.1) to obtain polarization. The first fitting parameter p_0 is the extracted polarization. Its uncertainty from the fitting is treated as statistical uncertainty. Figure 6.3 and 6.4 show the fitting results at each jet bin for Λ and $\overline{\Lambda}$ respectively.

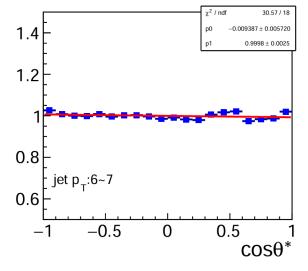


Figure 6.2: $\cos \theta^*$ distribution of Λ after acceptance correction and was fitted with a linear function (red line) to extract polarization

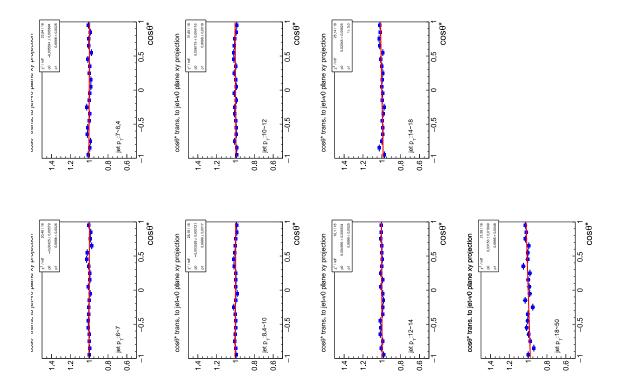


Figure 6.3: Extraction of transverse polarization of Λ as a function of jet p_T

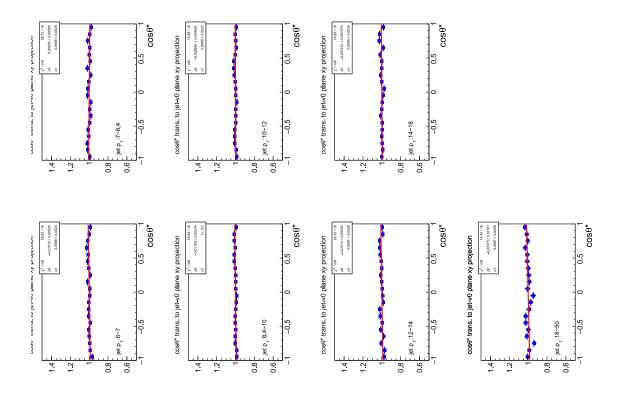


Figure 6.4: Extraction of transverse polarization of $\overline{\Lambda}$ as a function of jet p_T

Zero-test with K_s^0 6.2

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In order to confirm the validity of polarization extraction of Λ and $\overline{\Lambda}$, the K_s^0 particle with zero spin is used to make zero-test. If extracted polarizations of K_s^0 are consistent with 0, it 475 means the Λ and $\overline{\Lambda}$ polarizations extracted in this analysis are credible. The same procedure of polarization extraction is applied for K_s^0 particle. The transverse polarization of K_s^0 as a function of jet p_T is consistent with 0 as shown in Figure 6.5.

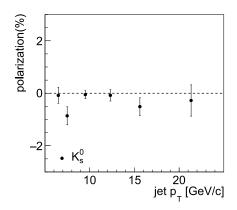


Figure 6.5: Transverse polarization of K_s^0 as a function of jet p_T

Besides, figure 6.6 and 6.7 present the transverse polarization of K_s^0 as a function of z and j_T .

They are all consistent with 0 as expected, which means the method of polarization extraction in this analysis is credible.

