Spectra and radial flow at RHIC with Tsallis distribution in Blast Wave model

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Thanks: Lijuan Ruan, Gene van Buren, Aihong Tang,
Mike Lisa

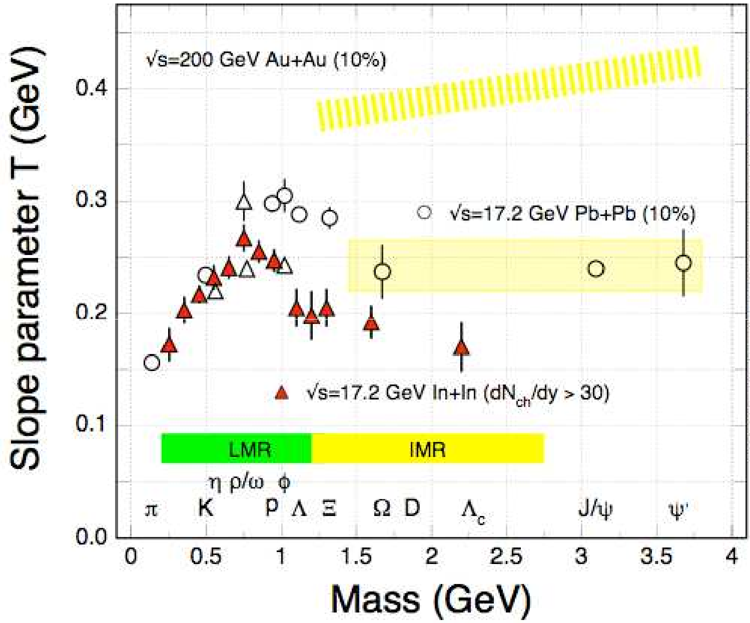
- What physics can spectra address?
- Why do we need a new BlastWave model
- How to implement Tsallis statistics in BlastWave framework
- Can spectra tell us about fluctuation and bulk viscosity?
- Who said p+p spectra are similar to Au+Au?
- Summary and Outlook

What physics can Spectra tell us?

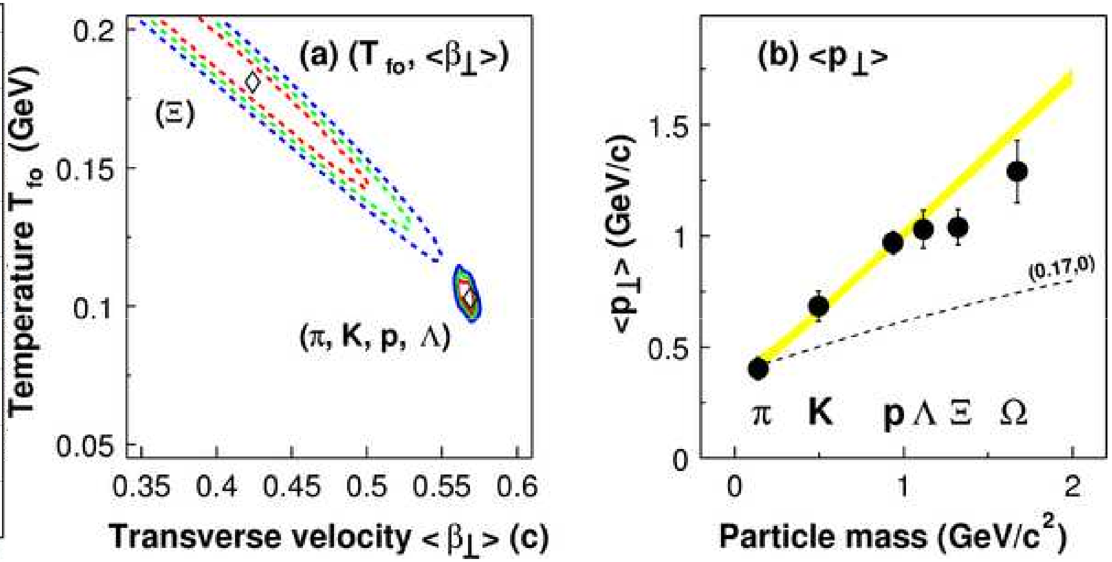
- Low p_T
  - Integrated particle yields (dN/dy)
  - Radial Flow and freeze-out temperature
- Intermediate p_T
  - Coalescence
- High p_T
  - Jet quenching
- What are the connections among them
  - Bulk medium interaction and pressure gradient drives thermalization and radial flow
  - Thermalization and quark degree of freedom provides quark coalescence
  - Jet quenching dissipates energy into the system
- Bulk Viscosity, Fluctuation?

m_T slope vs mass

Nu Xu's plot



Nu Xu, QM2008

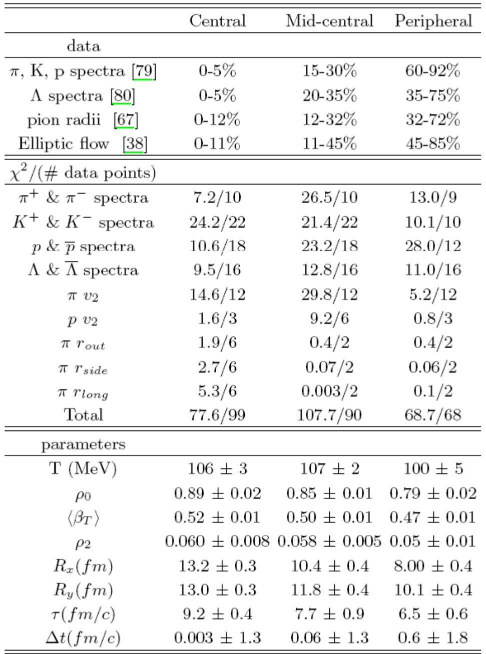


STAR whitepaper, PRL92(2004)

$T_{eff} = T+1/2m\beta^2$

Radial flow

Spectral shape depends on PID mass
Higher mass => larger inverse slope
More central => larger inverse slope



	Central	Mid-central	Peripheral
data			
π, K, p spectra [79]	0-5%	15-30%	60-92%
Λ spectra [80]	0-5%	20-35%	35-75%
pion radii [67]	0-12%	12-32%	32-72%
Elliptic flow [38]	0-11%	11-45%	45-85%
χ^2/(# data points)			
π^+ & π^- spectra	7.2/10	26.5/10	13.0/9
K^+ & K^- spectra	24.2/22	21.4/22	10.1/10
p & $\overline{p}$ spectra	10.6/18	23.2/18	28.0/12
Λ & $\overline{\Lambda}$ spectra	9.5/16	12.8/16	11.0/16
$\pi\ v_2$	14.6/12	29.8/12	5.2/12
$p\ v_2$	1.6/3	9.2/6	0.8/3
$\pi\ r_{out}$	1.9/6	0.4/2	0.4/2
$\pi\ r_{side}$	2.7/6	0.07/2	0.06/2
$\pi\ r_{long}$	5.3/6	0.003/2	0.1/2
Total	77.6/99	107.7/90	68.7/68
parameters			
T (MeV)	106 ± 3	107 ± 2	100 ± 5
ρ_0	0.89 ± 0.02	0.85 ± 0.01	0.79 ± 0.02
$\langle\beta_T\rangle$	0.52 ± 0.01	0.50 ± 0.01	0.47 ± 0.01
ρ_2	0.060 ± 0.008	0.058 ± 0.005	0.05 ± 0.01
$R_x(fm)$	13.2 ± 0.3	10.4 ± 0.4	8.00 ± 0.4
$R_y(fm)$	13.0 ± 0.3	11.8 ± 0.4	10.1 ± 0.4
$\tau(fm/c)$	9.2 ± 0.4	7.7 ± 0.9	6.5 ± 0.6
$\Delta t(fm/c)$	0.003 ± 1.3	0.06 ± 1.3	0.6 ± 1.8

TABLE II: Upper section: data used in the fit. Middle section: number of χ^2/ data points for each measure. Lower section: best fit parameters. Note that $\langle\beta_T\rangle$ is not a fit parameter, but it is calculated from ρ_0.

