A Critical Review of Thermalization Issue at RHIC

- Results from STAR

Aihong Tang for the STAR Collaboration

Outline

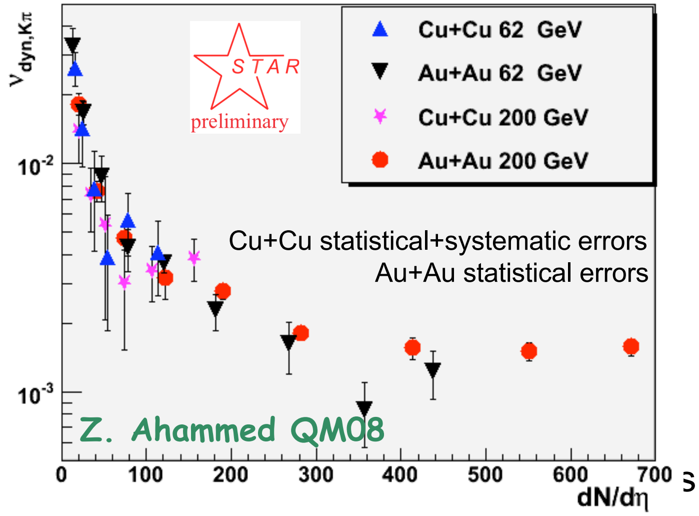
- Scaling Law and the magic of Knudsen number

