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Overview of STAR Measurements on Correlations and Fluctuations

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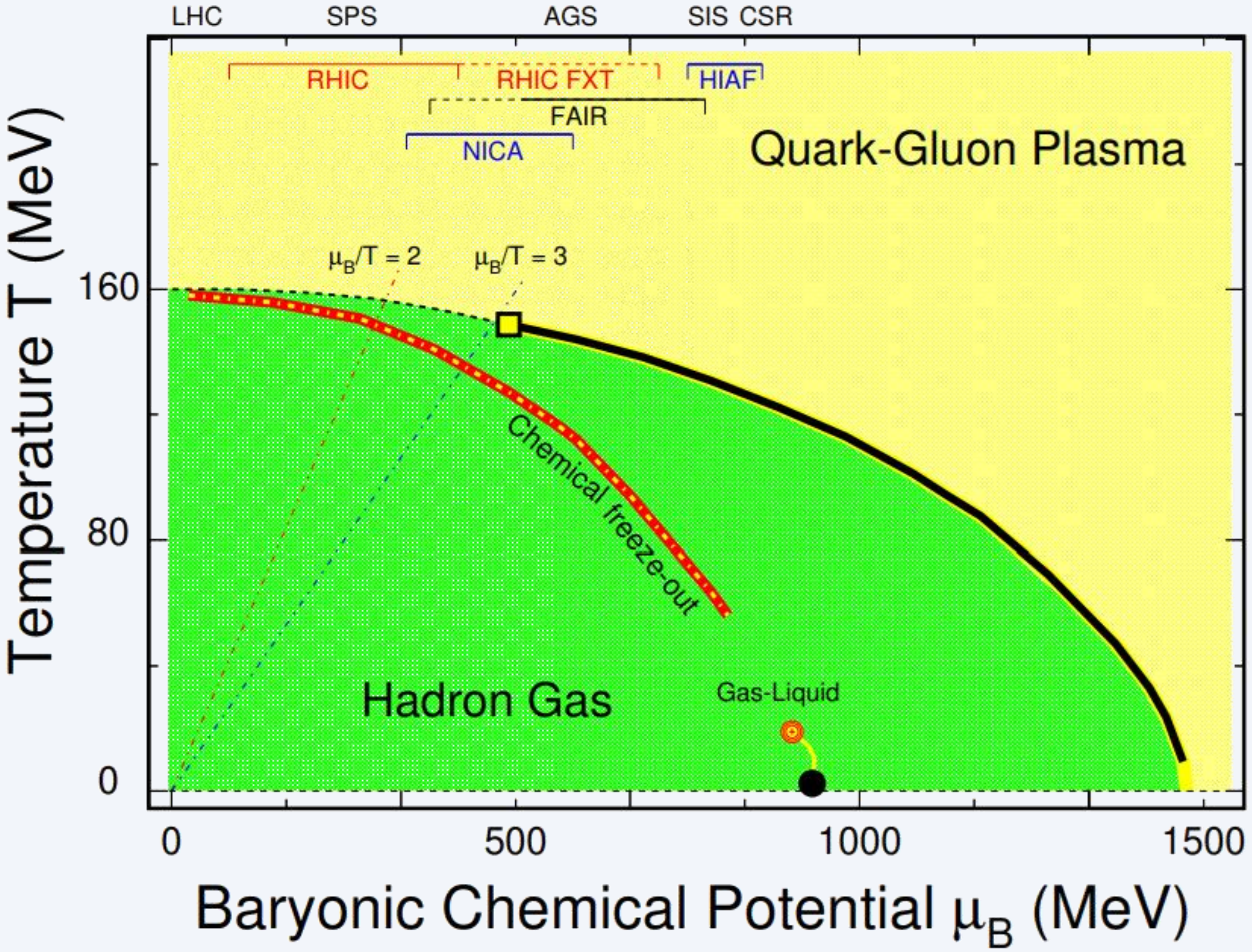
for the STAR collaboration

STAR

Supported in part by U.S. DEPARTMENT OF ENERGY | Office of Science

2024.08.28

QCD Phase Diagram



B. Mohanty and N. Xu, in Criticality in QCD and the Hadron Resonance Gas (2021) arXiv:2101.09210

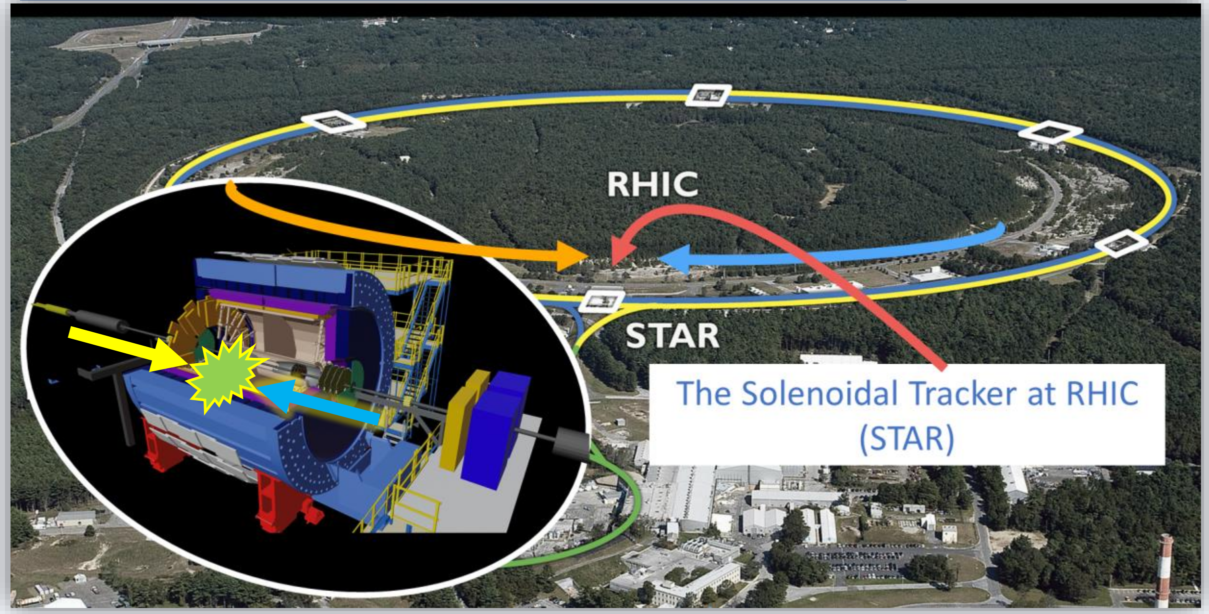
Phase structure:

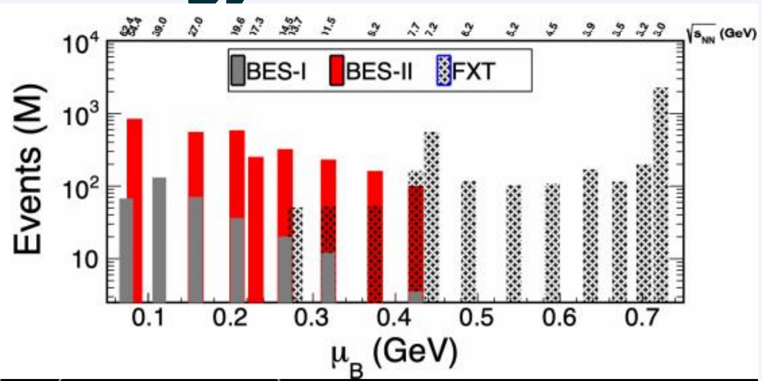
1. **QCD Critical End Point**
   - ❖ Crossover at small μ_B $\left(\frac{\mu_B}{T} < 3\right)$ **Lattice**
   - ❖ **1st order P.T. at large μ_B**
   - ❖ **Critical end point** **?**
   - ❖ **Recent prediction $\mu_B \sim 500 - 700$ MeV**

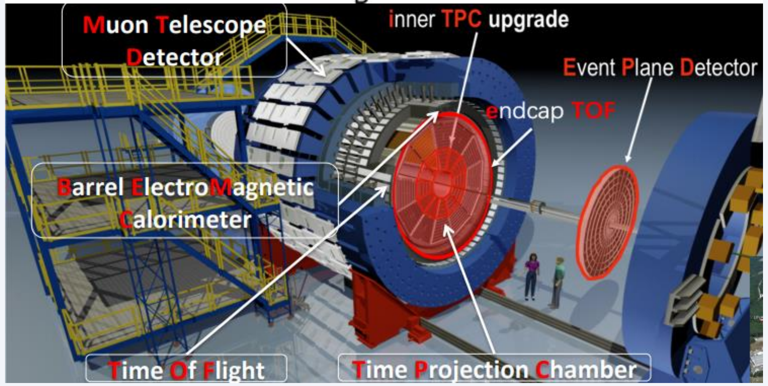
2. **Equation of State and interaction at high μ_B**
   - ❖ Structure of nuclear and hyper-nuclei matter
   - ❖ Mapping NN, YN, and NNY interactions

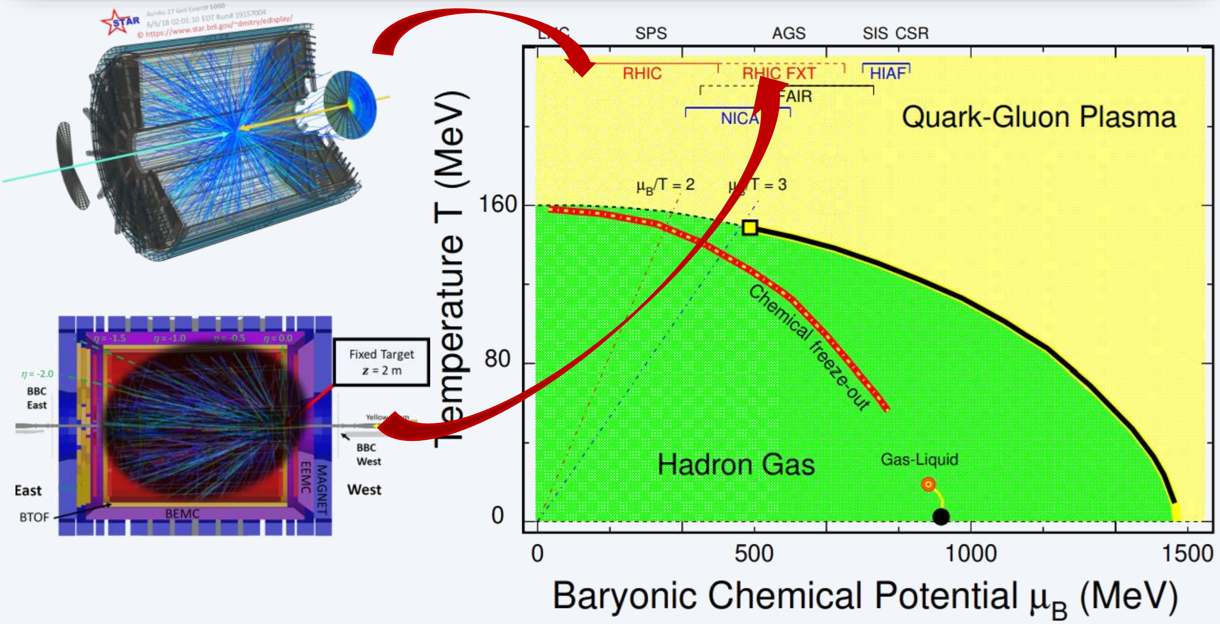
D. A. Clarke, et al. (2024), arXiv:2405.101966