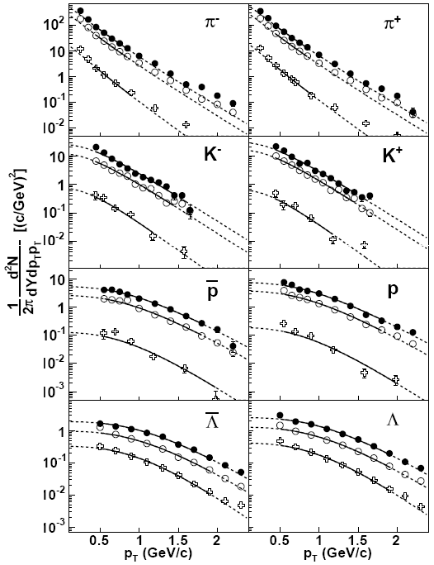
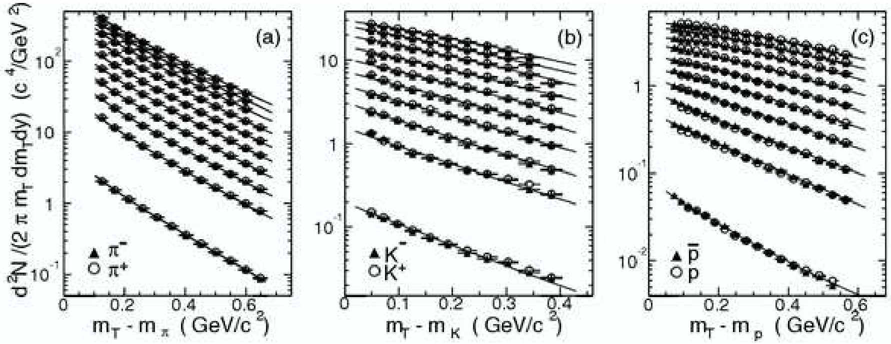
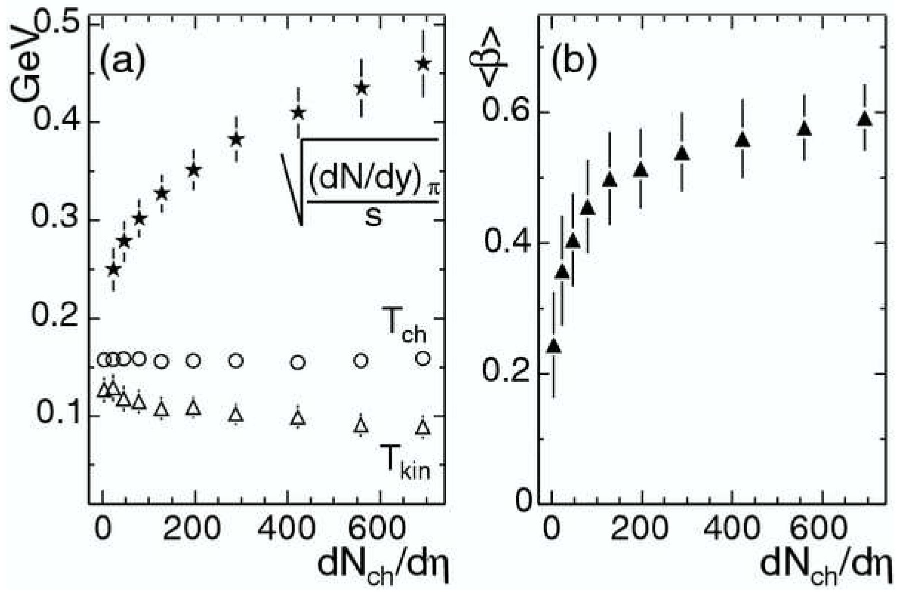


FIG. 49: Comparison of the data with the blast-wave calculations performed with the best fit parameters in three centrality bins. The closed circles are central data, the open circles are mid-central data and the crosses are peripheral data. The plain lines show the blast wave calculation within the fit range while the dash lines show the extrapolation over the whole range.



STAR PRL92



F. Retiere and M. Lisa PRC70; PHENIX PRL88

Blast Wave

Because of azimuthal symmetry we can integrate over ϕ making use of the modified Bessel function $I_0(z) = (2\pi)^{-1}\int_0^{2\pi} e^{z\cos\phi}\,d\phi$:

$$E\frac{d^3n}{d^3p} = \frac{g}{(2\pi)^2}\int_{-\mathcal{Z}}^{\mathcal{Z}} d\zeta\left[m_T\cosh y\frac{\partial z}{\partial\zeta} - m_T\sinh y\frac{\partial t}{\partial\zeta}\right] \times \int_0^R r\,dr\,\exp\left(-\frac{m_T\cosh\rho\cosh(y-\eta)-\mu}{T}\right) I_0\left(\frac{p_T\sinh\rho}{T}\right) \qquad (14)$$

For the transverse mass spectrum we integrate with the help of another modified Bessel function $K_1(z) = \int_0^\infty \cosh y\, e^{-z\cosh y}\,dy$:

$$\frac{dn}{m_T dm_T} = \frac{g}{\pi}m_T\int_{-\mathcal{Z}}^{\mathcal{Z}} d\zeta\left[\cosh\eta\frac{\partial z}{\partial\zeta} - \sinh\eta\frac{\partial t}{\partial\zeta}\right]\int_0^R r\,dr\,K_1\left(\frac{m_T\cosh\rho}{T}\right) I_0\left(\frac{p_T\sinh\rho}{T}\right) = \frac{2g}{\pi}m_T Z_{\hat{t}}\int_0^R r\,dr\,K_1\left(\frac{m_T\cosh\rho}{T}\right) I_0\left(\frac{p_T\sinh\rho}{T}\right) \qquad (15)$$

E. Schnedermann, J. Sollfrank, U. Heinz, nucl-th/9307020, PRC48 (cited 312)

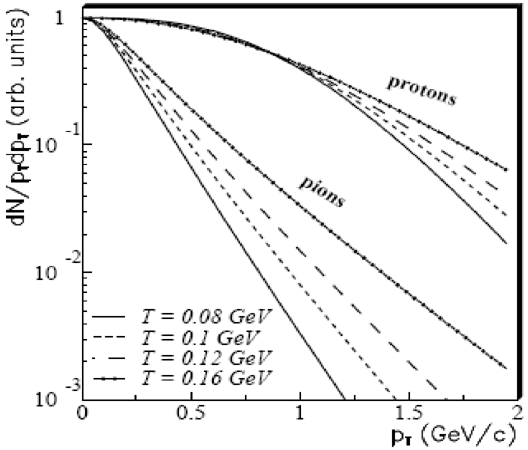


FIG. 4: Transverse momentum spectra for protons (upper curves) and pions (lower curves), as calculated by Equation 19, for several values of the temperature parameter T. Other parameters follow the "round" source defaults of Table I. All spectra are arbitrarily normalized to unity at $p_T = 0$.