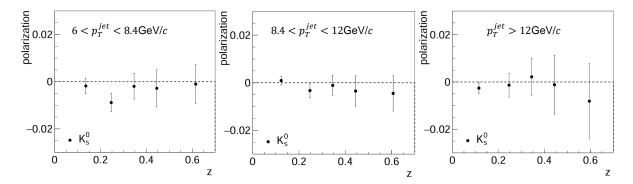


Figure 6.6: Transverse polarization of K_s^0 as a function of z at different jet p_T ranges

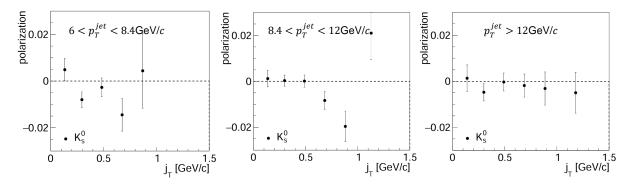


Figure 6.7: Transverse polarization of K_s^0 as a function of j_T at different jet p_T ranges

6.3 Comparison of results extracted by mixed events and MC

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To validate our analysis results, we performed a systematic cross-check by comparing the polarization results obtained from two independent methods: mixed events and MC simulation incorporating detector response. Here show our JP1 results with the same selection criteria , which were extracted by MC simulation in Fig6.8 and by the mixed-events method. The trend of polarization that extracted by these two different methods are consistent.

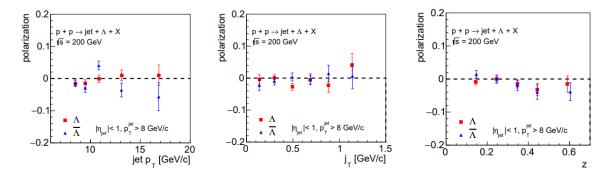


Figure 6.8: Transverse polarization extracted by MC

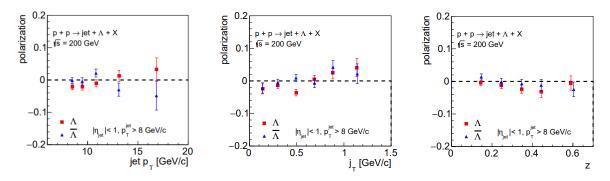


Figure 6.9: Transverse polarization extracted by mixed events

7 Correction for results

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7.1 Polarization direction correction

The angular resolution of jet axis reconstruction is degraded by detector acceptance effects, directly impacting the determination of the polarization direction through the spin vector definition:

$$\hat{\mathbf{S}} = \hat{\mathbf{p}}_{jet} \times \hat{\mathbf{p}}_{\Lambda} \tag{7.1}$$

where \hat{p}_{jet} and \hat{p}_{Λ} represent the unit momentum vectors of the jet and Λ hyperon, respectively.

As shown in the Fig.7.1, the angle $(\delta\theta)$ between true and detected polarization direction is

resulted by the shift of jet axis, which will dilute Λ polarization signal. where the finite angular

resolution can lead to:

- Misreconstruction of the polarization plane orientation
- Systematic shifts in the measured polarization magnitude
- Complete inversion of the polarization direction in extreme cases

This dilution effect becomes particularly significant for Λ candidates in close proximity to the jet axis. Fig.7.2 presents the ΔR dependence of $\cos \delta \theta$. Apparently, the broadening of $\cos \delta \theta$ distribution is large at samll ΔR ($\equiv \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2}$) range, which should be 1 theoritical. ΔR is required to exceed 0.05 to ensure sufficient resolution in determining the Λ polarization direction. Hence, the final extracted polarization is component of true poalrization signal, which should be corrected by:

$$P_{\Lambda} = P_{det}/\cos\delta\theta \tag{7.2}$$

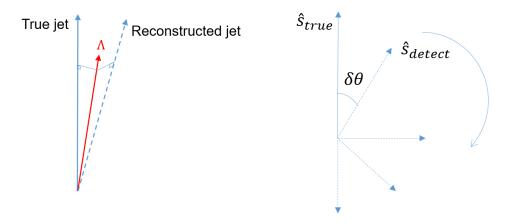


Figure 7.1: The resolution of reconstructed jet axis

We observed the significant jet p_T dependence of $\cos \delta \theta$ distribution in the Fig.8.4. Therefore, it is necessary to make correction for the Λ polarization as function of jet p_T bin by bin. Here, we used the average value of $\cos \delta \theta$ to make correction.