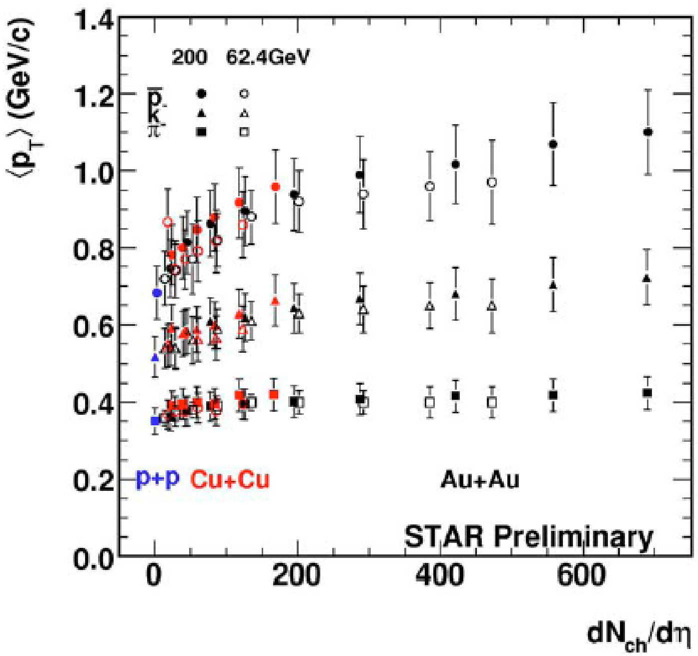
- Thermalization

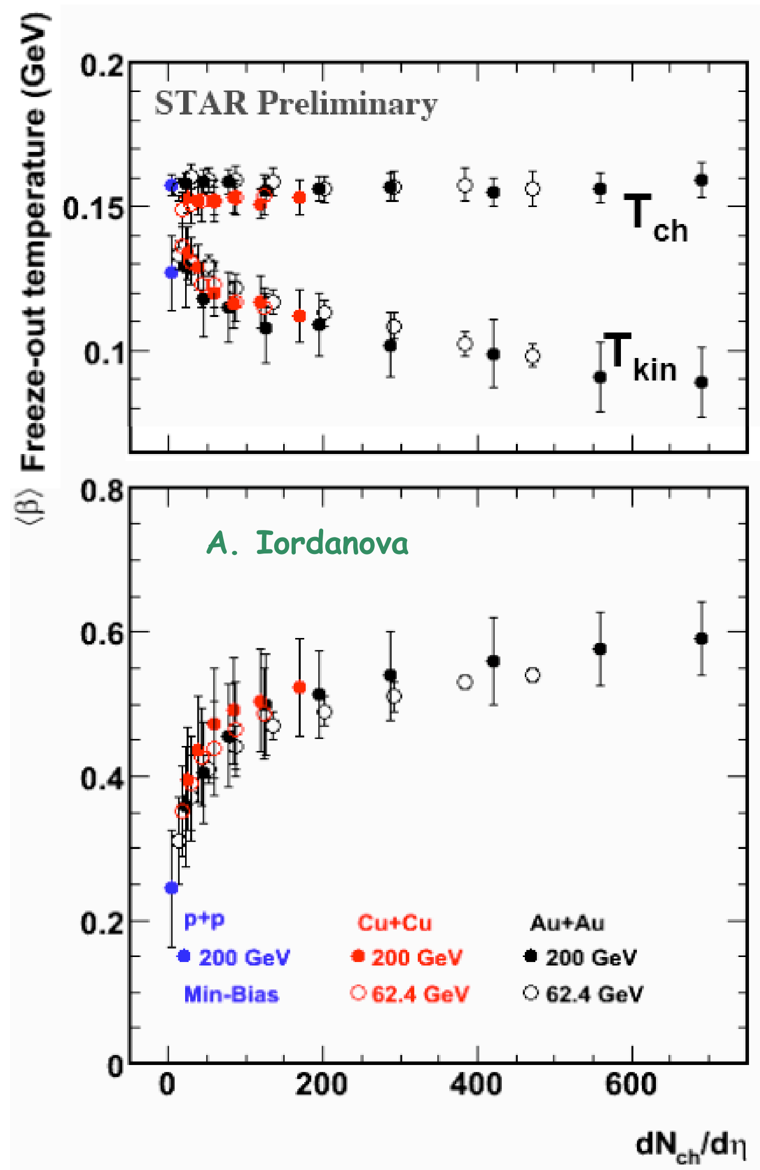
- Summary

Scaling Law and The Magic of Knudsen Number

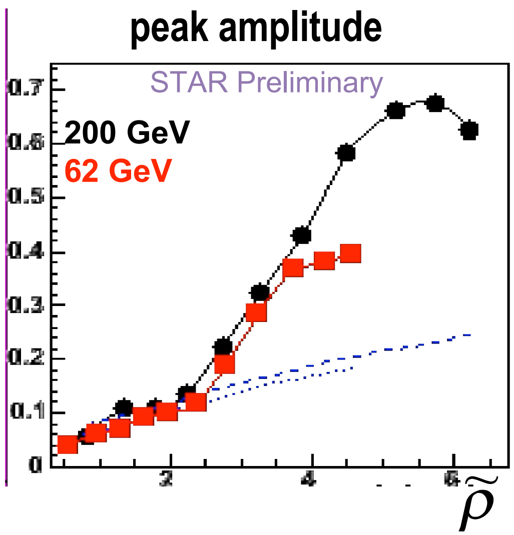
Scale with dN/dη

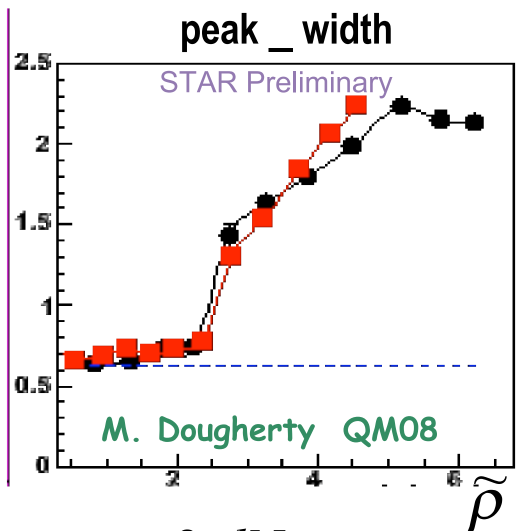






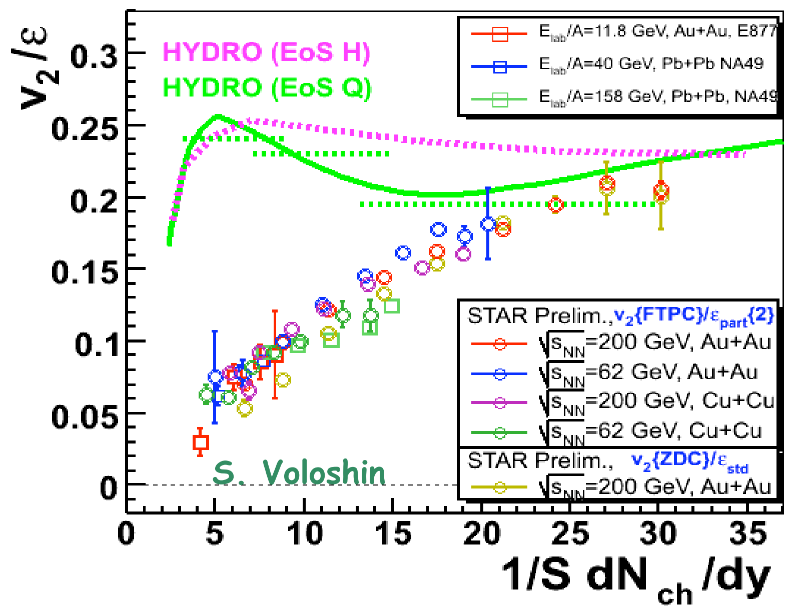
Scale with 1/S dN/dy



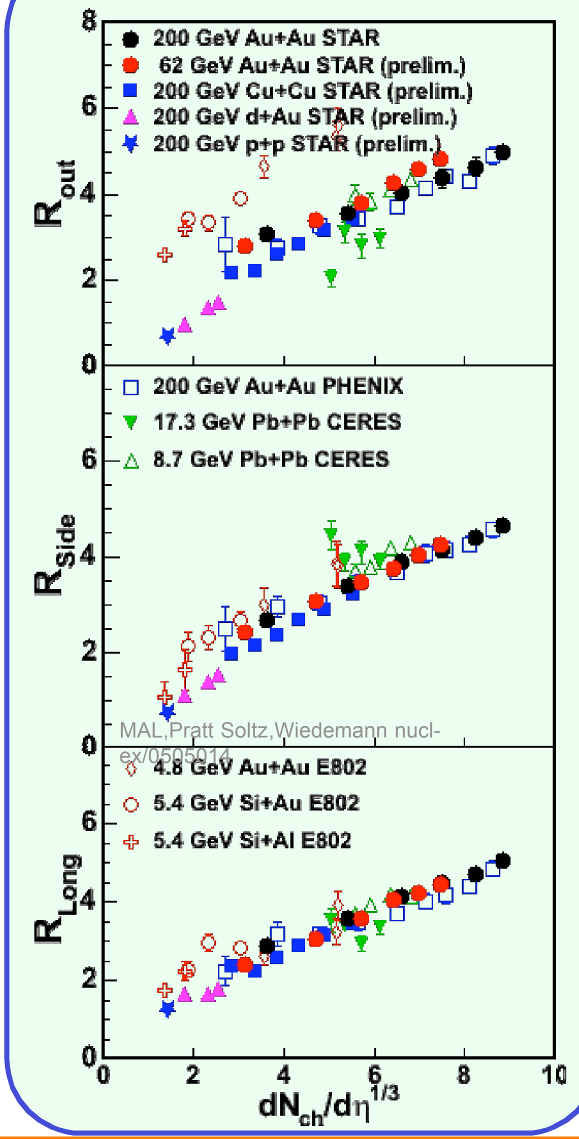


Transverse particle density

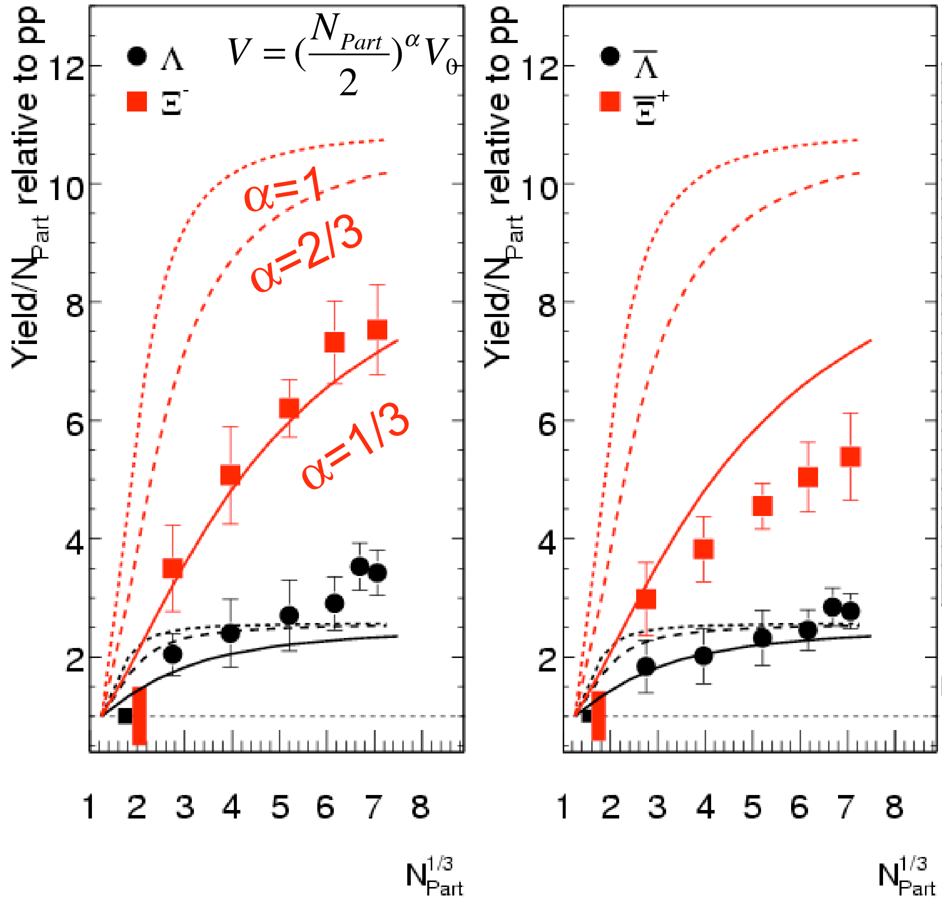
$$\tilde{\rho} = \frac{3}{2}\frac{dN_{ch}}{d\eta} / S$$



Scale with $X^{1/3}$



M. Lamont, SQM06. Curve : K. Redlich



D. Das & Z. Chajeck

Collecting Evidences and Connecting Pieces

Scaling variables I have shown so far:

$$\frac{1}{S}\frac{dN}{dY}, \frac{dN}{d\eta}, \left(\frac{dN}{d\eta}\right)^{1/3}, N_{part}^{1/3}$$

Albeit in different formats, they are sensitive to the same quantity :

$$\frac{1}{K} \equiv \frac{R}{\lambda}$$

← **Number of collisions.**
Local thermal equilibrium is achieved if $k^{-1} >> 1$ (will come back to this in the 2nd part of this talk)