Heavy Ion Collision Experiment & STAR Beam Energy Scan – II



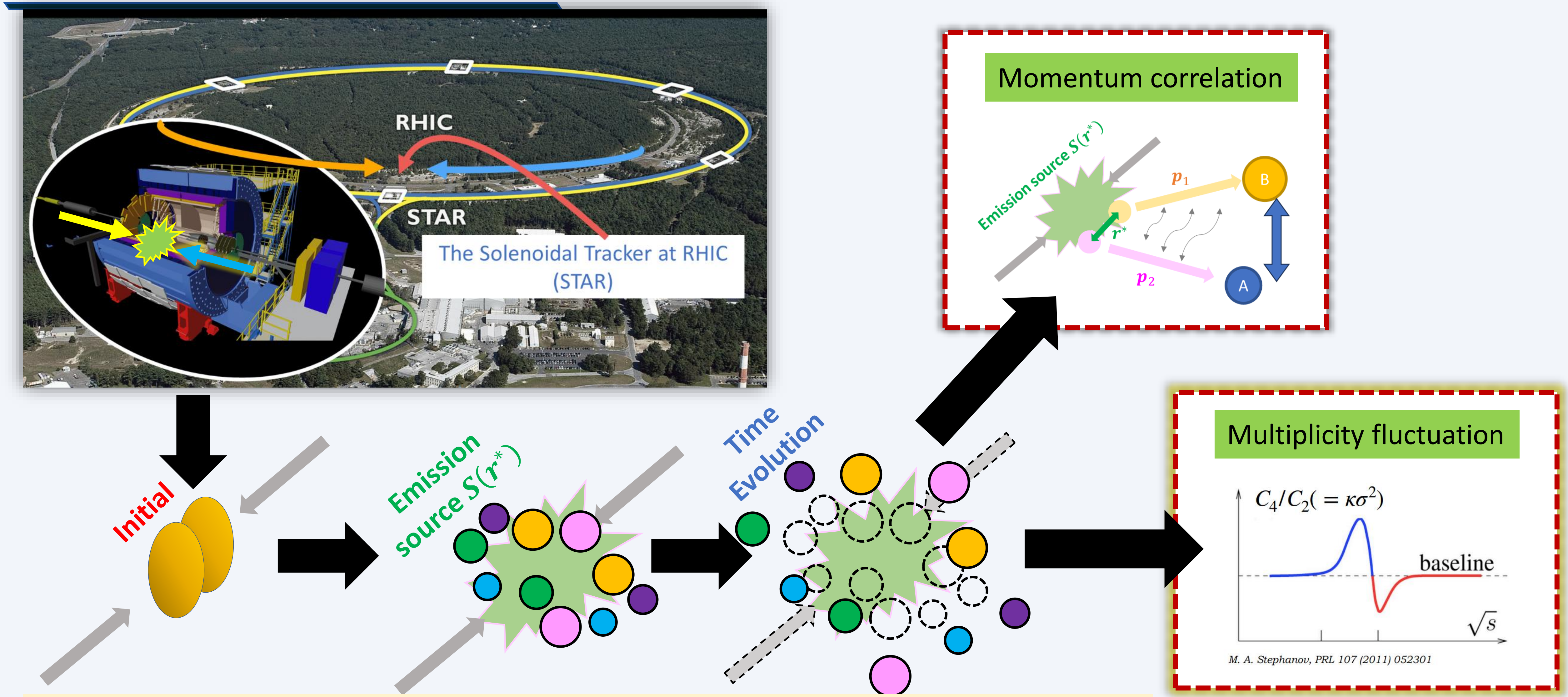






- ❖ **Varied collision energies: systematically explore high baryon density region $(25 < \mu_B < 750\ \mathrm{MeV})$**
- ❖ **Targeted detector upgrades:**
  - ❖ iTPC: $|\eta| < 1$ to $|\eta| < 1.6$, lower p_T reach, improved dE/dx
  - ❖ ETOF: PID at forward rapidity
  - ❖ EPD: Event plane determination & trigger

Heavy Ion Collision Experiment



❖ **Space and time evolution of particle-emitting source + final state interaction**

Net-Proton Distribution

Cumulants

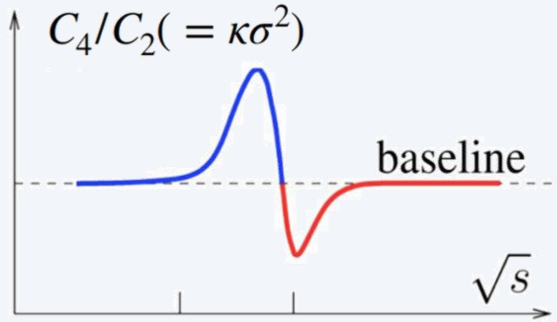
$$\delta n = n - \langle n \rangle$$

$$C_1 = \langle n \rangle$$

$$C_2 = \langle \delta n^2 \rangle$$

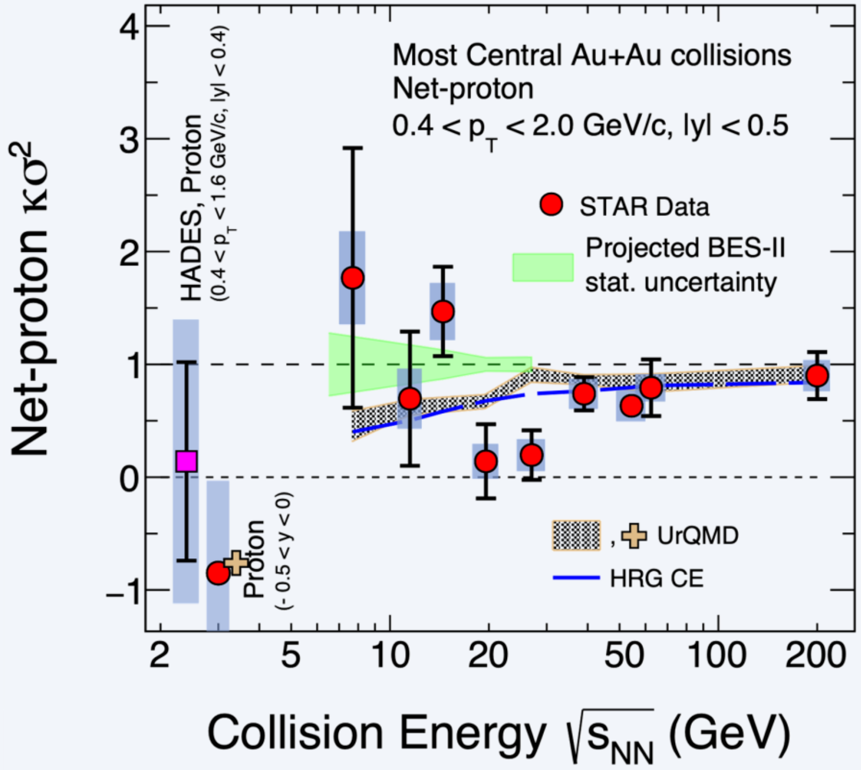
$$C_3 = \langle \delta n^3 \rangle$$

$$C_4 = \langle \delta n^4 \rangle - 3\langle \delta n^2 \rangle$$



M. A. Stephanov, PRL 107 (2011) 052301

Measurement @ BES-I

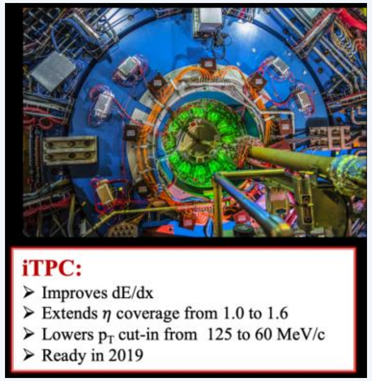


Assumption: Thermodynamic equilibrium
Non-monotonic $\sqrt{s_{NN}}$ dependence of C_4/C_2 of conserved quality – **Existence of a critical region**

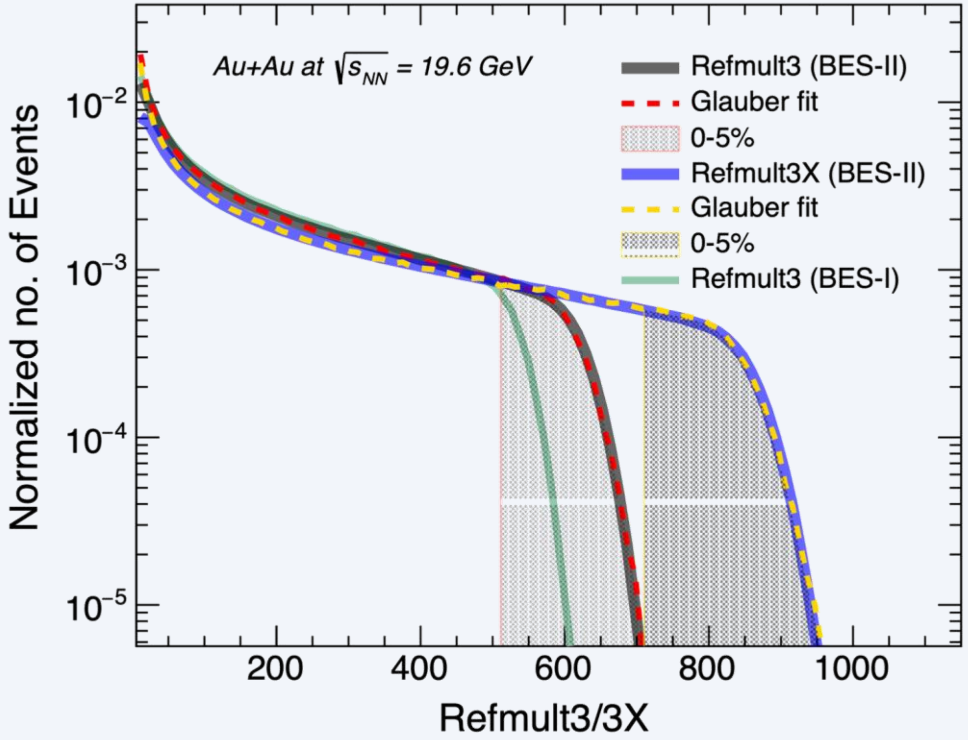
Observed **hint of non-monotonic trend** in BES-I (3σ)
Robust conclusion require confirmation from precision measurement from BES-II

STAR: PRL 127, 262301 (2021), PRC 104, 24902 (2021)
PRL 128, 202302 (2022), PRC 107, 24908 (2023)
HADES: PRC 102, 024914 (2020)

Precision Measurement of Net-proton Cumulants @ BES-II



$\sqrt{s_{NN}}$ (GeV)	Events BES-I (10^6)	Events BES-II (10^6)
7.7	3	**45**
9.2	-	**78**
11.5	7	**110**
14.5	20	**178**
17.3	-	**116**
19.6	15	**270**
27	30	**220**



❖ p and $\bar{p}$ are excluded to avoid self correlation

❖ Enlarged rapidity acceptance:
$|\eta| < 1$ to $|\boldsymbol{\eta}| < \mathbf{1.6}$

❖ Improved particle identification:
$p_T \geq 125$ MeV/c to $\boldsymbol{p_T} \geq$ **60 MeV/c**

❖ Enhanced centrality resolution
Refmult3 ($|\eta| < 1$) to **Refmult3X** ($|\boldsymbol{\eta}| < \mathbf{1.6}$)

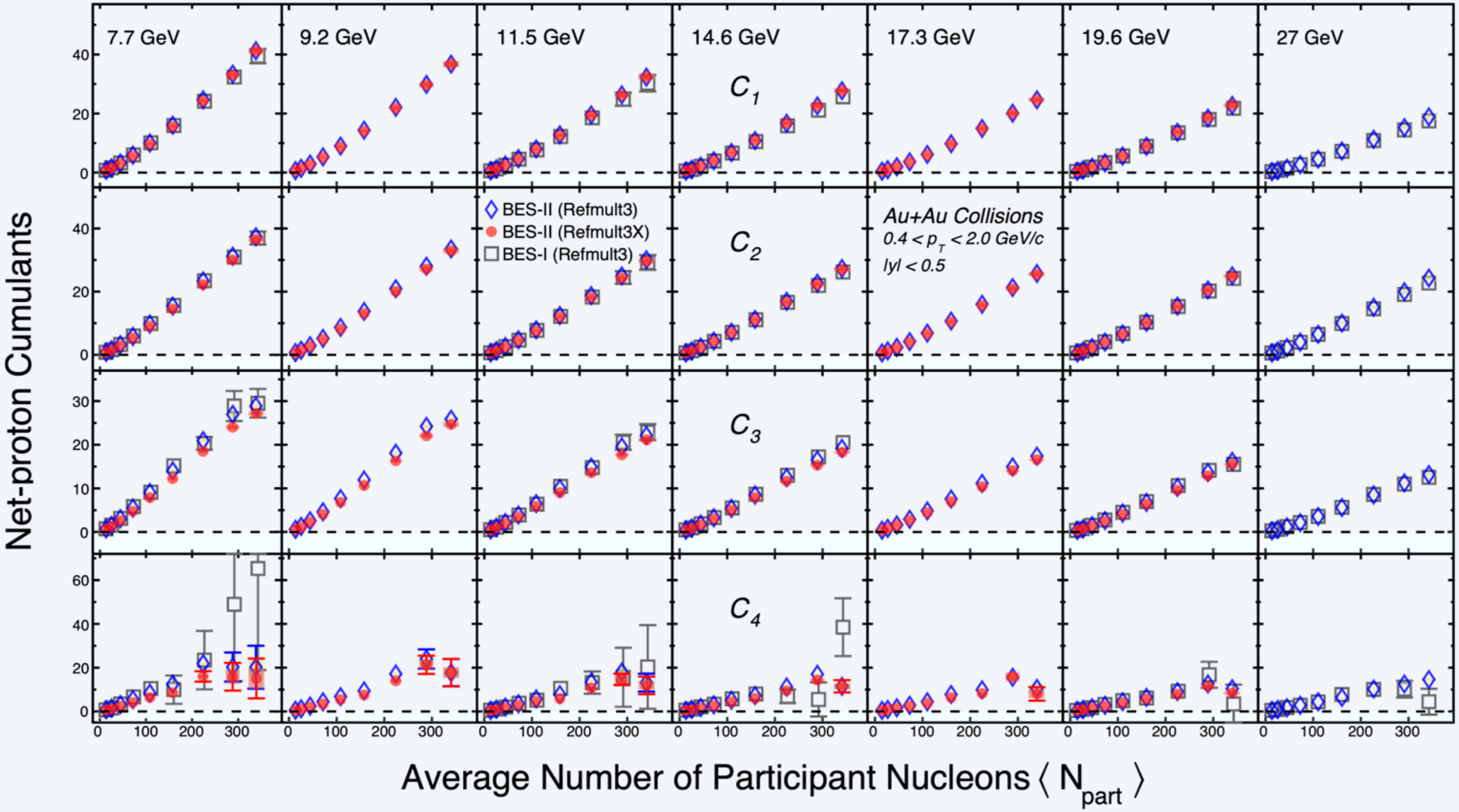
❖ Better control on uncertainty on efficiency:
5% to **2%**

