ABSTRACT

Studies of Strong Parity Violation and Correlations using Hyperons in Au+Au Collisions Measured with the STAR Detector

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In relativistic heavy ion collisions, metastable vacuum domains may be formed in the QCD vacuum in the vicinity of the deconfinement phase transition in which parity is spontaneously broken. The strong parity violation characterized by nonzero winding number Q_w leads to a difference between the number of left- and right-handed quarks, which may result in helicity correlation in $\Lambda\Lambda$ and $\bar{\Lambda}\bar{\Lambda}$ systems. We discuss the possibility of observing this effect for different Q_w configurations and Λ detection efficiencies.

Two identical particles are correlated at small relative momentum due to Bose-Einstein or Fermi-Dirac statistics. Analyses of such HBT correlations in relativistic heavy ion collisions have provided space-time characteristics of the production processes. $\Lambda\Lambda$ and $\bar{\Lambda}\bar{\Lambda}$ HBT correlations at the STAR experiment are studied. Λ and $\bar{\Lambda}$ hyperons are reconstructed through their decay modes $\Lambda \to p\pi^-$ and $\bar{\Lambda} \to \bar{p}\pi^+$ (branching ratio 63.9%). The idea of enhancing the HBT effect by selecting $\Lambda\Lambda$ and $\bar{\Lambda}\bar{\Lambda}$ pairs with identical spins is also presented.

After careful studies, we find out that there is not enough statistics to actually

measure the correlations with STAR Run 4 data. This thesis lays out the physics and analysis methods and estimates how many events are needed.

Studies of Strong Parity Violation and Correlations using Hyperons in Au+Au Collisions Measured with the STAR Detector

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By

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Chapter 1

Introduction

This thesis mainly studies

- 1. how to test the possible parity violation in relativistic heavy ion collisions using Λ hyperons,
- 2. the correlations of Λ hyperons at the STAR experiment.

The STAR Run 4 statistics is not sufficient to measure these effects. I will describe the analysis methods, present the current results, and estimate how many events are needed for these purposes.

The chapters in this thesis are listed as follows:

Chapter 2 gives a short review of the relevant physics. Section 1 briefly reviews the Quark Gluon Plasma (QGP) that heavy ion programs are searching for and three important RHIC results indicating that strongly interacting matter is created during the heavy ion collisions at RHIC. Section 2 reviews the history of parity violation, the strong CP problem, and possible parity violation in hot QCD. Section 3 reviews the history of the intensity interferometer in astronomy. Section 4 describes the theory and application of intensity interferometry in high energy physics. Section 5 explains the statistical model that successfully describes the particle ratios in heavy ion collisions. We will use the statistical model and the $\Lambda + \Sigma$ yield measured at STAR to estimate the primordial Λ yield. Section 6 discusses the properties of the Λ hyperon.

- Chapter 3 describes the STAR experiment where the data used in this thesis are obtained.
- Chapter 4 studies the number of events needed at STAR to detect the possible parity violation according to different assumptions of the magnitude of parity violation and the Λ detection efficiency.
- **Chapter 5** describes the analysis methods used to reconstruct Λ hyperons at STAR.
- **Chapter 6** studies Λ hyperon correlations at STAR.
- **Chapter 7** briefly discusses the implication of final state interaction and the H^0 dibaryon to the Λ correlation function and the outlook of the parity violation and Λ correlation studies at STAR.
- Appendix A lists some of the kinematic variables used in this thesis.
- Appendix B shows the current author list of the STAR Collaboration.

Chapter 2

Physics

2.1 High energy heavy ion physics

The theory that describes the strong interaction is Quantum Chromodynamics (QCD), with the Yang-Mills Lagrangian density

$$\mathcal{L}_{\text{QCD}} = \sum_{f} \bar{\psi}_f (i\gamma^{\mu} D_{\mu} - m_f) \psi_f - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a,$$

where the meanings of various symbols are

- ψ : 4-component Dirac spinors for quark fields,
- γ^{μ} : Dirac matrices,
- D_{μ} : covariant derivative defined by $D_{\mu} = \partial_{\mu} + \frac{1}{2} i g_s A^a_{\mu} \lambda^a$,
- g_s : QCD coupling constant,
- A^a_μ : gluon fields, $a = 1, \dots, 8$,

 λ^a : Gell-Mann matrices satisfying $[\lambda^a, \lambda^b] = 2i f^{abc} \lambda^c$,

 f^{abc} : structure constants,

$$F^a_{\mu\nu}$$
: gluon field strength tensors defined by $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$.

In 1973, Gross, Wilczek, and Politzer [1, 2] discovered that the strong interaction is asymptotically free. The Callan-Symanzik β function of an SU(3) gauge theory can be expanded in the following series

$$\beta(g_s) \equiv \mu \frac{\partial g_s}{\partial \mu} = -\beta_0 \frac{g_s^3}{(4\pi)^2} - \beta_1 \frac{2g_s^5}{(4\pi)^4} - \cdots$$

with

$$\beta_0 = 11 - \frac{2}{3}n_f,$$

$$\beta_1 = 51 - \frac{19}{3}n_f,$$

where n_f is the number of quarks with mass less than the energy scale μ [3]. Thus the running coupling constant can be written as

$$\alpha_s(\mu) \equiv \frac{g_s^2}{4\pi} = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)} \left[1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln[\ln(\mu^2/\Lambda^2)]}{\ln(\mu^2/\Lambda^2)]} + \cdots \right],$$

where Λ is the QCD scale parameter. For $n_f \leq 8$, β_0 and β_1 are positive, $\beta(g_s)$ is negative, and $\alpha_s(\mu)$ goes to zero as μ goes to infinity.

Fig. 2.1 shows the values of $\alpha_s(\mu)$ at the values of μ where they are measured.

At high temperature and/or high baryon density, it is expected that α_s becomes small, quarks and gluons are no longer confined inside hadrons, and a new state



Figure 2.1: Summary of the values of $\alpha_s(\mu)$ determined from μ , τ width, Υ decays, deep inelastic scattering, e^+e^- annihilation, and Z width [3].



Figure 2.2: Schematic QCD phase diagram [4].

of matter called quark gluon plasma (QGP) may be formed. Various lattice QCD (LQCD) calculations have been done to study this QGP phase transition. Recent results determine the transition temperature $T_c \simeq 170$ MeV at zero chemical potential with systematic errors of about 10% [4]. The generic form of the QCD phase diagram is shown in Fig. 2.2.

To search and study the properties of QGP, various experimental programs have been carried out for about 20 years. The Super Proton Synchrotron (SPS) at European Organization for Nuclear Research (CERN) and Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory (BNL) started relativistic heavy ion experiments in 1986. Since the year 2000, experiments at BNL's Relativistic Heavy Ion Collider (RHIC) have provided a wealth of data of pp, d+Au, Au+Au, and Cu+Cu collisions at various energies up to $\sqrt{s_{NN}} = 200$ GeV. Some of the RHIC results are reviewed as follows.

2.1.1 Anisotropic flow

In non-central nuclear collisions where the impact parameter b (the perpendicular distance between the center of the two colliding nucleons) is not zero, the overlap region is nearly elliptic with an almond shape. If enough rescatterings occur among particles created from this spatial anisotropic region, it can result in a momentum anisotropy. The observed anisotropy can be expanded in Fourier series [5],

$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{p_t dp_t dy} \left(1 + \sum_{n=1}^{\infty} 2v_n \cos[n(\phi - \Psi)]\right),$$



Figure 2.3: Flow measured by STAR in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. (a) The values of v_1 (stars) for charged particles for 10% to 70% centrality plotted as a function of pseudorapidity. Also shown are the results from NA49 (triangles) for pions from 158A GeV Pb+Pb midcentral (12.5% to 33.5%) collisions plotted as a function of rapidity. The open points have been reflected about midrapidity. The NA49 points have also been shifted (circles) plus or minus by the difference in the beam rapidities of the two accelerators. The dashed lines indicate midrapidity and RHIC beam rapidity. (b) The minimum bias values of v_2 , v_4 , v_6 with respect to the second harmonic event plane as a function of p_t for $|\eta| < 1.2$. [6].

where ϕ is the particle azimuthal angle, and Ψ is the reaction plane angle. v_1 is called directed flow, and v_2 is called elliptic flow. Fig. 2.3 shows some of the flow results at RHIC.

2.1.2 Hadron spectra

The RHIC experiments have measured the transverse momentum distribution for various hadron species, some of which are shown in Fig. 2.4. These results can be successfully described by ideal hydrodynamics, at least for $p_t < 1.5 - 2 \text{ GeV}/c$ [7, 8].



Figure 2.4: Mid-rapidity hadron spectra from $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions. The spectra are displayed for decreasing centrality from the top downwards within each frame, with appropriate scaling factors applied to aid visual comparison of the results for different centralities. For K^* only, the lowest spectrum shown is from 200 GeV pp collisions [9].

2.1.3 Hard probes — suppression of high p_t hadrons

In 1982, Bjorken pointed out that high energy partons might suffer significant collisional energy loss when propagating through QGP [10]. He also noted that "an interesting signature may be events in which the hard collision occurs near the edge of the overlap region, with one jet escaping without absorption and the other fully absorbed [10]." It was later found out that the dominant part of energy loss for light quarks and gluons is gluon Bremsstrahlung radiations. For a dense media with high gluon density, the radiative energy loss is so large that high p_t hadrons are suppressed.

To quantify the effects of suppression, we define $R_{AB}(p_t)$ as the ratio of the measured yield in the nuclear collision A + B (in the case of RHIC, d+Au or Au+Au) to



Figure 2.5: (a) $R_{AB}(p_t)$ for minimum bias and central d+Au collisions, and central Au+Au collisions, (b) Two-particle azimuthal distributions of d+Au, pp, and Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, where $4 < p_t^{\text{trigger}} < 6$ GeV/c and 2 GeV/c $< p_t < p_t^{\text{trigger}}$ [12].

the scaled pp yield, or

$$R_{AB}(p_t) \equiv \frac{d^2 N_{AB}/dp_t d\eta}{T_{AB} d^2 \sigma_{pp}/dp_t d\eta}$$

where $T_{AB} = \langle N_{\text{binary}} \rangle / \sigma_{\text{inelastic}}^{pp}$ accounts for the nuclear collision geometry. $\langle N_{\text{binary}} \rangle$, the equivalent number of binary pp collisions, is calculated using a Glauber model [11].

RHIC experiments showed that high p_t hadrons in central Au+Au collisions are indeed suppressed, as shown in Fig. 2.5(a). This effect can also be seen in dihadron azimuthal correlations — back-to-back high p_t hadron correlations are observed in ppand d+Au collisions, but disappear in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, as shown in Fig. 2.5(b). The fact that the suppression is not seen in d+Au collisions demonstrates that high p_t suppression is due to final state interactions.

2.2 Parity violation

2.2.1 History of parity violation (weak interactions)

In 1924, Laporte found that an atom's energy level changes from parity even to parity odd or vice versa when one photon is emitted or absorbed. In 1927, Wigner realized that Laporte's rule was a consequence of right-left symmetry (or mirror image symmetry) of the electromagnetic forces in the atom [13]. The conservation of parity was then taken for granted. In the early 1950's, two mesons, named θ and τ (now the kaon), were discovered. They had the same masses and lifetimes, but different parities — θ decays into two π and thus parity even, while τ decays into three π and parity odd. Lee and Yang examined the $\theta - \tau$ puzzle and questioned parity conservation in weak interactions [14]. Following Lee and Yang's advice, Wu et al. successfully performed the ⁶⁰Co β -decay experiment in 1957 and confirmed parity violation in weak interactions [15].

2.2.2 The U(1) problem in QCD

To illustrate the CP (C: charge conjugation, P: parity) problem in the strong interactions, let us first examine the U(1) problem in QCD. For QCD with 2 massless quarks (u and d), the QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \sum_{f=u,d} \bar{\psi}_f (i\gamma^\mu D_\mu) \psi_f$$

has a global symmetry $SU(2)_L \otimes SU(2)_R \otimes U(1)_L \otimes U(1)_R$, where L and R denote left and right, respectively.

The $SU(2)_V$ (V = R + L, vector) symmetry, or invariance under the transformation

$$\begin{pmatrix} u \\ d \end{pmatrix} \to e^{i\alpha^a \sigma^a/2} \begin{pmatrix} u \\ d \end{pmatrix},$$

where σ^a (a = 1, 2, 3) are the Pauli matrices, corresponds to the isospin conservation. The $U(1)_V$ symmetry, or invariance under the transformation

$$\begin{pmatrix} u \\ d \end{pmatrix} \to e^{i\beta} \begin{pmatrix} u \\ d \end{pmatrix},$$

corresponds to the baryon number conservation.

The $SU(2)_A$ (A = R - L, axial) symmetry is "eaten" by three Goldstone bosons — the pions — whose masses vanish in the massless quark limit [16].

The $U(1)_A$ symmetry, or invariance under the transformation

$$\begin{pmatrix} u \\ d \end{pmatrix} \to e^{i\gamma_5\theta} \begin{pmatrix} u \\ d \end{pmatrix},$$

however, poses a serious problem. It should correspond to a parity doubling of the hadron spectrum, which is not realized in nature [17]. A Goldstone boson is then needed for the $U(1)_A$ symmetry to be spontaneously broken. The isospin-zero η meson is expected to play the role, but its mass 547 MeV/ c^2 is too heavy, as an upper bound for this isoscalar boson is $\sqrt{3}m_{\pi}$, obtained by Weinberg [18]. ($m_{\pi} = 138$ MeV.) This is the "U(1) problem."

