Stony Brook University | The State University of New York

Supported in part by:

U.S. DEPARTMENT OF ENERGY | Office of Science

Imaging the shape of 238U via correlation between elliptic flow and radial flow

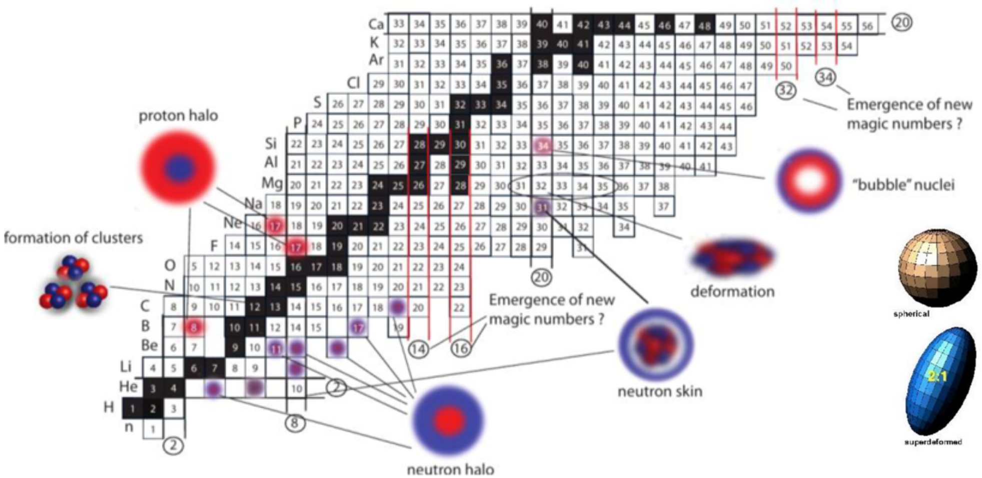
Jiangyong Jia for the STAR Collaboration

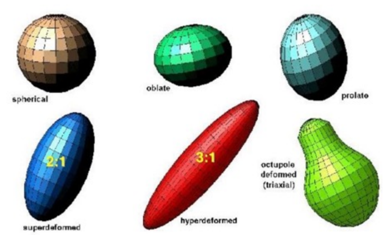
2023 IS INITIAL STAGES

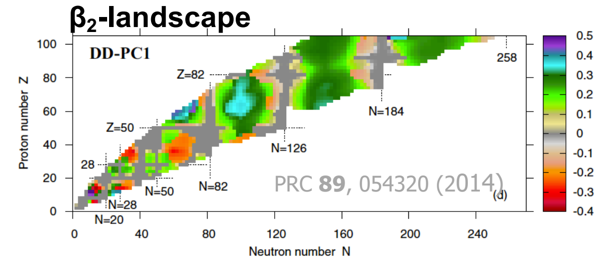
The 7th International Conference on the Initial Stages of High-Energy Nuclear Collisions : Initial Stages 2023

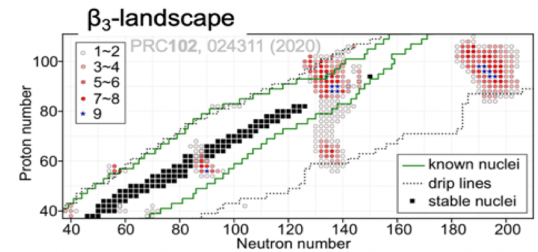
Shapes of atomic nuclei

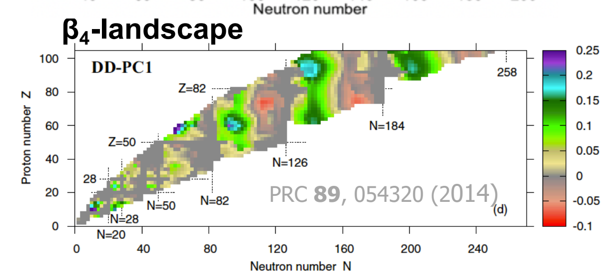
- Collective phenomena of many-body quantum system
  - clustering, halo, skin, bubble…
  - quadrupole/octupole/hexdecopole deformations
  - Non-monotonic evaluation with N and Z.









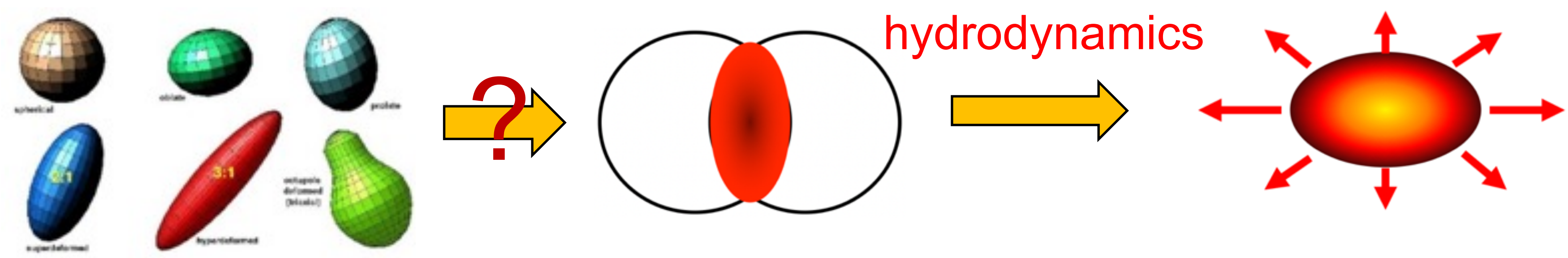


$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+e^{(r-R(\theta,\phi))/a_0}}$$

$$R(\theta,\phi) = R_0(1+\beta_2[\cos\gamma Y_{2,0}(\theta,\phi)+\sin\gamma Y_{2,2}(\theta,\phi)]+\beta_3 Y_{3,0}(\theta,\phi)+\beta_4 Y_{4,0}(\theta,\phi))$$

Flow assisted imaging in heavy ion collision

Nuclear structure | Initial condition | Final state



Shape and radial dis.