Net-proton cumulants C_4/C_2 at 0-5% centrality

7.7 GeV		19.6 GeV	
stat. error	sys. error	stat. error	sys. error
Percentage stat. and sys. error in net-proton cumulants			
61%	29%	22%	11%
Reduction factor in uncertainties, BES-II vs BES-I			
4.7	**3.2**	**4.5**	**4**

Both **statistical and systematical uncertainties** are significant reduced in BES-II results

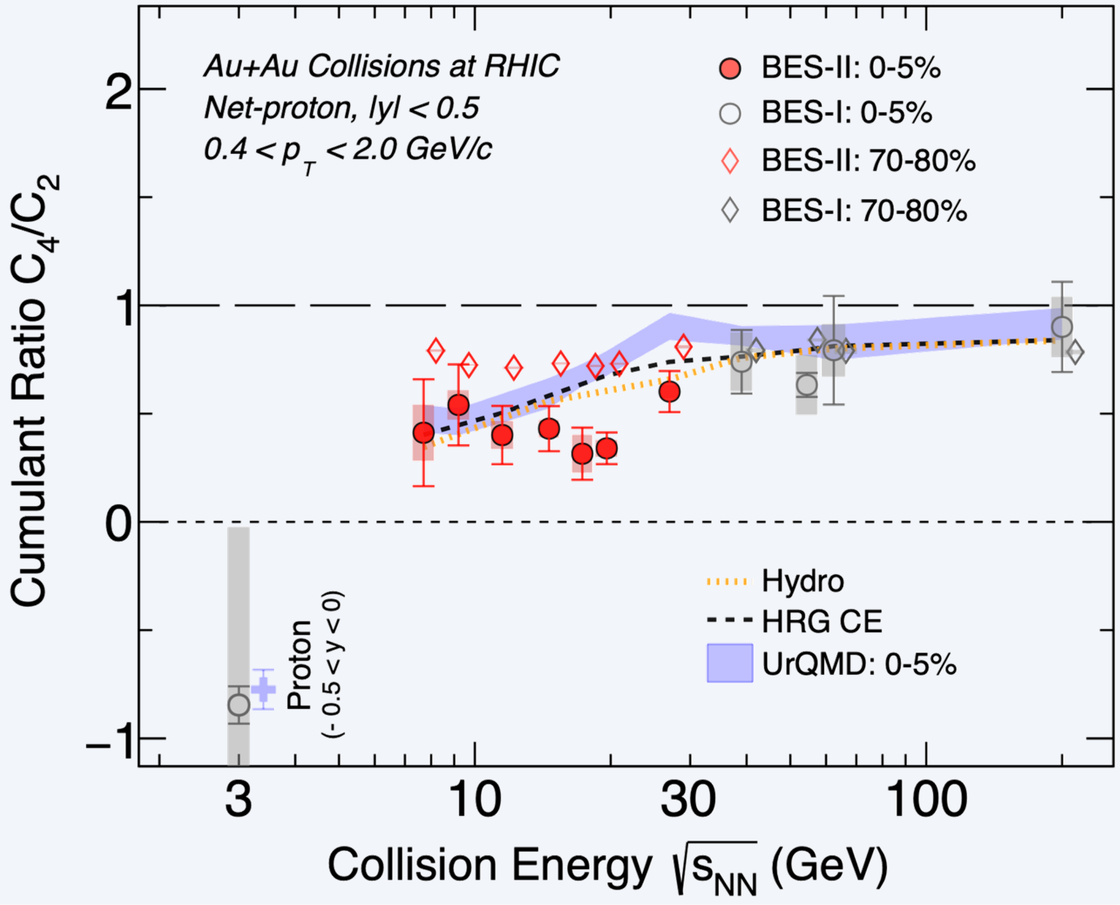
Precision Measurement of Net-proton Cumulants @ BES-II

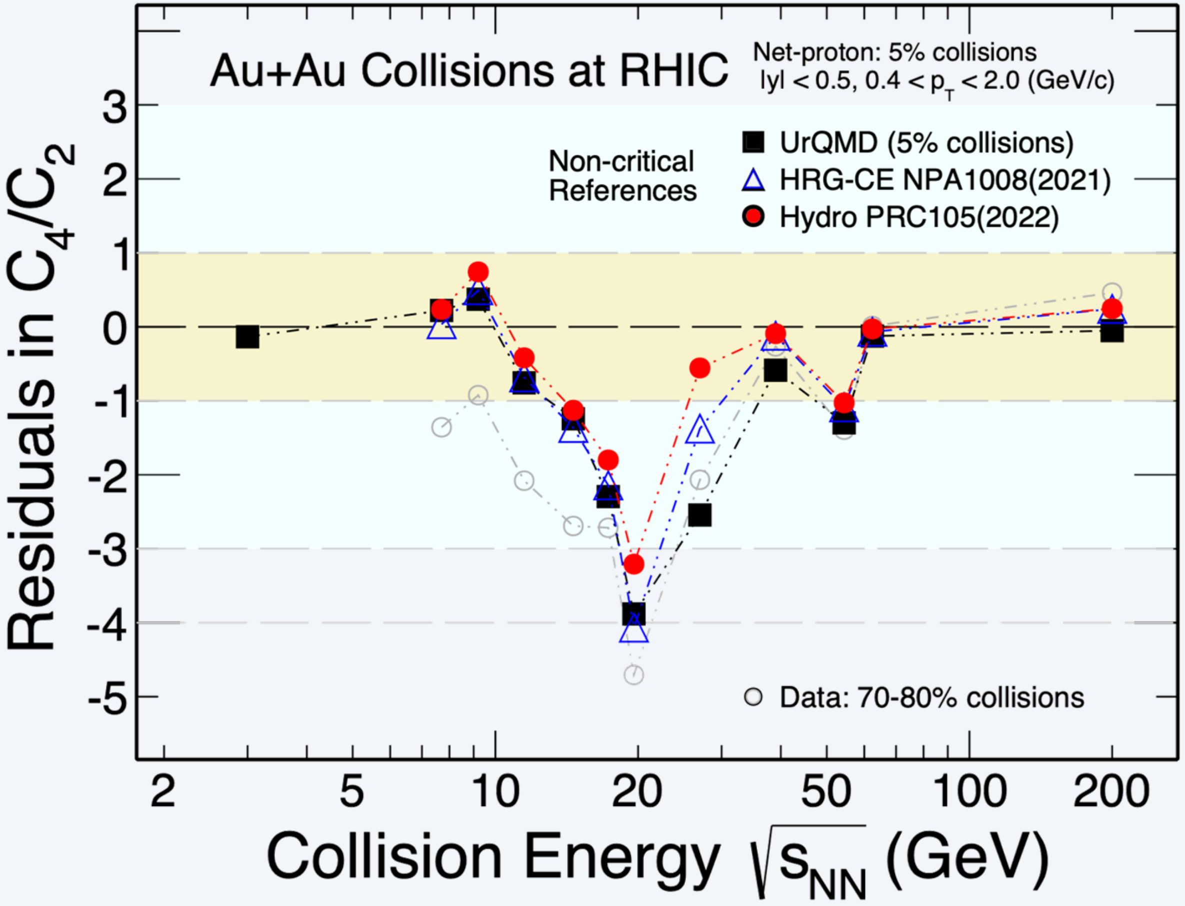


❖ The precision measurements from BES-II are consistent with the BES-I results
❖ A smooth variation across centrality and collision energy are observed

Ashish Pandav (STAR), CPOD 2024

CP sensitive observables: C_4/C_2

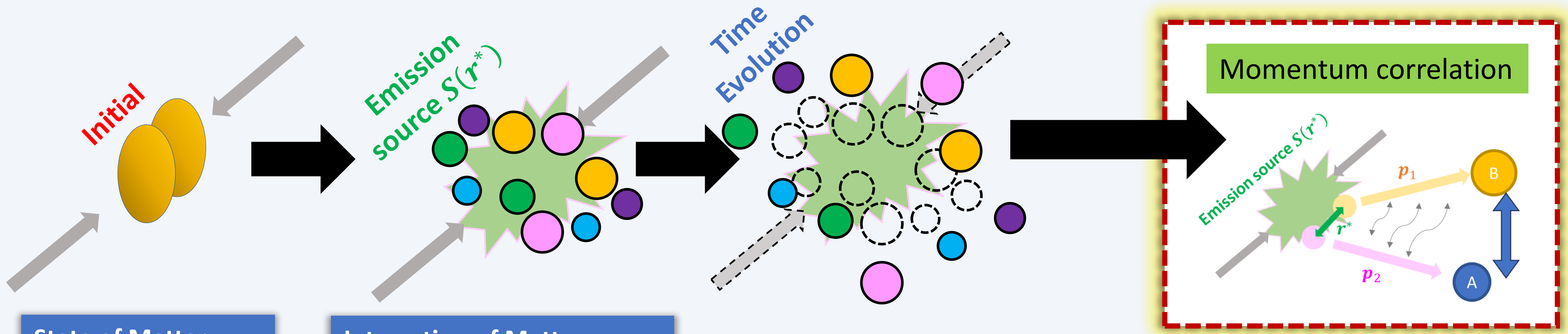




- ❖ C_4/C_2 shows minimum around ~ 20 GeV comparing to the non-CP models and 70 - 80% data
- ❖ Maximum deviation: **3.2 - 4.7σ** at $\sqrt{s_{NN}} \sim 20$ GeV (1.3 - 2σ at BES-I)

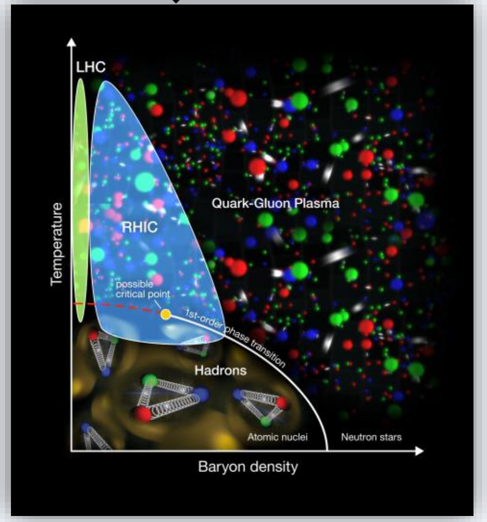
Ashish Pandav (STAR), CPOD 2024

Momentum correlation

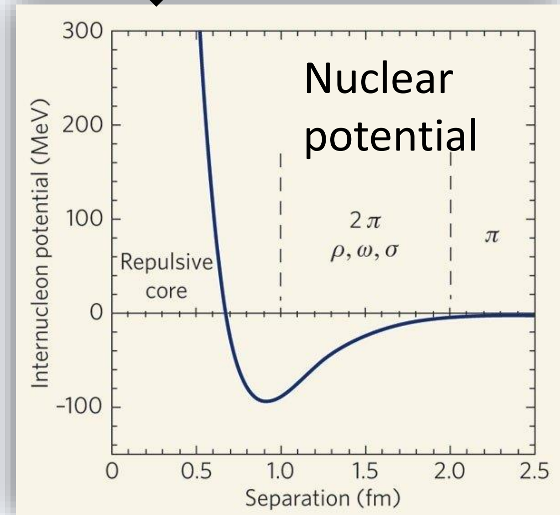


State of Matter

Interaction of Matter



Phase diagram



- Collision dynamics: source size and shape
- Structure of nuclear and hyper-nuclei matter
- Nucleon-Nucleon (N-N) and Hyperon-Nucleon (Y-N) interactions

https://www.bnl.gov/newsroom/news.php?a=219079
https://www.quora.com/What-does-the-potential-function-for-the-strong-nuclear-force-look-like