2.2.3 The strong CP problem

One way to solve the U(1) problem, pointed out by 't Hooft in 1976 [19, 20], is to include instantons in the path integral. However, this brings an additional θ -term [17]

$$\mathcal{L}_{\theta} = \theta_{\rm QCD} \cdot \frac{g_s^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu},$$

where $\theta_{\rm QCD}$ is a free parameter, $\tilde{F}^{a\mu\nu}$ is the dual of $F^a_{\mu\nu}$:

$$\tilde{F}^{a\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F^a_{\rho\sigma},$$

and ϵ is the Levi-Civita antisymmetric tensor.

Also, \mathcal{L}_{θ} is allowed by gauge invariance and renormalizability, and it is a fourdivergence therefore leaving the equations of motion unchanged on the classical level [21]. We can also get this term by performing a $U_A(1)$ chiral rotation of the fermion field on the massless QCD Lagrangian [22]

$$\psi \to \psi' = e^{i\theta\gamma_5/2}\psi.$$

Including the contributions from the weak interaction, we have

$$\theta_{\rm QCD} \to \bar{\theta} = \theta_{\rm QCD} + \arg \det(M_q),$$

where M_q is the complex quark mass matrix.

It can be shown that

$$F \cdot \tilde{F} \equiv F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \propto \sum_a \mathbf{E}_a \cdot \mathbf{B}_a,$$

where \mathbf{E}_a and \mathbf{B}_a are the color electric and magnetic fields, respectively. \mathbf{E}_a is odd under parity transformation and even under time reversal, while \mathbf{B}_a is even under parity transformation and odd under time reversal, thus $F \cdot \tilde{F}$ violates both parity and time reversal invariance [23]. In another word, \mathcal{L}_{θ} is \mathcal{P} -odd and \mathcal{T} -odd.

The theory does not have constraints on the value of $\bar{\theta}$. However, \mathcal{P} or \mathcal{CP} violation has not been observed in strong interactions, and the experimental limit on the neutron electric dipole moment leads to a tiny $\bar{\theta}$ ($|\bar{\theta}| < 3 \times 10^{-10}$) [17]. This is referred to — somewhat sloppily — as the strong \mathcal{CP} problem.

There are some propositions to solve the strong CP problem, for example, the dynamical solution of the axion scenario, suggested by Peccei and Quinn [24] in 1977.

2.2.4 Parity violation in hot QCD

2.2.4.1 Cause

Even if θ_{QCD} is equal to 0 for the true QCD ground state, \mathcal{P} and \mathcal{CP} still can be spontaneously violated in hot QCD due to \mathcal{P} - and \mathcal{CP} -odd metastable vacuum domains (or "bubbles") formed in the QCD vacuum in the vicinity of the deconfinement phase transition [25].

If the QGP phase transition is of second order, the large N limit of an SU(N)gauge theory seems to be the appealing model to solve the U(1) problem. The $U(1)_A$ symmetry is dynamically restored as the temperature T approaches the phase transition temperature T_d from below. Using a nonlinear sigma model, Kharzeev et al. showed that \mathcal{P} -odd bubbles can appear in the hadronic phase [26].

Some \mathcal{P} -odd observables, for example,

$$\mathbf{J} = \sum_{\pi^{\pm}} (\hat{p}_+ \times \hat{p}_-) \cdot \hat{z},$$

where the unit vectors \hat{p}_{\pm} denote the directions of the π^{\pm} momentum, and \hat{z} denote an arbitrary, fixed unit vector [27], are proposed to observe such violations [28].

2.2.4.2 Winding number (or topological charge)

When a gauge field configuration transforms from one classical vacuum to another, the integer winding number, or topological charge

$$Q_w = \frac{g_s^2}{32\pi^2} \int d^4x \, F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$$

is nonzero [29]. The change of chirality of the fermions (quarks and antiquarks) is

$$\Delta(N_L - N_R) = 2n_f Q_w, \qquad (2.1)$$

where N_L is the number of left-handed fermions, and N_R is the number of right-handed fermions. This equation links the change of chirality, hence \mathcal{P} - and \mathcal{CP} -violation, to the topology of the gluon fields [30]. For each flavor,

$$\Delta(N_L^f - N_R^f) = 2Q_w. \tag{2.2}$$

The magnitude of the topological charge is not certain though. A 2002 lattice

calculation [31] within the framework of a classical effective field theory shows that in the color glass condensate model, the root mean square of the topological charge created at RHIC is only one unit per two units of rapidity. However, in the later stages of the collision, boost invariance may be lost and substantial topological charge (root mean square $\sim 20 - 40$ per unit of rapidity) may be generated [31].

2.2.4.3 Charge separation

If there is a very large background (electromagnetic) magnetic field \mathbf{B} ($eB \gg p^2$), all particles are in their ground states, hence the spins of fermions with positive charge are aligned along the $-\hat{B}$ direction while the spins of fermions with negative charges are aligned along \hat{B} direction. From the definition of chirality, we can see that positively charged right-handed fermions and negatively charged left-handed fermions move along \hat{B} direction, while positively charged left-handed fermions and negatively charged right-handed fermions move in the opposite direction.

For $Q_w \neq 0$, according to Eq. (2.1), the gluon fields converts $2n_f Q_w$ right-handed (or left-handed if $Q_w < 0$) fermions into left-handed (or right-handed if $Q_w < 0$) fermions by reversing the direction of their momentum, and this causes charge separation — more charges in one side of a plane perpendicular to the magnetic field than the other side. This is called the chiral magnetic effect [29, 30, 32].

In a moderate or smaller magnetic field, not all spins are aligned along $\pm \hat{B}$. The chiral magnetic effect still exists but is reduced.

In relativistic heavy ion collisions, enormous magnetic fields are generated in the direction of angular momentum at the center of the collision, and charge separation might be observable [29].

2.3 Hanbury Brown and Twiss (HBT) interferometry in astronomy

In the early 1930's, cosmic radio waves were first discovered by Karl Jansky at Bell Telephone Laboratories. In the early 1950's, about 100 discrete radio sources in the sky were identified [33, 34]. They were believed to be invisible stars. Measuring the angular diameter of a star by use of a phase interferometer at radio frequency would require signals be transmitted in phase by thousands of kilometers, which was not doable then. To solve this problem, R. Hanbury Brown at the Jodrell Bank Experimental Station of the University of Manchester, collaborating with mathematician R. Q. Twiss, developed the theory of a new type of interferometer — the intensity interferometer [35]. With no need to keep the signals coherent, the intensity interferometer could be operated with very long baselines. After successfully measuring the angular diameters of two radio sources [36, 37], Hanbury Brown and Twiss extended the use of the intensity interferometer to visible stars [38, 39, 40].

2.3.1 The phase interferometer in the early days

2.3.1.1 Visible stars

The first successful measurement of the angular diameter of a star other than the Sun was done by Michelson and Pease [41] in 1920–1921. With a 20-foot (6.1-meter) stellar interferometer at a 100-inch (2.5-meter) telescope at the Mount Wilson Observatory, the angular diameter of the supergiant Betelgeuse (α Orionis), one of the largest known star in terms of angular size [42], was found to be 0.047" within 10%, which is consistent with modern measurements [43].





Figure 2.6: Simplified diagrams of different interferometers

The principle of Michelson's stellar interferometer is shown in Fig. 2.6(a). Two isolated pencils of rays from a star are made to interfere through the device. The angular diameter of the star as a uniformly distributed luminous disk is given by Airy's formula

$$\theta = \frac{1.22\lambda}{d},$$

where λ is the wavelength of light, and d is the length of separation between two beams when the interference fringes just disappear. For Michelson's experiment in 1920, $\lambda = 575$ nm, d = 121 in = 3.07 m.

Six stars were measured by this instrument. In order to measure fainter stars, a 50-foot (15-meter) interferometer at Mount Wilson was built in 1929 [44], but no successful results were obtained. Stellar diameter measurement became inactive until the intensity interferometer was invented in the 1950's.

2.3.1.2 Radio sources

The radio analogy of Michelson's stellar interferometer is shown in Fig. 2.6(b). Two spaced radio aerials are connected with a cable. The output is read from the center of the cable.

Using this type of instrument at 175 MHz, Ryle and Vonberg [45, 46] at the Cavendish Laboratory found the angular diameter of the high-intensity radiation source in the sun to be not greater than 10' (minute of arc), the same order as a sunspot. For radio sources Cygnus A and Cassiopeia A, however, only an upper limit of 6', the resolving power of the apparatus with a baseline of 500 m at a frequency of 80 MHz, could be placed [47].

2.3.2 The intensity interferometer in astronomy

2.3.2.1 Theory

The intensity interferometer differs from the phase interferometer in that the signals are measured first by a square-law detector without interference, then correlated. A simplified diagram of the intensity interferometer is shown in Fig. 2.6(c).

Let I_A and I_B denote the signals received at the two aerials that are linear to the intensities of the incoming wave, d the baseline length projected normal to the direction of the source, and c(d) the correlator output $\langle I_A I_B \rangle$ as a function of d. After pages of derivation, Hanbury Brown and Twiss [35] concluded that the normalized correlator output, or the normalized correlation function, is given by

$$\frac{\langle I_A I_B \rangle}{\langle I_A^2 \rangle^{1/2} \langle I_B^2 \rangle^{1/2}} = \frac{F_{\cos}^2(2\pi d/\lambda) + F_{\sin}^2(2\pi d/\lambda)}{F_{\cos}^2(0)} = \frac{c(d)}{c(0)},$$

where λ is the wave length, and the functions F's are the Fourier transform of the angular distribution of the intensity across the source, i.e.

$$F_{\cos}(x) = \int i(\alpha) \cos(\alpha x) \, d\alpha,$$
$$F_{\sin}(x) = \int i(\alpha) \sin(\alpha x) \, d\alpha.$$

Thus, the shape of the source can be determined by measuring the function c(d).

For a uniform disk of angular diameter θ , the explicit form of the normalized correlation function is given by [48]

$$\left[\frac{2J_1(\pi\theta d/\lambda)}{\pi\theta d/\lambda}\right]^2,\tag{2.3}$$

where J_1 is the Bessel function of the first kind.

To illustrate the cause of the correlation, let us consider a source made of a pair of points P_1 , P_2 with angular separation θ [49, 50]. The wave amplitude received at the two aerials A and B can be written as

$$E_{A} = E_{1} \sin(\omega_{1}t + \phi_{1}) + E_{2} \sin(\omega_{2}t + \phi_{2}),$$
$$E_{B} = E_{2} \sin(\omega_{1}t + \theta_{1} + \phi_{1}) + E_{2} \sin(\omega_{2}t + \theta_{2} + \phi_{2}),$$

where

$$\theta_i = \frac{\omega_i(r_{iB} - r_{iA})}{c} = \frac{\omega_i d_i}{c}, \qquad i = 1, 2,$$

and c is the speed of the light. In Hanbury Brown's experiments, low-pass filters were applied before the correlator, so we only need to keep the $\omega_1 - \omega_2$ components for the intensities,

$$I_A \propto E_A^2 \to E_1 E_2 \cos[(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)],$$

$$I_B \propto E_B^2 \to E_1 E_2 \cos[(\omega_1 - \omega_2)t + (\phi_1 - \phi_2) + (\theta_1 - \theta_2)]$$

The correlator output then becomes

$$c(d) = \langle I_A I_B \rangle \propto E_1^2 E_2^2 \cos(\theta_1 - \theta_2)$$
$$= E_1^2 E_2^2 \cos[(\omega/c)(d_1 - d_2)]$$
$$= E_1^2 E_2^2 \cos(2\pi d\theta/\lambda),$$

where $\omega_1 \approx \omega_2 = \omega$ and λ is the mean wavelength of the light.

2.3.2.2 HBT experiments in astronomy

In 1951, Hanbury Brown, Jennison, and Das Gupta completed a prototype of the intensity interferometer and measured the angular diameter of the sun at 125 MHz. Then a full instrument was built to measure two most intense radio sources: Cygnus A and Cassiopeia A. Data collected with four different baselines in the range of 0.3 km to 4 km indicated that Cygnus A was asymmetrical with angular diameter in the order of 35", and that the angular diameter of the Cassiopeia A was about 4' [36]. With more data collected in the following years, Jennison and Gupta [37] discovered that Cygnus A was a double radio source.

In 1956, Hanbury Brown and Twiss [38, 39] measured the angular diameter of Sirius (α Canis Majoris) A, the brightest star in the night sky. During 5 months, 18 hours' observations were made with four different baseline lengths up to 9.2 m,



Figure 2.7: Sirius measurement by Hanbury Brown and Twiss. The dashed line corresponds to the theoretical values for a star of angular diameter 6.3 mas. Reprinted by permission from Macmillan Publishers Ltd: Nature **178**:1046, copyright 1956.

as shown in Fig. 2.7. The result of uniform disk angular diameter 6.9 ± 0.4 (p.e.) mas (milliarcsecond, or 0.001'') was obtained by fitting data with Eq. (2.3). The angular diameter after limb-darkening correction was 7.1 ± 0.55 (p.e.) mas. Recent measurements [51] find the uniform disk value to be 5.936 ± 0.016 mas and limb darkened value 6.039 ± 0.019 mas.

In 1963, the Narrabri Stellar Intensity Interferometer (NSII) with a baseline ranged from 10 to 188 m was completed in Australia. During NSII's 12 years' operation, the angular diameters of 32 hot main-sequence stars with magnitude B < 2.5 were measured [40].