- $\beta_2 \rightarrow$ Quadrupole deformation
- $\beta_3 \rightarrow$ Octupole deformation
- $a_0 \rightarrow$ Surface diffuseness
- $R_0 \rightarrow$ Nuclear size

Volume, size and shape

N_{part}

$R_\perp^2 \propto \langle r_\perp^2 \rangle,$

$\mathcal{E}_n \propto \langle r_\perp^n e^{in\phi} \rangle$

Observables

$$\frac{d^2N}{d\phi dp_T} = N(p_T)\left(\sum_n V_n \, e^{-in\phi}\right)$$

Flow-assisted imaging relies on linear response: $N_{ch} \propto N_{part} \quad \frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp} \quad V_n \propto \mathcal{E}_n$

- Constrain the initial condition by comparing nuclei with known structure properties
- Reveal novel properties of nuclei by leveraging known hydrodynamic response.

Strategy for nuclear shape imaging

Flow observable = k $\otimes$ initial condition (structure)

QGP response, a smooth function of N+Z

Structure of colliding nuclei, non-monotonic function of N and Z

Compare two systems of similar size but different structure

---

^{238}U+^{238}U RUN12 193GeV ^{197}Au+^{197}Au RUN10/11 200GeV

^{238}U is strongly prolate : $\beta_{2\mathrm{U}} = 0.287 \pm 0.007$ $\gamma_{\mathrm{U}} = 6°–8°$

$$\beta_{2,\mathrm{rotor}} = \frac{4\pi}{5R_0^2 Z}\sqrt{\frac{B(\mathrm{E2})}{e^2}}$$

arXiv: 1312.5975 *PRC* 54, 2356 (1996)

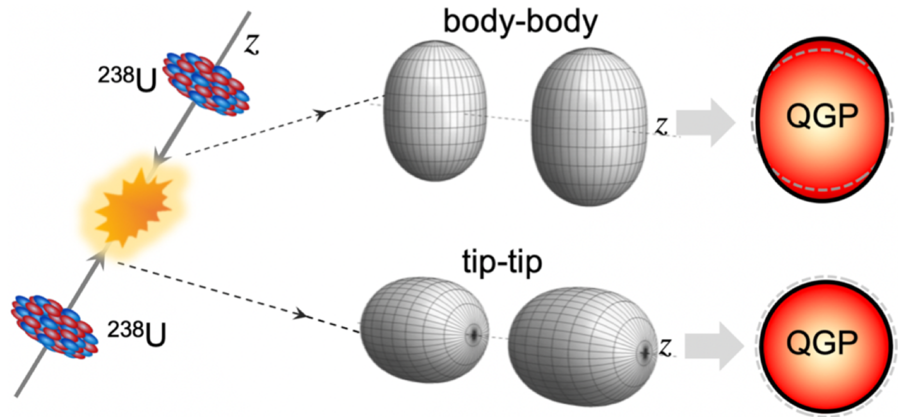
^{197}Au predicted to be slightly oblate: $\beta_{2\mathrm{Au}}$ =0.1–0.14 $\gamma_{\mathrm{Au}} \gtrsim 40°$

arXiv: 2301.02420

Use charged particles in $|\eta|<1$, 0.3-3 GeV/c with STAR TPC

Impact of quadrupole deformation in U+U

G. Giuliano 1910.04673 $\delta p_{\mathrm{T}} \propto -\delta R_{\perp}$



$\varepsilon_2 \nearrow R_{\perp} \nearrow$ $v_2 \nearrow p_T \searrow$

$\varepsilon_2 \searrow R_{\perp} \searrow$ $v_2 \searrow p_T \nearrow$

Random orientations increase flow fluctuations

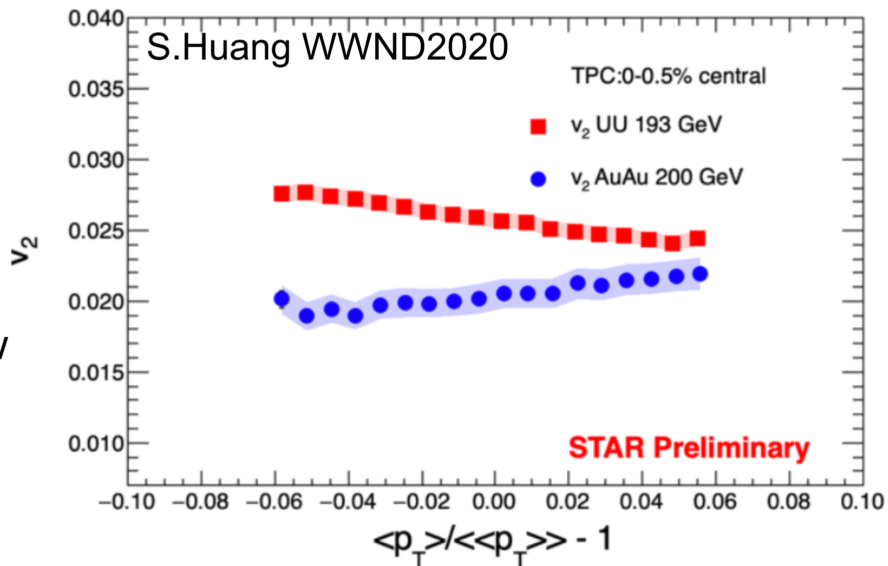
2109.00604

$$\langle v_2^2 \rangle \approx a_2 + b_2 \beta_2^2$$

$$\left\langle (\delta p_{\mathrm{T}})^2 \right\rangle \approx a_0 + b_0 \beta_2^2$$