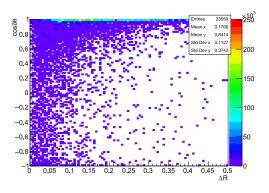


Figure 7.2: $\cos \delta \theta$ vs ΔR

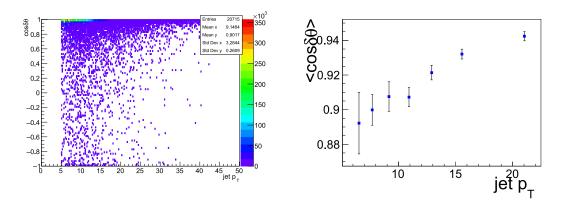


Figure 7.3: Left: $\cos\delta\theta$ vs jet p_T ; Right: the average of $\cos\delta\theta$ vs jet p_T

7.2 Kinematic quantities correction

The finite resolution of the jet axis reconstruction induces shifts in the jet p_T and related kinematic variables, such as the momentum fraction z and transverse momentum relative to jet axis j_T . Figure 7.4 presents the resolution of these quantities $(z, j_T, and jet p_T)$ as measured in the embedding sample. Here, the y-axis represents the particle-level (true) values, while the x-axis corresponds to the detector-level (reconstructed) measurements. We observe significant broadening in the distributions of these kinematic variables, indicating potential systematic shifts between the true and reconstructed values.

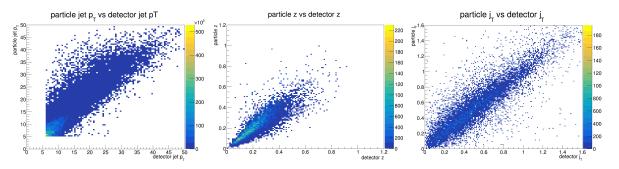


Figure 7.4: Particle-level versus detector-level

We performed linear fits to the mean particle-level values within corresponding detector-level (x-axis) ranges. Figure 7.5 demonstrates this linear correlation between particle-level and detector-level jet p_T . For the z and j_T variables, we divided their distributions into three distinct jet p_T regions matching those used in the final analysis. The fitting functions for these variables were more complex, as shown in Figure 7.6. These derived correction functions will be applied to account for reconstruction-induced shifts in the kinematic quantities.

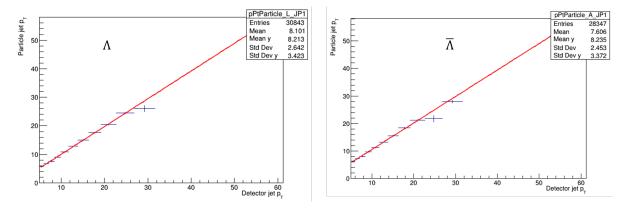


Figure 7.5: Shift correction for jet p_T

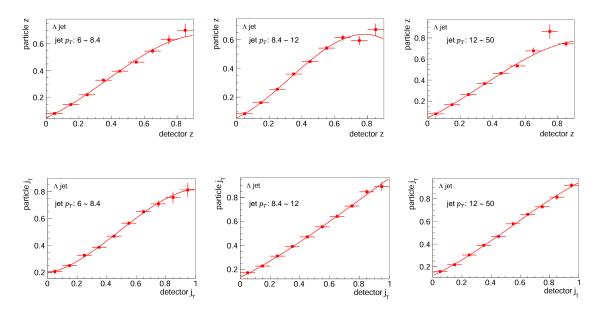


Figure 7.6: Shifts correction for z, j_T

8 Systematic uncertainties

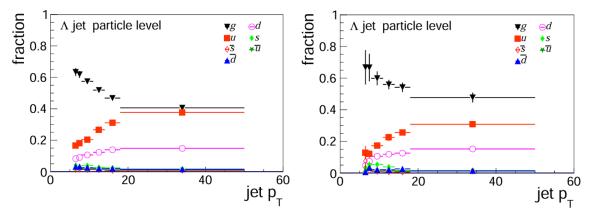
Five sources of systematic uncertainties are taken into account. The first one is resulted from trigger effects, which will impact jet flavor and transverse momentum. The next systematic uncertainty originates from the variation of side-band range for background subtraction. The precision of decay parameter of Λ and $\overline{\Lambda}$ also contribute to the systematic uncertainties. The last one is contributed from the mixed event method.