***K*: Knudsen number**
R: system size
λ: mean free path
n: particle density
σ: parton cross section

$$\lambda = \frac{1}{\sigma n}$$

$$n = \frac{1}{ct}\frac{1}{S}\frac{dN}{dy}$$

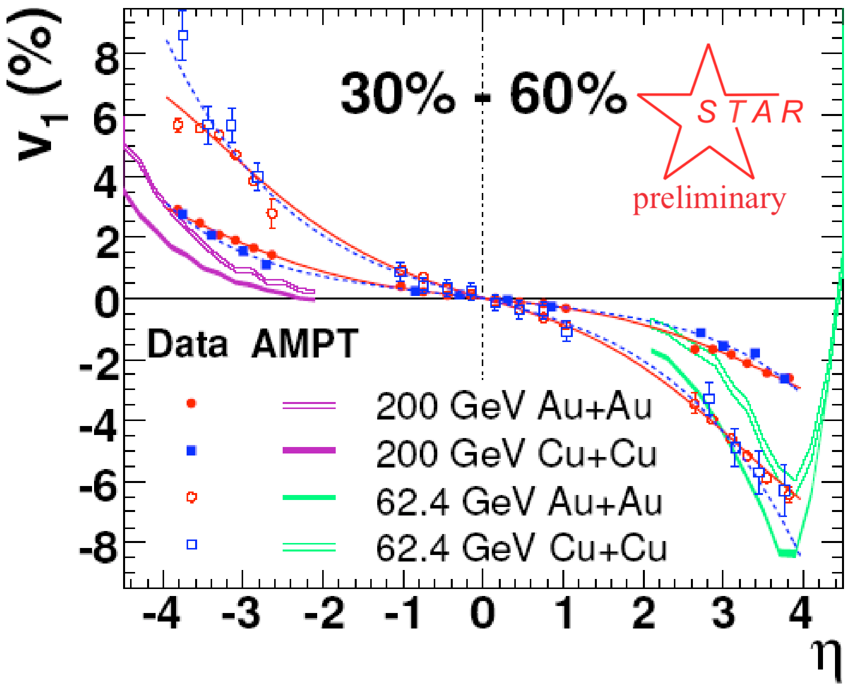
$$t \sim R / c_s$$

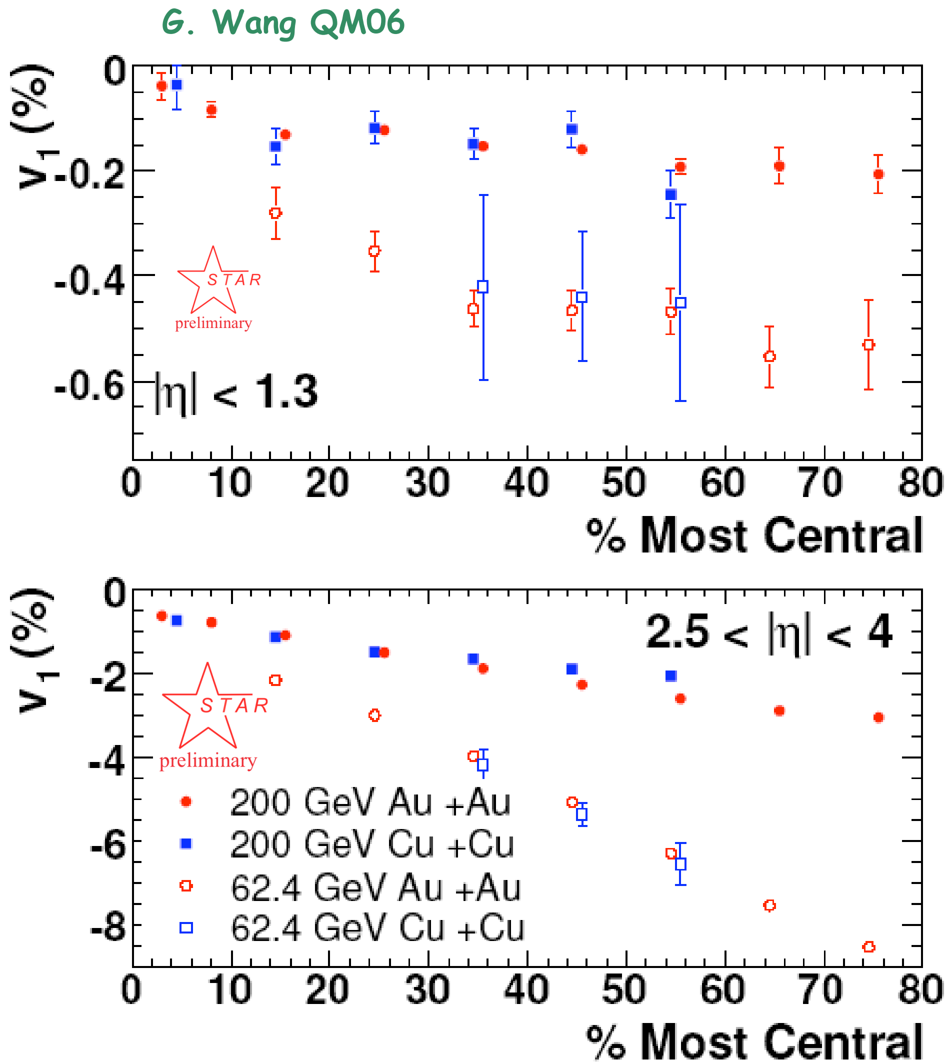
$$\frac{1}{K} = \sigma \boxed{\frac{1}{S}\frac{dN}{dy}} \frac{c_s}{c}$$

Lots (but not all) of physics seem to be driven by the number of collisions per particle encountered on its way out !

In many cases a better linearity is seen if plotted against $x^{1/3}$. Proportional to 1/K ?

Non-equilibrium physics does not scale with dN/dy

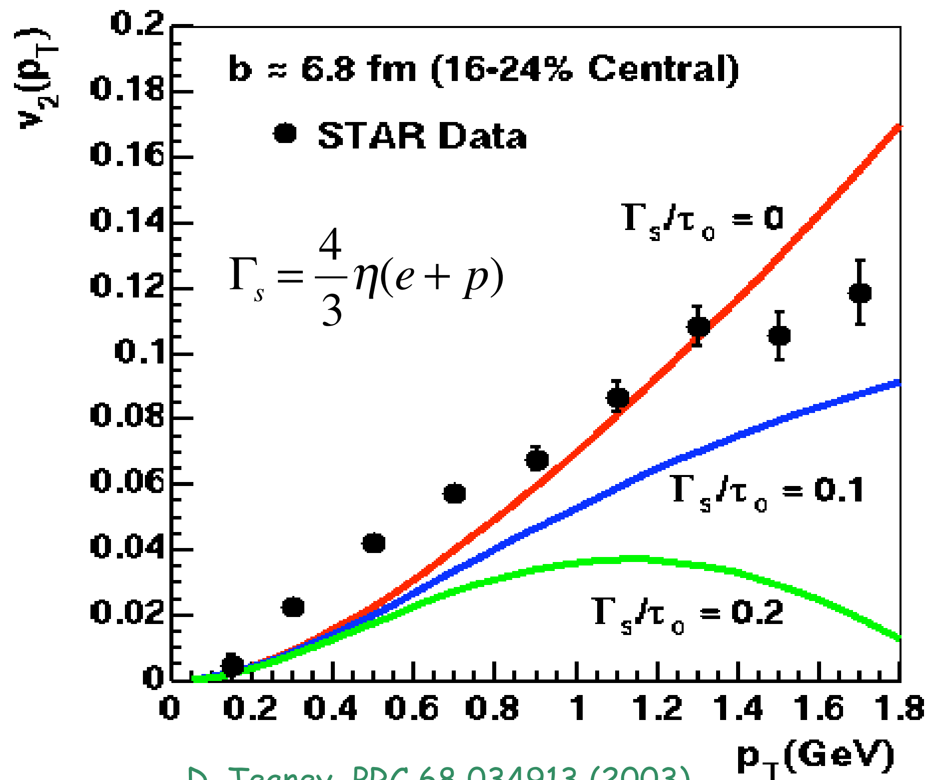




v_1 scales with the geometry but not dN/dy.

Thermalization

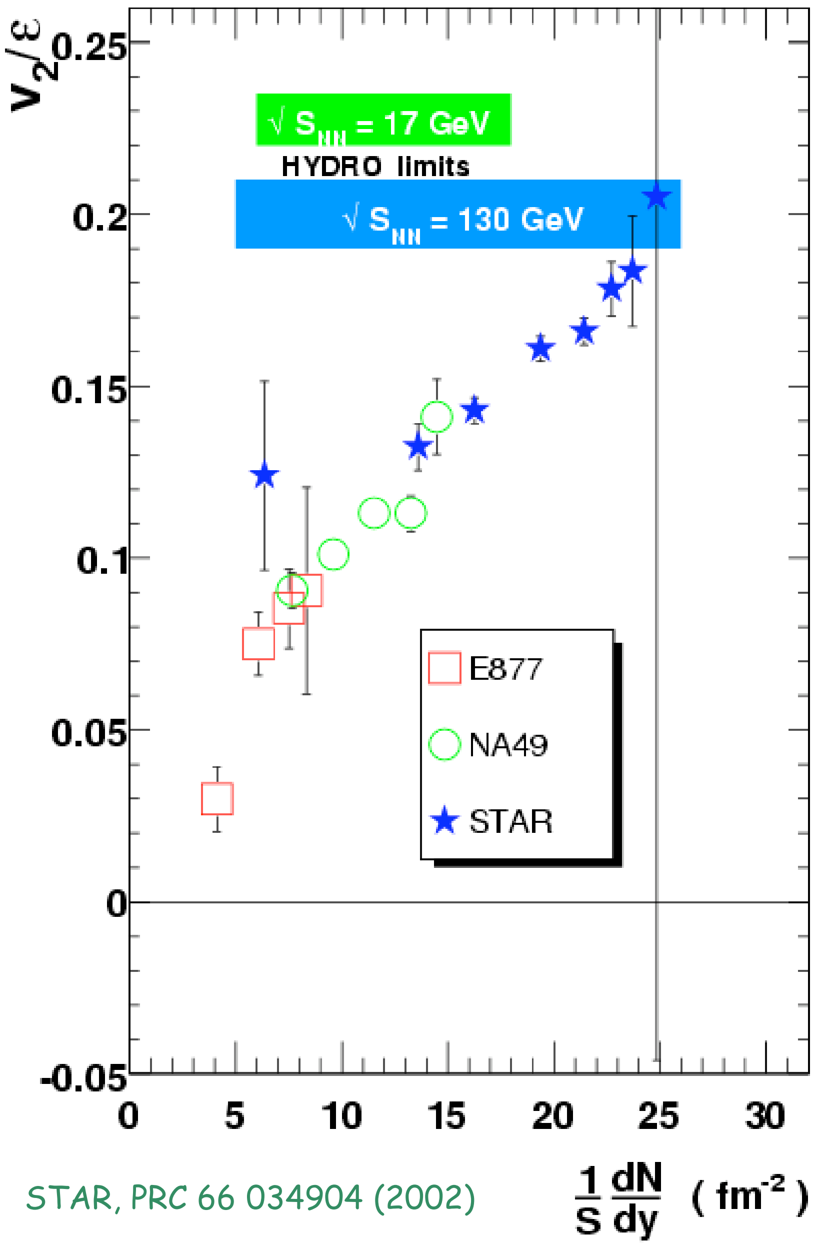
A Histrorical Review of Perfect Liquid



D. Teaney, PRC 68 034913 (2003)

Viscosity reduces v_2
Viscosity needs to be small in order to explain data

Revisit the v_2/ε plot



STAR, PRC 66 034904 (2002)

Part I :

- There are many v_2 methods, what is the relation among them?
- There are many ways to calculate the eccentricity, which one to choose?