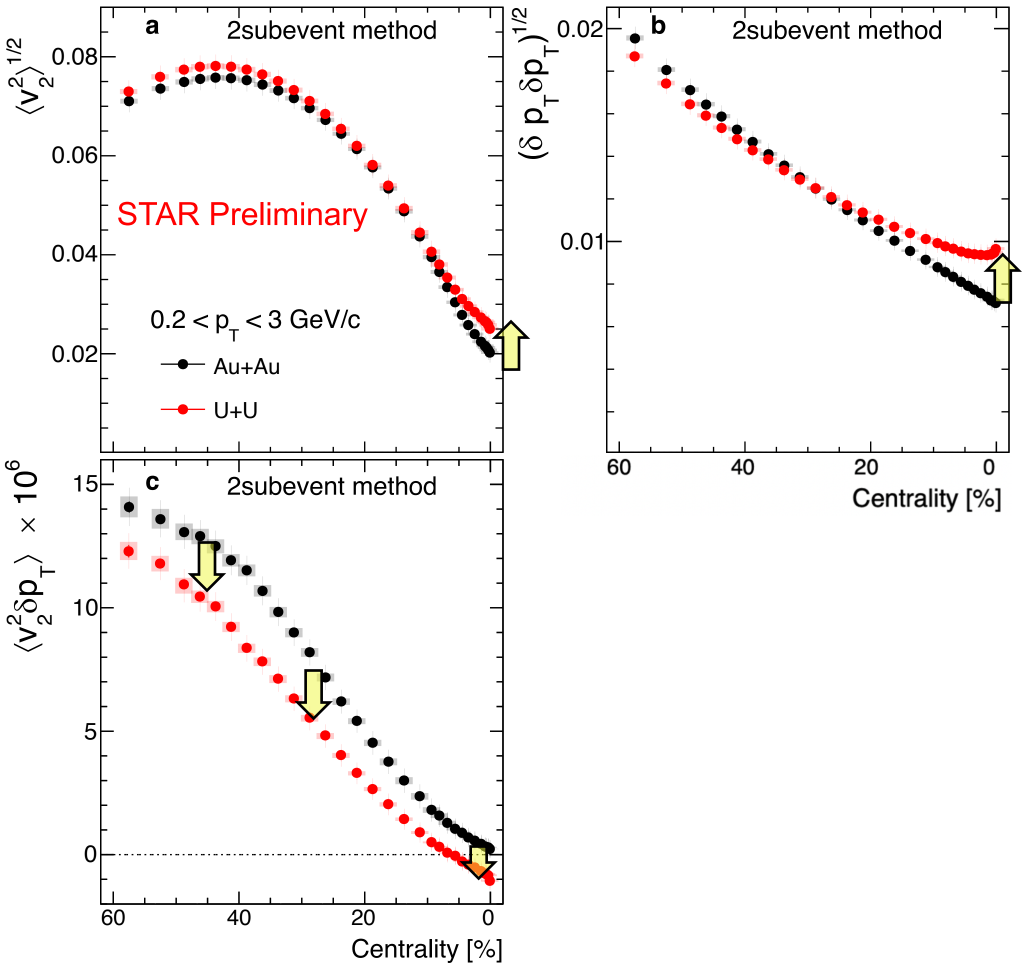
… anticorrelation between elliptic flow and radial flow

$$\langle v_2^2 \delta p_{\mathrm{T}} \rangle \approx a - b \cos(3\gamma) \beta_2^3$$



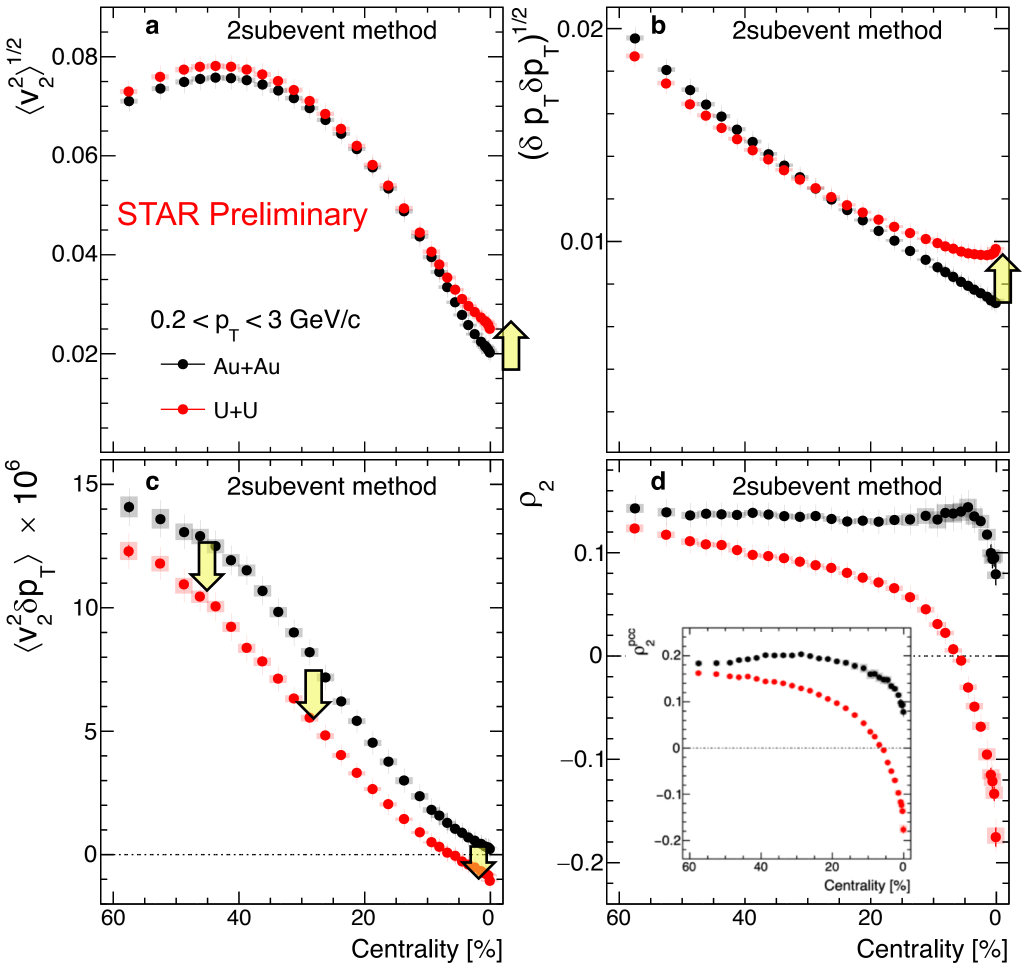
Three-particle correlation probes the triaxiality at leading order!