F. Retiere, M. Lisa, PRC70

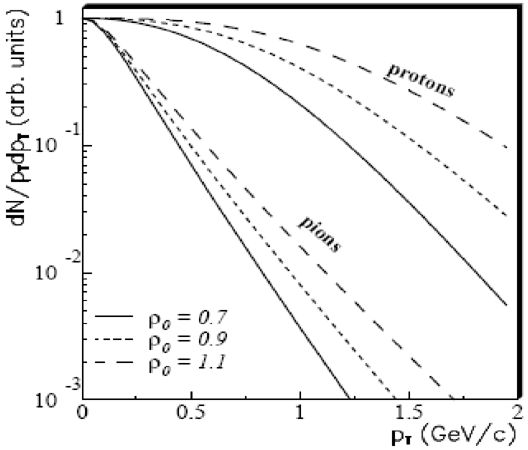


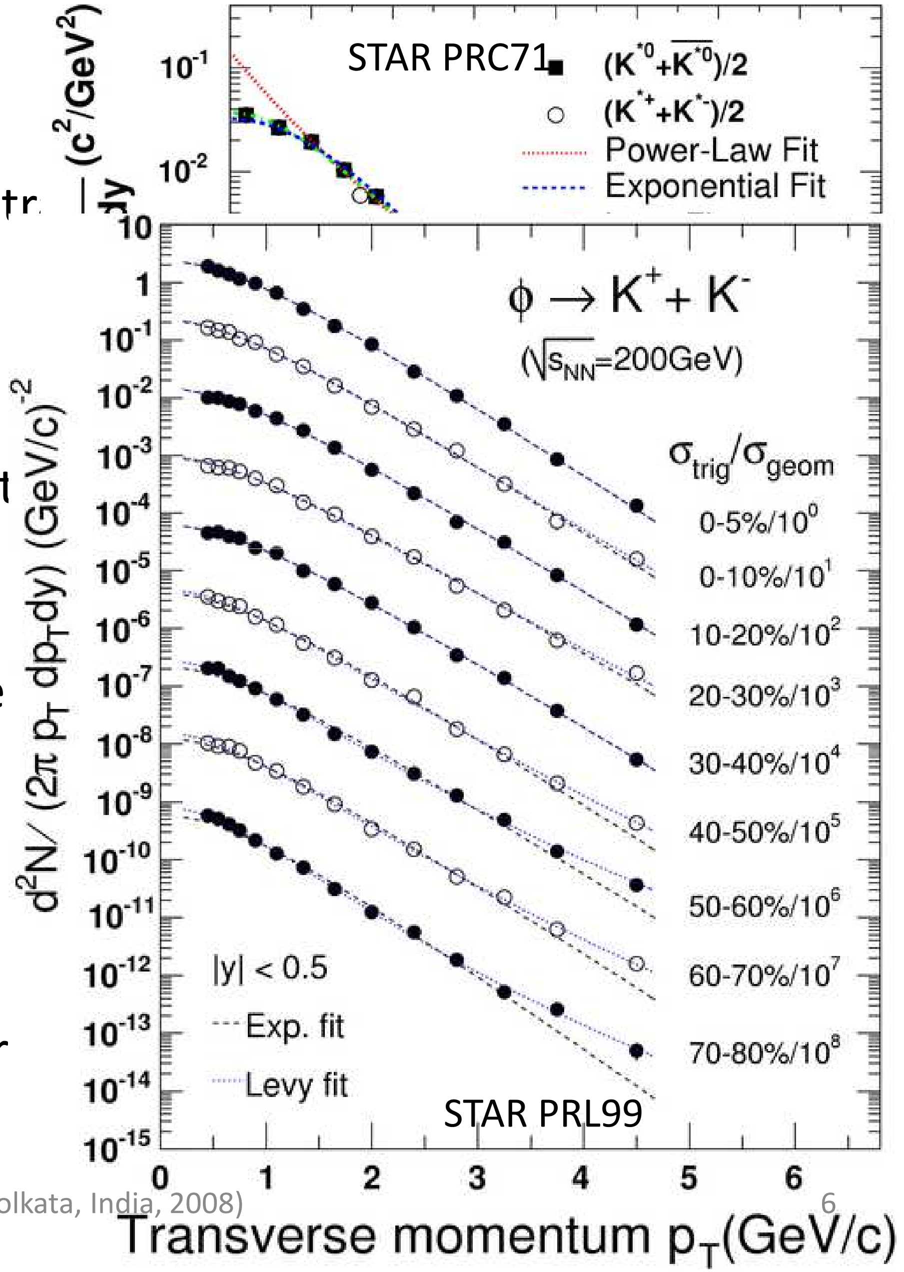
FIG. 5: Transverse momentum spectra for protons (upper curves) and pions (lower curves), as calculated by Equation 19, for several values of the radial flow parameter ρ_0. Other parameters follow the "round" source defaults of Table I. All spectra are arbitrarily normalized to unity at $p_T = 0$.

Assumptions:

1) Local thermal equilibrium ➔ Boltzmann distribution
2) Longitudinal and transverse expansions (1+2)
3) Radial flow profile $\rho(r) \propto \mathrm{Atanh}(\beta_m (r/R)^n)$, (n=1)
4) Temperature and <β> are global quantities

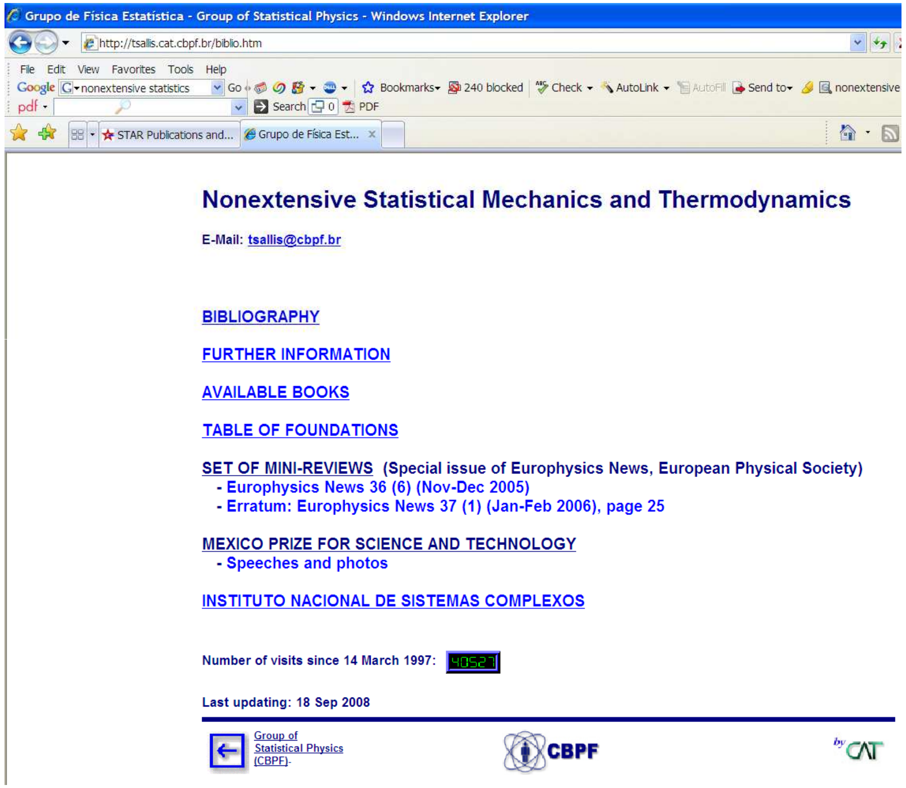
Limitations of THE BlastWave

- Strong assumption on local thermal equilibrium
- Arbitrary choice of p_T range of the spectr (low and high cuts)
- Flow velocity $<\beta>=0.2$ in p+p
- Lack of non-extensive quantities to describe the evolution from p+p to cent A+A collisions
- example in chemical fits: canonical to grand canonical ensemble
- m_T spectra in p+p collisions: Levy function or m_T power-law
- m_T spectra in A+A collisions: Boltzmann or m_T exponential
- What function can capture these featur



Tsallis Statistics

- **Nice web based notebooks: Tsallis Statistics, Statistical Mechar for Non-extensive Systems and Long-Range Interactions http://www.cscs.umich.edu/~crshalizi/notabene/tsallis.html**
- http://tsallis.cat.cbpf.br/biblio.htm



Nonextensive Statistical Mechanics and Thermodynamics

E-Mail: tsallis@cbpf.br

BIBLIOGRAPHY

FURTHER INFORMATION

AVAILABLE BOOKS

TABLE OF FOUNDATIONS

SET OF MINI-REVIEWS (Special issue of Europhysics News, European Physical Society)
- Europhysics News 36 (6) (Nov-Dec 2005)
- Erratum: Europhysics News 37 (1) (Jan-Feb 2006), page 25

