

# PRECISION MEASUREMENT OF NET-PROTON NUMBER FLUCTUATIONS IN Au+Au COLLISIONS AT RHIC

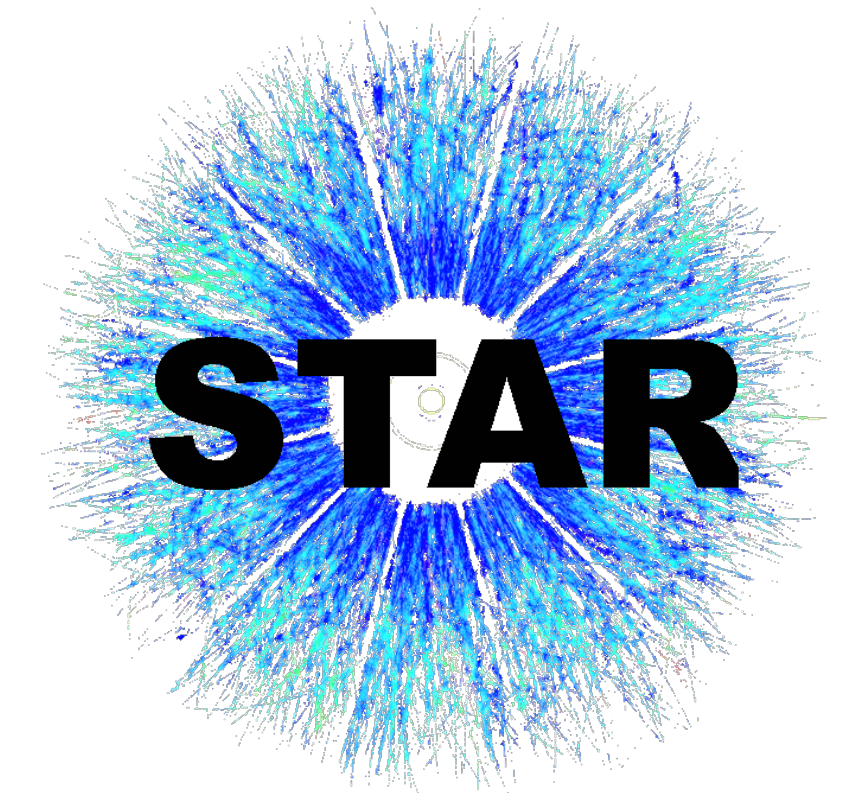
Ashish Pandav for STAR Collaboration  
Lawrence Berkeley National Laboratory  
May 28, 2024