Correlation Function (CF)

Statistical

Momentum correlation function:

$$C(\boxed{\boldsymbol{p}_1},\boxed{\boldsymbol{p}_2}) \equiv \frac{P(\boldsymbol{p}_1,\boldsymbol{p}_2)}{P(\boldsymbol{p}_1)\cdot P(\boldsymbol{p}_2)}$$

Single-particle momentum

Experimental

$$C(k^*) = \boxed{\mathcal{N}}\frac{\boxed{A(k^*)}}{\boxed{B(k^*)}}$$

Same events

Mixed events

Normalization factor

k^*: particle momentum in the pair rest frame

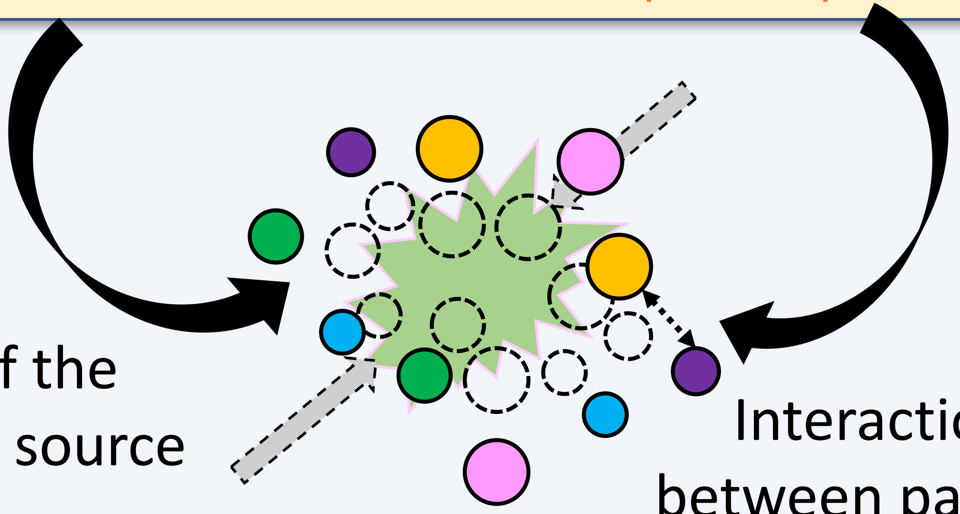
Modeling

Approximating the emission process and the momenta of the particles:

$$C(\boldsymbol{k}^*) = \int d^3r^* \boxed{S(\boldsymbol{r}^*)}\boxed{|\Psi(\boldsymbol{r}^*,\boldsymbol{k}^*)|^2}$$

Distribution of the relative distance of particle pair

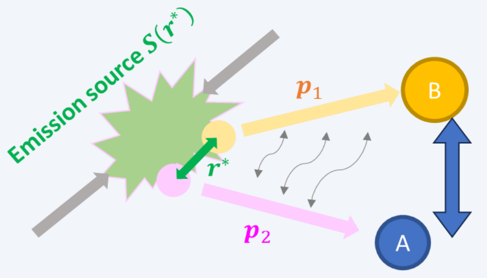
Relative wave function of the particle pair



Size of the emitting source

Interactions between particles

Kaon Correlation @ High μ_B



- Sinyukov-Bowler[1] approach used for K^+- K^+ and π^+- π^+ CF

$$CF(q_{inv}) = N[(1-\lambda) + \boxed{K_{coul}(q_{inv}, R_G)}\lambda(\boxed{e^{[-R_G^2 q_{inv}^2]}} + 1)]$$

Coulomb interaction part QS part

- N: normalize factor; λ: correlation strength

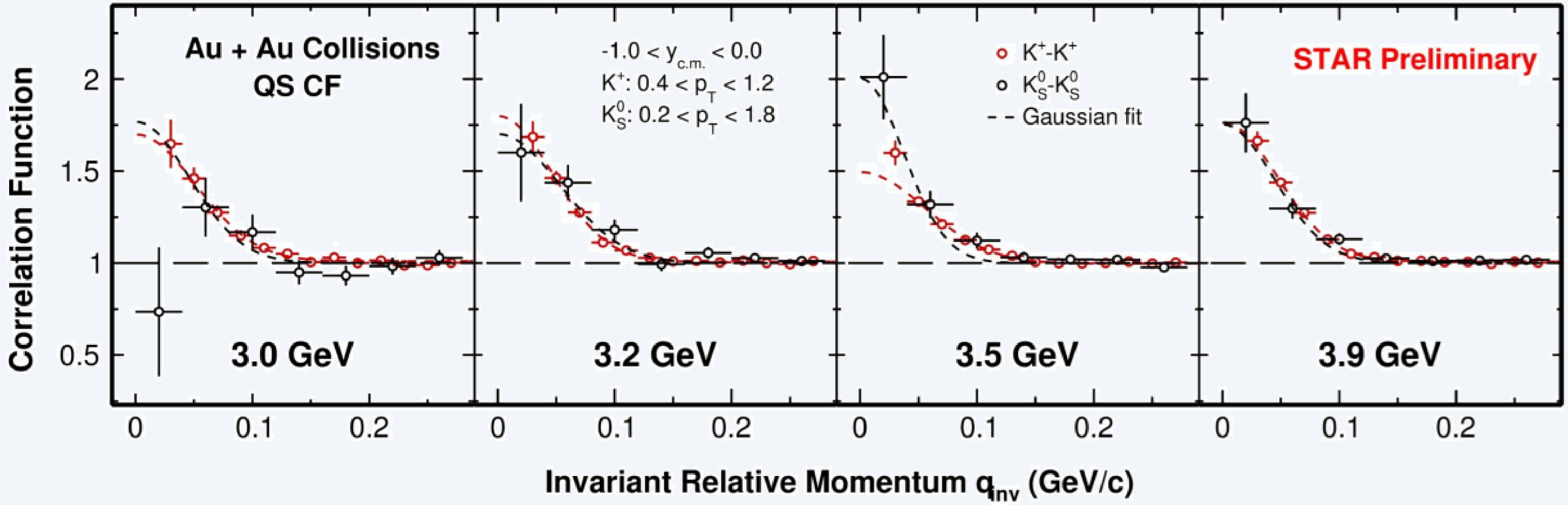
- Lednický-Lyuboshitz (L-L)[2] approach used for K_S^0-K_S^0 CF

$$CF(q) = 1 + \lambda\left(\boxed{e^{[-R_G^2 q_{inv}^2]}} + \boxed{\frac{1-\epsilon^2}{2}}\boxed{\left[\left|\frac{f(k^*)}{r_G}\right|^2 + \frac{4Re[f(k^*)]}{\sqrt{\pi}R_G}F_1(q_{inv}R_G) - \frac{2Im[f(k^*)]}{R_G}F_2(q_{inv}R_G)\right]}\right)$$

QS part

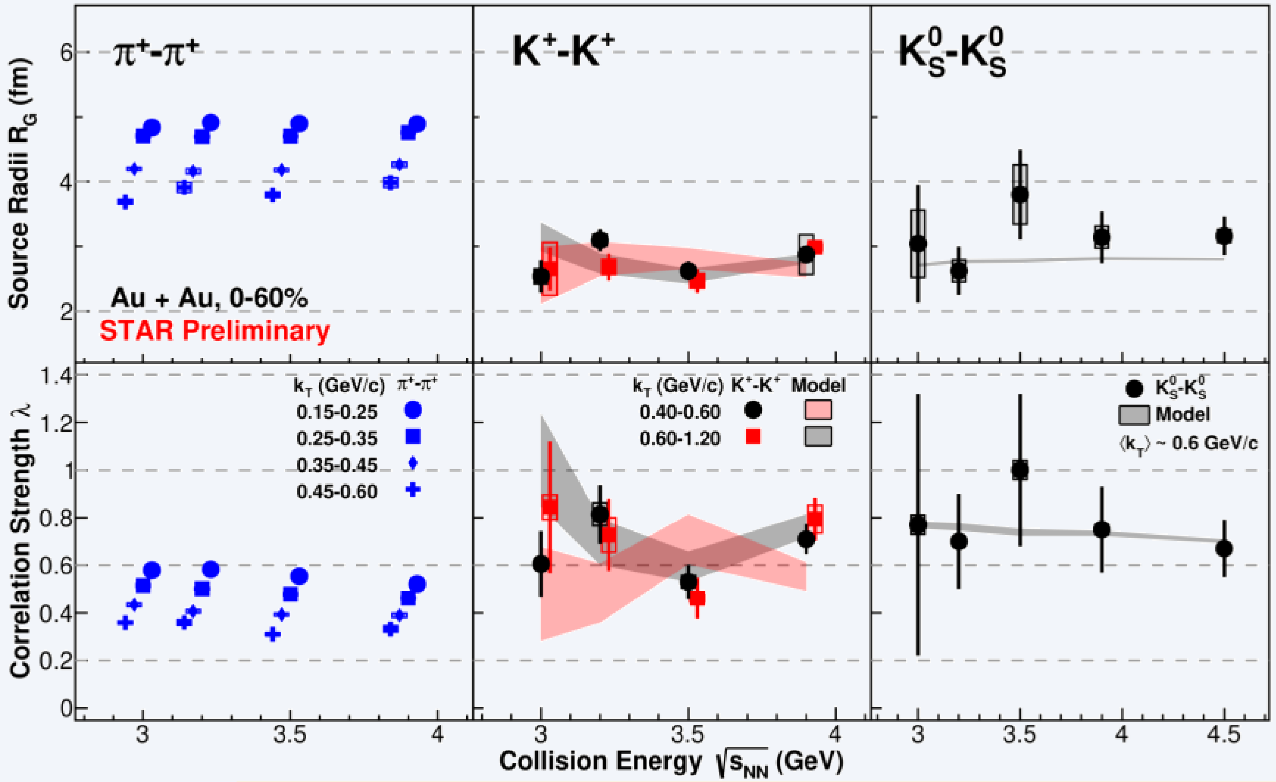
Strong interaction part

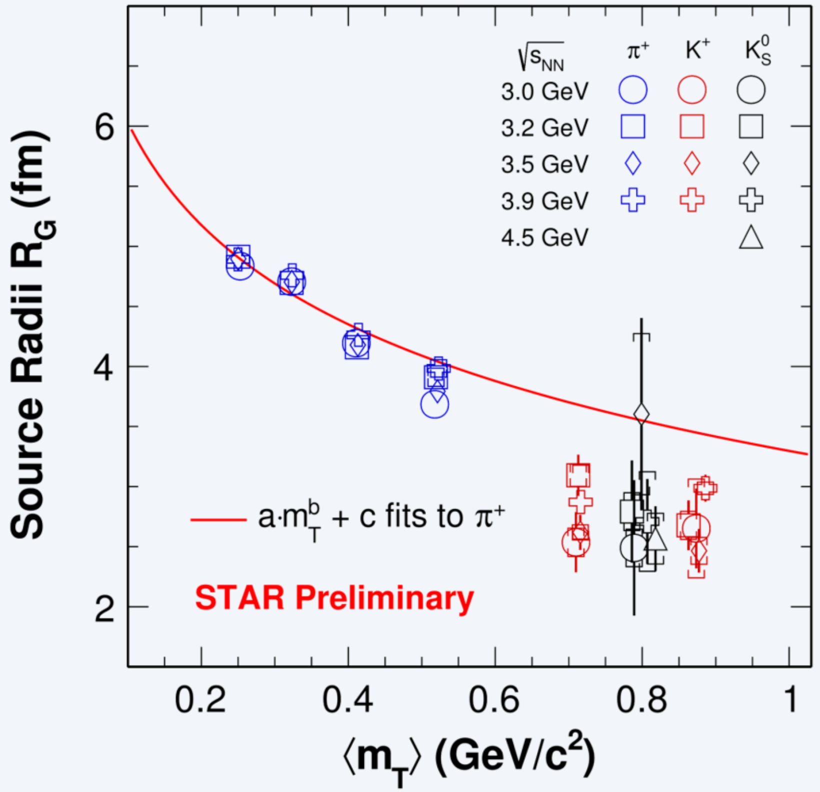
Kaon abundance asymmetry



- No significant energy dependence is observed for both K^+K^+ and $K_S^0K_S^0$
- CF for K^+K^+ and $K_S^0K_S^0$ are consistent under current precision