Quantify the correlation strength



Influence of deformation limited to 20% centrality for v_2 and p_T variances. But the full range for $<v_2^2 \delta p_T>$

Quantify the correlation strength



Influence of deformation limited to 20% centrality for v_2 and p_T variances. But the full range for $\langle v_2^2\, \delta p_T \rangle$

Pearson correlation coefficient

P. Bozek 1601.04513

$$\rho_2^{\mathrm{pcc}} \equiv \frac{\langle v_2^2 \delta p_{\mathrm{T}} \rangle}{\sqrt{\mathrm{var}(v_2^2)}\sqrt{\langle (\delta p_{\mathrm{T}})^2 \rangle}}$$

$$\mathrm{var}(v_2^2) = \langle v_2^2 \rangle^2 - v_2^4\{4\}$$

$$\approx (a_2 + b_2 \beta_2^2)^2 - c_2$$

Not ideal for deformation study

Adopt alternative normalization

$$\rho_2 \equiv \frac{\langle v_2^2 \delta p_{\mathrm{T}} \rangle}{\langle v_2^2 \rangle \sqrt{\langle (\delta p_{\mathrm{T}})^2 \rangle}}$$

ρ_2 in AuAu is a better baseline for spherical nuclei than ρ_2^{pcc}

Model comparison

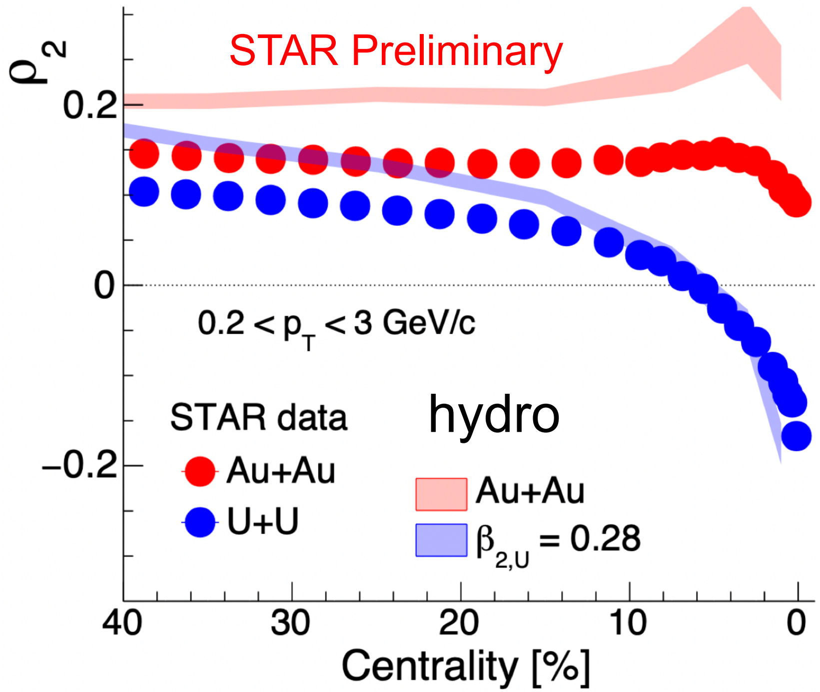
v_2-p_T is impacted by both initial state and final state effects.

IS: MC Quark Glauber model

IS+FS: IP-Glasma+Music+UrQMD from Chun Shen et.al.

Compare with hydro model

$$\rho_2 \equiv \frac{\langle v_2^2 \delta p_T \rangle}{\langle v_2^2 \rangle \sqrt{\langle (\delta p_T)^2 \rangle}}$$