*RHIC-BES  
Online Seminar*

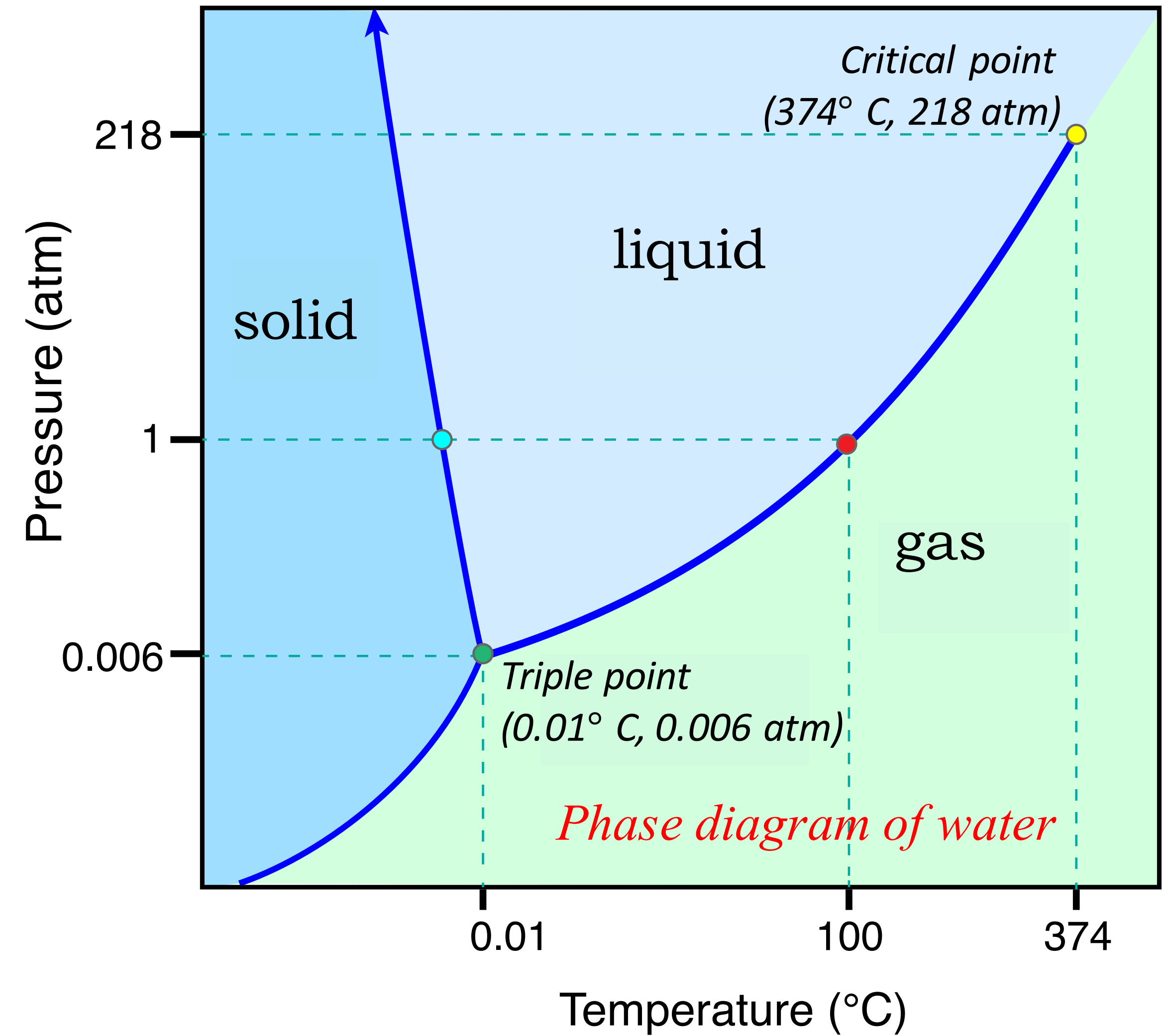
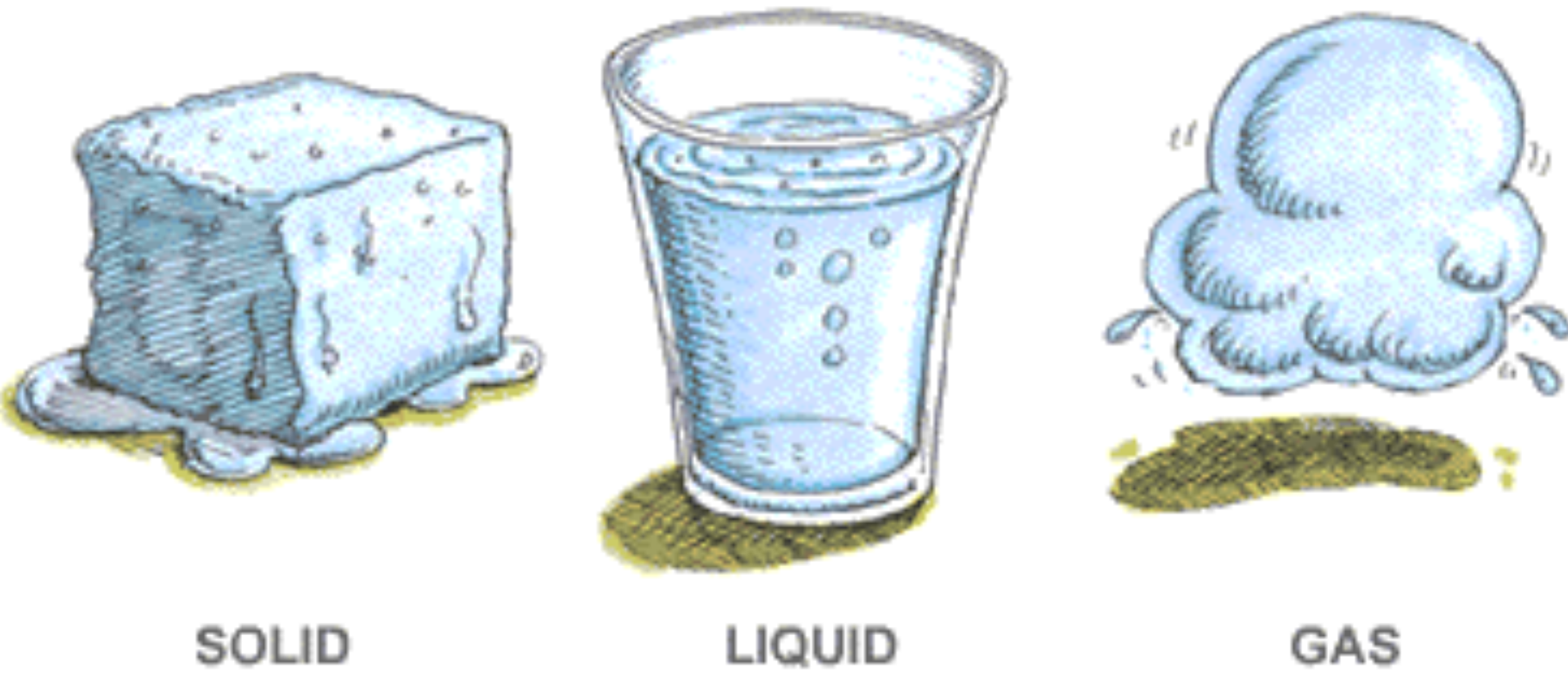