Pion & Kaon Correlation @ High μ_B



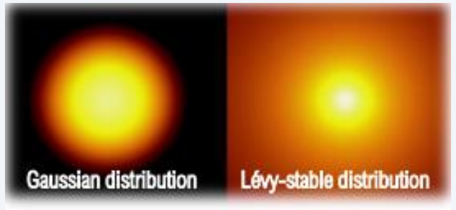


- Within $3.0 < \sqrt{s_{NN}} < 3.9$ GeV, no clear energy dependence was observed for both source radii and correlation strength
- $R_G(\pi) > R_G(\mathrm{K})$
- Different $\langle m_T \rangle$ depends for π and K

$m_T = (k_T^2 + m^2)^{1/2}$

Pion Correlation w. Lévy source

$$C = \int d^3r^* \boxed{S(\boldsymbol{r}^*)} |\Psi|^2$$



Lévy parametrization without final state effects:

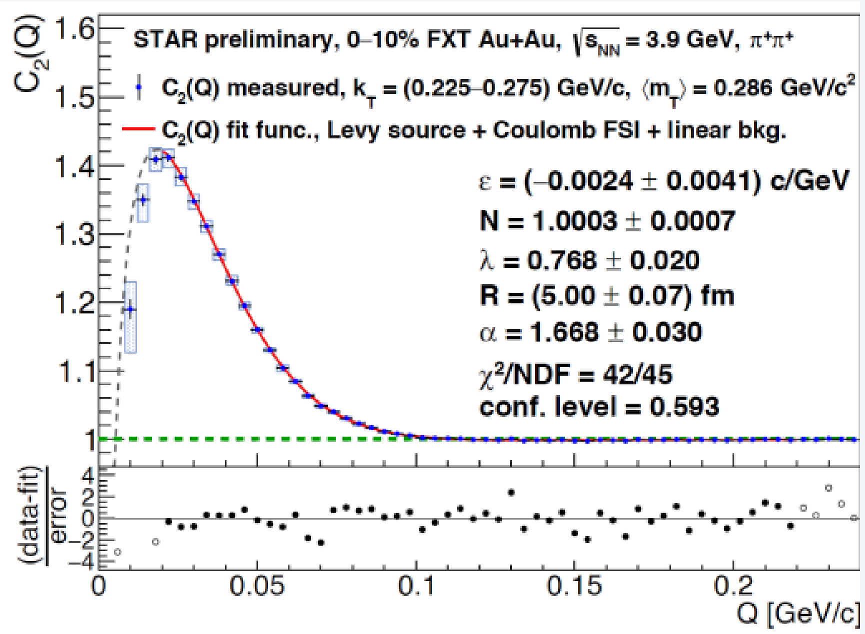
$$C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^\alpha}$$

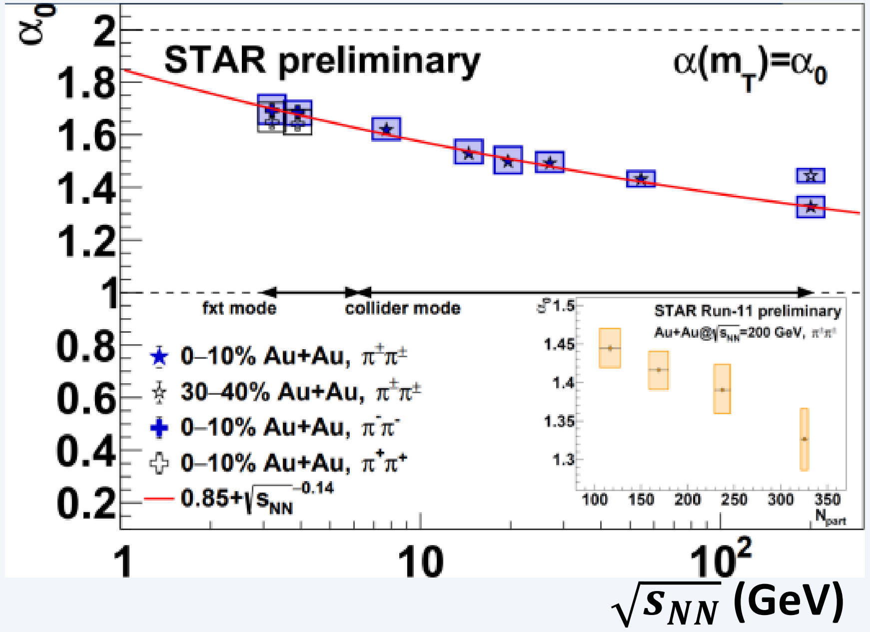
Q: LCMS three-momentum difference $Q = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q^2_{long,LCMS}}$

λ: correlation strength

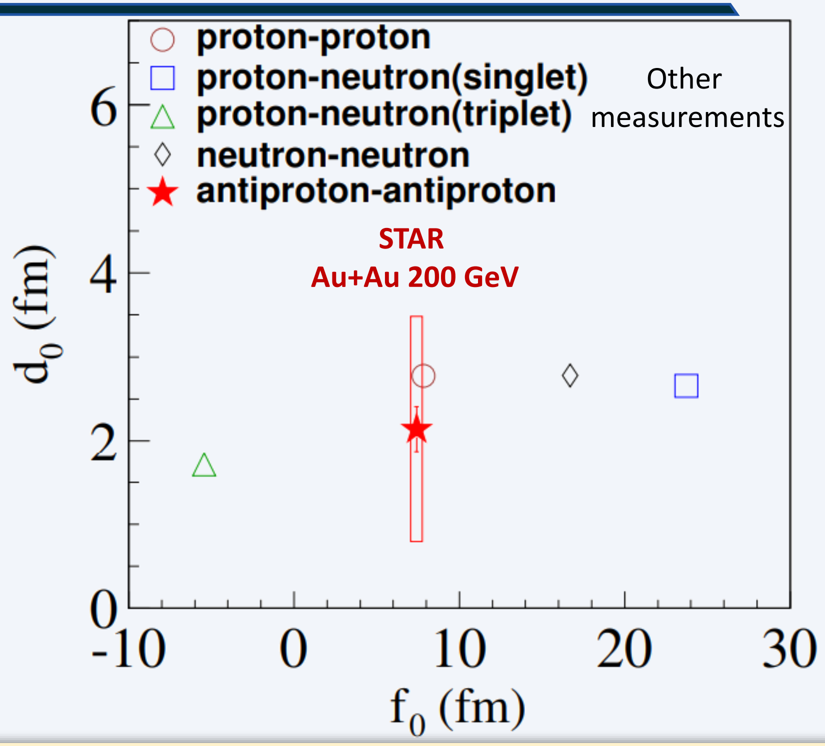
α: Lévy exponent

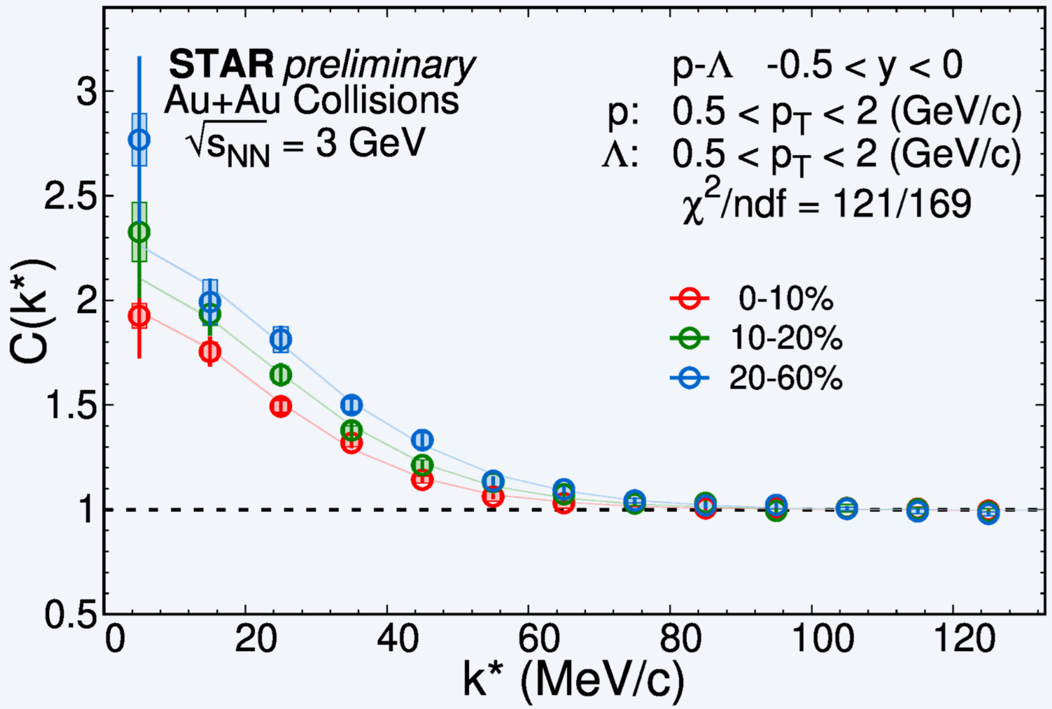
- ❖ Lévy-source + Coulomb FSI have a good description of the CF
- ❖ $\alpha < 2$ indicate a non-gaussian shape of the sources at all collision energies





p-p & p-Λ Correlation Functions





Scattering length (f_0) for $p-p$:

- Low-E experiment found $f_0 = 7.806 \pm 0.003$ fm
- Correlations in HIC with **Lednicky-Lyuboshitz (L-L) approach:** $f_0 \sim 7$ **fm**

Correlation functions for $p-\Lambda$. With L-L approach:

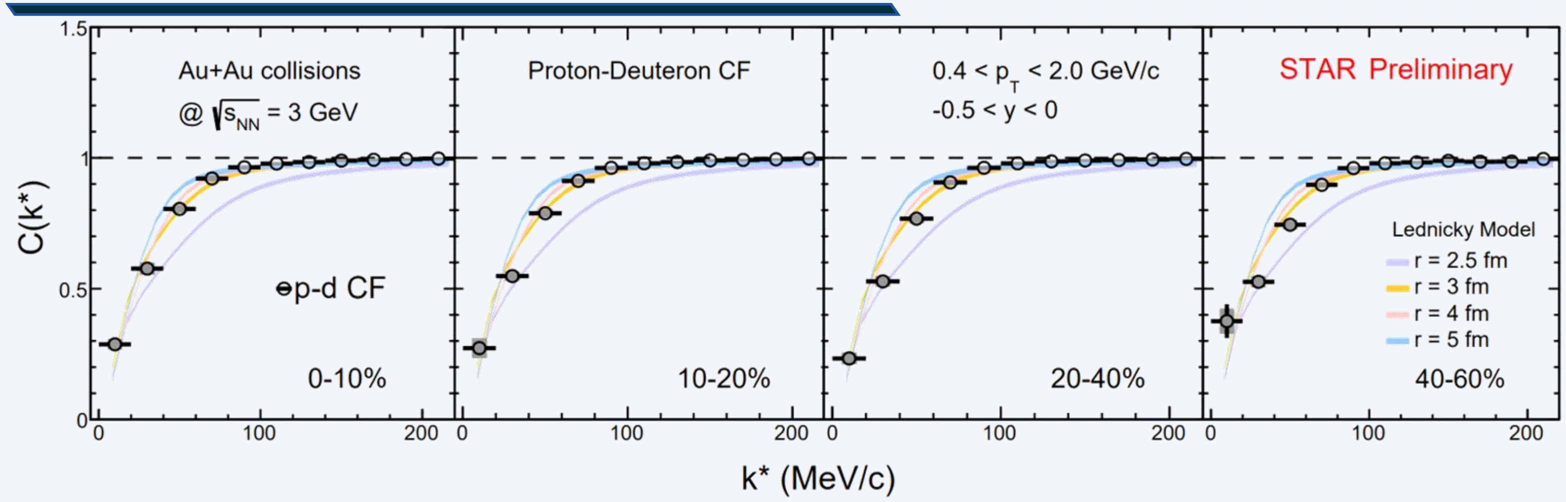
- Simultaneous fit to data in different centralities/rapidity
- Spin-avg scattering length (f_0) and effective range (d_0):