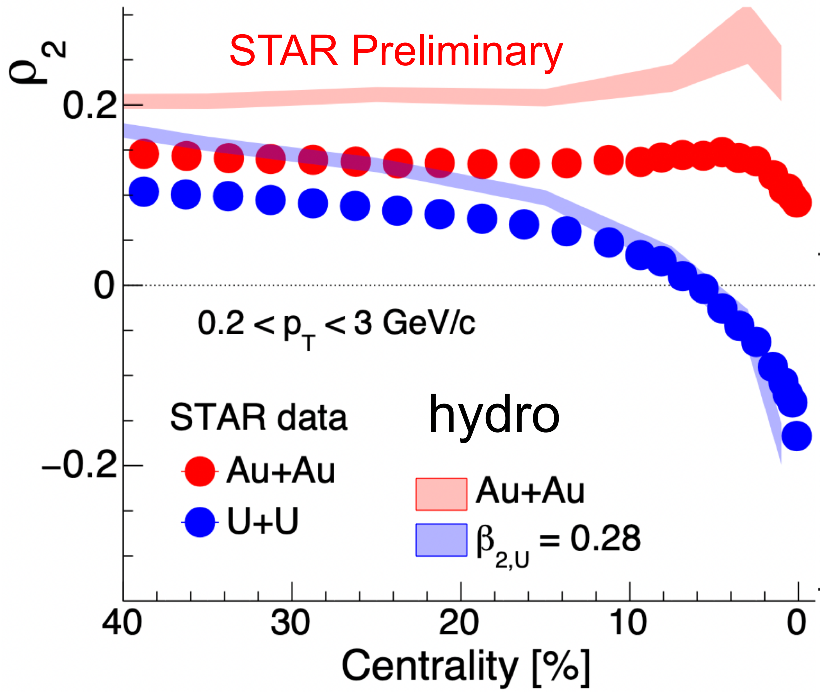


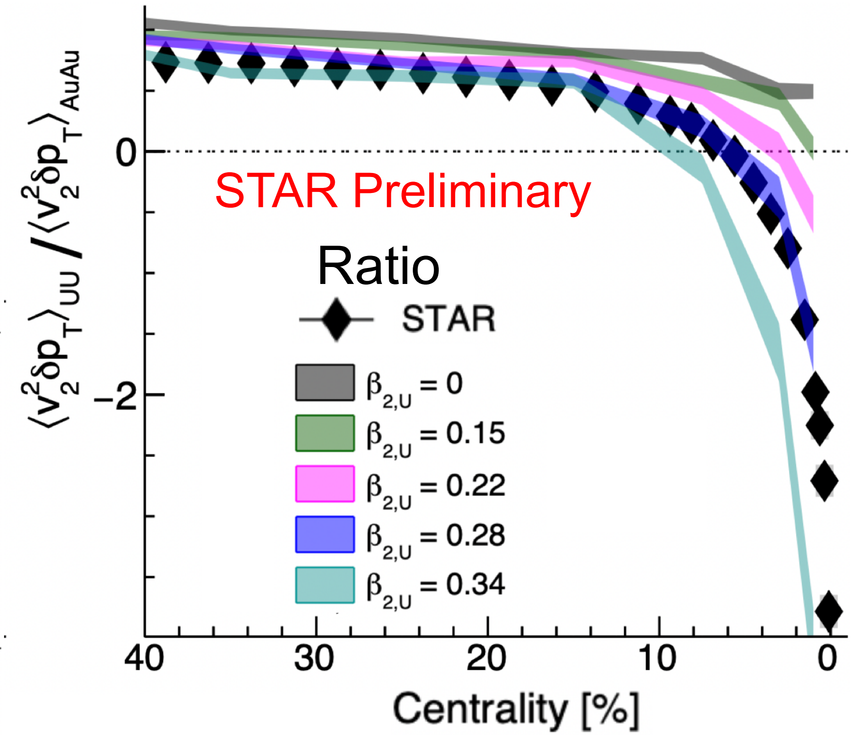
$$\langle v_2^2 \delta p_T \rangle = a - b\cos(3\gamma)\beta_2^3$$

- Hydro model describes trends of ρ_2 but not its absolute values

Compare with hydro model

$$\rho_2 \equiv \frac{\langle v_2^2 \delta p_{\mathrm{T}} \rangle}{\langle v_2^2 \rangle \sqrt{\langle (\delta p_{\mathrm{T}})^2 \rangle}}$$





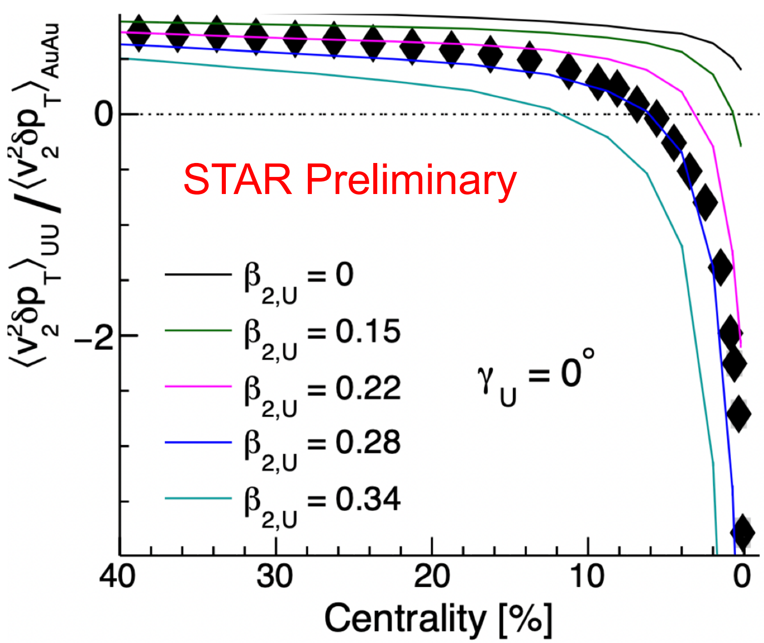
$$\langle v_2^2 \delta p_{\mathrm{T}} \rangle = a - b\cos(3\gamma)\beta_2^3$$

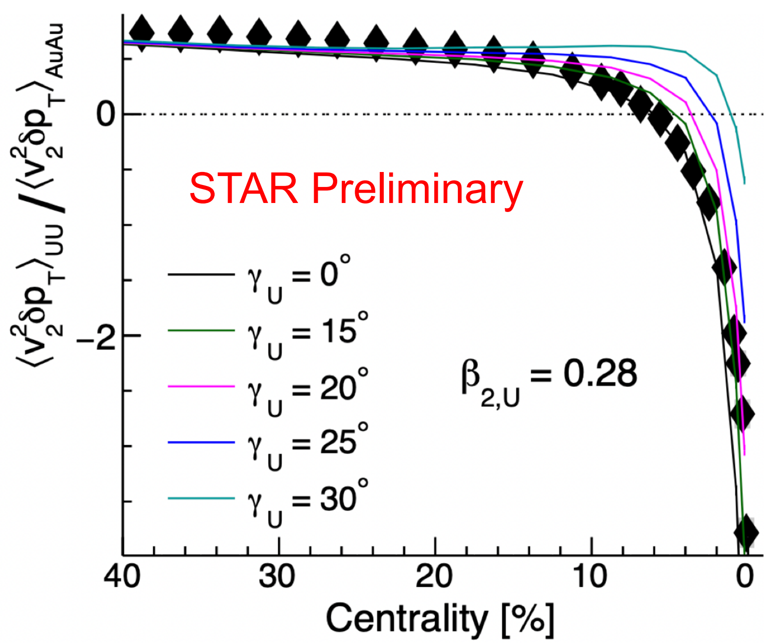
- Hydro model describes trends of ρ_2 but not its absolute values
- Achieves a better description of $\langle v_2^2 \delta p_T \rangle_U / \langle v_2^2 \delta p_T \rangle_{Au}$.
- The $\boldsymbol{\beta}_{2U,WS}$=0.28, γ_U=0 agrees with the data.