The intensity interferometer played an important role from the 1950's to 1970's. However, as technology developed, the phase interferometer became appealing again. In 1985, the angular diameter of Sirius was measured by a phase interferometer, and the result was in agreement with NSII's result [52]. Now, the intensity interferometer is non-mainstream, but the use of intensity interferometer in space [53] or with
Atmospheric Cherenkov Telescope arrays [54] is still discussed in the literature.

2.4 HBT interferometry in high energy physics

2.4.1 History

In 1959, Goldhaber et al. at the Lawrence Radiation Laboratory and University of California, Berkeley, studied the $\bar{p}p$ annihilation process with an antiproton beam of momentum 1.05 BeV/c in a propane bubble chamber, searching for the ρ^0 meson through the $\pi^+\pi^-$ decay channel. While there were not enough events to discover the ρ meson, they discovered that like-sign pion pairs ($\pi^+\pi^+$ and $\pi^-\pi^-$) were enhanced at smaller opening angles compared to unlike-sign pion pairs ($\pi^+\pi^-$), as shown in Fig. 2.8 [55]. For 4π reaction, $\gamma_{\text{like}} = 1.23 \pm 0.11$, and $\gamma_{\text{unlike}} = 2.06 \pm 0.12$, where

$$\gamma = \frac{\text{number of pion pairs with opening angle} > 90^{\circ}}{\text{number of pion pairs with opening angle} < 90^{\circ}}$$

In 1960, G. Goldhaber, S. Goldhaber, Lee, and Pais (GGLP) explained these results by Bose-Einstein statistics [56]. Since then, the pion HBT effect has been explored in hadron reactions (π^-p [57], π^+p [58], K^-p [58], pp [59], $\alpha\alpha$ [59], μp [60], νD [61]), e^+e^- annihilations [62], and heavy ion collisions.

2.4.2 Theory

2.4.2.1 Simple picture

The HBT effect in quantum physics is due to the Bose-Einstein or Fermi-Dirac statistics, i.e., the wave symmetrization for bosons and anti-symmetrization for fermions.



Figure 2.8: Angular distribution of pion pairs, Goldhaber et al. The curves correspond to the Lorentz-invariant phase-space model calculations [55].



Figure 2.9: Schematic picture for HBT effect

For two point sources A and B that are observed in detectors 1 and 2, as shown in Fig. 2.9, the wave function for bosons can be described as [63]

$$\psi(k_1, k_2) = \frac{1}{\sqrt{2}} (\psi_{1A} \psi_{2B} + \psi_{1B} \psi_{2A})$$

$$\propto \frac{1}{\sqrt{2}} (e^{ik_1 \cdot r_1 + i\phi_1} e^{ik_2 \cdot r_2 + i\phi_2} + e^{ik_1 \cdot r_2 + i\phi_2'} e^{ik_2 \cdot r_1 + i\phi_1'}).$$

The probability to observe 4-momenta k_1 , k_2 in detectors 1 and 2 then takes the form

$$P(k_1, k_2) = \langle |\psi(k_1, k_2)|^2 \rangle,$$

which, for chaotic sources, reduces to

$$P(k_1, k_2) = P(k_1)P(k_2)\{1 + \cos[(k_1 - k_2) \cdot (r_1 - r_2)]\},\$$

where $P(k_1)$ and $P(k_2)$ are the single detection probability. Thus the correlation function for two chaotic point sources is

$$C(k_1, k_2) \equiv \frac{P(k_1, k_2)}{P(k_1)P(k_2)} = 1 + \cos[(k_1 - k_2) \cdot (r_1 - r_2)].$$

For an extended source, integration over all possible pairs gives

$$C(k_1, k_2) = 1 + |\tilde{\rho}(q)|^2,$$

where $q = k_1 - k_2$ and $\tilde{\rho}$ is the Fourier transform of the source density

$$\tilde{\rho}(q) = \int e^{iq^{\mu}x_{\mu}} \rho(x) \, d^4x.$$

2.4.2.2 Parametrizations of the correlation function

Based on different source models, various parametrizations have been developed.

The Goldhaber parametrization

GGLP [56] used a static Gaussian source

$$\rho(\mathbf{r}) \propto e^{-\mathbf{r}^2/2R^2}$$

and derived the correlation function

$$C(q) = 1 + e^{-\mathbf{q}^2 R^2}$$

for bosons, but they used its relativistic counterpart

$$C(q) = 1 + e^{-Q^2 R^2}$$

for convenience anyway, where

$$Q^{2} \equiv -q^{\mu}q_{\mu} = -(k_{1} - k_{2})^{2} = -(E_{1} - E_{2})^{2} + (\mathbf{k}_{1} - \mathbf{k}_{2})^{2}.$$

This, in fact, corresponds to a Gaussian source of

$$\rho(x) \propto e^{-x^{\mu}x_{\mu}/2R^2}.$$

The Kopylov parametrization

In the 1970's, Kopylov and Podgoretskiĭ at the Joint Institute for Nuclear Research in Russia studied correlation between identical particles emitted by moving sources [64].

For a source disk of radius R that decays exponentially with mean life time τ ,

$$\tilde{\rho}(q) = \int e^{iq \cdot x} \rho(x) \, dx = \frac{1}{A\tau} \int_0^R \int_0^{2\pi} e^{-iq_\perp r \cos\theta} r dr \, d\theta \int_0^\infty e^{iq_0 t} e^{-t/\tau} \, dt,$$

where $A = \pi R^2$, $1/A\tau$ is the source normalization factor, and the Kopylov variables are defined as

$$\begin{split} q_{\parallel} &= \mathbf{q} \cdot \frac{\mathbf{k}_1 + \mathbf{k}_2}{|\mathbf{k}_1 + \mathbf{k}_2|}, \\ \mathbf{q}_{\perp} &= \mathbf{q} - \mathbf{q}_{\parallel}. \end{split}$$

The time part

$$\int_0^\infty e^{iq_0 t} e^{-t/\tau} dt = \int_0^\infty e^{(iq_0 - 1/\tau)t} dt = -\frac{1}{iq_0 - 1/\tau},$$

and the space part

$$\int_0^R \int_0^{2\pi} e^{-iq_\perp r \cos\theta} r \, dr \, d\theta = \int_0^R 2\pi J_0(q_\perp r) r \, dr$$
$$= 2\pi \frac{q_\perp R}{q_\perp^2} J_1(q_\perp R)$$
$$= \frac{2A}{q_\perp R} J_1(q_\perp R),$$

where we have used

$$\int_0^u u' J_0(u') \, du' = u J_1(u).$$

Thus

$$|\tilde{\rho}(q)| = \frac{1}{A\tau} \frac{2A}{q_{\perp}R} J_1(q_{\perp}R) \frac{\tau}{\sqrt{1+\tau^2 q_0^2}} = \frac{2J_1(q_{\perp}R)}{q_{\perp}R\sqrt{1+\tau^2 q_0^2}},$$

and the correlation function becomes [65]

$$1 + \frac{[2J_1(q_\perp R)/q_\perp R]^2}{1 + q_0^2 \tau^2} \approx 1 + \frac{\exp(-q_\perp^2 R^2/4)}{1 + q_0^2 \tau^2}.$$

For a uniform spherical source that decays exponentially [66], the correlation function

$$C(q) = 1 + \frac{I^2(|\mathbf{q}|R)}{1 + q_0^2 \tau^2} \approx 1 + \frac{\exp(-\mathbf{q}^2 R^2 / 2.15)}{1 + q_0^2 \tau^2}$$

where

$$I(x) \equiv \frac{3(\sin x - x\cos x)}{x^3}.$$

For a Gaussian source

$$\rho(\mathbf{r},t=0) \propto e^{-\mathbf{r}^2/2R^2}$$

which decays exponentially with life time $\tau,$

$$C(q) = 1 + \frac{\exp(-\mathbf{q}^2 R^2)}{1 + q_0^2 \tau^2}.$$

The Pratt-Bertsch parametrization

A popular way of three-dimensional HBT analysis is to decompose the vector \mathbf{q} into "out, side, long" directions, where the longitudinal direction is along the beam axis, the outward direction is parallel to the pair transverse momentum, and the sideward direction is along $\hat{\mathbf{q}}_l \times \hat{\mathbf{q}}_s$ [67, 68]. The correlation function is sometimes parametrized as

$$C(\mathbf{q}) = 1 + e^{-(q_o^2 R_o^2 + q_s^2 R_s^2 + q_l^2 R_l^2)}.$$
(2.4)

Ref. [69] argues that the correlation function should include an "out-longitudinal" cross term to measure the asymmetry of the source:

$$C(\mathbf{q}) = 1 + e^{-(q_o^2 R_o^2 + q_s^2 R_s^2 + q_l^2 R_l^2) - 2q_o q_l R_{ol}^2}.$$

Ref. [70] considers a more general form of

$$C(\mathbf{q}, \mathbf{K}) = 1 + \exp\left[-\sum_{i,j=o,s,l} q_i q_j R_{ij}^2(\mathbf{K})\right],$$

where $\mathbf{K} = \frac{1}{2}(\mathbf{p}_1 + \mathbf{p}_2)$, and studies the constraints that various symmetries put on the radii R_{ij}^2 .

The STAR 200 GeV pion interferometry paper [71] uses

$$C(\mathbf{q}) = 1 + e^{-(q_o^2 R_o^2 + q_s^2 R_s^2 + q_l^2 R_l^2) - 2q_o q_s R_{os}^2}$$
(2.5)

at midrapidity in the longitudinal comoving system (LCMS) frame with the knowledge of the second-order reaction plane. When integrated over all azimuthal angles, R_{os}^2 vanishes due to symmetry, and Eq. (2.5) reduces to Eq. (2.4).

2.4.2.3 The correlation strength parameter

Experimentally, $C(\mathbf{q} = 0)$ usually does not reach 2 for bosons. Thus, the correlation factor λ is introduced into the formula for better fit of the data. For example, in

one-dimensional analysis, the correlation function becomes

$$C(q) = 1 + \lambda e^{-Q^2 R^2}$$

Many factors, such as the incoherence of the source (hence the old name incoherence parameter), resolution of the detectors, purity of particle identification, incorrect correlation function model, may contribute to the fact that $\lambda < 1$ [72, 73].

2.4.3 HBT for fermions

Fermions obey the Fermi-Dirac statistics. The mechanism of HBT effect for fermions is similar to that for bosons, with one complication — the total spin of two spin 1/2baryons can be either S = 0 or S = 1:

$$\begin{split} |S &= 0, m = 0 \rangle &= \frac{1}{\sqrt{2}} (|+-\rangle - |-+\rangle), \\ |S &= 1, m = 1 \rangle &= |++\rangle, \\ |S &= 1, m = 0 \rangle &= \frac{1}{\sqrt{2}} (|+-\rangle + |-+\rangle), \\ |S &= 1, m = -1 \rangle = |--\rangle. \end{split}$$

The wave function of the two spin 1/2 baryons Ψ_{12} is the product of the orbital part ψ_{12} and the spin part $|S, m\rangle$. Ψ_{12} is anti-symmetric due to the Fermi-Dirac statistics, thus the orbital wave function is symmetric for the spin singlet state S = 0, and anti-symmetric for the spin triplet state S = 1, or

$$\psi_{12}(S=0) = \frac{1}{\sqrt{2}}(\psi_{1A}\psi_{2B} + \psi_{1B}\psi_{2A}),$$

$$\psi_{12}(S=1) = \frac{1}{\sqrt{2}}(\psi_{1A}\psi_{2B} - \psi_{1B}\psi_{2A}).$$

As shown in the above sections, we obtain

$$C(S = 0) = 1 + e^{-Q^2 R^2},$$

 $C(S = 1) = 1 - e^{-Q^2 R^2},$

for a static Gaussian source. We assume that at high Q values we face a spin mixture ensemble, so the $|S = 0, m = 0\rangle$, $|S = 1, m = 1\rangle$, $|S = 1, m = 0\rangle$, and $|S = 1, m = -1\rangle$ states are equally populated. The correlation function then becomes [74]

$$C = \frac{1}{4}(1 + e^{-Q^2R^2}) + \frac{3}{4}(1 - e^{-Q^2R^2}) = 1 - \frac{1}{2}e^{-Q^2R^2}.$$
 (2.6)

2.4.4 HBT at STAR

The STAR experiment at RHIC has measured π interferometry in Au+Au collisions at $\sqrt{s_{NN}} = 130$ GeV [75] and 200 GeV [71], three π HBT correlations [76], K_s^0 interferometry in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV [77], and azimuthally sensitive HBT in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV [78]. Some of the results are shown in Fig. 2.10 and Fig. 2.11.

2.5 The statistical model

The total particle yields in relativistic heavy ion collisions can be successfully described by statistical model. In the grand canonical ensemble, the partition function



Figure 2.10: (a) STAR π^- HBT results in Au+Au collisions at $\sqrt{s_{NN}} = 130$ GeV [75]. Filled circles are Coulomb-corrected, and open circles are uncorrected. (b) STAR K_s^0 interferometry in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV [77].



Figure 2.11: Squared HBT radii relative to the reaction plane angle for different centrality classes (a) and different k_T bins in 20%—30% centrality events (b) [78]. The correlation function of Eq. (2.5) is used.

can be written as

$$Z(T, V, \mu_i) = \operatorname{Tr}[e^{-(H - \sum_i \mu_i Q_i)/T}],$$

where H is the Hamiltonian of the system, T is the temperature, Q_i are the conserved charges (baryon charge and strangeness charge) that are fixed by

$$V \sum_{i} n_i B_i = Z + N,$$
$$V \sum_{i} n_i S_i = 0,$$

with Z and N being the proton and neutron numbers of the colliding nuclei, and μ_i the corresponding chemical potentials (μ_B and μ_S) [79, 80].