$f_0 = 2.32^{+0.12}_{-0.11}$ fm $\quad d_0 = 3.5^{+2.7}_{-1.3}$ fm

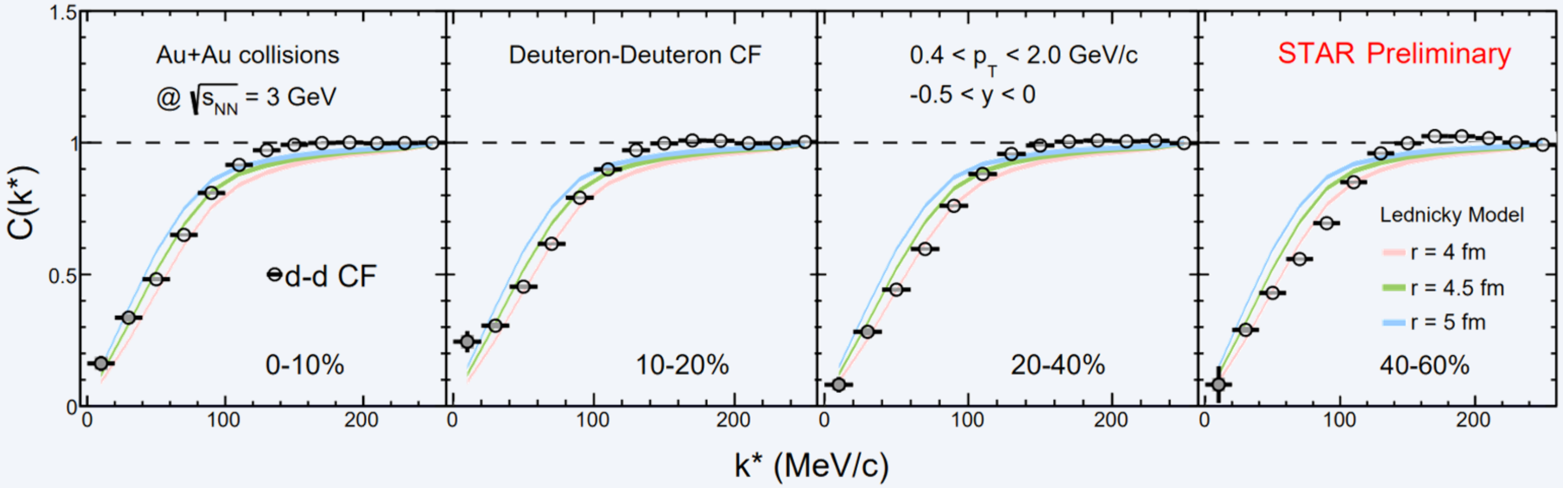
- **A valid method to study the interaction between baryons**
- **Scattering length is much larger in p-p compare with p-Λ**

I. ˇSlaus, Y. Akaishi, and H. Tanaka, Phys. Rept. 173, 257 (1989)
L. Mathelitsch and B. J. Verwest, Phys. Rev. C 29, 739 (1984).
L. Adamczyk et al. (STAR), Nature 527, 345 (2015), arXiv:1507.07158
Y. Hu (STAR), EPJ Web Conf. 296, 14010 (2024), arXiv:2401.00319

p-d & d-d Correlation Functions



Consistent with L-L model with Coulomb + repulsive interaction



Consistent with L-L model with Coulomb + QS + repulsive interaction

Model prediction

p-d	f_0 (fm)	d_0 (fm)
Doublet	-2.73	2.27
Quartet	-11.88	2.63

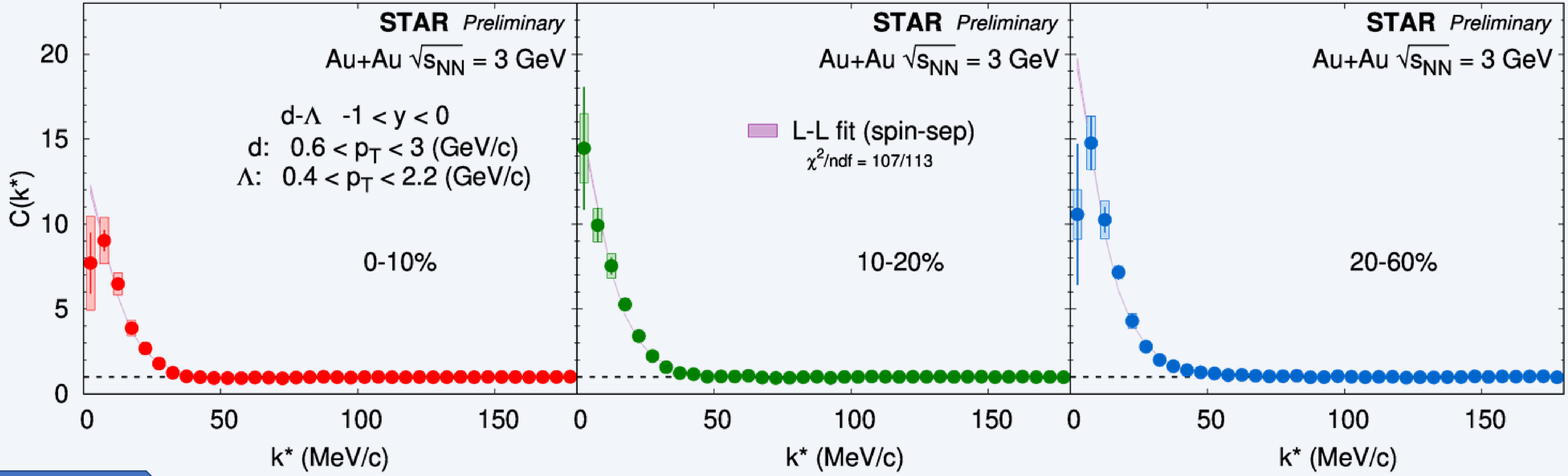
d-d	f_0 (fm)
Singlet	-10.2
Quintet	-7.5

- Models which only has two-body interactions can well describe our p-d and d-d data

J. Arvieux, NPA 221 (1974) 253
I.N. Filikhin and S.L. Yakovlev, Phys. Atom. Nucl. 63 (2000) 55 / 216
Robert B. Wiringa, et. al, Phys.Rev.C 51 (1995) 38-51

K. Mi (STAR), Acta Phys. Polon. Supp. 16, 1 (2023), arXiv:2208.05722

d-Λ Correlation Functions @ STAR



Corrections

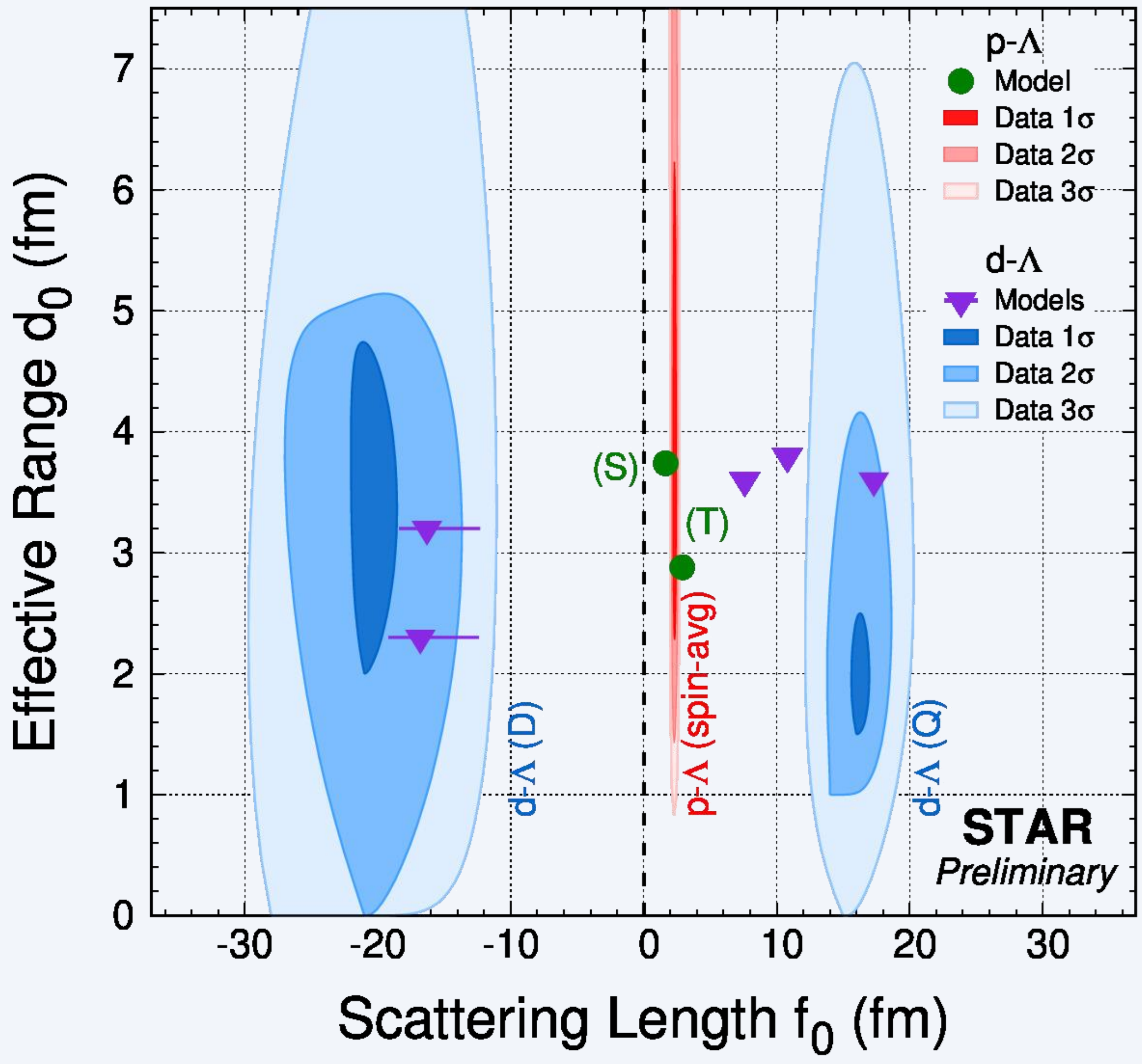
1. Purity correction
2. Track splitting & merging
3. Contamination from ${}^3_\Lambda\mathrm{H} \to \pi^- + p + d$ decay

- **First d-Λ correlation measurements in the heavy-ion collision experiment**
- Simultaneous fit to data in different centralities
  - R_G^i, $f_0(D)$, $d_0(D)$, $f_0(Q)$, and $d_0(Q)$ with Lednicky-Lyuboshitz approach

$$f_0(D) = -20^{+3}_{-3}\ \text{fm} \qquad d_0(D) = 3^{+2}_{-1}\ \text{fm}$$
$$f_0(Q) = 16^{+2}_{-1}\ \text{fm} \qquad d_0(Q) = 2^{+1}_{-1}\ \text{fm}$$

- Λ feed-down correction not applied due to unknown d-Σ/Ξ correlation
- Momentum smearing effect negligible

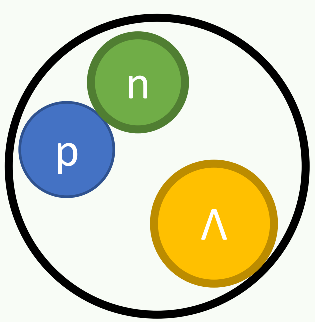
Scatterings Length (f_0) and Effective Range (d_0)