530 8.1 Trigger Bias

In the data taking of STAR, trigger sets will impact jet transverse momentum and flavor fraction, especially at the edge of trigger threshold. This effect was simulated using embedding sample to estimate how large variation of jet flavor resulted by it. The two flavor fraction distributions at different jet p_T are presented at Fig. 8.1. The left plot is for no-bias sample and right one is for triggered sample. By comparing these two distributions from Fig. 8.1, the variation of quark fraction are used to estimate the systematic uncertainty with the following formula:

$$\sigma_{\text{trig}} = \left| \frac{f_{\text{nobias}} - f_{\text{trigger}}}{f_{\text{nobias}}} \right| \times \max(P_{\Lambda}, \sigma_{\text{stat}}),$$
(8.1)

where f_{nobias} and f_{trigger} are the sum of all quark fraction of no-bias sample and trigger-bias sample, respectively. Here, P_{Λ} is measured Λ polarization and σ_{stat} is statistical error of Λ polarization. In case σ_{trig} is too small as the measured Λ polarization is closed to zero, the maximum of P_{Λ} and σ_{stat} is applied to calculation.



(a) Flavor fraction distribution without no-bias sam- (b) Flavor fraction distribution with triggered sample ple

Figure 8.1: Flavor fraction distribution of Λ at different jet p_T .

8.2 Mixed event method

The second source comes from the ME correction in correcting the detector acceptance. A closure test is performed with the MC sample by manually putting a polarization signal into the generator level and then extract the polarization at detector level using the ME method. The extracted results are consistent with input value as shown before. The following figure 8.2 shows the relative difference $(((P_{out} - P_{in})/P_{in}))$ and absolute difference $(P_{out} - P_{in})$ between them. We fitted 6 points from -0.07 to 0.07, which is close to the range of our polarization results, and obtain mean value of the relative differences up to 4.9% $\pm 4.4\%$. And average value of absolute viaration is 0.0009 \pm 0.0019. The higher value 0.19% as a scale uncertainty is taken as systematic uncertainty.

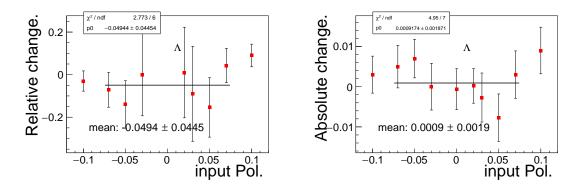


Figure 8.2: Relative change between inputted and extracted polarization.

8.3 Background estimation

The side-band method was applied to make background estimation and subtraction, as shown in Fig. 2.2. The background $dN/d(\cos\theta)$ distribution is subtracted from $dN/d(\cos\theta)$ distribution under Λ peak range. The estimated background varies with different choices of side-band region. Therefore, the choice of side-band will introduce a potential uncertainty to the measured

polarization. This uncertainty is estimated by varying the side-band region. The polarizations are calculated with the varied side-band region and the maximum of change of P_{Λ} resulted by variation of side-band window are treated as the systematic uncertainties.

$$\sigma_{\text{bkg}} = \Delta P_{\Lambda} = |\max(P_{\Lambda} - P_{bkg})| \tag{8.2}$$

where P_{bkg} is the extracted Λ polarization under varied side-band shift. And σ_{bkg} denotes background systematic uncertainty.