Compare with Glauber model

Linear response: $v_2 \propto \varepsilon_2$ $\delta p_\mathrm{T}/p_\mathrm{T} \propto \delta d_\perp/d_\perp$ $d_\perp = 1/R_\perp$

initial state estimator: $\langle \varepsilon_2^2 \delta d_\perp/d_\perp \rangle_\mathrm{UU} \,/\, \langle \varepsilon_2^2 \delta d_\perp/d_\perp \rangle_\mathrm{AuAu}$





Glauber model prefers $\boldsymbol{\beta}_{2U,WS}$=0.28, γ_U=0
→reflects initial state effects

Assuming $\boldsymbol{\beta}_{2U,WS}$=0.28, data prefers $\gamma_{U,WS} \lesssim 15^o$, larger $\gamma_{U,WS}$ spoils shape

Values are consistent with low-energy estimates based on rigid rotor model

$$\beta_{2,\mathrm{rotor}} = \frac{4\pi}{5R_0^2 Z}\sqrt{\frac{B(\mathrm{E2})}{e^2}}$$

$\beta_{2\mathrm{U}} = 0.287 \pm 0.007$ 1312.5975

$\gamma_\mathrm{U} = 6°\text{–}8°$ PRC54, 2356 (1996)

Another way to probe deformation

$$\langle v_2^2 \delta p_{\mathrm{T}} \rangle \approx a - b\cos(3\gamma)\beta_2^3 \quad \langle v_2^2 \rangle \approx a_2 + b_2\beta_2^2 \quad \left\langle (\delta p_{\mathrm{T}})^2 \right\rangle \approx a_0 + b_0\beta_2^2$$

Removing "a" terms by subtracting the system A (UU) and B (AuAu)

$$\rho_2^{\mathrm{sub}} = \frac{\langle v_2^2 \delta p_{\mathrm{T}} \rangle_{\mathrm{A}} - \langle v_2^2 \delta p_{\mathrm{T}} \rangle_{\mathrm{B}}}{(\langle \mathrm{v}_2^2 \rangle_{\mathrm{A}} - \langle \mathrm{v}_2^2 \rangle_{\mathrm{B}})\sqrt{\left(\left\langle (\delta p_{\mathrm{T}})^2 \right\rangle_{\mathrm{A}} - \left\langle (\delta p_{\mathrm{T}})^2 \right\rangle_{\mathrm{B}}\right)}} \approx -\frac{b_2\left(\cos(3\gamma_{\mathrm{A}})\beta_{\mathrm{A}}^3 - \cos(3\gamma_{\mathrm{B}})\beta_{\mathrm{B}}^3\right)}{b_2\sqrt{b_0}\left(\beta_{\mathrm{A}}^2 - \beta_{\mathrm{B}}^2\right)^{3/2}} \overset{\beta_{2\mathrm{Au}}\to 0}{\equiv} -\frac{b}{b_2\sqrt{b_0}}\cos(3\gamma_{\mathrm{U}})$$