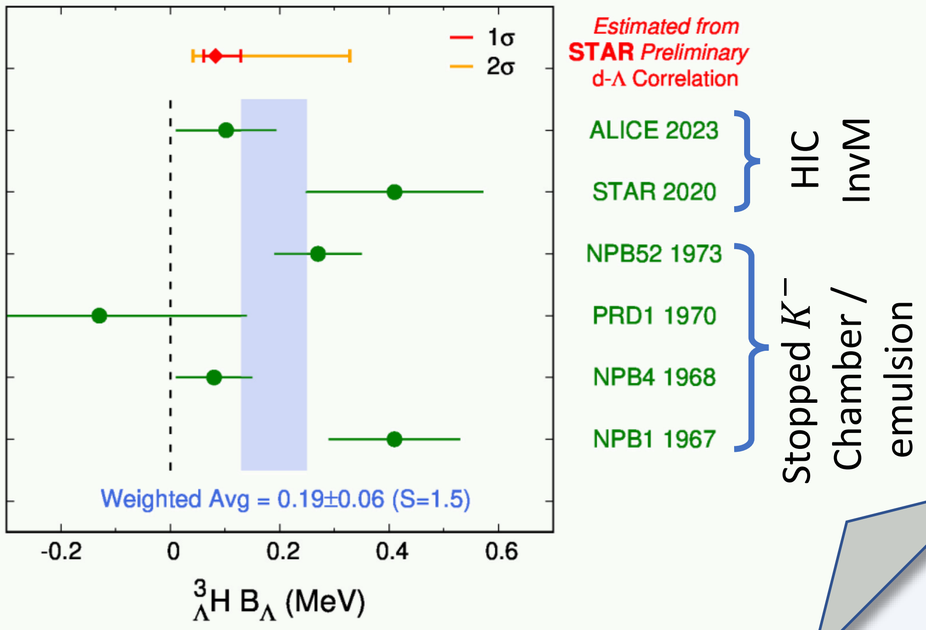


Λ separation energy of $^{3}_{\Lambda}\mathrm{H}$

$$\frac{1}{-f_0} = \gamma - \frac{1}{2} d_0 \gamma^2$$

- $\mathrm{B}_\Lambda = \frac{\gamma^2}{2\mu_{d\Lambda}}$
- $\mu_{d\Lambda}$: reduced mass
- γ: binding momentum

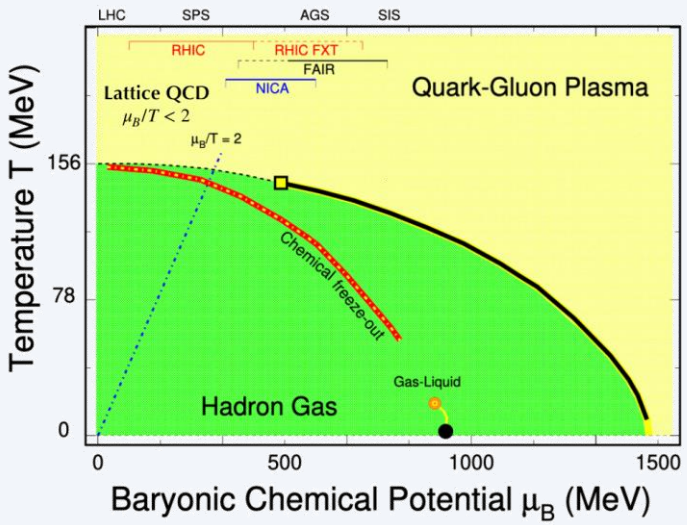




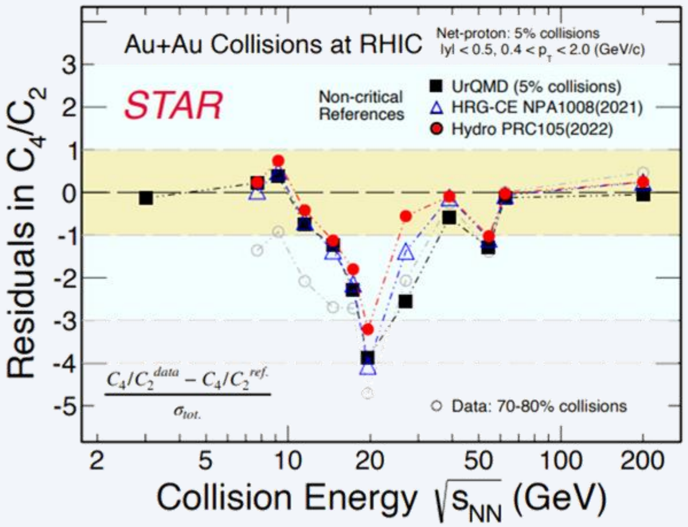
- Constraint fit for d-Λ, require $f_0(\mathrm{D}) < 0$
- Edge of d-Λ contours are shown with Bezier smooth to improve the visibility

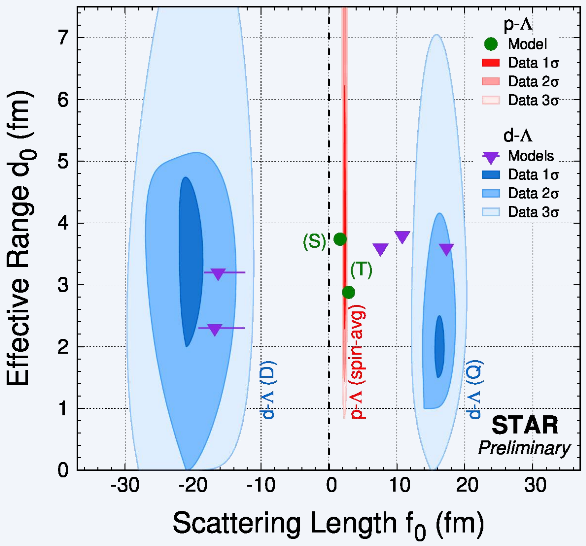
H. W. Hammer, Nucl. Phys. A 705 (2002) 173
F. Wang, et al. Phys.Rev.Lett. 83 (1999) 3138
M. Schäfer, et al. Phys.Lett.B 808 (2020) 135614
A. Cobis, et al. J. Phys. G 23 (1997) 401
G. Alexander, et al. Phys. Rev. 173 (1968) 1452
J. Haidenbauer, Phys.Rev.C 102 (2020) 3, 034001
J. Haidenbauer, et al. Nucl. Phys. A 915 (2013) 24

Summary



B. Mohanty, N. Xu, arXiv:2101.09210





- **QCD Critical End Point**
  - Precision measurement of net-proton fluctuations in BES-II significantly reduced the uncertainty
  - $3.2 - 4.7\sigma$ maximum deviation for net-proton C_4/C_2 w.r.to non-CP model/70-80% data is observed
    – **Need more theory input**
  - Stay tuned for the measurements at FXT energies

- **Equation of State**
  - A large scope of meson-meson and baryon-baryon correlations is studied at STAR
  - First experimental measurement of d–Λ correlation function
  - FXT program: a unique probe to NN, and YN interactions

Thank you!

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BERKELEY LAB

Supported in part by U.S. DEPARTMENT OF ENERGY

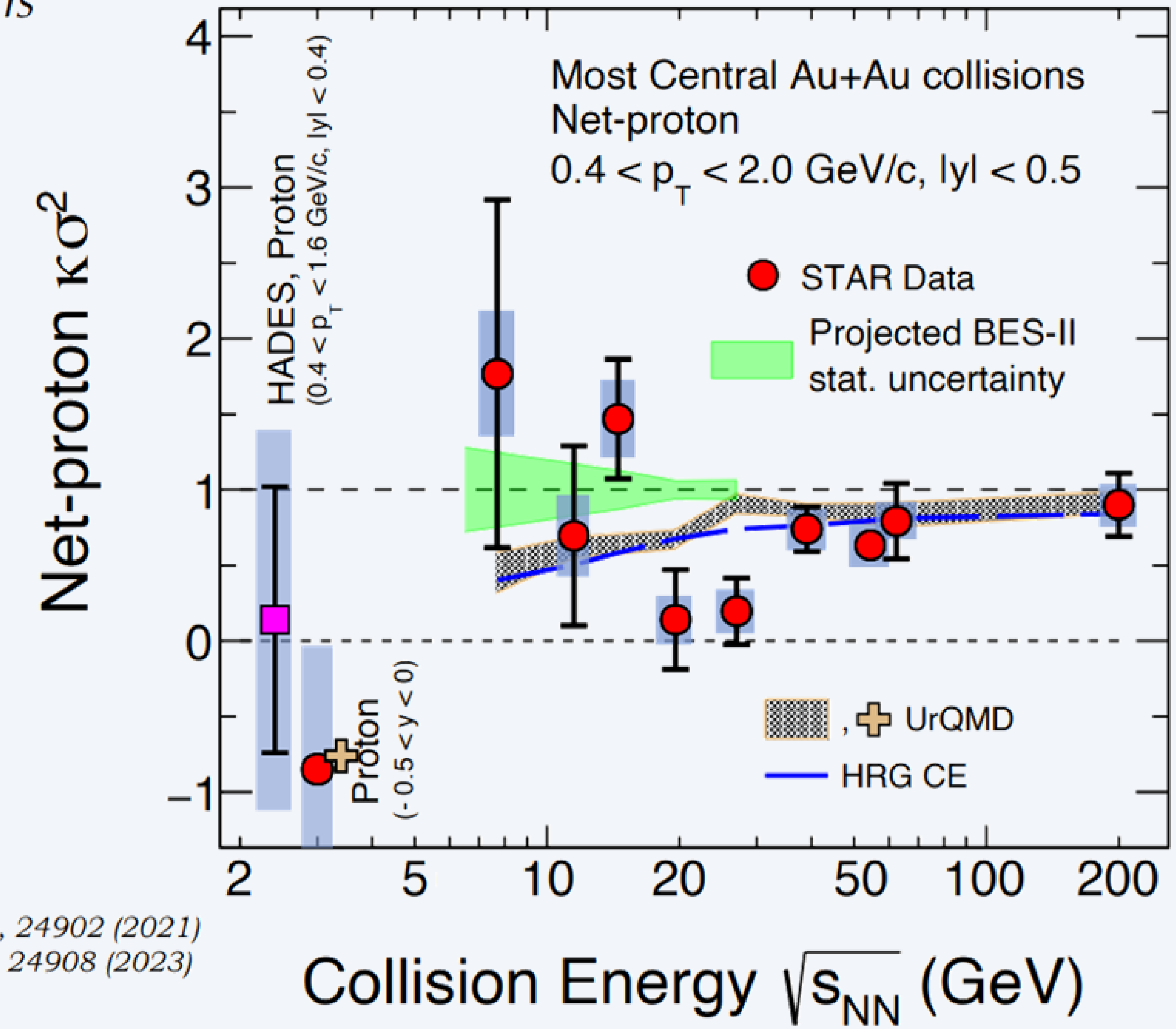
2024.08.28

Higher order Cumulants Measurements at BES-I

Phase I of BES program (BES-I): *Au+Au collisions*

J. Cleymans, et. al, PRC. 73, 034905 (2006)

$\sqrt{s_{NN}}$ (GeV)	Events (10^6)	μ_B (MeV)
200	220	25
62.4	43	75
54.4	550	85
39	92	112
27	31	156
19.6	14	206
14.5	14	262
11.5	7	316
7.7	2.2	420
3.0	140	750



STAR : PRL 127, 262301 (2021), PRC 104, 24902 (2021)
: PRL 128, 202302 (2022), PRC 107, 24908 (2023)
HADES: PRC 102, 024914 (2020)

Low–E scattering experiment & Effective Range Expansion

Low energy elastic scatterings:

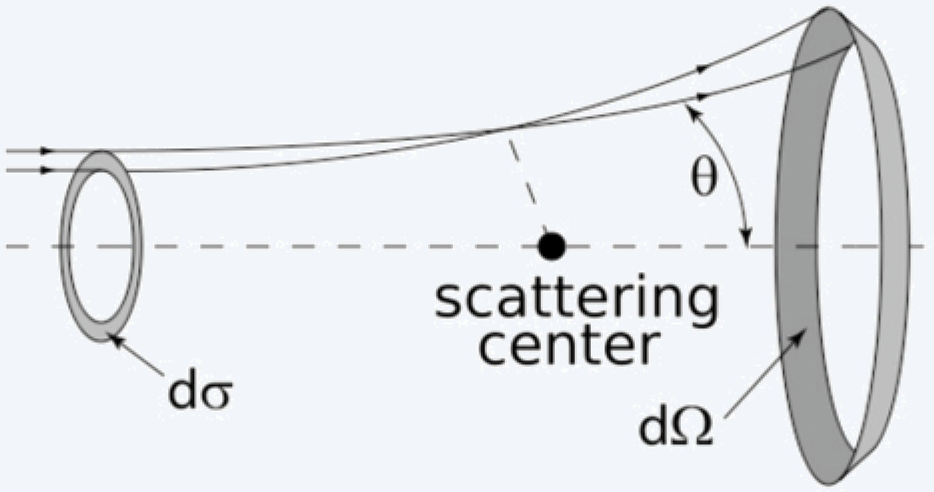
$$k\cot\big(\delta(k)\big) = -\frac{1}{a} + \frac{1}{2} r_0 k^2 + O(k^4)$$