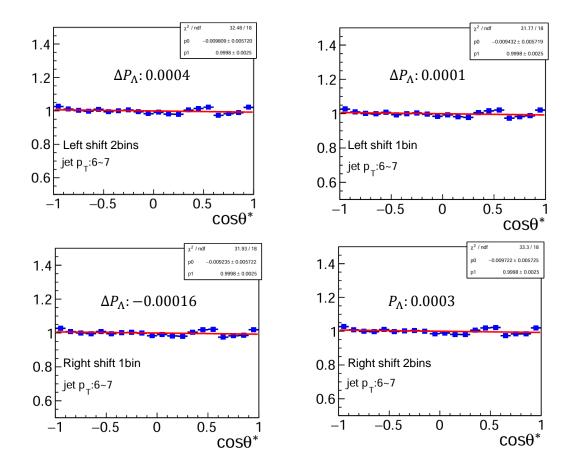


Figure 8.3: The extracted Λ polarization under varied side-band shift. The top two panels show polarization extraction under left shift of side-band, the bottom two panels show the polarization extraction under right shift of side-band.

8.4 The polarization direction correction

The precision of $\cos \delta \theta$, which is limited by the statistic of embedding sample, is also taken into account. The systematic uncertainties is calculated by the

$$\sigma_{\delta\theta} = \sigma_{\theta} / \langle \cos \delta\theta \rangle \times |P_{\Lambda}| \tag{8.3}$$

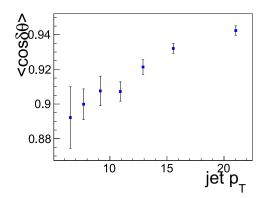


Figure 8.4: Left: $\cos \delta \theta$ vs jet p_T ; Right: the average of $\cos \delta \theta$ vs jet p_T

564 8.5 Decay parameter

The last source of systematic uncertainties is from the precision of weak decay constants of Λ and $\overline{\Lambda}$. In this analysis, the weak decay constant α of Λ and $\overline{\Lambda}$ are: 0.747 ± 0.009 and -0.757 ± 0.004 respectively [9]. The systematic uncertainties from decay parameter relative to P_{Λ} is calculated by the following equation:

$$\sigma_{\alpha} = 0.009/0.747 \times |P_{\Lambda}| \tag{8.4}$$

The total systematic uncertainty $\sigma_{\rm sys}$ is calculated through following formula:

$$\sigma_{\rm sys} = \sqrt{\sigma_{\rm trig}^2 + \sigma_{\rm bkg}^2 + \sigma_{\alpha}^2 + \sigma_{\rm mix}^2 + \sigma_{\delta\theta}^2}$$
 (8.5)

The systematic uncertainties $\sigma_{\rm sys}$ at different jet p_T range for Λ and $\overline{\Lambda}$ are summarized in Table 8.1 and 8.2 respectively. The systematic uncertainties for the polarization as the function of z and j_T are estimated with the same procedure.

Λ										
jet p_T [GeV]	P_{Λ}	$\sigma_{ m stat}$	$\sigma_{ m bkg}$	σ_{lpha}	$\sigma_{ m trig}$	$\sigma_{ m mixed}$	$\sigma_{ m sys}$			
6-7	-0.0098	0.0058	0.0007	0.0001	0.0009	0.0005	0.0012			
7-8.4	-0.0089	0.0057	0.001	0.0001	0.0008	0.0004	0.0014			
8.4-10	-0.0051	0.0038	0.0005	0.0001	0.0000	0.0003	0.0005			
10-12	0.0025	0.0042	0.0012	0.0000	0.0007	0.0001	0.0014			
12-14	-0.0002	0.0057	0.0004	0.0000	0.0002	0.0000	0.0005			
14-18	0.0154	0.0065	0.001	0.0002	0.0021	0.0008	0.0025			
18-50	0.0246	0.0113	0.0009	0.0003	0.003	0.0012	0.0033			

Table 8.1: The table of Λ extracted polarization, statistical uncertainties and summary of systematic uncertainties at different jet p_T ranges

9 Results and conclusion

In this analysis, we measure the dependence of Λ and $\overline{\Lambda}$ transverse polarization on jet p_T, z and j_T .