expected to work well only in central region 2109.00604

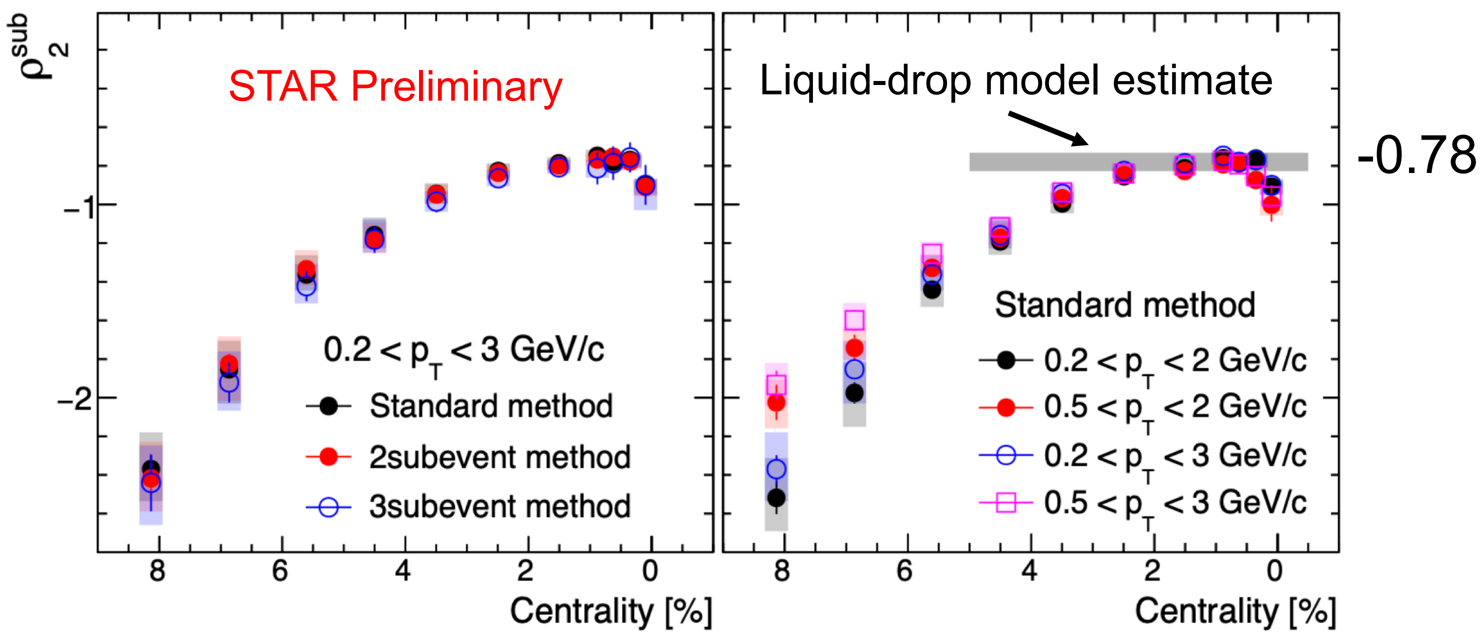
Another way to probe deformation

$$\left\langle v_2^2 \delta p_T \right\rangle \approx a - b\cos(3\gamma)\beta_2^3 \quad \left\langle v_2^2 \right\rangle \approx a_2 + b_2\beta_2^2 \quad \left\langle (\delta p_T)^2 \right\rangle \approx a_0 + b_0\beta_2^2$$

Removing "a" terms by subtracting the system A (UU) and B (AuAu)

$$\rho_2^{\rm sub} = \frac{\left\langle v_2^2\delta p_T\right\rangle_A - \left\langle v_2^2\delta p_T\right\rangle_B}{\left(\left\langle v_2^2\right\rangle_A - \left\langle v_2^2\right\rangle_B\right)\sqrt{\left(\left\langle(\delta p_T)^2\right\rangle_A - \left\langle(\delta p_T)^2\right\rangle_B\right)}} \approx -\frac{b_2\left(\cos(3\gamma_A)\beta_A^3 - \cos(3\gamma_B)\beta_B^3\right)}{b_2\sqrt{b_0}\left(\beta_A^2 - \beta_B^2\right)^{3/2}} \overset{\beta_{2\rm Au}\to 0}{\equiv} -\frac{b}{b_2\sqrt{b_0}}\cos(3\gamma_U)$$

expected to work well only in central region 2109.00604

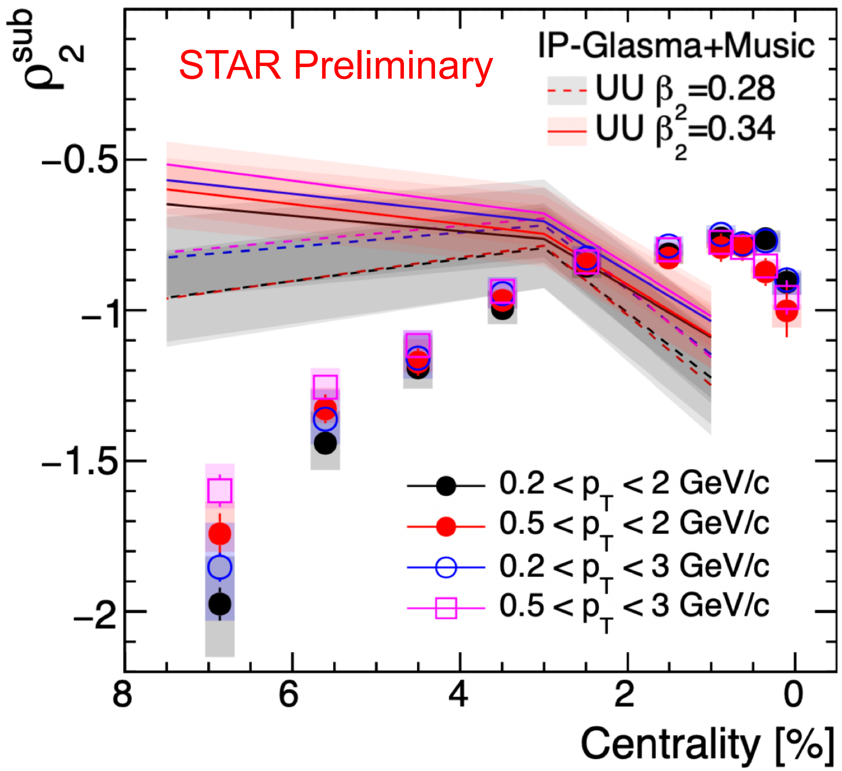


No dependence on methods, very little dependence on p_T ranges

Compare "subtracted" ratio to models

$$\rho_2^{\rm sub} = \frac{\langle v_2^2 \delta p_{\rm T}\rangle_{\rm A} - \langle v_2^2 \delta p_{\rm T}\rangle_{\rm B}}{(\langle v_2^2\rangle_{\rm A} - \langle v_2^2\rangle_{\rm B})\sqrt{\left(\left\langle(\delta p_{\rm T})^2\right\rangle_{\rm A} - \left\langle(\delta p_{\rm T})^2\right\rangle_{\rm B}\right)}} \approx -\frac{b_2\left(\cos(3\gamma_{\rm A})\beta_{\rm A}^3 - \cos(3\gamma_{\rm B})\beta_{\rm B}^3\right)}{b_2\sqrt{b_0}\left(\beta_{\rm A}^2 - \beta_{\rm B}^2\right)^{3/2}} \overset{\beta_{2\rm Au}\to 0}{\equiv} -\frac{b}{b_2\sqrt{b_0}}\cos(3\gamma_{\rm U})$$