MEXICO PRIZE FOR SCIENCE AND TECHNOLOGY
- Speeches and photos

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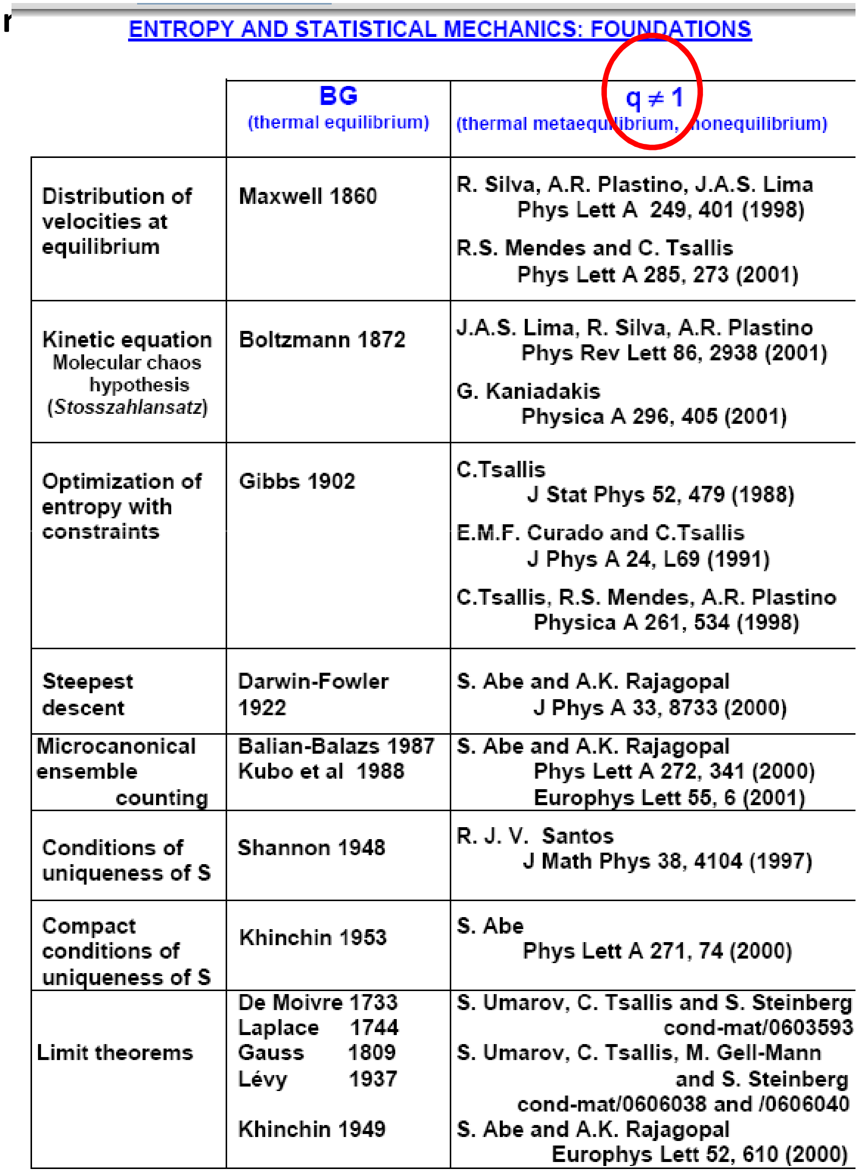
Negative Binomial Distribution: κ=1/(q-1)

Temperature fluctuation: $$\frac{\langle 1/T^2\rangle - \langle 1/T\rangle^2}{\langle 1/T\rangle^2} = 1 - q$$

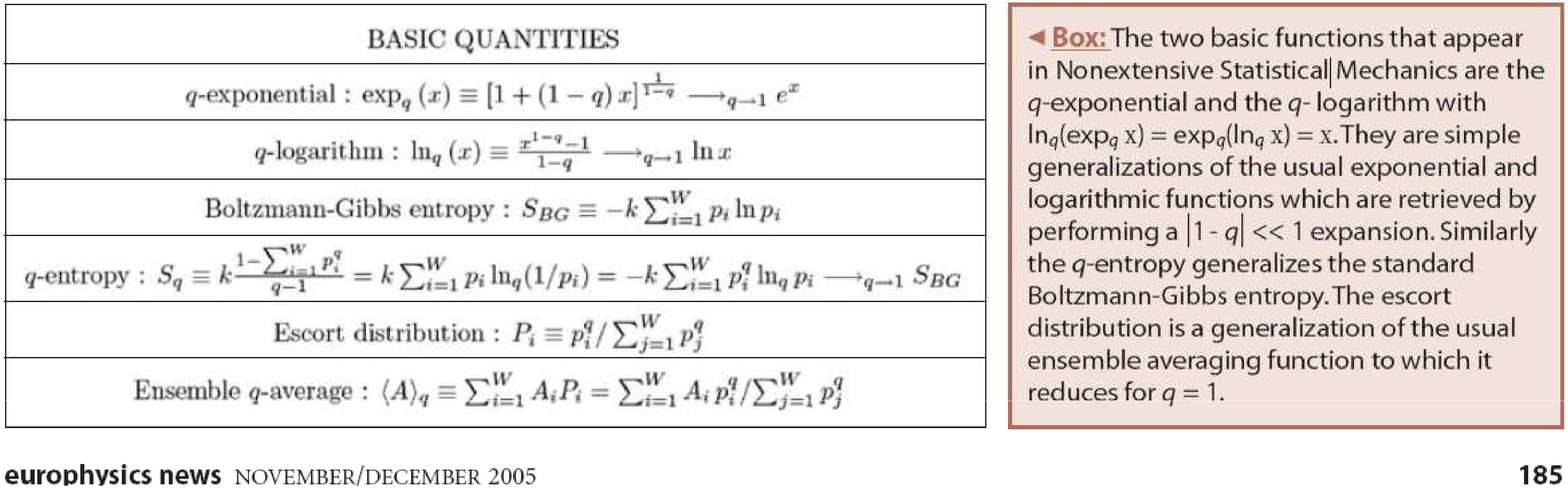
G. Wilk: arXiv: 0810.2939



ENTROPY AND STATISTICAL MECHANICS: FOUNDATIONS

	BG (thermal equilibrium)	$q \neq 1$ (thermal metaequilibrium, nonequilibrium)
Distribution of velocities at equilibrium	Maxwell 1860	R. Silva, A.R. Plastino, J.A.S. Lima Phys Lett A 249, 401 (1998) R.S. Mendes and C. Tsallis Phys Lett A 285, 273 (2001)
Kinetic equation Molecular chaos hypothesis (*Stosszahlansatz*)	Boltzmann 1872	J.A.S. Lima, R. Silva, A.R. Plastino Phys Rev Lett 86, 2938 (2001) G. Kaniadakis Physica A 296, 405 (2001)
Optimization of entropy with constraints	Gibbs 1902	C.Tsallis J Stat Phys 52, 479 (1988) E.M.F. Curado and C.Tsallis J Phys A 24, L69 (1991) C.Tsallis, R.S. Mendes, A.R. Plastino Physica A 261, 534 (1998)
Steepest descent	Darwin-Fowler 1922	S. Abe and A.K. Rajagopal J Phys A 33, 8733 (2000)
Microcanonical ensemble counting	Balian-Balazs 1987 Kubo et al 1988	S. Abe and A.K. Rajagopal Phys Lett A 272, 341 (2000) Europhys Lett 55, 6 (2001)
Conditions of uniqueness of S	Shannon 1948	R. J. V. Santos J Math Phys 38, 4104 (1997)
Compact conditions of uniqueness of S	Khinchin 1953	S. Abe Phys Lett A 271, 74 (2000)
Limit theorems	De Moivre 1733 Laplace 1744 Gauss 1809 Lévy 1937 Khinchin 1949	S. Umarov, C. Tsallis and S. Steinberg cond-mat/0603593 S. Umarov, C. Tsallis, M. Gell-Mann and S. Steinberg cond-mat/0606038 and /0606040 S. Abe and A.K. Rajagopal Europhys Lett 52, 610 (2000)