Part II :

- Is the hydrodynamic limit really saturated?
- What is the trend we should expect if the requirement on local equilibrium is relaxed?

v_2 methods

Define: $\varepsilon = \{\varepsilon_x, \varepsilon_y\} = \{\left\langle \frac{\sigma_y^2 - \sigma_x^2}{\sigma_x^2 + \sigma_y^2} \right\rangle_{part}, \left\langle \frac{2\sigma_{xy}}{\sigma_x^2 + \sigma_y^2} \right\rangle_{part}\}$ we have

$$\varepsilon_x \equiv \varepsilon_{RP}$$
$$\langle \varepsilon_x \rangle \approx \varepsilon_{optical}$$
$$\varepsilon_{part} \equiv \varepsilon_{PP} = \sqrt{\varepsilon_x{}^2 + \varepsilon_y{}^2}$$

Pdf of ε_{part} :

$$\frac{dn}{d\varepsilon_{part}} = \frac{\varepsilon_{part}}{\sigma_\varepsilon^2} I_0\left(\frac{\varepsilon_{part}\langle \varepsilon_{RP} \rangle}{\sigma_\varepsilon^2}\right) \exp\left(-\frac{\varepsilon_{part}^2 + \langle \varepsilon_{RP} \rangle^2}{2\sigma_\varepsilon^2}\right) \equiv BG\left(\varepsilon_{part}; \langle \varepsilon_{RP} \rangle, \sigma_\varepsilon\right)$$

Pdf of participant v_2 :

$$\frac{dn}{dv_2{}'} = \frac{v_2{}'}{\sigma_{v_2,dyn}^2} I_0\left(\frac{v_2{}'\langle v_2 \rangle}{\sigma_{v_2,dyn}^2}\right) \exp\left(-\frac{v_2{}'^2 + \langle v_2 \rangle^2}{2\sigma_{v_2,dyn}^2}\right)$$

Pdf of q :

$$\frac{dn}{dq} = \frac{q}{\sigma^2} I_0\left(\frac{v_2 q\sqrt{M}}{\sigma^2}\right) \exp\left(-\frac{q^2 + M v_2^2}{\sigma^2}\right)$$

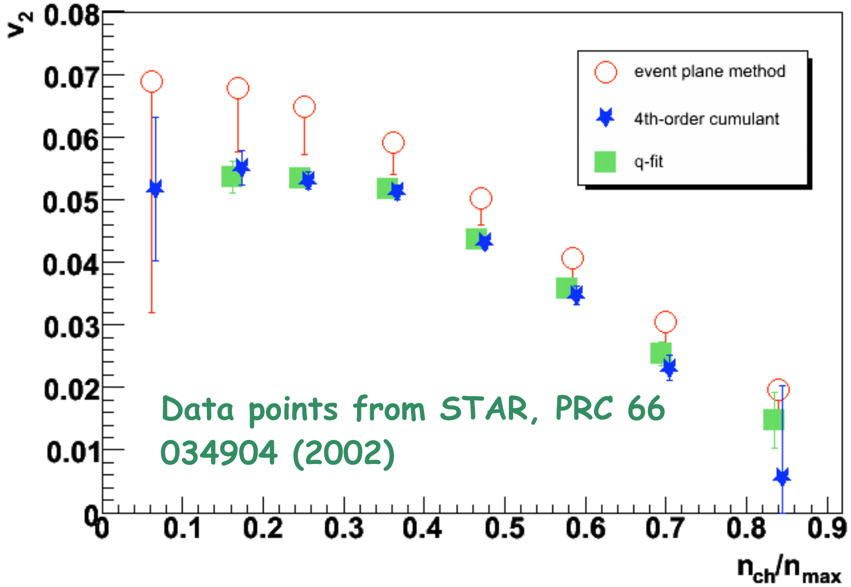
where

$$q_{n,x} = \frac{1}{\sqrt{M}} \sum_{i=1}^{M} \cos(n\varphi_i)$$
$$q_{n,y} = \frac{1}{\sqrt{M}} \sum_{i=1}^{M} \sin(n\varphi_i)$$

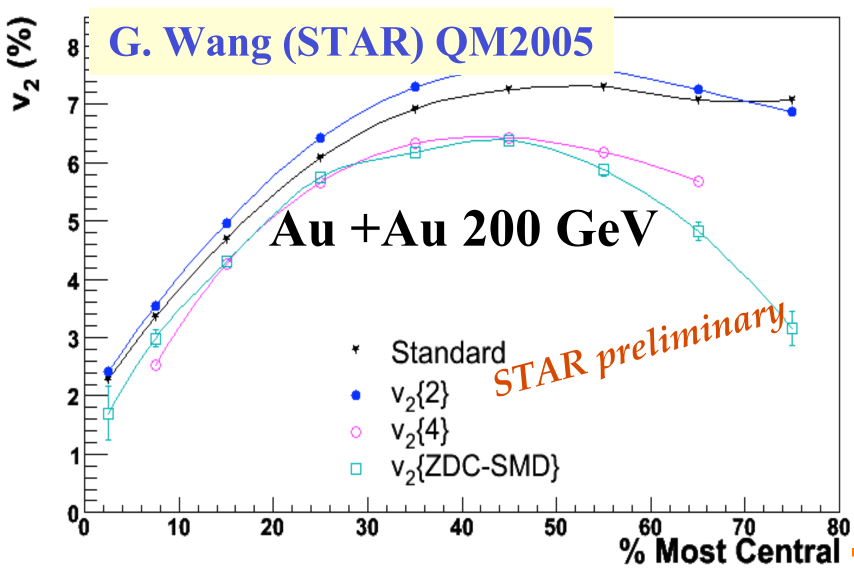
$$2\langle x^2 \rangle^2 - \langle x^4 \rangle = \bar{x}^4$$
$$\langle x^6 \rangle - 9\langle x^4 \rangle \langle x^2 \rangle + 12\langle x^2 \rangle^3 = 4\bar{x}^6$$

R.Bhalerao and J-Y. Ollitrault, Phys. Lett. B 614 (2006) 260
S.Voloshin, A.Poskanzer, A.Tang and G.Wang, Phys. Lett. B 659 (2008) 537

v_2 methods



$v_2\{4\}=v_2\{qDist\}$



$v_2\{4\}=v_2\{ZDC\text{-}SMD\}$

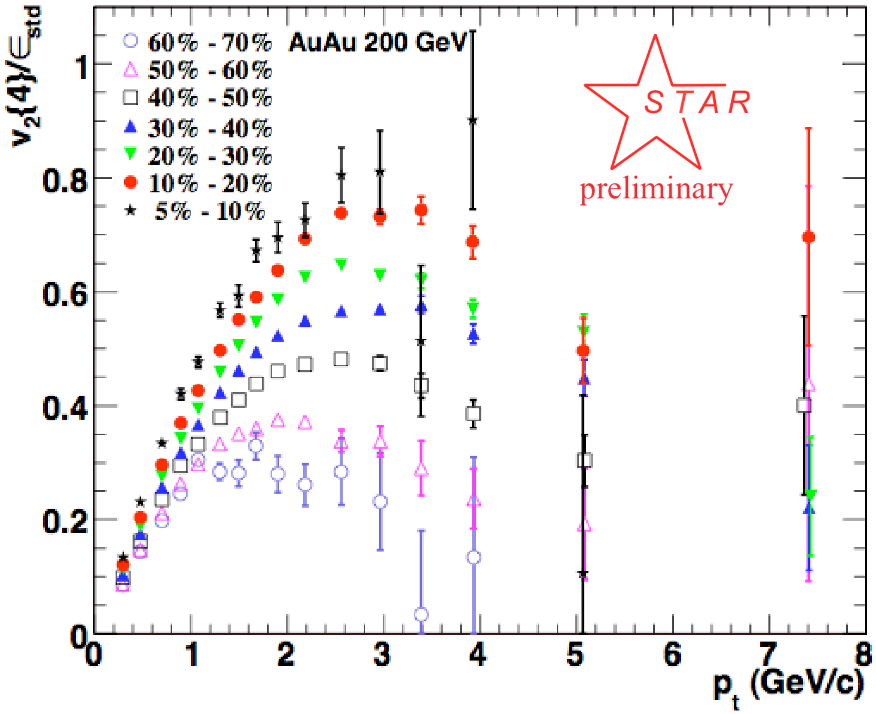
Choose the right $\{v_2, \varepsilon\}$ pairs

v_2 that are sensitive to anisotropy w.r.t. the **Reaction Plane v_2:** $v_2\{4\}$, $v_2\{qDist\}$, $v_2\{qCumulant4\}$, $v_2\{ZDCSMD\}$	v_2 that are sensitive to anisotropy w.r.t. the **Participant Plane :** $v_2\{2\}$,$v_2\{EP\}$,$v_2\{uQ\}$ etc.
ε that are sensitive to anisotropy w.r.t. the **Reaction Plane**: $\varepsilon\{std\}$, $\varepsilon\{4\}$	ε That are sensitive to anisotropy w.r.t. the **Participant Plane:** $\varepsilon\{part\}$ $\varepsilon\{2\}$