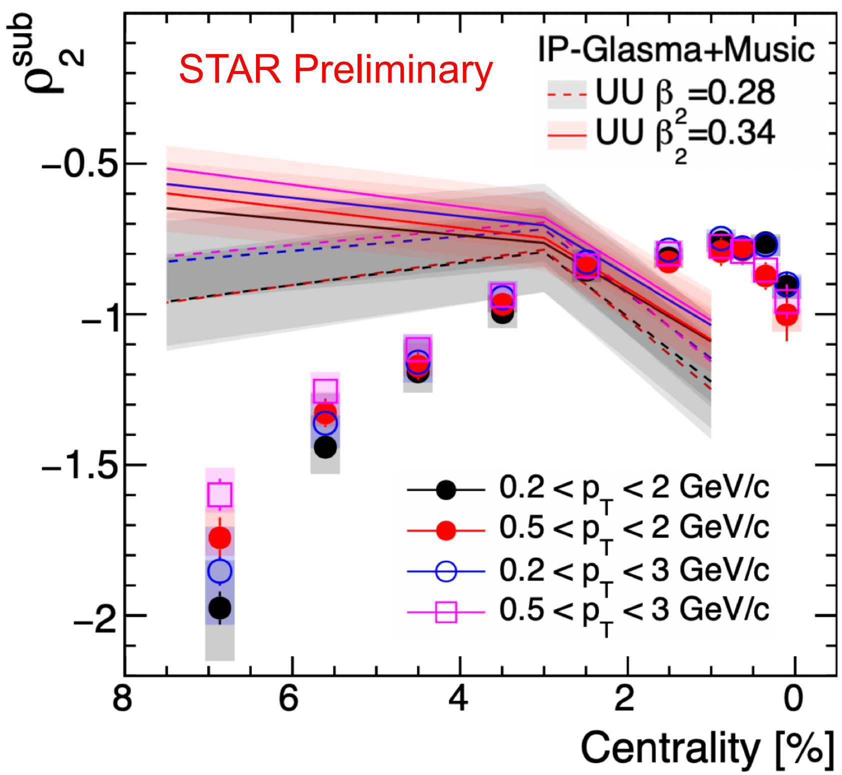
Hydro model predicts the same limit in UCC region for large $\boldsymbol{\beta}_{2\rm U}$ ($\gtrsim$0.28)



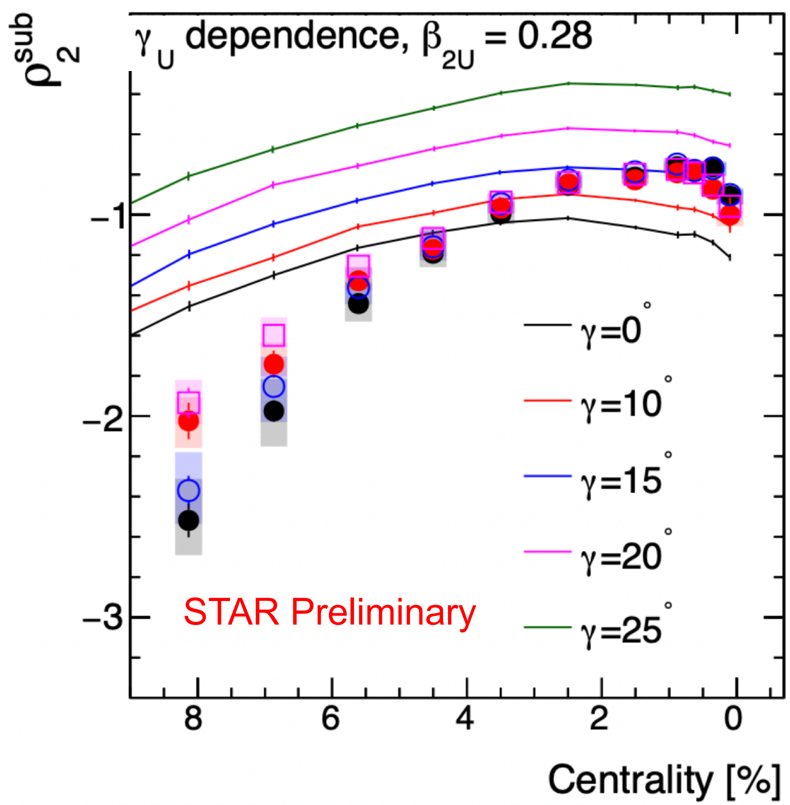
Compare "subtracted" ratio to models

$$\rho_2^{\rm sub} = \frac{\langle v_2^2 \delta p_{\rm T}\rangle_{\rm A} - \langle v_2^2 \delta p_{\rm T}\rangle_{\rm B}}{(\langle v_2^2\rangle_{\rm A} - \langle v_2^2\rangle_{\rm B})\sqrt{\left(\left\langle(\delta p_{\rm T})^2\right\rangle_{\rm A} - \left\langle(\delta p_{\rm T})^2\right\rangle_{\rm B}\right)}} \approx -\frac{b_2\left(\cos(3\gamma_{\rm A})\beta_{\rm A}^3 - \cos(3\gamma_{\rm B})\beta_{\rm B}^3\right)}{b_2\sqrt{b_0}\left(\beta_{\rm A}^2 - \beta_{\rm B}^2\right)^{3/2}} \overset{\beta_{2\rm Au}\to 0}{\equiv} -\frac{b}{b_2\sqrt{b_0}}\cos(3\gamma_{\rm U})$$

Hydro model predicts the same limit in UCC region for large $\boldsymbol{\beta}_{2\rm U}$ ($\gtrsim$0.28)



Glauber model prefers $\gamma_{\rm U} \lesssim 15^{\rm o}$



Summary and outlook

- Correlations of elliptic flow and radial flow carry imprint of the quadrupole deformation β_2 and triaxiality γ. Strong suppression in central U+U collisions but not Au+Au provides direct evidence that ^{238}U has a pronounced prolate shape.
- Comparison with hydro & Glauber models leads to $\beta_{2U} \approx 0.28$ and $\gamma_U \lesssim 15^o$ within a Woods-Saxon parameterization. Shape imaged at shorter-time scale at high $\sqrt{s}$ is consistent with low energy
- Our study shows that initial condition is sensitive to and can be probed by nuclear shape.
- The flow-assisted imaging approach provides a new tool for studying the collective, many-body structure of atomic nuclei across the nuclide chart.