It is all about the q-statistics



BASIC QUANTITIES
q-exponential : $\exp_q(x) \equiv [1+(1-q)\,x]^{\frac{1}{1-q}} \longrightarrow_{q\to 1} e^x$
q-logarithm : $\ln_q(x) \equiv \frac{x^{1-q}-1}{1-q} \longrightarrow_{q\to 1} \ln x$
Boltzmann-Gibbs entropy : $S_{BG} \equiv -k\sum_{i=1}^{W} p_i \ln p_i$
q-entropy : $S_q \equiv k\frac{1-\sum_{i=1}^{W} p_i^q}{q-1} = k\sum_{i=1}^{W} p_i \ln_q(1/p_i) = -k\sum_{i=1}^{W} p_i^q \ln_q p_i \longrightarrow_{q\to 1} S_{BG}$
Escort distribution : $P_i \equiv p_i^q / \sum_{j=1}^{W} p_j^q$
Ensemble q-average : $\langle A\rangle_q \equiv \sum_{i=1}^{W} A_i P_i = \sum_{i=1}^{W} A_i\, p_i^q / \sum_{j=1}^{W} p_j^q$

◀ **Box:** The two basic functions that appear in Nonextensive Statistical Mechanics are the q-exponential and the q- logarithm with $\ln_q(\exp_q x) = \exp_q(\ln_q x) = x$. They are simple generalizations of the usual exponential and logarithmic functions which are retrieved by performing a $|1 - q| << 1$ expansion. Similarly the q-entropy generalizes the standard Boltzmann-Gibbs entropy. The escort distribution is a generalization of the usual ensemble averaging function to which it reduces for $q = 1$.

europhysics news NOVEMBER/DECEMBER 2005 **185**

- Why is this relevant to us (Heavy-ion physics)?
  - We have dealt with Boltzmann distribution
    But the spectra are clearly non-Boltzmann
  - It is easy to make a change
  - It is easy to compare
  - Change m_T exponential to m_T power law

Tsallis statistics in Blast Wave model

Because of azimuthal symmetry we can integrate over ϕ making use of the modified Bessel function $I_0(z) = (2\pi)^{-1} \int_0^{2\pi} e^{z\cos\phi} d\phi$:

$$E\frac{d^3n}{d^3p} = \frac{g}{(2\pi)^2} \int_{-\mathcal{Z}}^{\mathcal{Z}} d\zeta \left[m_T \cosh y \frac{\partial z}{\partial \zeta} - m_T \sinh y \frac{\partial t}{\partial \zeta} \right] \times \int_0^R r\,dr\, \exp\left(-\frac{m_T \cosh\rho \cosh(y-\eta) - \mu}{T} \right) I_0\left(\frac{p_T \sinh\rho}{T} \right) \qquad (14)$$

For the transverse mass spectrum we integrate with the help of another modified Bessel function $K_1(z) = \int_0^\infty \cosh y\, e^{-z\cosh y} dy$:

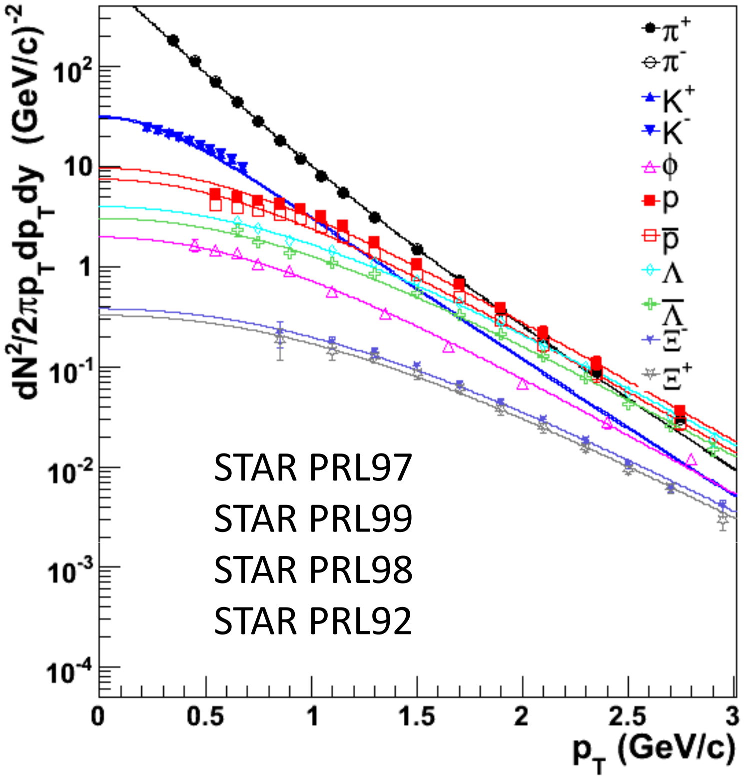
$$\begin{aligned} \frac{dn}{m_T dm_T} &= \frac{g}{\pi} m_T \int_{-\mathcal{Z}}^{\mathcal{Z}} d\zeta \left[\cosh\eta \frac{\partial z}{\partial \zeta} - \sinh\eta \frac{\partial t}{\partial \zeta} \right] \int_0^R r\,dr\, K_1\left(\frac{m_T \cosh\rho}{T} \right) I_0\left(\frac{p_T \sinh\rho}{T} \right) \\ &= \frac{2g}{\pi} m_T Z_{\tilde{t}} \int_0^R r\,dr\, K_1\left(\frac{m_T \cosh\rho}{T} \right) I_0\left(\frac{p_T \sinh\rho}{T} \right) \qquad (15) \end{aligned}$$

With Tsallis distribution, the BlastWave equation is:

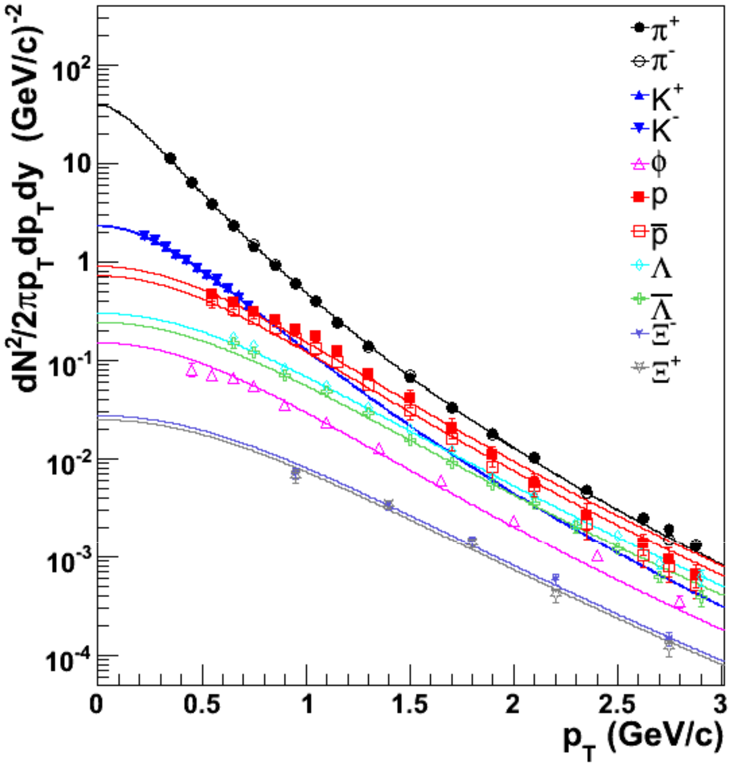
$$\frac{dN}{m_T dm_T} \propto m_T \int_{-Y}^{+Y} \cosh(y) dy \int_{-\pi}^{+\pi} d\phi \int_0^R r dr \left(1 + \frac{q-1}{T}(m_T \cosh(y)\cosh(\rho) - p_T \sinh(\rho)\cos(\phi))\right)^{-1/(q-1)}$$

Where $\rho = \mathrm{Atanh}(\beta_m (r/R)^n)$, n=1 ; any of the three integrals is HypergeometryF1