$\overline{\Lambda}$									
jet p_T [GeV]	$P_{\overline{\Lambda}}$	$\sigma_{ m stat}$	$\sigma_{ m bkg}$	σ_{lpha}	$\sigma_{ m trig}$	$\sigma_{ m mixed}$	$\sigma_{ m sys}$		
6-7	-0.0165	0.0052	0.0007	0.0001	0.0015	0.0008	0.0019		
7-8.4	0.0028	0.0056	0.001	0.0000	0.0005	0.0002	0.0012		
8.4-10	-0.0125	0.0037	0.0005	0.0001	0.0001	0.0006	0.0008		
10-12	-0.0057	0.0045	0.0012	0.0000	0.0009	0.0003	0.0015		
12-14	-0.0208	0.0063	0.0004	0.0001	0.0008	0.001	0.0014		
14-18	-0.0104	0.0075	0.001	0.0001	0.0015	0.0006	0.0019		
18-50	-0.0299	0.0134	0.0009	0.0002	0.0036	0.0015	0.0040		

Table 8.2: The table of $\overline{\Lambda}$ extracted polarization, statistical uncertainties and summary of systematic uncertainties at different jet p_T ranges

9.1 $P_{\Lambda/\overline{\Lambda}}$ vs jet p_T

Figure 9.1 shows the results of transverse polarization of Λ as a function of jet p_T . The red and blue markers denote Λ and $\overline{\Lambda}$ respectively. We can observe the significant transverse polarization of both Λ and $\overline{\Lambda}$ and clear jet p_T dependence. The Λ polarization increases with jet p_T and changes its sign from negative to positive at jet $p_T \sim 12$ GeV. The $\overline{\Lambda}$ polarization also increases with jet p_T but is always negative. In this figure, the vertical bars denote statistical uncertainties, and open boxes denote systematic uncertainties. The numerical values of the results are summarized in Tab. 9.1.

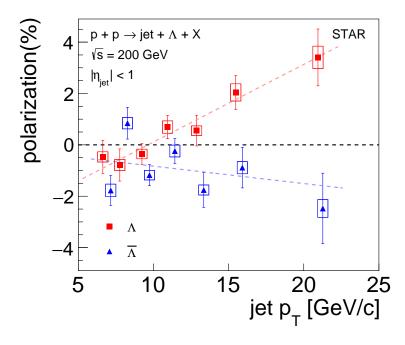


Figure 9.1: Transverse polarization of Λ and $\overline{\Lambda}$ as a function of jet p_T in unpolarized pp collisions at \sqrt{s} =200 GeV at STAR. Statistical uncertainties are shown as vertical bars. Systematic uncertainties are shown as boxes.

		1	1	$\overline{\Lambda}$					
jet p_T [GeV]	P_{Λ}	$\sigma_{ m stat}$	$\sigma_{ m sys}$	$\mid \text{jet } p_T \text{ [GeV]} \mid$	$P_{\overline{\Lambda}}$	$\sigma_{ m stat}$	$\sigma_{ m sys}$		
6.4821	-0.0098	0.0058	0.0012	6.47	-0.0165	0.0052	0.0019		
7.6453	-0.0089	0.0057	0.0014	7.627	0.0028	0.0056	0.0012		
9.1596	-0.0051	0.0038	0.0005	9.1422	-0.0125	0.0037	0.0008		
10.9155	0.0025	0.0042	0.0014	10.8958	-0.0057	0.0045	0.0015		
12.9024	-0.0002	0.0057	0.0005	12.8898	-0.0208	0.0063	0.0014		
15.586	0.0154	0.0065	0.0025	15.5532	-0.0104	0.0075	0.0019		
21.2445	0.0246	0.0113	0.0033	21.1216	-0.0299	0.0134	0.0040		

Table 9.1: The table of Λ and $\overline{\Lambda}$ extracted polarization, statistical uncertainties and summary of systematic uncertainties at different jet p_T ranges