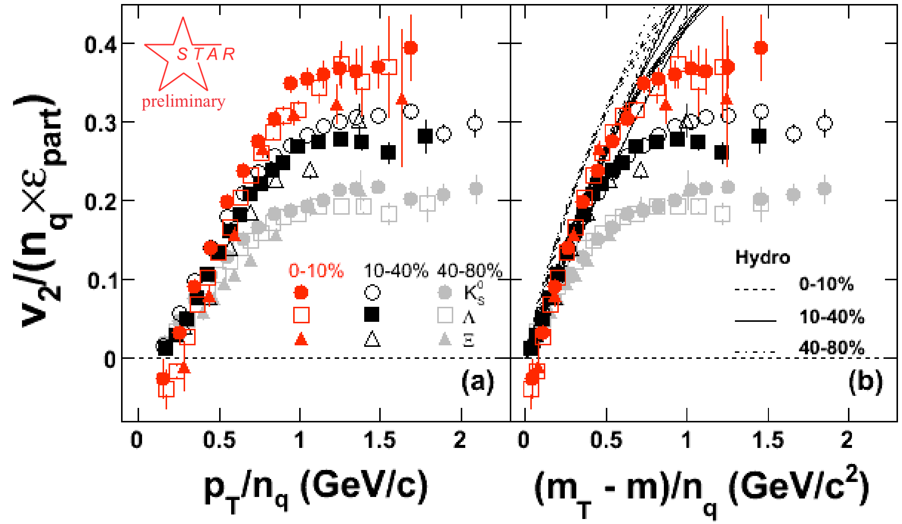
In this slide, I assume that nonflow has been suppressed by external techniques (such as pseudorapidity gap etc.) in v_2 measurements that are based on two particle correlations ($v_2\{2\}$,$v_2\{EP\}$,$v_2\{uQ\}$).

R.Bhalerao and J-Y. Ollitrault, Phys. Lett. B 614 (2006) 260
S.Voloshin, A.Poskanzer,A.Tang and G.Wang, Phys. Lett. B 659 (2008) 537

Flow Increases



Y. Bai, Ph.D. Thesis, STAR.



Y. Lu, QM08

$v_2\{4\}/\varepsilon_{std}$ (or $v_2\{EP\}/\varepsilon_{part}$) increases with centrality over large p_t range
Peak position of $v_2\{4\}$ moves to higher transverse momentum with increasing centrality

How to view the hydro behavior better ? - Move away from it

- Ideal fluid and low viscosity ⇔ local equilibrium (small λ or large σ)

- **To study the local equilibrium, we have to move away from it**, say, check what if we relax the constraint of local equilibrium

- How to get a complete view? Study Boltzman equation for diluted system. It recovers Hydro when λ becomes small.

"To have a complete view of Lu Mountain, one has to move away from it."
- Shi Su (1037~1101)

Transport Theory and Hydrodynamics

Transport Theory	Hydrodynamics
Microscopic	Macroscopic
Applicable out of equilibrium	Local equilibrium
Cannot describe phase transition	Can treat phase transition
D<<1	K<<1

D (Dilution parameter) = $\frac{\text{Typical distance between two particles}}{\text{Mean free path}}$

K (Knudsen number) = $\frac{\text{Mean free path}}{\text{System size}}$

Boltzmann Equation will be reduced to Hydrodynamics when both D<<1 and K<<1

How much deviation from ideal hydro ?

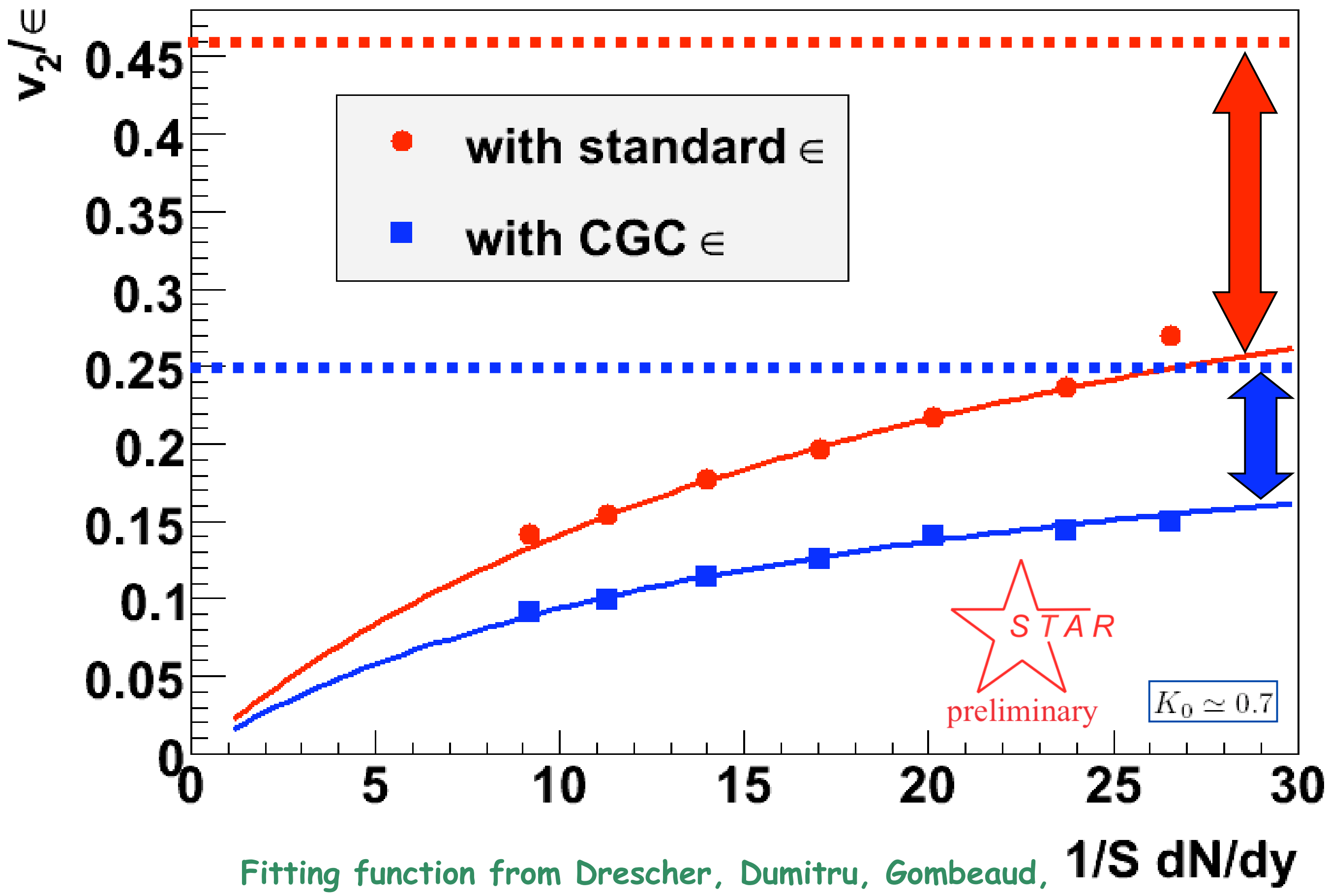
$$\frac{v_2}{\varepsilon} = \left[\frac{v_2}{\varepsilon}\right]_{Perfect} \frac{1}{1 + K / K_0}$$

$$K = \lambda / R$$

$$\frac{1}{K} = \frac{\sigma}{S}\frac{dN}{dy}C_s$$

For the case with Standard ε :
σ=4.3mb, v_2/ε=0.46.
For 20-30% K=0.85

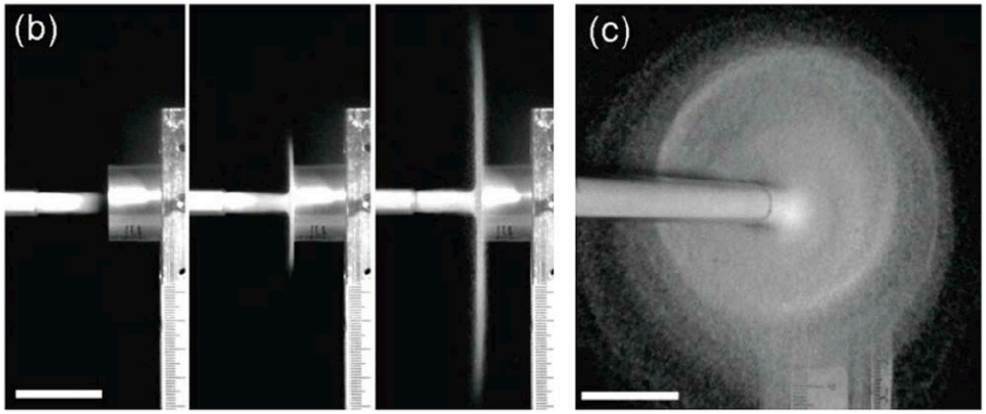
For the case with CGC ε :
σ=5.7mb, v_2/ε=0.25.
For 20-30% K=0.56