Fit results in Au+Au collisions

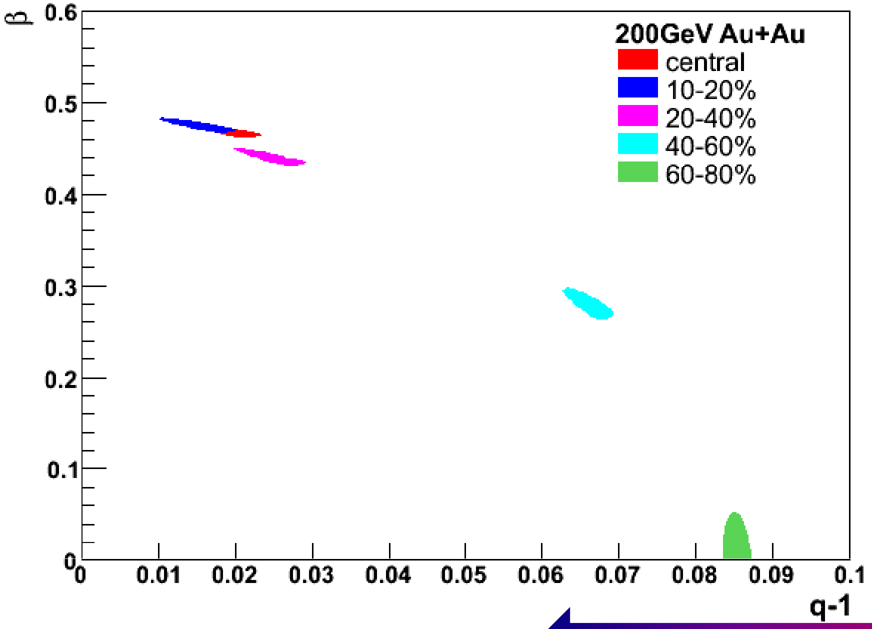


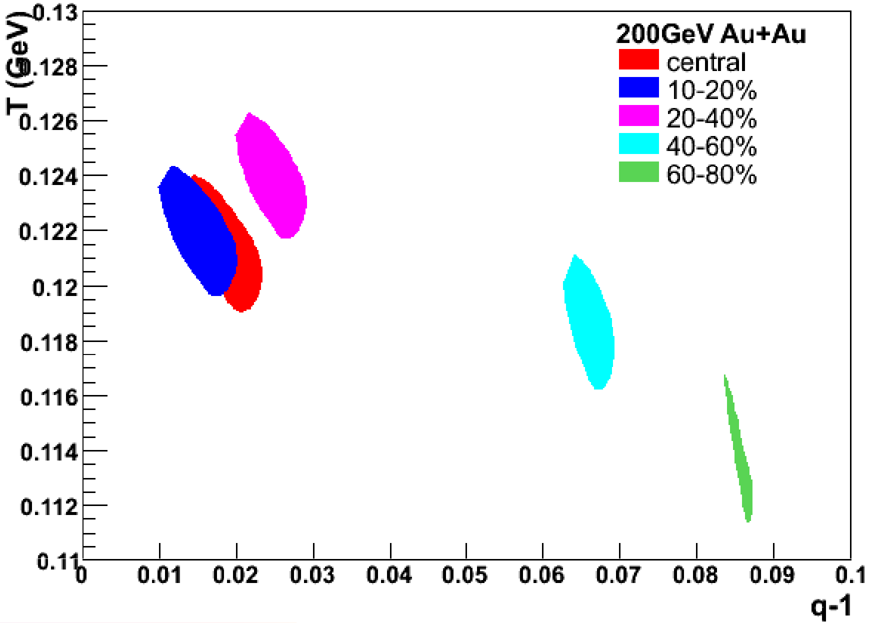
Au+Au 0—10%:
<β> = 0.470+- 0.009
T = 0.122 +- 0.002
q = 1.018 +- 0.005
chi^2/nDof = 130 / 125



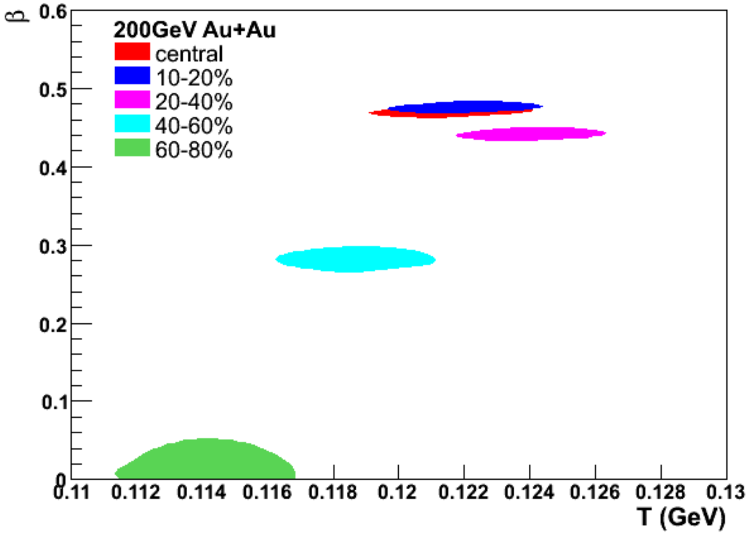
Au+Au 60—80%:
<β>=0
T = 0.114 +- 0.003
q = 1.086 +- 0.002
chi^2/nDof = 138/ 123

Dissipative energy into flow and heat





More thermalized



1. Decrease of q➔1, closer to Boltzmann
2. Increase of radial flow (0➔0.5)
3. Increase of temperature
4. T, $\beta \propto (q-1)^2$, NOT linear (q-1)

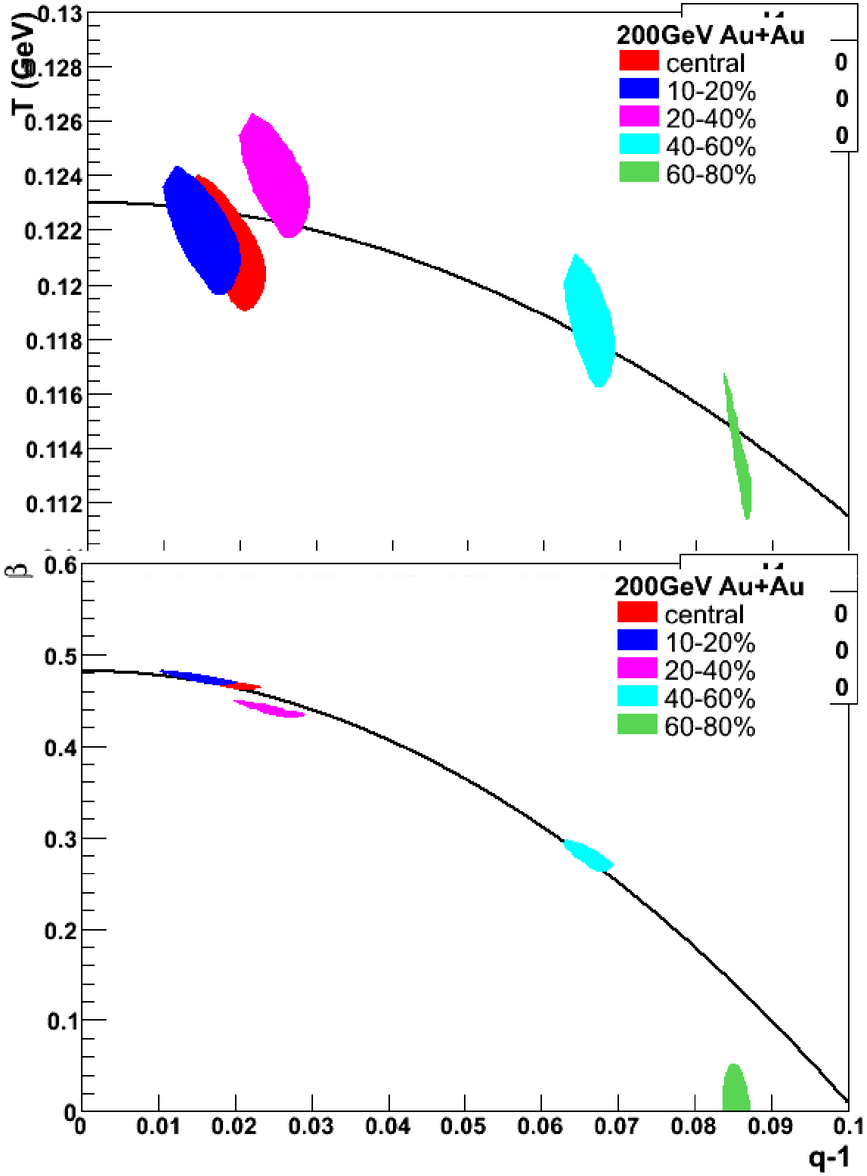
Related to bulk viscosity (ξ)