9.2 $P_{\Lambda/\overline{\Lambda}}$ vs z and j_T

To provide further constraints for the pFFs, the transverse polarizations of Λ and $\overline{\Lambda}$ are also measured as functions of z and j_T , as shown in Figure 9.2 and 9.3. Because the Λ polarization as a function of jet p_T cross zero from negative to positive. There might be different z and j_T dependence of polarization at different jet p_T ranges. Hence, We separate jet p_T into three different ranges of: $6 < p_T^{jet} < 8.4$ GeV, $8.4 < p_T^{jet} < 12$ GeV and $p_T^{jet} > 12$ GeV, respectively. The polarizations of Λ and $\overline{\Lambda}$ show different z dependence at different jet p_T ranges. At low jet p_T range of $0 < p_T^{jet} < 0.2$ GeV, no clear z dependence of z of z polarization is observed. The polarization trend with z of z is similar to z at z dependence with z GeV range. At high jet z range, the polarization of z and z become opposite and increase with z. But no z dependence of polarization is observed at these three jet z range.

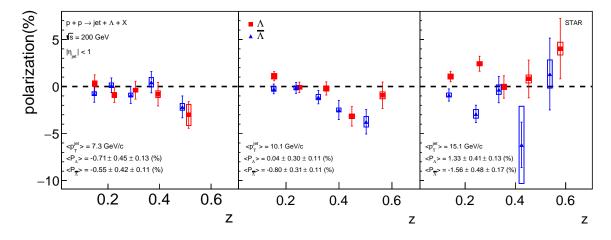


Figure 9.2: Transverse polarization of Λ , and $\overline{\Lambda}$ as a function of z at different jet p_T ranges of $6 < p_T^{jet} < 8.4$ GeV (left), $8.4 < p_T^{jet} < 12$ GeV (middle) and $p_T^{jet} > 12$ GeV (right). Statistical uncertainties are shown as vertical bars. Systematic uncertainties are shown as boxes.

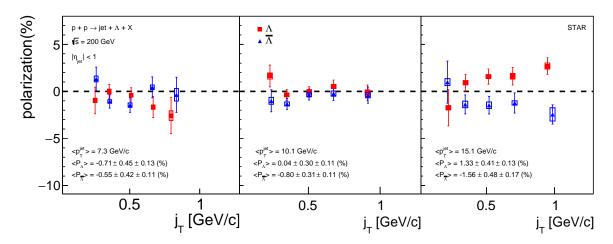


Figure 9.3: Transverse polarization of Λ , and $\overline{\Lambda}$ as a function of j_T at different jet p_T ranges of $6 < p_T^{jet} < 8.4$ GeV (left), $8.4 < p_T^{jet} < 12$ GeV (middle) and $p_T^{jet} > 12$ GeV (right). Statistical uncertainties are shown as vertical bars. Systematic uncertainties are shown as boxes.

$6 < p_T^{jet} \le 8.4$					$8.4 < p_r^3$	$T^{jet} \leq 12$		$p_T^{jet} > 12$			
z	P_{Λ}	$\sigma_{ m stat}$	$\sigma_{ m sys}$	z	P_{Λ}	$\sigma_{ m stat}$	$\sigma_{ m sys}$	z	P_{Λ}	$\sigma_{ m stat}$	$\sigma_{ m sys}$
0.1528	0.0006	0.0085	0.0022	0.1426	0.0077	0.0044	0.0018	0.1243	0.0067	0.0053	0.0018
0.2484	-0.0099	0.0072	0.0020	0.2449	-0.004	0.0051	0.0008	0.2463	0.015	0.008	0.0012
0.3448	-0.0072	0.0087	0.0009	0.3459	-0.0036	0.0065	0.002	0.3432	0.0053	0.012	0.0023
0.4457	-0.0056	0.0113	0.003	0.4434	-0.0227	0.0094	0.0019	0.4424	0.0254	0.0193	0.0049
0.6033	-0.0251	0.0124	0.0103	0.5926	-0.0164	0.0124	0.0040	0.5908	0.0253	0.0291	0.0059
z	$P_{\overline{\Lambda}}$	$\sigma_{ m stat}$	$\sigma_{ m sys}$	z	$P_{\overline{\Lambda}}$	$\sigma_{ m stat}$	$\sigma_{ m sys}$	z	$P_{\overline{\Lambda}}$	$\sigma_{ m stat}$	$\sigma_{ m sys}$
0.157	-0.0076	0.0083	0.0010	0.1229	0.0035	0.0019	0.0022	0.1251	-0.0104	0.0061	0.0011
0.2491	-0.0042	0.007	0.0017	0.247	-0.0017	0.003	0.0009	0.2463	-0.0288	0.0088	0.0047
0.3458	-0.016	0.0081	0.0012	0.3454	0.0001	0.0043	0.0020	0.343	-0.019	0.0133	0.0063
0.4464	0.0037	0.0101	0.0047	0.4445	-0.0023	0.0064	0.0014	0.442	-0.0556	0.022	0.0397
0.6198	-0.013	0.01	0.0022	0.6065	-0.0044	0.0074	0.0047	0.6071	-0.0415	0.0343	0.0172