The primordial particle density is then given by

$$n_{i} = \frac{N_{i}}{V}$$

$$= g_{i}\gamma_{S}^{|S_{i}|} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{1}{e^{(E_{i}-\mu_{i}Q_{i})/T} \pm 1}$$

$$= \frac{g_{i}}{2\pi^{2}}\gamma_{S}^{|S_{i}|} \int_{0}^{\infty} \frac{p^{2}dp}{e^{(E_{i}-\mu_{i}Q_{i})/T} \pm 1},$$
(2.7)

where γ_S is the phenomenological strangeness suppression (or saturation) factor that characterizes the incomplete equilibration of strangeness [81], E_i is the energy of the particle, g_i is the spin degeneracy factor, + for fermions and - for bosons.

In the Boltzmann approximation, Eq. (2.7) becomes [82]

$$n_i = \frac{g_i}{2\pi^2} \gamma_S^{|S_i|} m_i^2 T K_2(m_i/T) e^{\mu_q q_i/T} e^{\mu_S S_i/T}, \qquad (2.8)$$

where K_2 is the second-order modified Bessel function, $\mu_q = \mu_B/3$ is the light (up



Figure 2.12: RHIC particle ratios and the statistical model fit [83].

and down) quark potential.

The four free parameters, T, μ_q , μ_S , γ_S , can be obtained by fitting various experimental particle ratios, as in Fig. 2.12.

2.6 The Λ particle

The Λ particle is a spin-1/2 strange baryon with quark composition of uds, mass of 1115.683 \pm 0.006 MeV/ c^2 , and mean life time of $(2.631 \pm 0.020) \times 10^{-10}$ s. There are two dominant decay channels $\Lambda \to p\pi^-$ and $\Lambda \to n\pi^0$, with branching ratios of $(63.9 \pm 0.5)\%$ and $(35.8 \pm 0.5)\%$, respectively. Unfortunately, the latter decay mode is currently not detectable at STAR.

Now let us focus on the parity-violating $\Lambda \to p\pi^-$ decay mode. Set up a coordinate system with the \hat{z} direction as the Λ polarization direction. Thus $J = J_z = 1/2$. Since π^- is a spin-0 boson, and p is spin-1/2, the relative momentum between them must be either l = 0 (s-wave) or l = 1 (p-wave) to make up a total angular momentum of 1/2. Let f_s and f_p be the complex amplitudes of the *s*- and *p*- waves, χ^+ be the proton spin-up state, and χ^- be the proton spin-down state, then the total wave function is

$$\psi = f_s Y_0^0 \chi^+ + f_p \left(\sqrt{\frac{2}{3}} Y_1^1 \chi^- - \sqrt{\frac{1}{3}} Y_1^0 \chi^+\right), \tag{2.9}$$

where we have used the Clebsch-Gordan coefficients.

Plugging in the values of Y_l^m and multiplying Eq. (2.9) by its complex conjugate, we obtain

$$4\pi |\psi|^2 = |f_s|^2 + |f_p|^2 - 2\Re\{f_s^* f_p\} \cos\theta,$$

where $\Re\{z\}$ is the real part of a complex number z. The proton angular distribution is then

$$dw(\theta) = \frac{1}{2}(1 - \alpha \cos \theta) d(\cos \theta), \qquad (2.10)$$

where

$$\alpha = \frac{2\Re\{f_s^*f_p\}}{|f_s|^2 + |f_p|^2}.$$

 α is experimentally measured to be 0.642 ± 0.013 .

Chapter 3

The STAR experiment

3.1 The RHIC accelerator

The Brookhaven National Laboratory (BNL), located in Upton, New York on Long Island with 21 km² area, is a multi-program United States national laboratory operated for the United States Department of Energy. The BNL accelerator complex, shown in Fig. 3.1, includes the Tandem Van de Graaff for heavy ions, Linear Accelerator (Linac) for protons, the Booster synchrotron, Alternating Gradient Synchrotron (AGS), the AGS-To-RHIC (ATR) transfer line, and the Relativistic Heavy Ion Collider (RHIC).

For Au beams, negative Au⁻¹ ions are first accelerated in the Tandem Van de Graaff. The ions are accelerated from ground potential to +14 MV at the center of the Tandem where they pass through a thin carbon stripping foil and become positively charged. Stripped to charge state +32 by passing another carbon foil downstream of the Tandem with a kinetic energy of 0.925 MeV per nucleon or v = 0.05c, the Au³²⁺ ions are then transported to the 202 m Booster ring through the Tandem-to-Booster



Figure 3.1: Overall layout of the RHIC complex [84].

line (TTB). Accelerated by two radio-frequency (RF) cavities at the Booster and further stripped to +77 charge state by another foil, the Au⁷⁷⁺ ions at v = 0.37c are injected into the AGS ring, which is four times as long as the Booster ring. Accelerated to kinetic energy of 8.86 GeV per nucleon or v = 0.997c, $\gamma = 10.5$ at AGS, the Au⁷⁷⁺ ions are transported down the ATR where all electrons are stripped, and enter RHIC at the 6 o'clock position. [85]

The RHIC collider consists of two rings, denoted as "blue" and "yellow" rings, each 3834 m long in circumference. RHIC uses superconducting magnets operated at 4 K to achieve high magnetic fields. The top energy at RHIC is 100 GeV per nucleon for heavy ions. RHIC uses two RF systems, 28 MHz for capture and acceleration and 198 MHz for collisions [84].

There are six possible interaction points at RHIC, at four of which experiments were set up — the STAR (Solenoidal Tracker at RHIC) experiment at 6 o'clock, the PHENIX (Pioneering High Energy Nuclear Ion eXperiment) experiment at 8 o'clock, the PHOBOS experiment at 10 o'clock, and the BRAHMS (Broad RAnge Hadron Magnetic Spectrometers) experiment at 2 o'clock. The two larger detector systems, STAR and PHENIX, are still active, while PHOBOS and BRAHMS have completed their operation in 2005 and 2006 respectively.

3.2 The STAR detector

On the first page of the 1992's STAR Conceptual Design Report reads "The <u>S</u>olenoidal <u>T</u>racker <u>At RHIC</u> (STAR) will search for signatures of quark-gluon plasma (QGP) formation and investigate the behavior of strongly interacting matter at high energy density." With this purpose, STAR was designed to be able to track and identify most

of the high-density charged particle tracks produced at relativistic heavy ion collisions at midrapidity over a large pseudorapidity range with full azimuthal coverage ($\Delta \phi = 2\pi$). [86]

The layout of the STAR detector system is shown in Fig. 3.2 and Fig. 3.3. A solenoidal magnet operated at room temperature provides a uniform magnetic field of up to 0.5 T for charged particle momentum analysis. The Silicon Vertex Tracker (SVT) provides charged particle tracking close to the interaction region. The main detector is the large volume Time Projection Chamber (TPC), located at a radial distance from 50 to 200 cm from the beam axis. The TPC provides charged particle tracking and particle identification with a pseudorapidity coverage of $-1.8 < \eta < 1.8$ and full azimuthal coverage. Other detectors include the Forward TPC (FTPC) that extends the tracking to the forward region (2.5 < $|\eta| < 4$), some Time-Of-Flight (TOF) patches, endcap electromagnetic calorimeter (EEMC) on the east side and a full-barrel electromagnetic calorimeter (BEMC) installed over the years.

3.2.1 The STAR trigger system

The minimum bias trigger for Au+Au collisions is provided by the combination of signals from fast detectors: the Central Trigger Barrel (CTB) in the pseudorapidity range $|\eta| < 1$ and 2π in the azimuthal angle ϕ , and two Zero-Degree Calorimeters (ZDC east and ZDC west) located in the forward direction at ±18 m along the beam direction from the TPC center. The scintillator CTB surrounding the outer cylinder of the TPC measures the charged particle multiplicity within $|\eta| < 1$, while the ZDC's measure neutral energy in a small solid angle near zero degrees [87, 88].

The peripheral events, or collisions at large impact parameters, are characterized



Figure 3.2: Layout of the STAR experiment, with a cutaway for viewing inner detector systems [87].



Figure 3.3: Cutaway side view of the STAR detector as configured in 2001 [87].

by large ZDC pulse heights and small pulse heights in the CTB, while the central events, or collisions at small impact parameters, are characterized by small ZDC pulse heights but large pulse heights in the CTB [87].

The central trigger for Au+Au collisions is constructed by imposing an upper cut on the ZDCs' signal with a modest minimum CTB cut to exclude contamination from very peripheral events. It corresponds to approximately 12% of the total cross-section [88].

These triggers are essentially 100% efficient in Au+Au collisions.

The EMC also serves as a trigger detector for triggering high tower events (barrel EMC with a high tower > 3 GeV or endcap EMC with a high tower > 4.25 GeV in Au+Au collisions).

3.2.2 The STAR TPC

The TPC is the primary tracking device at STAR, shown schematically in Fig. 3.4. It is 4.2 m long and 4 m in diameter, filled with P-10 gas (90% argon and 10% methane) regulated at 2 mbar above atmospheric pressure. Its acceptance is $-1.8 < \eta < 1.8$ with full azimuthal angle. The central membrane is operated at -28 kV while both ends of the TPC are at ground. This provides a uniform drift electric field of ~ 135 V/cm along the beam direction, or the z-direction. The TPC ends are divided into twelve equal-size bisectors, and are equipped with read-out pads and front end electronics. Multi-Wire Proportional Chambers (MWPC) are installed close to the end pads inside the TPC. [88, 89]

When the charged particles produced from the collisions traverse the TPC gas volume, they ionize the gas atoms along the track. Ionization electrons drift towards



Figure 3.4: A schematic figure of the STAR TPC [89].

the TPC ends at a constant drift velocity of 5.45 cm/ μ s, and avalanche in the high fields at the 20 μ m MWPC anode wires, providing an amplification of 1000–3000. The positive ions created in the avalanche induce a temporary image charge on the pads measured by a preamplifer/shaper/waveform digitizer system. The original track positions (hits) are formed from the signals on each padrow (a row of read-out pads) by the hit reconstruction algorithm. There are a total of 136,608 pads in the read-out system. The x and y coordinates of a hit can be reconstructed to a small fraction of a pad width because the induced charge from an avalanche is shared over several adjacent pads. The position resolution across the pad rows of the TPC is 0.3–2.1 mm at full magnetic field (0.5 T). The z coordinate of a hit is determined by the drift time and the average drift velocity, with a resolution of 0.7–3 mm at full magnetic field. Tracks can have a maximum of 45 hits. [88, 89]



Figure 3.5: Beam's eye view of a central Au+Au collision event in the STAR Time Projection Chamber [87].



Figure 3.6: Pion tracking efficiency in STAR for Au+Au collisions at 0.25 T [89].



Figure 3.7: STAR TPC pad plane with one full sector shown [89].

The hits are then reconstructed to particle tracks (the global tracks) by a pattern recognition program with a helix fit. The pion tracking efficiency in Au+Au collisions at half magnetic field (0.25 T) is shown in Fig. 3.6. The efficiency is about 80% for $p_t > 300 \text{ MeV}/c$ in central Au+Au collisions. It drops rapidly below 300 MeV/c because the low momentum primary particles do not reach the outer field cage and the interaction with the beam pipe and the inner field cage before entering the TPC tracking volume is more significant for low momentum particles. [88, 89]

The primary interaction vertex is then fit using the global tracks with at least 10 hits. The primary vertex resolution is within 350 μ m when there are more than 1000 tracks. The global tracks are then refit with the primary vertex position to improve momentum resolution. If the refitting works well, the refit track becomes a primary track. [88, 89] The root mean square of the closest distance of approach (DCA) of global tracks that are later refit as primary tracks to the primary vertex is about 0.9 cm.

The transverse momentum resolution of a track is measured to be [90]

$$\sigma_{\delta p_t} = 0.01 + \frac{p_t}{200 \text{ GeV}/c}$$

3.2.3 Particle identification by dE/dx

The particle species is identified by its ionization energy loss (called dE/dx) in the TPC gas, extracted from the energy loss measured on up to 45 padrows. The measured dE/dx sample for a given track length follows the Landau distribution with a long tail. To reduce fluctuation, the truncated mean $\langle dE/dx \rangle$, determined from 70% of the samples with the lowest dE/dx along a track, is used. The resolution of the obtained $\langle dE/dx \rangle$ is measured to be 8–9% in central Au+Au collisions. [91, 89, 88]

With the measured particle transverse momentum and $\langle dE/dx \rangle$, the particle type can be determined by comparing the measurements against the Bethe-Bloch expectation [3]

$$-\frac{dE}{dx} = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\text{max}}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right],$$

where K = 0.307075 MeV mol⁻¹ cm², ze is the charge of incident particle, Z is the atomic number of the absorber, A is the atomic mass of the absorber, $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$, m_e is the electron mass, T_{max} is the maximum kinetic energy that can be imparted to a free electron in a single collision, I is the mean excitation energy, and $\delta(\beta\gamma)$ is the density effect correction to ionization energy loss. For a particle with mass M and momentum $M\beta\gamma c$, T_{max} is given by

$$T_{\rm max} = \frac{2m_ec^2\beta^2\gamma^2}{1+2\gamma m_e/M+(m_e/M)^2}$$



Figure 3.8: The energy loss distribution in the STAR TPC as a function of p_t at 0.25 T [89].