$$T_{eff} = T_0 + \frac{\xi}{a} f(\beta)$$

$$= T_0 + (q-1)\frac{(\xi/\rho)(c_p\rho/a)}{(c_p/c_V)} f(\beta)$$

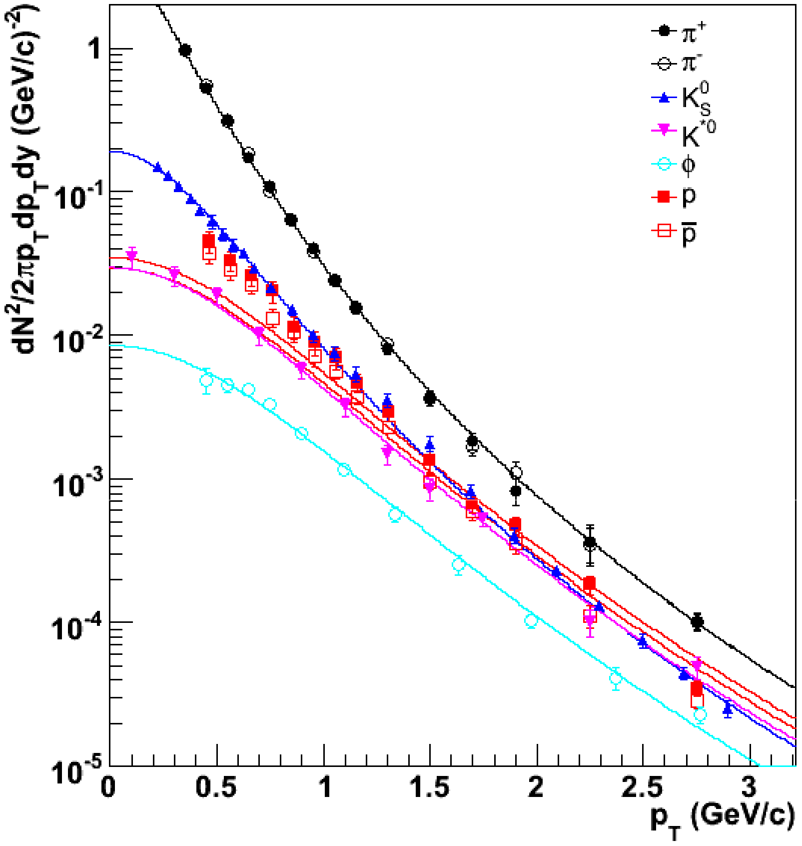
$$= T_0 + (q-1)^2 \frac{(\xi/\rho)}{(c_p/c_V)D} f(\beta)$$

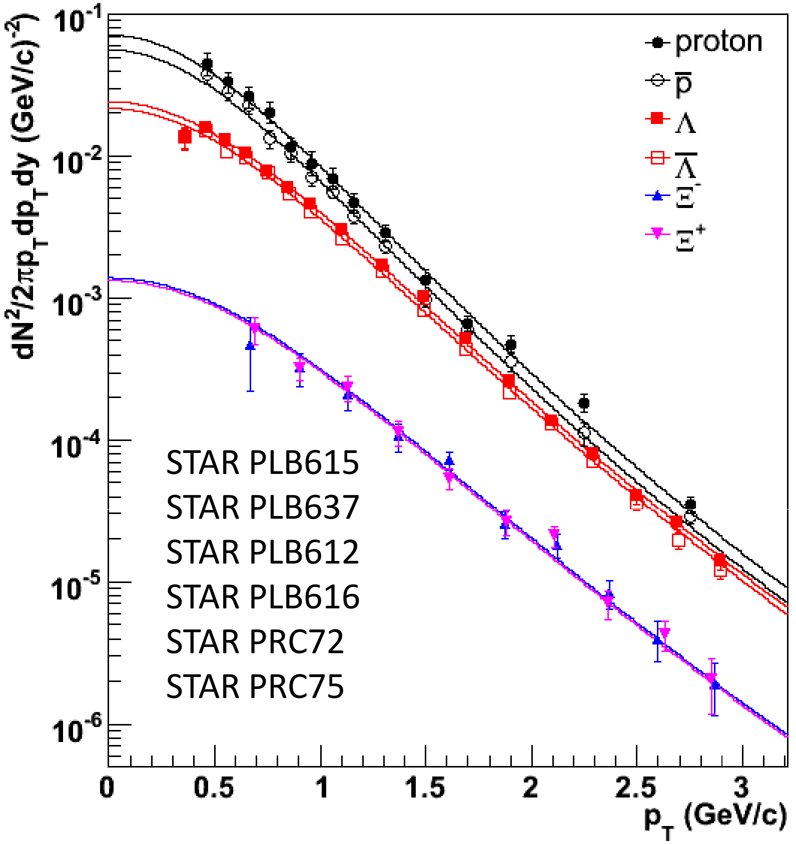
c_p, ρ and *a* are, respectively,
the specific heat under
constant pressure,
density and
the coefficient of external conductance