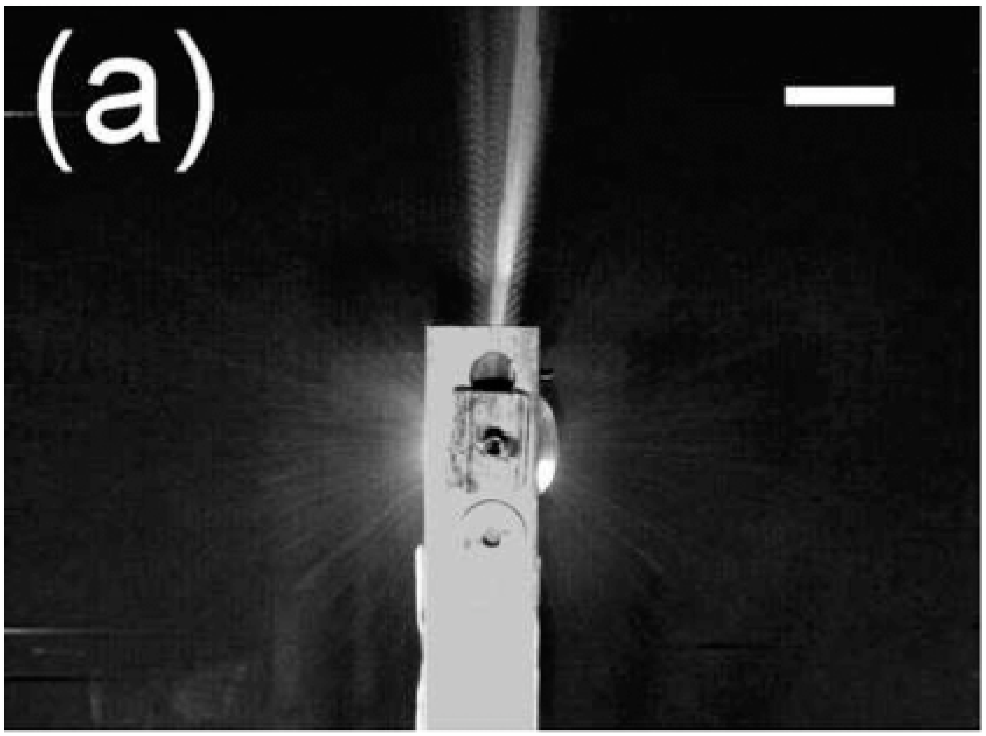
Fitting function from Drescher, Dumitru, Gombeaud, J.Ollitrault, Phys. Rev. C76, 024905(2007)
CGC ε obtained from A.Adil, H-J Drescher, A.Dumitru, A.Hayashigaki and Y.Nara, Phys. Rev. C 74 044905 (2006)

Dashed lines are hydro limit from fitting the data (as opposed to a pure theoretical calculation as as adopted before)
~40% away from ideal hydro even in central collisions

Is hydro limit saturated ? Let's check a classical example



A jet of sand deforms into an extraordinarily thin symmetric granular sheet clearly resembling a spreading liquid.

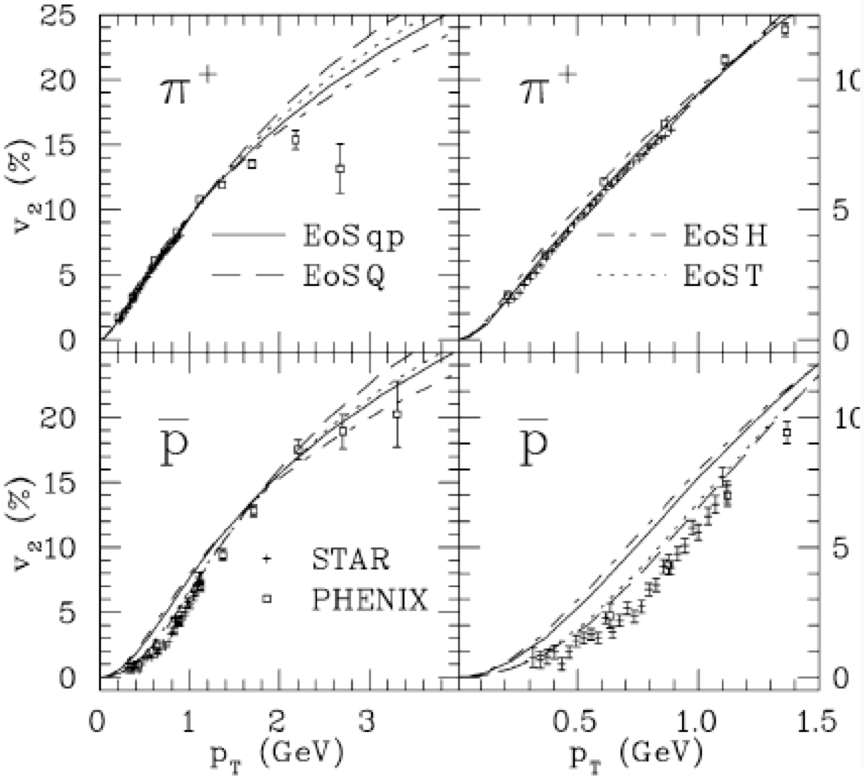


A sharply focused azimuthal pattern is seen if the target has a rectangular shape.

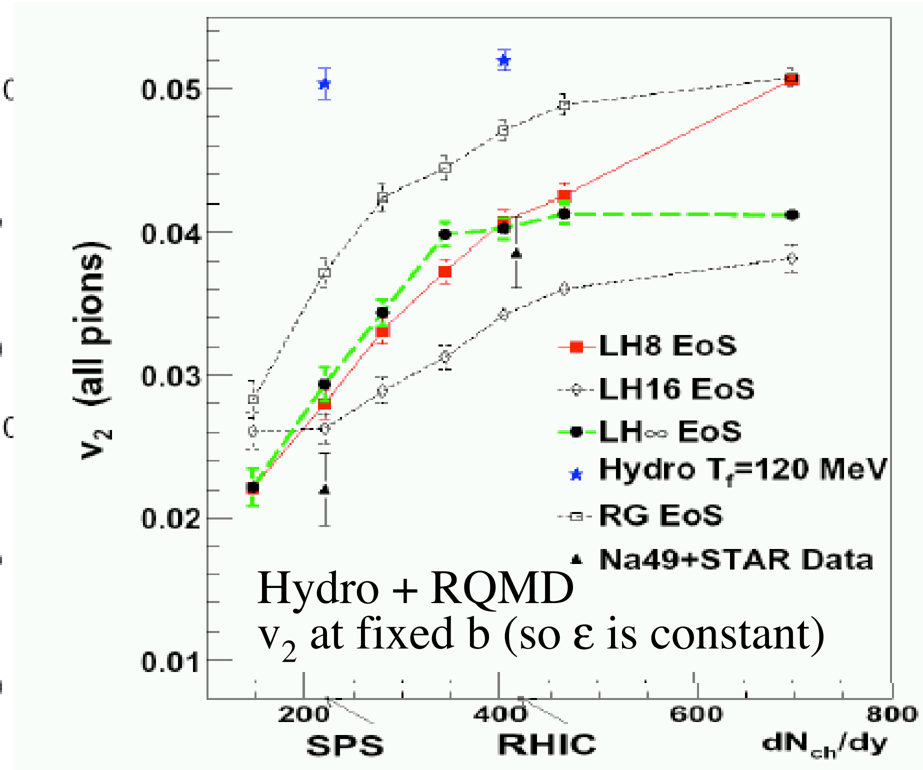
v_2/ε = 0.26 ~ comparable to central AuAu collisions at RHIC ! (shall we believe that it behaves like ideal hydro as well ? ☺)