Fig. 3.8 shows the measured $\langle dE/dx \rangle$ versus the transverse momentum at 0.25 T. Various bands correspond to different mass particles. Pions and protons can be separated from each other up to 1 GeV/c [89].

For thin materials the more precise Bichsel formula is used [92].

Chapter 4

Study of parity violation in strong interactions

4.1 Λ polarization

The Λ hyperon is a member of the SU(3) S = 1/2 octet $(n, p, \Sigma^{-}, \Sigma^{0}, \Lambda, \Sigma^{+}, \Xi^{-}$ and Ξ^{0}), based on up (u), down (d), and strange (s) quarks. In the naive quark model, the u and d quarks are coupled to a spinless state $(\Delta u_{\Lambda} = \Delta d_{\Lambda} = 0)$, and the spin of the Λ particle is entirely carried by the s quark $(\Delta s_{\Lambda} = 1)$, where Δq_{Λ} denotes the contribution of quark q to the spin of the Λ [93, 94]. Predictions for the Λ spin composition from several models are summarized in Tab. 4.1.

Ref. [98] studies the longitudinal Λ polarization in the target fragmentation region in deep-inelastic $\bar{\nu}N$ collisions, and finds that the polarization transfer from the remnant s quark to Λ in the WA59 experiment seems to be 70% efficient. The authors think the dilution may be largely due to the decays of heavier hyperon resonances.

Ref. [99, 100] consider several Λ hadronization scenarios. For exclusive $qqq\to\Lambda$

	$\Delta u_{\Lambda} = \Delta d_{\Lambda}$	Δs_{Λ}
Quark model	0	1
Quenched lattice calculation [95]	-0.02(4)	0.68(4)
Valence quark contribution [96]	-0.07(4)	0.73(4)
Statistical model with NOMAD data [97]	0.10	0.74

Table 4.1: The contributions from quarks to the Λ spin from various models.

recombination process, P_{Λ} , the polarization of Λ , is the same as the polarization of its *s* quark P_s by assuming that polarized hyperons contain the initial polarized leading quark in its SU(6) wave function. For fragmentation process $q \to \Lambda + X$, similar calculations lead to $P_{\Lambda} = n_s P_s/(n_s + 2f_s)$, where n_s and f_s are the strange quark abundances relative to up and down quarks in QGP and quark fragmentation, respectively. If $f_s = n_s$, $P_{\Lambda} = P_s/3$ in fragmentation process.

In this thesis, we will assume that the Λ polarization is 100% correlated with its s quark.

4.2 Strangeness production at STAR

In order to estimate the effect caused by the possible parity violation in Au+Au collisions at STAR, we need to know the yields of strange particles.

Ref. [101] measured the $\Lambda(\overline{\Lambda})$, $\Xi^{-}(\overline{\Xi}^{+})$, and $\Omega^{-}(\overline{\Omega}^{+})$ spectra, as shown in Fig 4.1. These strange particles were reconstructed from their charged decay modes ($\Lambda \rightarrow p + \pi, \Xi \rightarrow \Lambda + \pi, \Omega \rightarrow \Lambda + K$) in the TPC. The signal was obtained by plotting invariant mass distributions and subtracting the linear background after various topology cuts. To calculate the reconstruction efficiency, Monte Carlo simulated tracks were embedded into real Au+Au collision events. The efficiency correction was based on



Figure 4.1: Transverse momentum distributions of $\Lambda(\overline{\Lambda})$ for |y| < 1.0, $\Xi^{-}(\overline{\Xi}^{+})$ for |y| < 0.75, and $\Omega^{-}(\overline{\Omega}^{+})$ for |y| < 0.75 in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV as a function of centrality. The dashed lines correspond to Boltzmann fits.

the probability of reconstructing these embedded tracks by applying the same cuts as used in reconstructing the real tracks.

The kaons' spectra were measured in Ref. [102] and [91]. Charged kaons were identified by the energy loss $\langle dE/dx \rangle$ in the TPC gas, as explained in Sec. 3.2.3; or by the kink decay topology $(K \to \mu\nu \text{ or } K \to \pi\pi^0)$, as shown in Fig. 4.2. Neutral kaons were reconstructed via their decay $K_S^0 \to \pi^+\pi^-$.

The total yield of strange particles at $\sqrt{s_{NN}} = 200$ GeV and 130 GeV, and their strangeness are shown in Tab. 4.2. We have made the following assumptions:

- The K_S^0 200 GeV yield is scaled from 130 GeV data, by the same factor according to charged kaons' data;
- K_L^0 and K_S^0 have the same yield;



Figure 4.2: A kink decay at STAR [103].

• The yield of $\Xi^0 + \overline{\Xi}^0$ is the same as $\Xi^- + \overline{\Xi}^+$.

4.3 Estimation of the number of polarized Λ hyperons

For the yield of $\Lambda + \overline{\Lambda}$, dN/dy = 16.7 + 12.7 = 29.4, which is already corrected for Ξ , Ξ^0 , and Ω feed-down (15% contribution altogether, $\Xi^- \to \Lambda \pi^-$ with branching ratio 99.9%, $\Xi^0 \to \Lambda \pi^0$ with branching ration 99.5%, $\Omega^- \to \Lambda K^-$, $\Xi^0 \pi^-$ or $\Xi^- \pi^0$ with branching ratio 100%), but includes Σ feed-down ($\Sigma^0 \to \Lambda \gamma$, branching ratio 100%). To calculate the primordial Λ yield, we need to know the Σ/Λ ratio.

According to the statistical model in the Boltzmann approximation, the particle density is given by Eq. (2.8), or

$$n \equiv \frac{N}{V} = \frac{g}{2\pi^2} m^2 T \gamma_S^{|n_s|} \exp\left(\frac{n_q \mu_q}{T}\right) \exp\left(\frac{n_s \mu_s}{T}\right) K_2\left(\frac{m}{T}\right),$$

		dN/dy	dN/dy
		(200 GeV, 0-5% most central)	(130 GeV, 0-6% most central)
Λ	1	$16.7 \pm 0.2 \pm 1.1$	
$\overline{\Lambda}$	1	$12.7 \pm 0.2 \pm 0.9$	
K^+	1	51.3 ± 7.7	$46.2 \pm 0.6 \pm 6.0$
K^-	1	49.5 ± 7.4	$41.9 \pm 0.6 \pm 5.4$
K_S^0	1	39.0	$33.9 \pm 1.1 \pm 5.1$
K_L^0	1	39.0	
Ξ-	2	$2.17 \pm 0.06 \pm 0.19$	
Ξ^+	2	$1.83 \pm 0.05 \pm 0.20$	
$\Xi^0 + \overline{\Xi}^0$	2	4.0	
Ω	3	$0.53 \pm 0.04 \pm 0.04$	
total $ S $		225.8	

Table 4.2: Yield of strange particles in Au+Au Collisions at $\sqrt{s_{NN}} = 200$ GeV and 130 GeV.

where T is the temperature of the system and K_2 is the second-order modified Bessel function. Since Σ and Λ have the same quark contents (*uds*), the Σ/Λ ratio only depends on their masses ($m_{\Sigma} = 1193$ MeV, $m_{\Lambda} = 1116$ MeV) and the temperature T. For $T \sim 157$ MeV,

$$\frac{n_{\Sigma}}{n_{\Lambda}} = \frac{m_{\Sigma}^2}{m_{\Lambda}^2} \frac{K_2(m_{\Sigma}/T)}{K_2(m_{\Lambda}/T)} = 0.67.$$

Hence, the primordial $\Lambda + \bar{\Lambda}$ yield is 29.4/1.67 = 17.6, and the total $\Lambda + \bar{\Lambda}$ yield (including Σ , Ξ , and Ω feed-down) is 29.4 + 8.0 + 0.5 = 37.9. In two units of rapidity (|y| < 1), the measurable primordial $\Lambda + \bar{\Lambda}$ yield through $p\pi$ decay (63.9% branching ratio) is 17.6 × 2 × 0.639 = 22.5, while the total measurable $\Lambda + \bar{\Lambda}$ yield is $37.9 \times 2 \times 0.639 = 48.4$, as summarized in Tab. 4.3.

The strangeness in primordial $\Lambda + \overline{\Lambda}$ hyperons accounts for

$$\frac{22.5}{225.8 \times 2} = 5.0\%$$



Figure 4.3: Σ/Λ ratio as a function of T.

	dN/dy	$ y < 1$, measurable $(p\pi \text{ decay})$
primordial Λ	17.6	22.5
Σ feed-down	11.8	15.1
Ξ feed-down	8.0	10.2
Ω feed-down	0.5	0.6
total Λ	37.9	48.4

Table 4.3: $\Lambda + \overline{\Lambda}$ yield at $\sqrt{s_{NN}} = 200$ GeV at STAR.

of the total strangeness produced in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

According to Eq. (2.2), or

$$\Delta (N_L^f - N_R^f) = 2Q_w,$$

each winding number creates two more left-handed strange quarks than right-handed ones. 5.0% of these extra left-handed strange quarks end up in primordial Λ and $\bar{\Lambda}$

hyperons. If we assume that the helicity of Λ is dominated by the chirality of its s quark, then the net handedness of the Λ and $\overline{\Lambda}$ hyperons is $0.1Q_w$.

The spin of the Λ hyperon may be estimated by the direction of its decay proton in the Λ rest reference system. The proton angular distribution is given by Eq. (2.10), or

$$dw(\theta) = \frac{1}{2}(1 - \alpha \cos \theta) d(\cos \theta),$$

thus the probability of correctly measuring the spin direction or the helicity of Λ or $\bar{\Lambda} (\alpha_{\Lambda} = -\alpha_{\bar{\Lambda}} = 0.642)$ is

$$\int_{-1}^{0} \frac{1}{2} (1 - \alpha t) \, dt = \frac{1}{2} + \frac{\alpha}{4} = 0.66.$$

Let κ denote the efficiency of finding a Λ or $\overline{\Lambda}$ hyperon experimentally. Then on average, $48.4\kappa \Lambda + \overline{\Lambda}$ hyperons are detected in an Au+Au collision within $|\eta| < 1$ at $\sqrt{s_{NN}} = 200$ GeV, $0.1\kappa Q_w$ of which have definite helicity (+1 for $Q_w > 0$ and -1for $Q_w < 0$), while the helicity of all other Λ and $\overline{\Lambda}$ hyperons is randomly distributed according to the Binomial distribution. A Binomial distribution is defined as

$$f(r; N, p) = \frac{N!}{r!(N-r)!} p^r (1-p)^{N-r}, \quad r = 0, 1, 2, \cdots, N; \quad 0 \le p \le 1$$

Its mean is Np and variance $\sigma^2 = Np(1-p)$.

4.4 Simulation

In this section, Λ should be taken to mean $\Lambda + \overline{\Lambda}$.

Suppose there are N events (Au+Au collisions). For each event, we generate m

"reconstructed" Λ hyperons $(m = 0, 1, \dots n_{\Lambda})$ according to the Binomial distribution

$$m \sim \operatorname{Bi}(n_{\Lambda}, \kappa)$$

where n_{Λ} is total number of Λ hyperons produced in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and set to be 48 in our analysis, and κ is the efficiency of finding a Λ hyperon experimentally. For a given Q_w configuration, we set $0.1 \kappa Q_w$, on average, of these m Λ hyperons with definite helicity (+1 for $Q_w > 0$ and -1 for $Q_w < 0$), and set other Λ hyperons with random helicity. Then we go through all these Λ hyperons and assign a possibility of 66% of measuring the helicity correctly.

After N events generated, we use the χ^2 -test to check for the effect of parity violation. Since the number of reconstructed Λ hyperons in each event is different, we first group all these N events according to the number of reconstructed Λ hyperons generated. Let N_m denote the number of events with m ($m = 0, 1, ..., n_\Lambda$) reconstructed Λ hyperons per event,

$$\sum_{m} N_m = N_{\cdot}$$

For each group of N_m (m = 2, 3, ...) events, we calculate its $\chi^2(m)$

$$\chi^2(m) = \sum_{i=0}^m \frac{(n_i - \bar{n}_i)^2}{\sigma_i^2},$$

where n_i is the number of events with *i* left-handed Λ hyperons, and \bar{n}_i is the expected value of n_i

$$\bar{n}_i = N_m f(i; N_m, p) = N_m \frac{N_m!}{i!(N_m - i)!} p^i (1 - p)^{N - i},$$

and the variance $\sigma_i^2 = \bar{n}_i$. In this analysis, we ignore the case where σ_i^2 is less than

10 so that the $\chi^2(m)$ constructed above follows the χ^2 probability density with the number of degrees of freedom equal to the number of measurements m minus the number of fitted parameters [3]. The χ^2 for all N events is the weighted sum of all $\chi^2(m)$:

$$\chi^{2} = \sum_{m} w_{m} \chi^{2}(m) = \sum_{m} \frac{N_{m}}{N} \chi^{2}(m).$$

We assume that there is no global polarization when there is no parity violation, i.e., equal probability for the Λ hyperons to be left-handed or right-handed, or p = 0.5.

 χ^2 calculated above is a random variable, thus we have to repeat the above procedure hundreds of times in order to obtain the distribution of χ^2 .

We have analyzed a few configurations with different Q_w distribution and different Λ efficiency κ . For each configuration, a control sample with no parity violation is also generated to test the validity of the simulation.

Let us take the following example where $Q_w \sim N(0, 20^2)$ $(N(\mu, \sigma^2)$ stands for a Gaussian distribution with mean μ and variance σ^2), and the Λ efficiency κ is set to be 10%, which is realistic for the STAR upgrade in the near future.