G. Wilk: arXiv: 0810.2939

Results in p+p collisioins

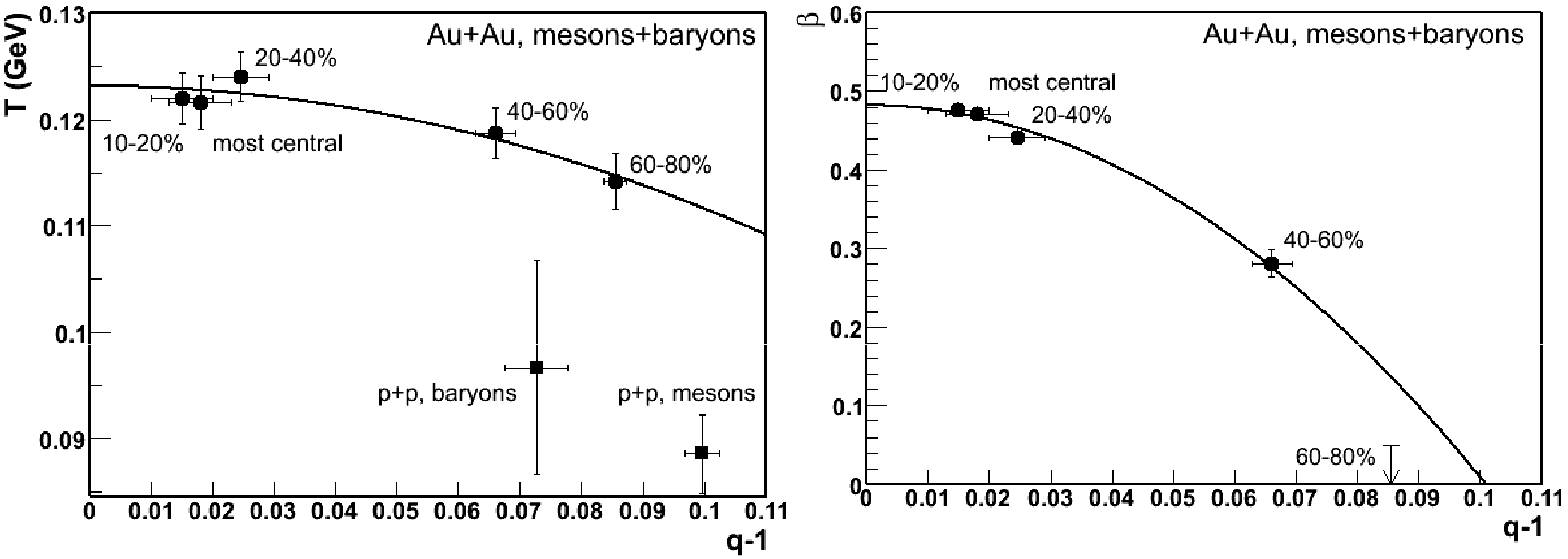




<β> = 0
T = 0.0889+- 0.004
q = 1.100 +- 0.003
chi^2/nDof = 53 / 66

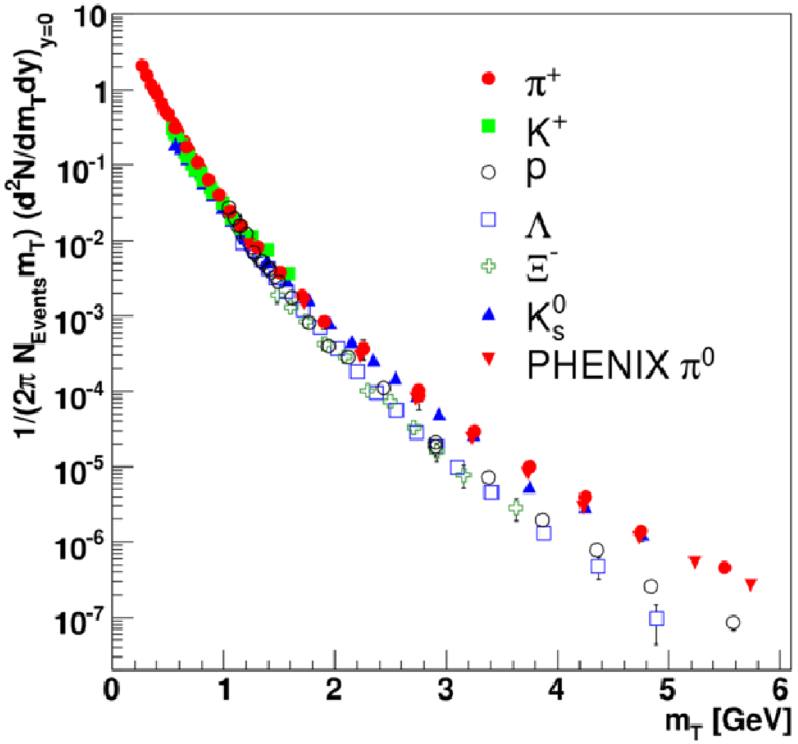
<β> = 0
T = 0.097+- 0.010
q = 1.073 +- 0.005
chi^2/nDof = 55 / 73

Evolution from p+p to Au+Au

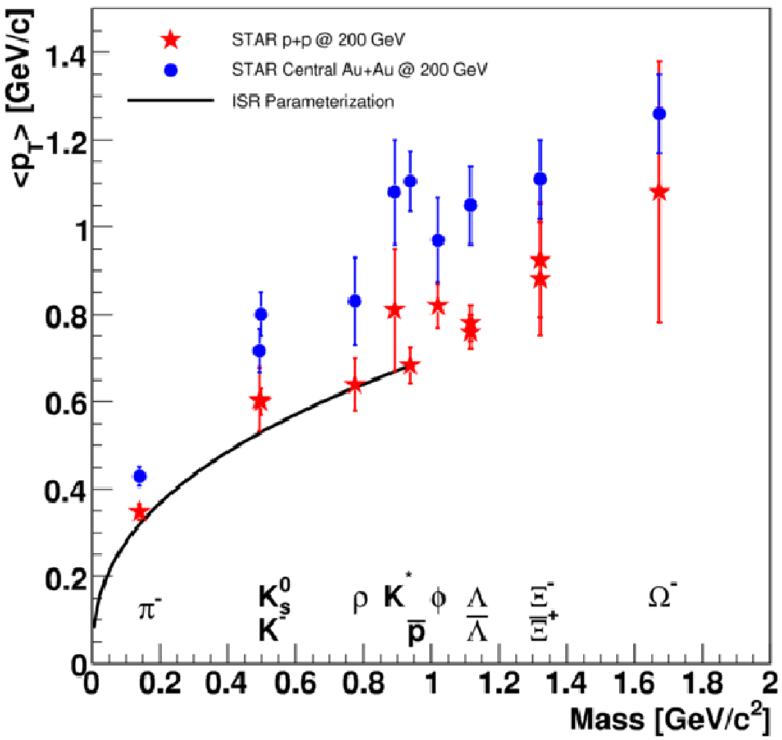


- Sharp increase of <T> from p+p to peripheral Au+Au
- Similar q from p+p to peripheral Au+Au
- Radial flow is zero at p+p and peripheral Au+Au

Baryon and meson are different classes



STAR PRC75



In p+p collisions, the m_T spectra of baryons and mesons are in two groups
However, equilibrated toward more central Au+Au collisions

Observations from the q-statistics

- Fit spectra well for all particles with p_T<~ 3 GeV/c
- Radial flow increases from 0 to 0.5c
- Kinetical freeze-out temperature increases from 90 (110) to 130 MeV
- q-1 decreases from 0.1 to 0.01
- T and β depend on $(q-1)^2$
- p+p collisions are very different, split between mesons and baryons

- Tsallis statistics describes the data better than Boltzmann-Gibbs statistics
- Radial flow is zero in p+p and peripheral Au+Au collisions
- Evolution from peripheral to central Au+Au collisions:
  hot spots (temperature fluctuation) are quenched toward a more uniform Boltzmann-like distribution
- dissipative energy into heat and flow, related to bulk viscosity
- Energy conservation is a built-in requirement in any statistical model (that is where you get the temperature)

Outlook

- Search for critical point:
  - large bulk viscosity at phase transition
  - PID spectra to 3 GeV/c
  - Study T, β vs q-1 with centrality and energy AGS➔SPS➔RHIC

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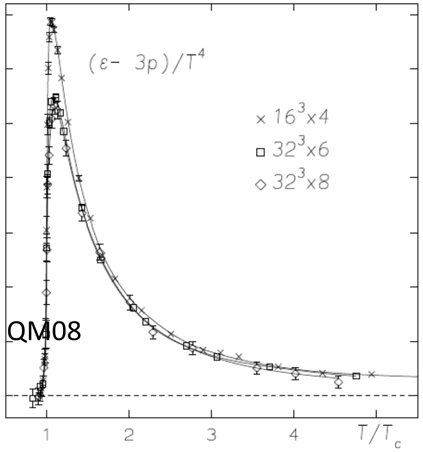

SU(3),
pure gauge

Talks by
J. Ellis,
S. Gupta,
F. Karsch,
Z. Fodor,
…

D. Kharzeev et al., QM08

The lattice data from G.Boyd, J.Engels, F.Karsch, E.Laermann, C.Legeland, M.Lutgeimer, B.Petersson, hep-lat/9602007

- Higher energy at LHC:
  - Large power-law tail due to semi-hard processes
  - Without Tsallis distribution, it is likely impossible to extract radial flow from spectra
  - Good (large) non-extensive effect and easy to extract bulk viscosity

Application of Tsallis statistics has a long history at RHIC

1) **Non-extensive thermodynamics, heavy ion collisions and particle production at RHIC energies.**
Bhaskar De (Maulana Azad Coll.) , S. Bhattacharyya (Indian Statistical Inst., Calcutta) , Goutam Sau (Unlisted) , S.
Published in **Int.J.Mod.Phys.E16:1687-1700,2007.**

LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Cited 1 time
Journal Server
Bookmarkable link to this information

2) **Nonextensive hydrodynamics for relativistic heavy-ion collisions.**
T. Osada (Musashi Inst. Tech.) , G. Wilk (Warsaw, Inst. Nucl. Studies) . Feb 2008. 23pp.
Published in **Phys.Rev.C77:044903,2008.**
e-Print: **arXiv:0710.1905** [nucl-th]

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3) **Signals of non-extensive statistical mechanics in high-energy nuclear collisions.**
W.M. Alberico (Turin U. & INFN, Turin) , P. Czerski (Cracow, INP) , A. Lavagno (Turin Polytechnic & INFN, Turin) ,
CERN & INFN, Turin) , V. Soma (Turin U.) . Oct 2005. 13pp.
e-Print: **hep-ph/0510271**

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Abstract and Postscript and PDF from arXiv.org (mirrors: au br cn de es fr il in it jp kr ru tw uk za aps lanl)
Bookmarkable link to this information

4) **A Nonextensive model for quark matter produced in heavy ion collisions.**
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