X. Cheng, G. Varas, D. Citron, H. Jaeger and S. Nagel, Phys. Rev. Lett. 99 188001 (2007)

Is hydro limit saturated ? Let's check different EoSs



P. Houvine Nucl. Phys. A 761 296 (2005)

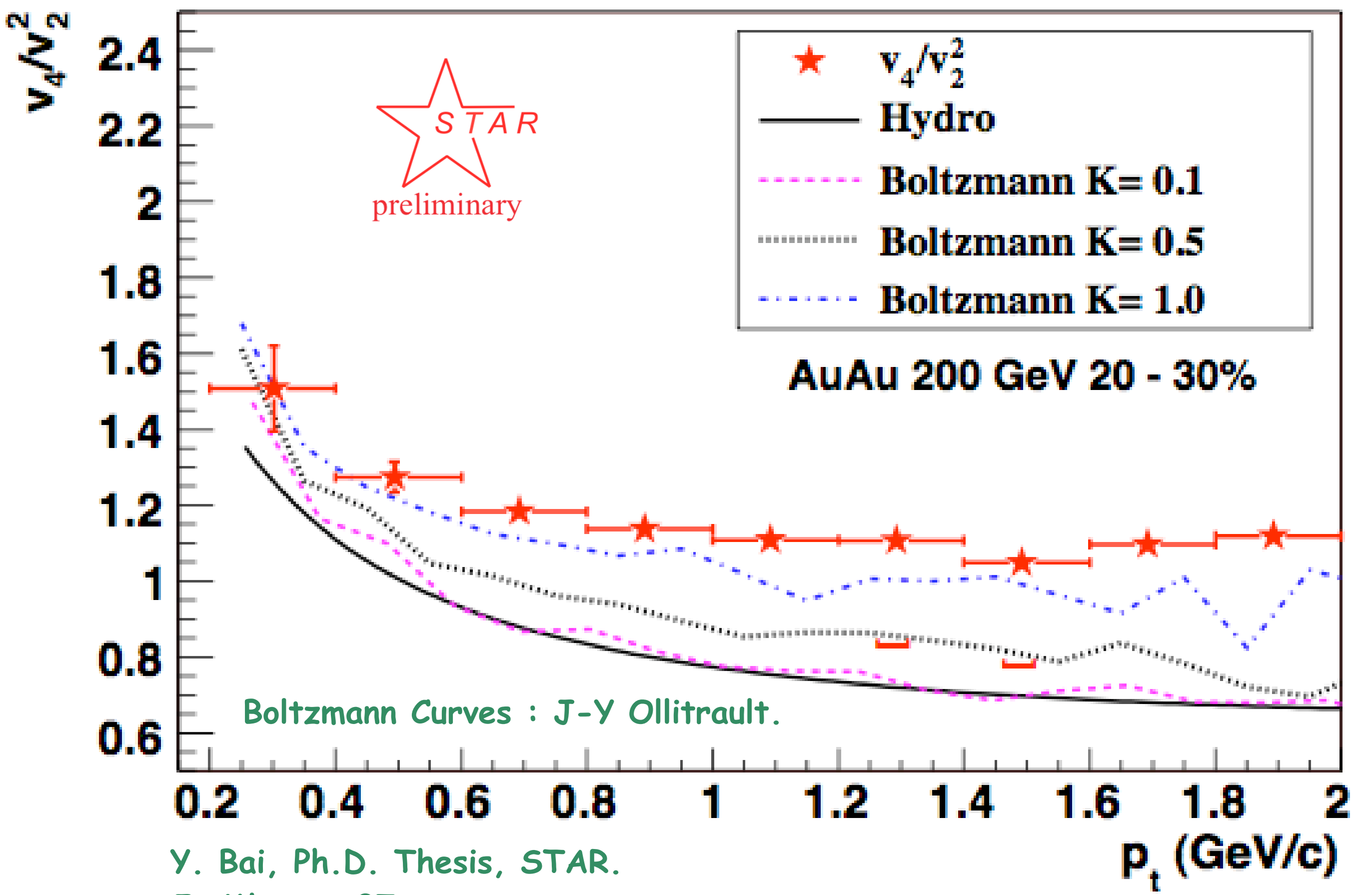


D. Teaney, J.Lauret and E.Shuryak nucl-th/0110037

An EoS with a rapid crossover over predicted the flow

Hydro with a non-const speed of sound v_2/ε increases with dN/dy. Sensitive to different EoSs

How much deviation from ideal hydro ?



Y. Bai, Ph.D. Thesis, STAR.
J. Mlynarz 07

An improved analysis since QM06
Considerable deviation from ideal Hydro
Hints of incomplete thermalization ?

v_4 Systematics From v_2

v_4 of particle i at a certain p_t can be obtained by three-particle (i, j, k) correlations:

$$\langle \cos(4\phi_i - 2\phi_j - 2\phi_k) \rangle = v_4(p_t) v_2^2, \quad (4.2)$$

where the average is taken over all the particles and events. The dominant non-flow contribution to the three particle correlation can be estimated as follows: if particle i is correlated with particle j by non-flow and correlated with particle k by flow, the three-particle non-flow correlations can be written like:

$$g_2 \times \langle \cos(2\phi_i - 2\phi_k) \rangle = g_2 \times v_2\{4\}(p_t) v_2 \quad (4.3)$$

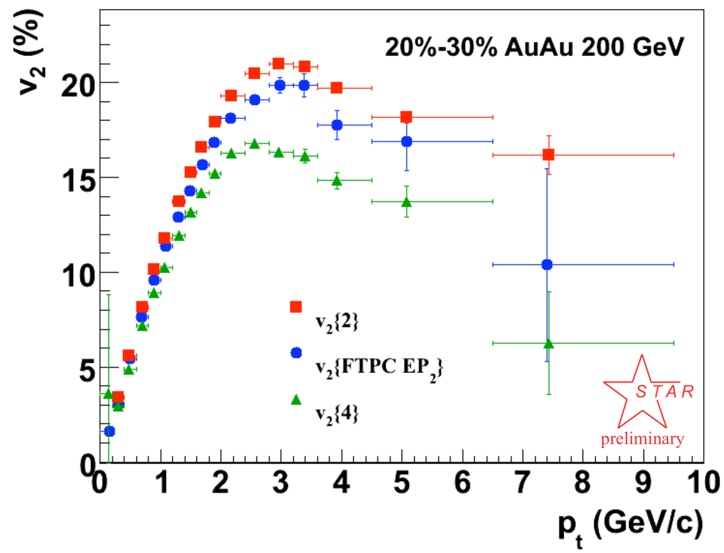
where g_2 is the non-flow contribution from two-particle correlations. It is shown that $g_2 \propto v_2^2\{2\}(p_t) - v_2^2\{4\}(p_t)$ [49, 95]. Therefore, the non-flow contributions to $v_4(p_t)$ is obtained by:

$$\frac{g_2 \times v_2\{4\}(p_t) v_2}{v_2^2} = \frac{(v_2^2\{2\}(p_t) - v_2^2\{4\}(p_t)) \times v_2\{4\}(p_t)}{v_2}. \quad (4.4)$$

The non-flow contributions to v_4/v_2^2 is then estimated by

$$\frac{v_2^2\{2\}(p_t) - v_2^2\{4\}(p_t)}{v_2 v_2\{4\}(p_t)}. \quad (4.5)$$

Y. Bai, Ph.D. Thesis, STAR.



The first order systematics in v_4 is from flow*nonflow term

The nonflow term is from v_2 nonflow (not v_4)

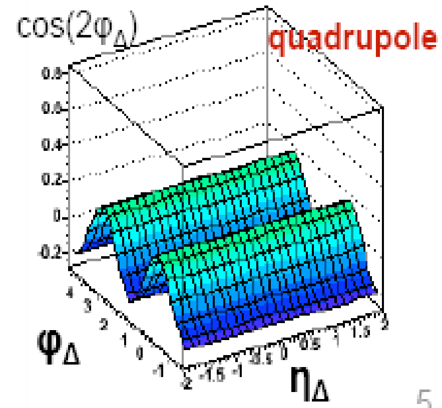
The difference between v_2\{FTPC\} and v_2\{4\} is used in the estimation of nonflow of v_4.

Insight from pair-wise particle correlations

M. Dougherty QM08

Au-Au fit function

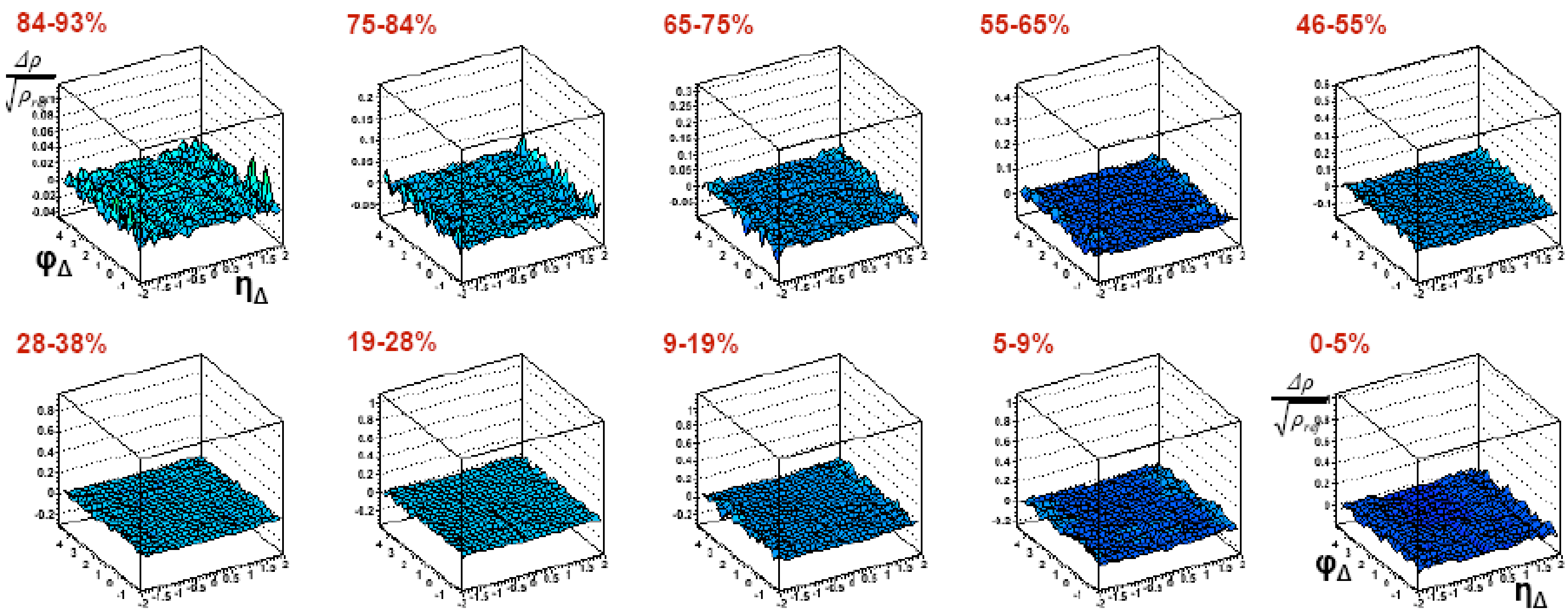
Use proton-proton fit function + $\cos(2\varphi_\Delta)$ quadrupole term ("flow").
This gives the ***simplest possible*** way to describe Au+Au data.