Fig. 4.4 shows the distribution of the number of extra detected left-handed Λ hyperons caused by Q_w . It has a mean of 0, since Q_w can be positive or negative with equal possibilities, and a RMS value of 0.44. For an event to be useful for a parity violation signal, we need at least two extra detected left-handed (or right-handed) Λ hyperons.

Fig. 4.5 shows the χ^2 distribution for 10,000 runs of 1 M events. The black dots correspond to the simulation, and the curve corresponds to the theoretical χ^2 curve of Binomial distribution with no parity violation. The dots and the theoretical curve agrees pretty well. Thus 1 M events are not enough to detect the parity violation



Figure 4.4: Event-by-event distribution of the number of extra left-handed Λ hyperons detected due to \mathcal{P} -odd bubble formed in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. $Q_w \sim N(0, 20^2)$ and $\kappa = 10\%$.

effect in this configuration. The dotted vertical line (on the left) is the line of 95% confidence level, which means that only 5% of the no-signal data have χ^2 larger than this value (on the right of this line), i.e., if we have measured a χ^2 that is larger than this value, there is a 95% possibility that the Λ hyperons are not randomly distributed. The dashed line (on the right) is 99% confidence level.

Fig. 4.6 shows the χ^2 distribution for 181 runs of 55 M events with $Q_w \sim N(0, 20^2)$ and $\kappa = 10\%$. We can see that the measured values greatly deviate from the theoretical curve according to random distribution. 95% of the time, the measured χ^2 values are larger than the 95% confidence level, and 84% of the time they are larger than the 99% confidence level. At the same time, the control sample with no parity violation built in shows no effect, as shown in Fig. 4.7.

The case for $Q_w \sim N(0, 10^2)$ and $\kappa = 20\%$ is shown in Fig. 4.8. No effect is seen



Figure 4.5: Parity violation observation through Λ helicity distribution. For a set of 1 M events with $Q_w \sim N(0, 20^2)$ and $\kappa = 10\%$, the measured χ^2 distribution follows the theoretical curve. No effects observed.



Figure 4.6: Parity violation observation through Λ helicity distribution. For a set of 55 M events with $Q_w \sim N(0, 20^2)$ and $\kappa = 10\%$, most of the runs have very large χ^2 : 95% of the runs deviate from the hypothesis of Λ helicity being randomly distributed with 95% confidence level, 84% deviate with 99% confidence level.



Figure 4.7: Control case for a set of 55 M events with no topological charge. The χ^2 distribution follows the theoretical curve pretty well.

for 10 M events, but every set of 850 M events shows an effect above 99% confidence level.

The results are summarized in Tab. 4.4. N_{events} is the number of events needed so that there is 95% possibility to observe a 95% confidence level of deviating from random distribution. For Q_w with a spread of 20 and $\kappa = 10\%$, we probably need 55 M events to observe the effect. The number of events increases dramatically as the spread of Q_w becomes smaller, since the chance to have at least two Λ hyperons with definite handedness in an event becomes much smaller.

With current STAR statistics (~24 M central events) and Λ detection efficiency about 3.5%, Q_w with standard deviation larger than 60 can be ruled out in the 95% confidence level, if we assume that the Λ polarization is 100% correlated with the chirality of its *s* quark.


Figure 4.8: Parity violation observation through Λ helicity distribution for a set of (a) 10 M events and (b) 850 M events with $Q_w \sim N(0, 10^2)$ and $\kappa = 20\%$.

Q_w	κ	$N_{\rm events}$
$N(0, 20^2)$	10%	$55 \mathrm{M}$
$N(0, 20^2)$	20%	$30 \mathrm{M}$
$N(0, 10^2)$	20%	$550 \mathrm{M}$
$N(0, 10^2)$	50%	$170~{\rm M}$

Table 4.4: Number of events needed for 95% of Monte Carlo runs to have a 95% confidence level of deviating from the Binomial distribution.

4.5 Background study

Two possible backgrounds are studied in [104]:

- Λ efficiency may differ for each event according to the reaction plane, primary vertex position, or other factors. An event-by-event variation of 2.5% may cause fake signal. This could be solved by dividing the events in z-vertex and reaction plane bins.
- 2. The systematic error from Ξ or Σ feed-down. A Λ produced by Ξ decay is in general longitudinally polarized in the Ξ rest frame. But it is not much of a problem either, as it shows no effect in 200 M Monte Carlo events with 100% Λ efficiency.

Chapter 5

Analysis methods

5.1 Event selection

This thesis uses the STAR Run-4 (from October 2003 to April 2004) Au+Au data at $\sqrt{s_{NN}} = 200$ GeV. The RHIC Run-4 Au+Au operation luminosity is shown in Fig. 5.1.

The STAR Au+Au events can be roughly divided into two categories: the central events and minimum bias events, as described in Sec. 3.2.1. The central trigger in 2004 corresponds to CTB sum > 3500 and ZDC hardware sum < 131 (in arbitrary units), as shown in Fig. 5.2.

The minimum bias Au+Au events are divided into nine centrality classes based on measured charged particle multiplicity within pseudorapidity $|\eta| < 0.5$. These classes correspond to, from central to peripheral, (0-5)%, (5-10)%, (10-20)%, (20-30)%, (30-40)%, (40-50)%, (50-60)%, (60-70)%, and (70-80)% of the measured total cross section, as shown in Fig. 5.3.

There are three trigger setup names associated with each data file (a file contains



Figure 5.1: RHIC integrated luminosity [105].



Figure 5.2: STAR central trigger in 2004 Au+Au collisions [106].



Figure 5.3: Uncorrected charge particle multiplicity distribution measured in the TPC in $|\eta| < 0.5$ for Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The shaded areas indicate the centrality bins used in the analysis. From right to left: (0-5)%, (5-10)%, (10-20)%, (20-30)%, (30-40)%, (40-50)%, (50-60)%, (60-70)%, and (70-80)% [107].

many events) and recorded in the data base [106]:

- The productionHigh trigger setup is mainly high tower running for highest luminosity, with some minimum bias events mixed in. About 31 thousand files are recorded under the productionHigh trigger.
- The productionMid trigger setup is a mixture of high tower, central, and minimum bias id's, optimized for medium luminosity. About 35 thousand files are recorded under the productionMid trigger.
- The productionLow trigger setup is a mixture of high tower, central, and minimum bias id's, optimized for low luminosity. About 78 thousand files are recorded under the productionLow trigger.

A trigger id is recorded in each event for event selection, for example, trigger id 15105 corresponds to central events and 15007 corresponds to minimum bias events.

For STAR Au+Au Run-4 at $\sqrt{s_{NN}} = 200$ GeV, there are roughly 24 M central events and 25 M minimum bias events for physics analysis.

5.2 V0 reconstruction

After the physical run (about three months each year), an official real data production at STAR starts. It converts the raw data collected by the detectors to usable data for physics analysis. This process is CPU heavy and can take several months.

First, all hits, primary tracks, and global tracks in an event are reconstructed. Then the reconstruction for V0's — the neutral strange particles K_S^0 , Λ and $\bar{\Lambda}$ takes place. These particles have such a long life time — $c\tau = 2.68$ cm for K_S^0 and 7.89 cm for Λ and $\bar{\Lambda}$ — that their decay vertices can often be distinguished from the primary vertex. (K_L^0 , with $c\tau = 15.3$ m, mostly does not decay inside the TPC.) Although V0s cannot be detected directly by the TPC as they are neutral and do not ionize the TPC gas, they all have decay modes to a positively charged particle and a negatively charged particle ($K_S^0 \to \pi^+\pi^-$ with branching ratio 69.2%, $\Lambda \to p\pi^-$ and $\bar{\Lambda} \to \bar{p}\pi^+$ with branching ratio 63.9%), and hence can be reconstructed through their daughter tracks.

The V0 finding algorithm, called StVOFinderMaker, combines each positively charged global track with each negatively charged global track. Each of the daughter tracks is required to have at least 11 hits. The following geometrical cuts, as shown in Fig. 5.5, are applied to each TPC pair:

• The two tracks are close enough to each other, i.e., the DCA (distance of closest approach) of them should be less than 0.8 cm.



Figure 5.4: A schematic diagram of a V0 decay.

- The position of the V0 decay vertex is then determined. The decay length (distance between the V0 vertex and the primary vertex of the event) should be larger than 2 cm to reduce background.
- The momentum of the V0 is determined by adding the momenta of its two daughter tracks at the decay vertex. The V0 momentum should point back to the primary vertex within 0.8 cm.
- The daughter tracks should not come from the primary vertex, or the DCA between the primary vertex and each daughter track should be larger than 0.7 cm.
- The Podolanski-Armenteros p_t should be less than 0.3 GeV/c and the magnitude of Podolanski-Armenteros α should be less than 1.2.

The Podolanski-Armenteros p_t is the momentum of the daughter track projected

to the direction that is perpendicular to the parent V0 momentum direction. The Podolanski-Armenteros α is defined as

$$\alpha = \frac{p_L^+ - p_L^-}{p_L^+ + p_L^-}$$

where p_L^+ is the momentum of the positive daughter projected to the direction of the V0 momentum, and p_L^- is the momentum of the negative daughter projected to the direction of the V0 momentum. The Podolanski-Armenteros plot is a useful visual tool for distinguishing ambiguous decays independent of the mass hypothesis [108].

If a pair passes the above cuts, a V0 is found and stored in the data file. We will tighten these cuts when reconstructing the Λ and $\overline{\Lambda}$ hyperons to get a cleaner sample.

On average, there are about 600 TPC V0's found in each central Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV. The distribution of number of V0's in central events is shown in Fig. 5.6.

5.3 Λ and $\overline{\Lambda}$ reconstruction

Fig. 5.7 shows the invariant mass of all V0's around the Λ mass range ($m_{\Lambda} = 1115.683 \pm 0.006 \text{ MeV}/c^2$). The momenta of the daughter tracks are measured in the TPC. The invariant mass is calculated by assuming the positively charged daughter track is a p and the negatively charged daughter track is a π^- :

$$m_{\rm inv} = \frac{1}{c^2} \sqrt{E^2 - \mathbf{p}^2 c^2} = \frac{1}{c^2} \sqrt{(E_{\rm pos} + E_{\rm neg})^2 - (\mathbf{p}_{\rm pos} + \mathbf{p}_{\rm neg})^2 c^2},$$



Figure 5.5: Some V0 cuts in the production chain. (a) DCA between V0 daughters is less than 0.8 cm; (b) V0 decay length is larger than 2.0 cm; (c) DCA between V0 and primary vertex is less than 0.8 cm; (d) DCA between V0 daughter tracks and the primary vertex is larger than 0.7 cm.



Figure 5.6: Distribution of number of V0's in $|\eta| < 1.8$ found in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

where E_{pos} , E_{neg} are the energy of the positively and negatively charged daughter, respectively:

$$E_{\text{pos}} = \sqrt{m_p^2 c^4 + \mathbf{p}_{\text{pos}}^2 c^2},$$
$$E_{\text{neg}} = \sqrt{m_\pi^2 c^4 + \mathbf{p}_{\text{neg}}^2 c^2},$$

and $\mathbf{p}_{\rm pos},\,\mathbf{p}_{\rm neg}$ are their momenta. There is only a small visible Λ peak in this figure.

Particle identification can be done by the track's energy loss $\langle dE/dx \rangle$ in the TPC (see Sec. 3.2.3). $\langle dE/dx \rangle$ is not normally distributed, but the new variable

$$z = \ln\left(\frac{\langle dE/dx\rangle}{\langle dE/dx\rangle^{\text{theory}}}\right)$$

follows a Gaussian distribution [109]. Fig. 5.8 shows the distribution for z_{π} , where



Figure 5.7: Invariant mass distribution of Λ candidates, with V0 default cuts only.

the pion band is located around 0. After requiring $\langle dE/dx \rangle$ of the positively charged charged track within 3σ of the proton band, and the negatively charged track within 3σ of the pion band, the Λ invariant mass spectrum looks much cleaner, as shown in Fig. 5.9. The background beneath the Λ peak is dominated by combinatoric pairs of charged particles. Decays of $K_S^0 \to \pi^+\pi^-$ also contribute to the smooth background due to pions misidentified as protons [110]. We take V0s of invariant mass within the range of $|m - m_{\Lambda}| < 5$ MeV as Λ or $\bar{\Lambda}$ hyperons. The signal to background ratio is about 0.5.

To obtain a clean sample of Λ hyperons, we apply some of the geometrical cuts used in Ref. [110]:

- the proton candidate tracks miss the primary vertex by at least 0.9 cm;
- π^- candidate tracks miss the primary vertex by at least 2.85 cm;



Figure 5.8: Distribution of z_{π} in 200 GeV minimum bias pp collisions [88].



Figure 5.9: Invariant mass distribution of Λ candidates, with $3\sigma~\langle dE/dx\rangle$ cut for both daughters.



Figure 5.10: Invariant mass distribution of Λ candidates, with PID and geometrical Cuts I. The thick line represents a fit of double Gaussian signal plus an exponential background. The circles are the data points that within the mass range cut ($|m - m_{\Lambda}| < 5$ MeV).

• the DCA between V0 and the primary vertex is less than 0.5 cm.