Table 9.2: The table of Λ and $\overline{\Lambda}$ extracted polarization, statistical uncertainties and summary of systematic uncertainties at different z ranges

9.3 Conclusions

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- Our analysis is the first measurement of transverse polarization of Λ and $\overline{\Lambda}$ within jet in unpolarized pp collisions at $\sqrt{s} = 200$ GeV.
- Significant polarizations of Λ and $\overline{\Lambda}$ are observed with clear dependence on jet p_T .
- The z and j_T dependence of polarization are measured, and visible z dependencies are observed for medium to high jet p_T .
- These measurements provide important constraints on polarizing Fragmentation Functions.

$6 < p_T^{jet} \le 8.4$				$8.4 < p_T^{jet} \le 12$				$p_T^{jet} > 12$			
j_T	P_{Λ}	$\sigma_{ m stat}$	$\sigma_{ m sys}$	j_T	P_{Λ}	$\sigma_{ m stat}$	$\sigma_{ m sys}$	j_T	P_{Λ}	$\sigma_{ m stat}$	$\sigma_{ m sys}$
0.1345	-0.0159	0.0105	0.0016	0.136	0.011	0.0083	0.0025	0.1381	-0.0068	0.0134	0.0014
0.2993	-0.0004	0.0069	0.0018	0.3006	-0.0067	0.0052	0.0009	0.3014	0.0086	0.0081	0.0014
0.49	-0.0047	0.0073	0.0004	0.4926	-0.0032	0.0051	0.0016	0.494	0.0112	0.0076	0.0010
0.6821	-0.0204	0.0103	0.0017	0.6873	0.0026	0.0063	0.0012	0.6901	0.0119	0.0087	0.0023
0.9026	-0.0238	0.0176	0.0050	0.9506	-0.008	0.0078	0.0028	1.006	0.017	0.0084	0.0021
j_T	$P_{\overline{\Lambda}}$	$\sigma_{ m stat}$	$\sigma_{ m sys}$	j_T	$P_{\overline{\Lambda}}$	$\sigma_{ m stat}$	$\sigma_{ m sys}$	j_T	$P_{\overline{\Lambda}}$	$\sigma_{ m stat}$	$\sigma_{ m sys}$
0.1349	0.0047	0.0092	0.0019	0.1366	-0.0129	0.0083	0.0038	0.1376	-0.0139	0.0155	0.0032
0.2985	-0.0125	0.0063	0.0008	0.3009	-0.0114	0.0053	0.0013	0.3023	-0.0212	0.0091	0.0048
0.4898	-0.0153	0.007	0.0019	0.4928	-0.0065	0.0053	0.0019	0.4947	-0.0164	0.0087	0.0036
0.6831	0.0015	0.0102	0.0017	0.6882	-0.0059	0.0066	0.0021	0.6909	-0.0162	0.01	0.0017
0.9057	-0.0072	0.0171	0.0065	0.9597	-0.0103	0.0079	0.0026	1.0123	-0.0247	0.0097	0.0075

Table 9.3: The table of Λ and $\overline{\Lambda}$ extracted polarization, statistical uncertainties and summary of systematic uncertainties at different z ranges

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Appendices

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