Small residual indicates goodness of fit

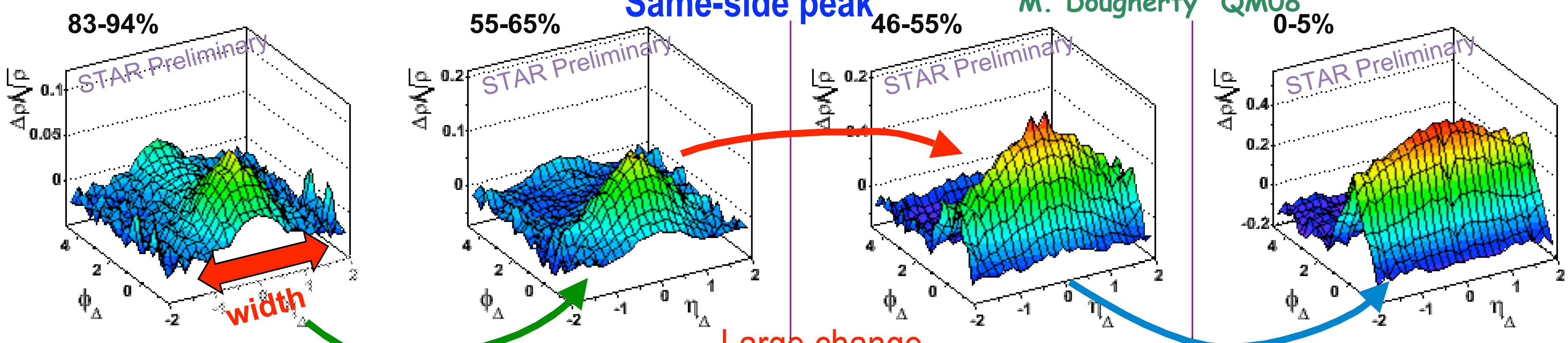
Fit residual = data - model

STAR Preliminary



Study of minijet correlations in Au+Au collisions

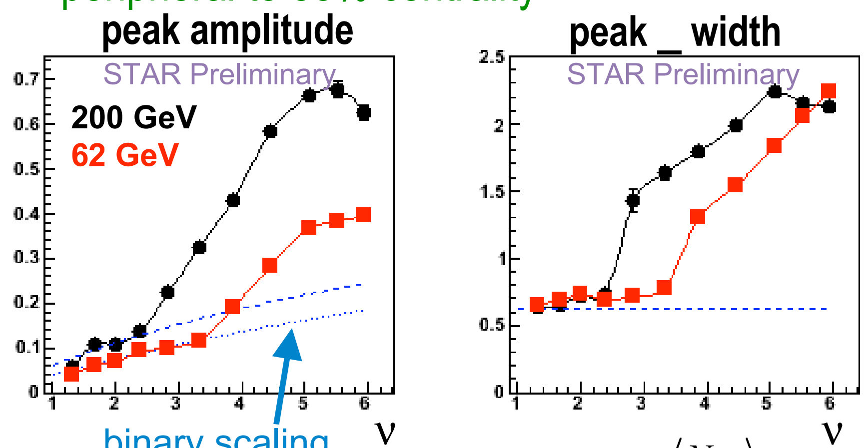
Same-side peak — M. Dougherty QM08



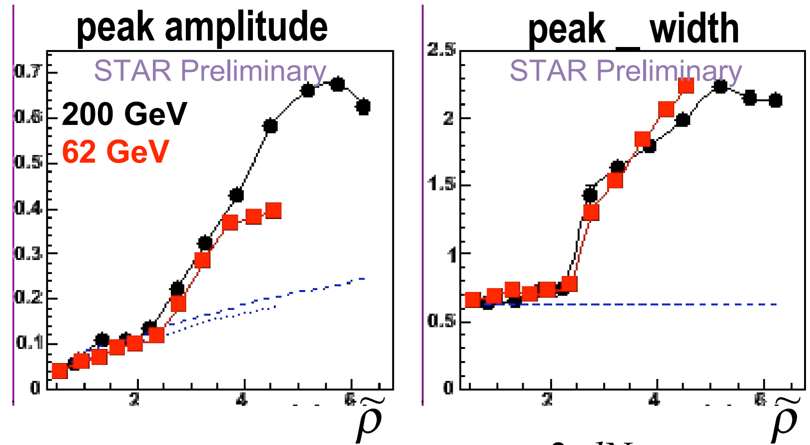
Little shape change from peripheral to 55% centrality

Large change within ~10% centrality

Smaller change from transition to most central



$$\nu \equiv \frac{\langle N_{bin} \rangle}{\langle N_{part}/2 \rangle}$$



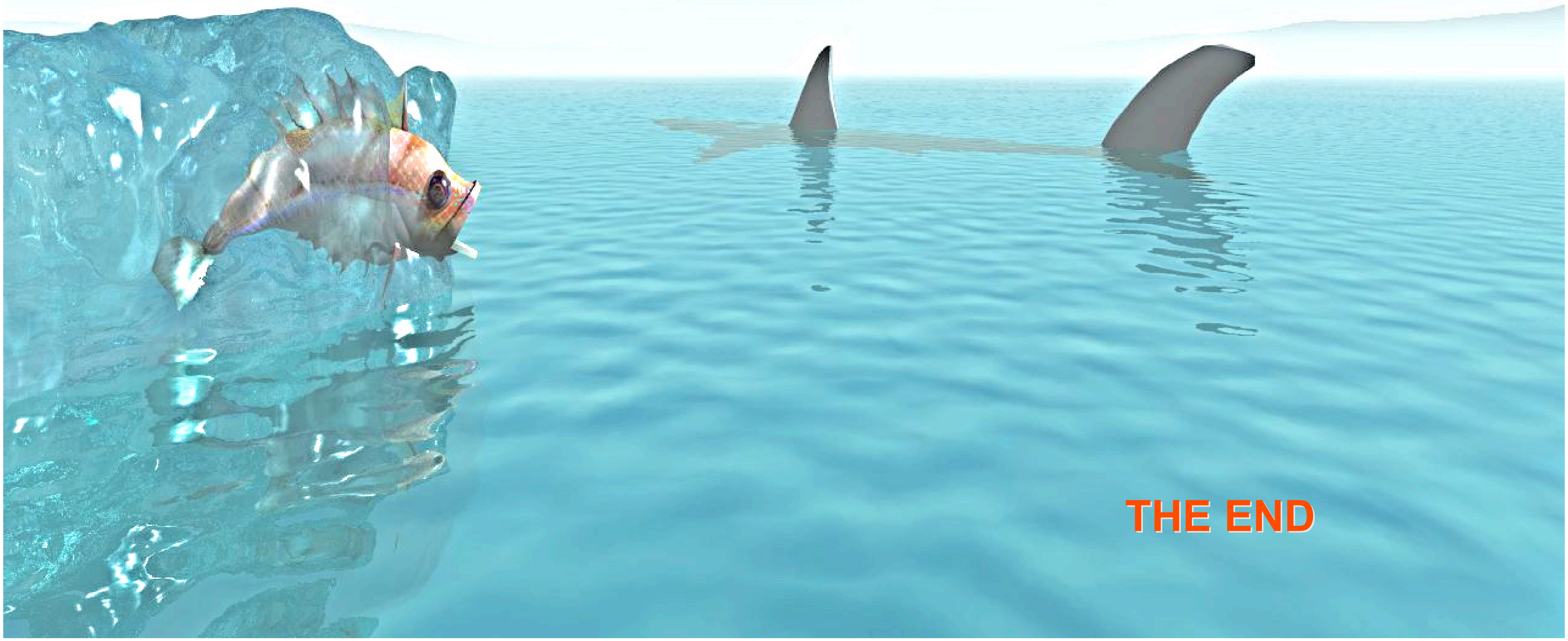
Transverse particle density $\tilde{\rho} = \frac{3}{2}\frac{dN_{ch}}{d\eta}/S$

Binary scaling reference followed until sharp transition at ν~2.5.
Minijet fragments comprise ~20% of the total yield in central AuAu collisions - **can it co-exist with a complete thermalization ?**

An Inconvenient Truth

(not really related to global warming)

- Many physics are driven by the Knudsen number, which when small, a thermal equilibrium is considered reached. While it is generally accepted that Hydrodynamics did a good job, for the first time, in describing RHIC's data, there are features that are not consistent with a complete thermalization, and they cannot be easily dismissed.



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