We also require the number of hits on each daughter to be larger or equal to 23 to avoid track splitting effects. We will refer the above set of cuts as "Cuts I". The invariant mass distribution using Cuts I is shown in Fig. 5.10. The peak is fit by a double Gaussian parametrization

$$Ae^{-(x-\mu_1)^2/2\sigma_1^2} + Be^{-(x-\mu_2)^2/2\sigma_2^2}$$

plus an exponential background. The fit parameters are $\mu_1 = \mu_2 = 1.115 \text{ GeV}/c^2$, $\sigma_1 = 1.1 \text{ MeV}/c^2$, $\sigma_2 = 2.4 \text{ MeV}$, A/B = 1.4. The signal to background ratio in the range of $|m - m_{\Lambda}| < 5$ MeV is about 17.

5.4 Cuts studies

5.4.1 Podolanski-Armenteros cut

The Podolanski-Armenteros plot for K_S^0 , Λ and $\overline{\Lambda}$ is shown in Fig. 5.11. The K_S^0 candidates are constructed with V0s that satisfy the following cuts:

- the daughters should be near the $\langle dE/dx \rangle$ pion bands (within 3σ);
- each daughter should have at least 15 hits;
- DCA between V0 and primary vertex should be less than 0.6 cm;
- DCA between each daughter and primary vertex should be larger than 1.2 cm.

The K_S^0 invariant mass plot is shown in Fig. 5.12. The Λ and $\bar{\Lambda}$ hyperons are selected by Cuts I. In Fig. 5.11 the top black band corresponds to K_S^0 , the lower left band corresponds to $\bar{\Lambda}$, and the lower right band corresponds to Λ . Fig. 5.13 shows the one dimensional Podolanski-Armenteros p_t plot for selected Λ and K_S^0 candidates. Fig. 5.14 shows the one dimensional Podolanski-Armenteros α plot for selected Λ and $\bar{\Lambda}$ candidates. The peak on the left corresponds to $\bar{\Lambda}$, and the peak on the right corresponds to Λ .

Although it seems that we may reduce the K_S^0 contamination by applying some cut on the Podolanski-Armenteros variables, Fig. 5.15 shows that the K_S^0 contamination is not severe (about 1%) in the Λ candidates with Cuts I, and cut on the Podolanski-Armenteros variables ($p_t < 0.11$ and $0.1 < \alpha < 0.9$) is not effective when Λ Cuts I have been applied.



Figure 5.11: Podolanski-Armenteros plot for selected K_S^0 , Λ and $\bar{\Lambda}$ candidates.



Figure 5.12: K_S^0 invariant mass plot.



Figure 5.13: Podolanski-Armenteros p_t for selected Λ and K_S^0 candidates. The red plus points correspond to Λ and the black circles correspond to K_S^0 .



Figure 5.14: Podolanski-Armenteros α for selected Λ and $\overline{\Lambda}$ candidates.



Figure 5.15: K_S^0 invariant mass distribution from the Λ candidates. (a) Two curves (with or without Podolanski-Armenteros cut) overlap with each other; (b) K_S^0 invariant mass peak with a Gaussian fit after polynomial background subtraction.

	Cuts I	Cuts II	
$\langle dE/dx \rangle$	3σ	3σ	
DCA: p to primary vertex	$> 0.9 {\rm ~cm}$	(> 0.7 cm)	
DCA: π to primary vertex	> 2.85 cm	> 2.0 cm	
DCA: V0 to primary vertex	$< 0.5 {\rm ~cm}$	< 0.6 cm	
DCA: daughters	$< 0.8 {\rm ~cm}$	$< 0.7 { m cm}$	
decay length	(> 2.0 cm)	(> 2.0 cm)	
Podolanski-Armenteros α	$(-1.2 < \alpha < 1.2)$	$(-1.2 < \alpha < 1.2)$	
Podolanski-Armenteros p_t	(< 0.3 GeV/c)	(< 0.3 GeV/c)	
Number of hits on track	≥ 23	≥ 15	
Mass range $ m - m_{\Lambda} $	$< 5 \ { m MeV}/c^2$	$< 6 \text{ MeV}/c^2$	
m_K	No cut	$ m - m_K > 18 \text{ MeV}/c^2$	

Table 5.1: Two sets of Λ cuts used in HBT analysis. Cuts in parentheses denote the V0 production cuts.

	Λ		$\bar{\Lambda}$	
	number per event	signal / noise	number per event	signal / noise
V0 cut	1.6	0.5	1.2	0.5
Cuts I	0.96	15	0.74	16
Cuts II	1.04	3.0	0.78	3.0

Table 5.2: Measured Λ and $\overline{\Lambda}$ number per event and signal-over-noise ratio with various cuts. The $\Lambda/\overline{\Lambda}$ ratio agrees with the published data 16.7/12.7 = 1.3 [101].

5.4.2 Two different sets of cut used in HBT analysis

Besides the cuts mentioned in Sec. 5.3, we will also use a similar set of cuts to those used in Ref. [111]. This set of cuts are looser than Cuts I, and provide a relatively clean Λ sample and more Λ pairs. These two sets of cuts are listed in Tab. 5.1. The Λ invariant mass distribution for Cuts II is shown in Fig. 5.16.



Figure 5.16: A invariant mass distribution for Cuts II. The thick line represents a fit of double Gaussian signal plus an exponential background. The circles are the data points that within the mass range cut $(|m - m_{\Lambda}| < 6 \text{ MeV})$.

5.5 Mixed event technique in HBT studies

To measure the HBT correlation function

$$C(q) \equiv \frac{P(k_1, k_2)}{P(k_1)P(k_2)} = 1 \pm \lambda e^{-Q^2 R^2},$$

experimentally, where $q = k_1 - k_2$ and $Q^2 = -q^{\mu}q_{\mu}$, Kopylov suggested to use pairs from mixed events as the reference sample for HBT studies [65], i.e.

$$C(q) \propto rac{N(q)_{
m both \ particles \ from \ the \ same \ event}}{N(q)_{
m pairs \ from \ different \ events}}.$$

The idea is that pairs from different events do not exhibit the HBT correlation, and they are supposed to have the same correlations due to energy and momentum conservation etc. as the same-event pairs.

Chapter 6

$\Lambda\Lambda$ correlations

6.1 $\Lambda\Lambda$ and $\bar{\Lambda}\bar{\Lambda}$ HBT results

The statistics for STAR Run 4 is not sufficient to measure the $\Lambda\Lambda$ and $\bar{\Lambda}\bar{\Lambda}$ correlation functions. This chapter will give an example of how the future analysis will go.

We have used various cuts in studying $\Lambda\Lambda$ and $\Lambda\Lambda$ HBT effects in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV at STAR. We will present the results with Λ Cuts I and II listed in Tab. 5.1.

The one dimensional correlation function is plotted using the mixed event technique mentioned in Sec. 5.5. Namely, we first select Λ (or $\bar{\Lambda}$) candidates by Cuts I or II in each event, then calculate the Q value for every pair in an event. The Qdistribution for $\Lambda\Lambda$ (or $\bar{\Lambda}\bar{\Lambda}$) pairs is the numerator. To increase statistics, each Λ is mixed with ten other Λ s from different events (or $\bar{\Lambda}$ with other $\bar{\Lambda}$ s) and ten Qvalues are obtained. This Q distribution serves as the denominator. The correlation function is defined as

$$C(q) = \text{normalization factor} \times \frac{N(q)_{\text{both particles from the same event}}}{N(q)_{\text{pairs from different events}}}$$

One important detector effect which may alter the measured correlation function is that two particles may be reconstructed as only one track when they are close to each other. This effect is called merged tracks. If there are a lot of merged tracks, then pairs at low Q are reduced in the same event since particles that are close to each other have higher probability to have low Q, as shown in Fig. 6.1. The pairs in the mixed event sample do not suffer from this problem, hence the measured correlation function may be lower than the real value. To estimate the effect of merged tracks, we plot the distribution of

normalization factor
$$\times \frac{\text{Number of pairs in the same event}(\theta)}{\text{Number of pairs in the mixed event}(\theta)}$$

where θ is the opening angle of a pair, with all Q integrated. The normalized ratio is flat, as shown in Fig. 6.2, thus we do not suffer much from merged tracks in our analysis.

The distribution of the number of Λ and Λ per event is shown in Fig. 6.3. On average there are 0.47 $\Lambda\Lambda$ pair and 0.28 $\bar{\Lambda}\bar{\Lambda}$ pair per event for Cuts I, 0.55 $\Lambda\Lambda$ pair and 0.31 $\bar{\Lambda}\bar{\Lambda}$ pair per event for Cuts II, as shown in Tab. 6.1. The HBT plots are shown in Fig. 6.4. No HBT signal is observed in current data set with these two cuts.



Figure 6.1: The pair distribution of Q and the opening angle with Cuts I.



Figure 6.2: The normalized opening angle same event / mixed event ratio with Cuts I.



Figure 6.3: Distribution of n_{Λ} per event in Au+Au collisions.



Figure 6.4: $\Lambda\Lambda$ and $\bar{\Lambda}\bar{\Lambda}$ HBT plots.

	Cuts I	Cuts II
number of Λ per event	0.96	1.04
number of $\Lambda\Lambda$ pairs per event	0.47	0.55
number of $\Lambda\Lambda$ pairs with $Q < 100 \text{ MeV}/c$ per event	8.2×10^{-5}	1.2×10^{-4}
number of $\bar{\Lambda}$ per event	0.74	0.78
number of $\overline{\Lambda}\overline{\Lambda}$ pairs per event	0.28	0.31
number of $\overline{\Lambda}\overline{\Lambda}$ pairs with $Q < 100 \text{ MeV}/c$ per event	5.0×10^{-5}	7.2×10^{-5}

Table 6.1: Statistics for HBT results. About 20 M events are used in this analysis.

6.2 $\Lambda\Lambda$ and $\bar{\Lambda}\bar{\Lambda}$ HBT simulation

6.2.1 Transverse mass dependence of HBT radii

Fig. 6.5 shows that the one-dimensional HBT radii measured at STAR and PHENIX for various systems depend on the mean transverse mass $\langle m_t \rangle$, where

$$m_t = \sqrt{k_t^2 + m^2},$$

$$k_t = \frac{1}{2}(\mathbf{p}_1 + \mathbf{p}_2)_t,$$

 \mathbf{p}_1 and \mathbf{p}_2 are the momenta of a correlated pair. The particle species are listed in Fig. 6.5.

This thesis will use a fit of

$$R(\text{fm}) = \frac{3.83}{\sqrt{m_t(\text{GeV}/c^2)}}$$
 (6.1)

to estimate the HBT radii for $\Lambda\Lambda$ and $\bar{\Lambda}\bar{\Lambda}$ systems.



Figure 6.5: (a) m_t dependence of R_{inv} for different particles [112]. (b) m_t dependence of R_{inv} for pions and $p\Lambda$, the curve shows the $\langle m_t \rangle^{-1/2}$ dependence [111].



Figure 6.6: $A\langle m_t \rangle^{-1/2}$ fit of the HBT radii for several systems.

6.2.2 Residual HBT from feed-down

Since the Λ and $\overline{\Lambda}$ hyperons are spin 1/2 fermions, the $\Lambda\Lambda$ and $\overline{\Lambda}\overline{\Lambda}$ one-dimensional HBT correlation function takes form

$$C(q) = 1 - \frac{1}{2}e^{-Q^2R^2},$$

where

$$Q^{2} \equiv -q^{\mu}q_{\mu} = -(k_{1} - k_{2})^{2} = -(E_{1} - E_{2})^{2} + (\mathbf{k}_{1} - \mathbf{k}_{2})^{2},$$

for a chaotic static Gaussian source and a statistical spin mixture ensemble. However, we need to consider the effect of feed-down from Σ and Ξ . If two Λ hyperons in an event are both from Σ feed-down or from Ξ feed-down, they are still correlated because their parents are HBT correlated. To study this effect, we generate Σ pairs and Ξ pairs. Take Ξ pairs for example. We generate the Ξ spectra

$$\frac{1}{N_{\text{event}}2\pi} \frac{d^2 N}{p_t dp_t dy}$$

according to the Boltzmann fit at midrapidity (T = 335 MeV [101]). For each pair, we calculate its m_t and assign a weight according to the HBT correlation function for

$$1 - \frac{1}{2}e^{-Q^2R^2},$$

where R is determined by Eq. (6.1):

$$R = \frac{3.83}{\sqrt{m_t}}.$$

Then we let them decay, and calculate Q for both Ξ pairs and decay Λ pairs. The correlation function is plotted by the mixed event technique and fit by the formula

$$C(Q) = A(1 - \lambda e^{-Q^2 R^2}).$$

Fig. 6.7 shows the $\Lambda\Lambda$ residual HBT effect from Σ feed-down. The effective $\lambda = -0.13$ and R = 2.0 fm. Fig. 6.8 shows that the $\Lambda\Lambda$ residual HBT effect from Ξ feed-down is negligible.

6.2.3 Estimate number of events to observe the $\Lambda\Lambda$ HBT effect

From Tab. 4.3, the composition of all detected Λ hyperons is: 22.4/45.9 = 49%primordial Λ s, 15.0/45.9 = 33% from Σ feed-down, and 8.0/45.9 = 17% from Ξ



Figure 6.7: (a) HBT effect from the parent $\Sigma\Sigma$ pairs; (b) $\Lambda\Lambda$ residual HBT effect from Σ feed-down.



Figure 6.8: (a) HBT effect from the parent $\Xi\Xi$ pairs; (b) $\Lambda\Lambda$ residual HBT effect from Ξ feed-down.

feed-down.

To estimate how many events are needed to observe the $\Lambda\Lambda$ HBT effect, we do the following simulation. We first generate Λ according to the measured number of Λ hyperons per event and its p_t distribution. For any Λ generated, we assign a probability of 5% being background. For true Λ hyperons, we assign a probability of 49% being primordial Λ , 33% being Σ feed-down, and 17% being Ξ feed-down.