$\delta(k)$: phase shift

a: Fermi scattering length at zero energy

r_0: effective range

O: higher order contribution



H. A. Bethe, Phy. Rev. 76 (1949) 38

Cross section:

$$\lim_{k\to 0} \sigma_e = 4\pi a^2$$

Binding energy:

$$\frac{1}{a} = \gamma - \frac{1}{2} r_0 \gamma^2$$

- ❖ $\mathrm{B} = \frac{\gamma^2}{2\mu}$
- ❖ μ: reduced mass
- ❖ γ: binding momentum

For the n-p scattering:

S_0:	$a = -23.714$ fm	$r_0 = 2.73$ fm
S_1:	$a = 5.425$ fm	$r_0 = 1.749$ fm

$\Rightarrow$ $\mathrm{B_d} = 2.2$ MeV

Lednicky-Lyuboshitz (L-L) Approach

R. Lednicky, et al. Sov.J.Nucl.Phys. 35 (1982) 770
J. Haidenbauer, Phys.Rev.C 102 (2020) 3, 034001
L. Fabbietti, et al., Ann.Rev.Nucl.Part.Sci. 71 (2021) 377-402
Michael Annan Lisa, et al., Ann.Rev.Nucl.Part.Sci. 55 (2005) 357-402

Modeling

Approximating the emission process and the momenta of the particles:

$$C(\boldsymbol{k}^*) = \int d^3r^* \boxed{S(\boldsymbol{r}^*)} \boxed{|\Psi(\boldsymbol{r}^*, \boldsymbol{k}^*)|^2}$$

Distribution of the relative distance of particle pair

Relative wave function of the particle pair

Major Assumptions

Source

- Smoothness approximation for source function*
- Static and spherical Gaussian source
  - Single particle source: $S_i(x_i, p_i^*)$
  - Pair source (radius R_G): $S(x, p^*) \propto e^{-x^2/2R_G{}^2}\delta(t - t_0)$

Wave function

- S-wave scattering wave
- Effective range expansion for $\Psi(\boldsymbol{r}^*, \boldsymbol{k}^*)$
- Approximate the wave function by its asymptotic form

Gaussian source approximation:

$$S(\boldsymbol{r}^*) = (2\sqrt{\pi}R_G)^{-3} e^{-\boldsymbol{r}^{*2}/4R_G^2}$$

Scattering amplitude:

Consider only S-wave $\Psi(r^*) = e^{-i\boldsymbol{r}^*\cdot\boldsymbol{k}^*} + \frac{f(\boldsymbol{k}^*)}{\boldsymbol{r}^*} e^{i\boldsymbol{r}^*\cdot\boldsymbol{k}^*}$

$$f(\boldsymbol{k}^*) \approx \left(\frac{1}{\boldsymbol{f_0}} + \frac{\boldsymbol{d_0}\boldsymbol{k}^{*2}}{2} - i\boldsymbol{k}^*\right)^{-1}$$

Scattering length:
$a \to -f_0$
Effective range:
$r_0 \to d_0$

Lednicky-Lyuboshitz (L-L) approach

R_G : spherical Gaussian source of pairs

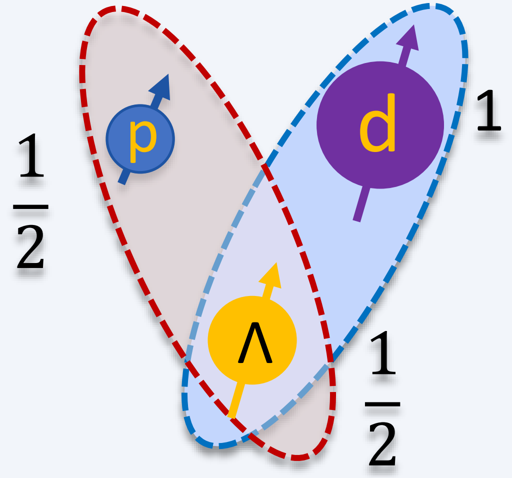
f_0 : scattering length

d_0 : effective range

*The smoothness approximation has been checked for expanding thermal sources, found to be very reasonable for large (RHIC-like) sources, but still questionable for smaller sources

Modeling with Separated Spin States

Singlet State	$^1S_0$	(S)
Triplet State	$^3S_1$	(T)



Doublet State	$^2S_{1/2}$	(D)
Quartet State	$^4S_{3/2}$	(Q)

Modeling

Approximating the emission process and the momenta of the particles:

$$C(\boldsymbol{k}^*) = \int d^3r^* \underbrace{S(\boldsymbol{r}^*)}_{\text{Source}} \underbrace{|\Psi(\boldsymbol{r}^*, \boldsymbol{k}^*)|^2}_{\text{Wave function}}$$

Spin averaged

$|\Psi(\boldsymbol{r}^*, \boldsymbol{k}^*)|^2$ expanded with averaged parameters: $\overline{f_0}$ and $\overline{d_0}$

$$\boxed{|\Psi(\boldsymbol{r}^*, \boldsymbol{k}^*)|^2} \rightarrow \boxed{\boldsymbol{f_{S1}}|\Psi_{S1}(\boldsymbol{r}^*, \boldsymbol{k}^*)|^2 + \boldsymbol{f_{S2}}|\Psi_{S2}(\boldsymbol{r}^*, \boldsymbol{k}^*)|^2}$$

Spin separated

$$C(\boldsymbol{k}^*) = \int d^3r^* S(\boldsymbol{r}^*)\left(\frac{1}{3}|\Psi_{1/2}(\boldsymbol{r}^*, \boldsymbol{k}^*)|^2 + \frac{2}{3}|\Psi_{3/2}(\boldsymbol{r}^*, \boldsymbol{k}^*)|^2\right)$$

For separated spin states in d-Λ

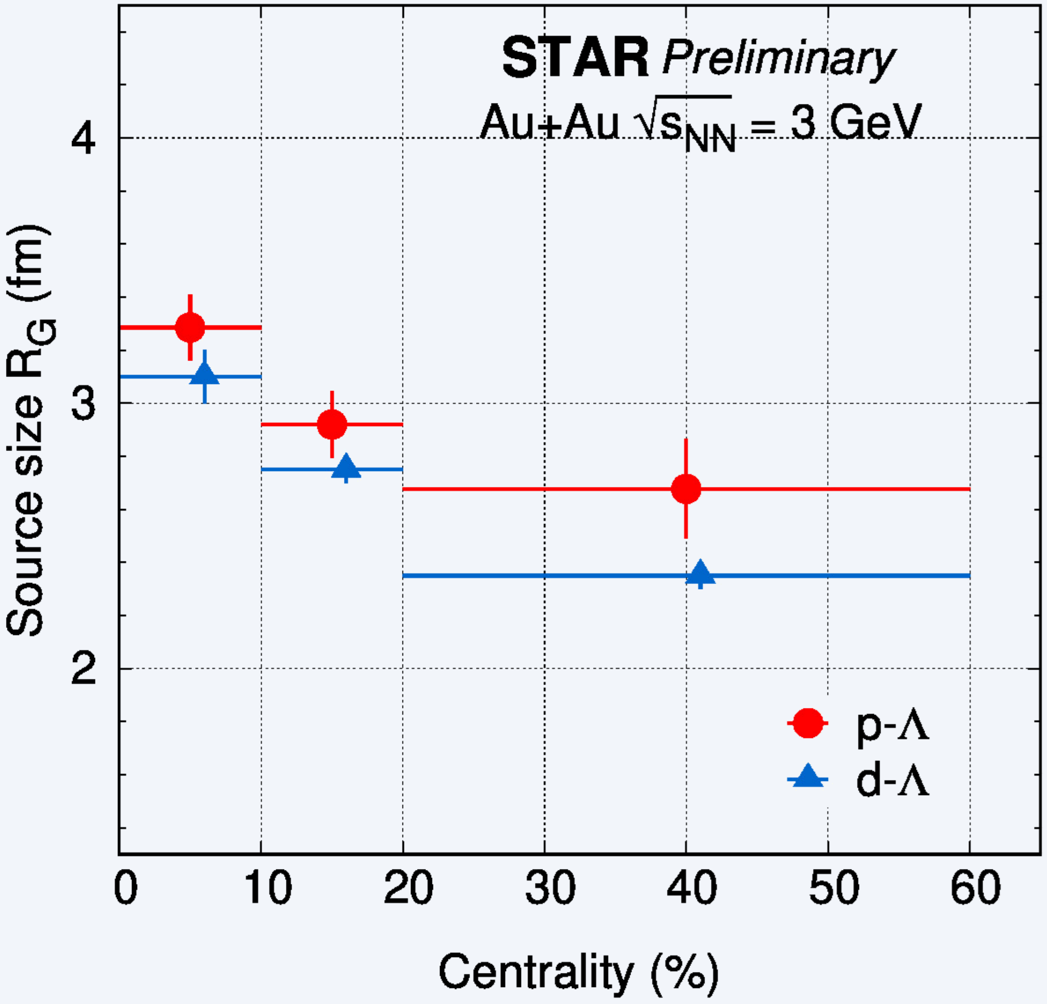
$$\boxed{\begin{matrix} f_0(D) \\ d_0(D) \end{matrix}} \qquad \boxed{\begin{matrix} f_0(Q) \\ d_0(Q) \end{matrix}}$$

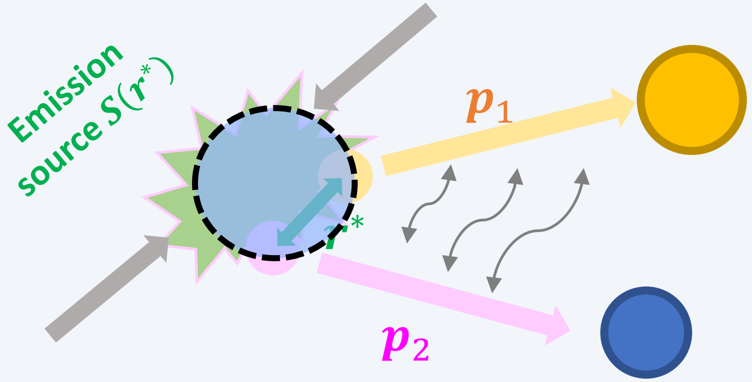
R. Lednicky, et al. Sov.J.Nucl.Phys. 35 (1982) 770

L. Michael, et al. Ann.Rev.Nucl.Part.Sci. 55 (2005) 357-402

J. Haidenbauer, Phys.Rev.C 102 (2020) 3, 034001

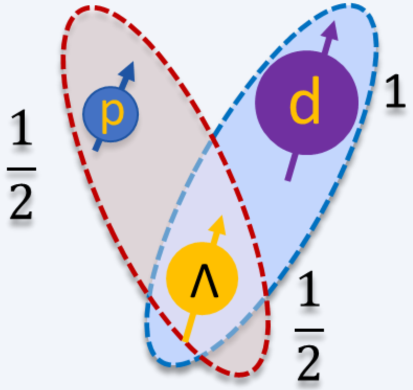
Source Size with L-L approach





- R_G: **spherical Gaussian source of pairs** by Lednicky-Lyuboshits approach
- Separation of emission source from final state interaction
- Collision dynamics as expected:
  - $R_G^{central} > R_G^{peripheral}$
  - $R_G\,(p-\Lambda) > R_G\,(d-\Lambda)$

Correlation Function & Spin States



Singlet State	$^1S_0$	(S)
Triplet State	$^3S_1$	(T)

Doublet State	$^2S_{1/2}$	(D)
Quartet State	$^4S_{3/2}$	(Q)

d-Λ: $|\psi(r,k)|^2 \rightarrow \frac{1}{3}\left|\psi_{1/2}(r,k)\right|^2 + \frac{2}{3}\left|\psi_{3/2}(r,k)\right|^2$

- Different spin states with different f_0 and d_0 parameters
- **p-Λ correlation:** current statistics is not enough to separate two spin states → **spin-averaged fit**
- **d-Λ correlation:** very different f_0 for (D) and (Q) are predicted → **Spin-separated fit**