Only primordial Λ pairs and Λ pairs both from Σ feed-down are given an HBT effect. We assign a weight of

$$1 - \frac{1}{2}e^{-Q^2 R^2(m_t)}$$

for primordial Λ pairs. The weight of Λ pairs both from Σ feed-down is from results in Sec. 6.2.2.

The simulation results are shown in Fig. 6.9. For 20 M events, it is hard to measure the HBT effect, while 120 M events seem to be enough to measure the HBT effect. The correlation function is fit by

$$C(Q) = 1 - \lambda e^{-Q^2 R^2}$$

6.3 $\Lambda\Lambda$ and $\bar{\Lambda}\bar{\Lambda}$ spin-selected HBT correlations

The $\Lambda\Lambda$ and $\Lambda\Lambda$ HBT effect differs from the bosons' HBT effect in that even for a purely chaotic source and perfect conditions, the magnitude of the correlation strength is 0.5 instead of 1. We think we may increase this correlation strength for $\Lambda\Lambda$ and $\bar{\Lambda}\bar{\Lambda}$ systems based on spin selection.

A loose argument for the reduced correlation strength in fermion pairs is that spin



Figure 6.9: Simulation of $\Lambda\Lambda$ HBT effect for (a) 20 M events; (b) 120 M events.



Figure 6.10: HBT effect for spin 1/2 baryons.

1/2 particles may be in two states. If the pair are both spin up or spin down, then they are identical and interference may happen, as shown in Fig. 6.10(a). However, if they are of different spins, they are not identical particles any more, and no interference can happen, as shown in Fig. 6.10(b) and (c). In the real world, we expect that these two conditions are equally populated, therefore the correlation strength for fermion pairs is reduced by a factor of two.

If there is a way to select the Λ spins, we may select only Λ pairs with identical spins, and then the magnitude of the HBT correlation strength should be restored to 1. We call this spin-selected HBT correlations.

As in Sec. 4.3, we utilize the fact that the Λ weak decay is parity-violating. We can estimate the Λ 's spin direction by the direction of its decay proton. We then place a cut on the angle between the estimated spin direction of a $\Lambda\Lambda$ pair to enhance the HBT effect. In this section, we consider $\Lambda\Lambda$ (or $\bar{\Lambda}\bar{\Lambda}$) pairs with only their decayed protons in the same hemisphere by making the cut of $\theta_{pp} < \pi/2$. It is worth noting that we should first transfer all momenta to the center of mass system of the $\Lambda\Lambda$ (or $\Lambda\Lambda$) pairs, since that is the relevant reference frame.

The results are shown in Fig. 6.11. Black full circles represent HBT without spin selection, and red open circles represent HBT with a cut of $\theta_{pp} < \pi/2$. Due to limited statistics, there is no visible effect.

6.4 $\Lambda\Lambda$ and $\Lambda\Lambda$ spin correlations

Let us consider the $\Lambda\Lambda$ system. Define θ_{pp} as in the previous section and $T = \cos \theta_{pp}$. Ref. [113] shows that the distribution of number of pairs N is linear as a function of T for both S = 0 and S = 1 states at or very near its threshold:

$$\left. \frac{dN}{dT} \right|_{S=0} \propto 1 - 0.4122T,$$
(6.2)

$$\left. \frac{dN}{dT} \right|_{S=1} \propto 1 + 0.1374T.$$
 (6.3)

For a $\Lambda\Lambda$ pair at its threshold, or Q = 0, only s-wave (l = 0) state exists. The l = 0, S = 1 state is forbidden by the Pauli exclusion principle, thus dN/dT has a negative slope of -0.4122 at Q = 0, according to Eq. 6.2. At Q > 0, both S = 0 and S = 1states exist, and

$$\frac{dN}{dT} = (1 - \epsilon) \left. \frac{dN}{dT} \right|_{S=0} + \epsilon \left. \frac{dN}{dT} \right|_{S=1}, \tag{6.4}$$

where ϵ is the fraction of the S = 1 state. For a statistical spin mixture, $\epsilon = 3/4$ and dN/dT is flat. The authors of [113] also assume that $\epsilon(Q)$ may be parametrized as

$$\epsilon(Q) = \frac{3}{4}(1 - e^{-R^2Q^2}) \tag{6.5}$$

for a spherical Gaussian source.



Figure 6.11: $\Lambda\Lambda$ and $\bar{\Lambda}\bar{\Lambda}$ spin-selected HBT correlations. Black full circles represent HBT without selection, and red open circles represent HBT with a cut of $\theta_{pp} < \pi/2$.



Figure 6.12: The distribution of dN/dT as a function of T of the $\Lambda\Lambda$ system for (a) Q < 50 MeV; (b) Q < 100 MeV.

The study of dN/dT as a function of Q according to Eqs. (6.4) and (6.5) — dN/dT starts with negative slope at very low Q and gradually becomes flat as Q increases — may provide an alternative way to measure the HBT radius. The results measured at STAR using the mixed event technique are shown in Fig. 6.12. The dN/dT distribution is flat within statistical errors for Q < 50 MeV and Q < 100 MeV.

6.5 ΛΛ and ΛΛ spin correlation due to global polarization in noncentral collisions

The large orbital angular momentum created in noncentral Au+Au collisions may lead to global quark polarization due to spin-orbital coupling. The global polarization of the produced quarks may result in a global Λ polarization [99]. Ref. [114] measured the Λ polarization directly and found out the $P_{\Lambda} = (-23.3 \pm 11.2) \times 10^{-2}$ for 3.3 <


Figure 6.13: Global polarization of Λ hyperons as a function of Λ transverse momentum p_t^{Λ} . Filled circles represent Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV in centrality region 20–70%, and open squares represent Au+Au collisions at $\sqrt{s_{NN}} = 62.4$ GeV in centrality region 0–80% [114].

 $p_t^{\Lambda} < 4.5$ GeV at $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions in centrality region 20–70%, as shown in Fig. 6.13.

We use a similar set of cuts as those used in [114], shown as Cuts III in Tab. 6.2. The Λ invariant mass distribution is shown in Fig. 6.14. Unfortunately no $\Lambda\Lambda$ pairs are found for $3.3 < p_t^{\Lambda} < 4.5$ GeV. For $p_t^{\Lambda} < 4.5$ GeV, we transform each proton in its parent Λ rest frame and plot $S \equiv \cos \theta_{pp}$ normalized by pairs from mixed events. Note that S is different from T in the previous section — the spin correlation is relevant in the $\Lambda\Lambda$ pair's center of mass frame, thus we need to do a Lorentz transformation to the $\Lambda\Lambda$ pair's center of mass frame first, but we should not do this transformation for global polarization studies, as the relevant reference frame is the center of mass frame of two colliding gold nucleons, i.e., the lab frame.

The results are shown in Fig. 6.15. The distribution of S is flat. No spin correlation of $\Lambda\Lambda$ pairs in the lab frame for $p_t^{\Lambda} < 4.5$ GeV is found.

	Cuts I	Cuts II	Cuts III
$\langle dE/dx \rangle$	3σ	3σ	3σ
DCA: p to primary vertex	$> 0.9 {\rm ~cm}$	(> 0.7 cm)	$> 1.0 {\rm ~cm}$
DCA: π to primary vertex	> 2.85 cm	$> 2.0 {\rm ~cm}$	$> 2.5 \mathrm{~cm}$
DCA: V0 to primary vertex	$< 0.5 {\rm ~cm}$	$< 0.6 {\rm ~cm}$	$< 0.5 {\rm ~cm}$
DCA: daughters	$< 0.8 {\rm ~cm}$	$< 0.7 {\rm ~cm}$	(< 0.8 cm)
decay length	(> 2.0 cm)	(> 2.0 cm)	$> 6.0 {\rm ~cm}$
Number of hits on track	≥ 23	≥ 15	≥ 15
Mass range $ m - m_{\Lambda} $	$< 5 \text{ MeV}/c^2$	$< 6 \text{ MeV}/c^2$	$< 5 \text{ MeV}/c^2$
$ m-m_K $	No cut	$> 18 \text{ MeV}/c^2$	No cut

Table 6.2: Comparison of the Λ cuts used in this section (Cuts III) with two sets of cuts previously used. Cuts in parentheses denote the V0 production cuts.



Figure 6.14: Λ invariant mass spectra with Cuts III.



Figure 6.15: Distribution of $S \equiv \cos \theta_{pp}$ normalized by pairs from mixed events for $\Lambda\Lambda$ system in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV in centrality region 20–70%. The histogram is fit with a linear function $p_0 + p_1 S$.

Chapter 7

Speculations and discussion

7.1 Final state interaction (FSI)

So far we have neglected the possible final state strong interaction between a $\Lambda\Lambda$ pairs. There is a poor knowledge of the $\Lambda\Lambda$ strong interaction, so a study of HBT correlations actually might be a reasonable way to study it [115].

We characterize the strong interaction by scattering length a. For an attractive potential, a is negative and increases the correlation function. The correlation function at q = 0 is found to be [116]

$$C(q=0) = \frac{1}{2} \left(1 + \frac{a^2}{2R^2} - \frac{2a}{\sqrt{\pi}R} \right)$$

for a Gaussian source

$$\rho(\mathbf{r}) \propto e^{-\mathbf{r}^2/2R^2}.$$

The correlation function for spin 1/2 fermions for R = 4 fm as a function of the scattering length is shown in Fig. 7.1 [116]. The curves correspond to scattering



Figure 7.1: $\Lambda\Lambda$ correlation function as a function of the scattering length.

length 0, -1, -2, -3, -4, -5, and -10 fm, from bottom to top. The flat distribution of the $\Lambda\Lambda$ correlation function measured at STAR favors a short scattering length (|a| < 10 fm).

7.2 The H^0 dibaryon

Using the MIT bag model, Jaffe [117] predicted a six-quark dibaryon $H^0 = |uuddss\rangle$ in 1976. If its mass is smaller than $2m_{\Lambda}$ (2.231 GeV/ c^2), it is stable against strong decays and will undergo a $\Delta S = 1$ weak decay. If its mass is above the $2m_{\Lambda}$ threshold, the H^0 could be a strong interaction resonance and decay to $\Lambda\Lambda$, and would therefore affect the $\Lambda\Lambda$ correlation function.

Fig. 7.2 [116] shows different $\Lambda\Lambda$ correlation functions with various possible H^0 masses $(m_H - 2m_\Lambda = 10 \text{ to } 30 \text{ MeV})$, and possible H^0 widths of 0.5 MeV and 1 MeV.



Figure 7.2: $\Lambda\Lambda$ correlation function with existence of H^0 . (a) Peaks from left to right correspond to $m_H - 2m_\Lambda = 10$, 20, 30 MeV/ c^2 , respectively, H^0 width is set to be $1 \text{ MeV}/c^2$; (b) $m_H - 2m_\Lambda = 10 \text{ MeV}/c^2$, H^0 width is set to 0.5 MeV (the upper peak) and 1 MeV (the lower peak).

If $m_H > 2m_{\Lambda}$ and its mass width is narrow, the H^0 dibaryon may be observable by measuring $\Lambda\Lambda$ correlation functions.

7.3 The future

The position resolution is expected to be greatly improved with STAR upgrades by installing new detectors inside the TPC. With better tracking resolution, one could relax the V0 cuts and the Λ reconstruction cuts, thus obtain more Λ hyperons in Au+Au collisions.

The DAQ (data acquisition) rate may reach 1k Hz with new DAQ1000 TPC readout and thus an order of magnitude more events could be recorded.

Both factors are crucial in Λ correlation measurements. With more Λ hyperons and more events, we may observe the parity violation or put an upper limit to the winding number Q_w generated per event. With an order of magnitude more statistics, $\Lambda\Lambda$ HBT measurement looks very promising. $\Lambda\Lambda$ scattering length may be obtained.

7.4 Conclusions

The strong parity violation effect may be observable for 55 M central Au+Au events at $\sqrt{s_{NN}} = 200$ GeV for 10% Λ detection efficiency and Q_w with a Gaussian width of 20. For Q_w with Gaussian width 10, the number of events needed rise to as many as 550 M for Λ detection efficiency of 20%.

 $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ correlation functions are found to be flat under the STAR Run 4 central Au+Au collisions of about 20 M events. Simulation shows that 120 M events may be enough to measure these correlation functions. With even more events, different

cuts on the angle between decay protons in its parent Λ rest frame spin θ_{pp} may be applied to study the enhanced spin-selected HBT effect.

The residual HBT effect of Λ feed-down from Σ is found to correspond to a radius of 2.0 ± 0.1 fm with a correlation strength of -0.13 ± 0.01 . Λ feed-down from Ξ shows no visible residual HBT effect.

Appendix A

Kinematic variables

In this appendix, we will use the natural units $c = \hbar = 1$. The conversion constant $\hbar c = 197$ MeV fm is useful.

We designate the beam axis as the z-axis, or longitudinal axis. The transverse directions are perpendicular to the beam axis.

The transverse momentum p_t is defined as

$$p_t = \sqrt{p_x^2 + p_y^2}.$$

The transverse mass of a particle with rest mass m is defined as

$$m_t = \sqrt{m^2 + p_t^2}.$$

The rapidity y is defined as

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}.$$

The pseudo-rapidity η is defined as

$$\eta = -\ln[\tan(\theta/2)] = \frac{1}{2}\ln\frac{p+p_z}{p-p_z},$$

where θ is the angle between the particle momentum **p** and the beam axis. $\eta \approx y$ when $p \gg m$.

Appendix B

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