## Outline

1. Introduction
2. Experimental analysis
3. Results
4. Summary

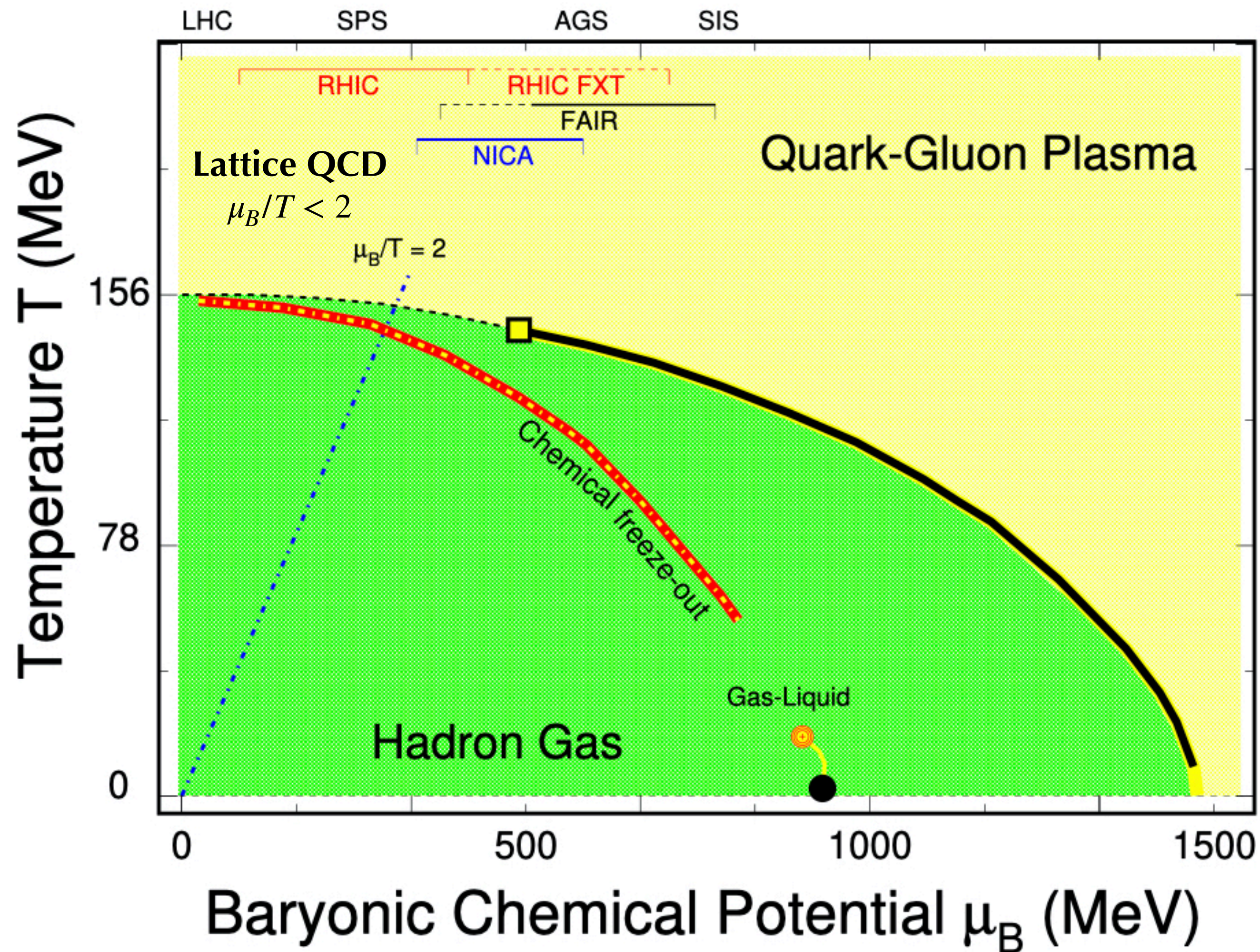


# INTRODUCTION: PHASE TRANSITION IN WATER



- Underlying interaction electromagnetic
- Precise understanding available

# INTRODUCTION: QCD PHASE DIAGRAM



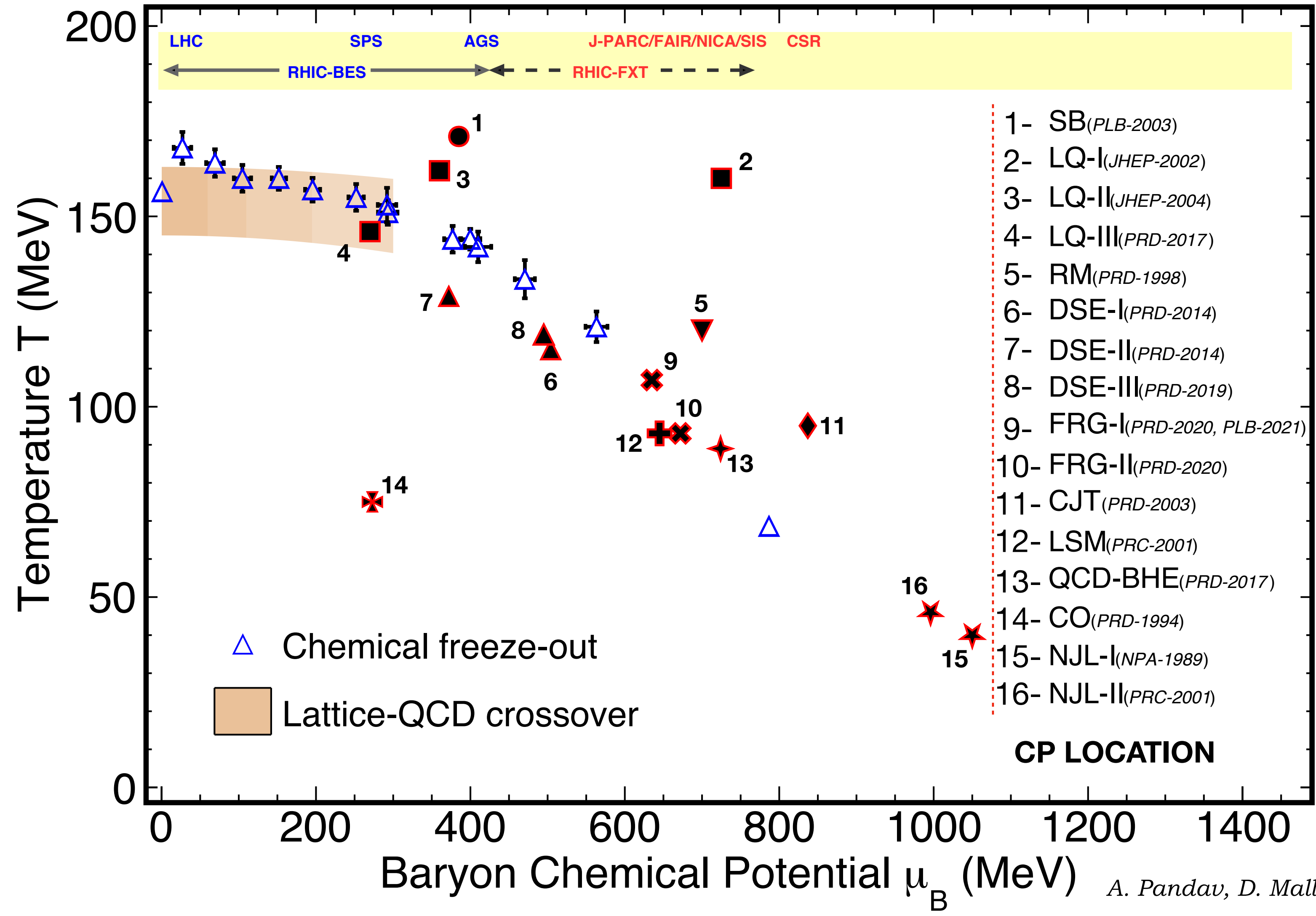
*B. Mohanty, N. Xu, arXiv:2101.09210*

## Phase structure:

- QGP and hadronic phase ✓
  - Transition temperature ( $T_c$ ) ✓
  - Crossover at small  $\mu_B$  ( $\frac{\mu_B}{T} < 2$ ) ✓
  - 1st order P.T. at large  $\mu_B$  ?
  - Critical end point ?
- Lattice QCD →
- Models →

- Phase diagram of strongly interacting matter
- Largely conjectured

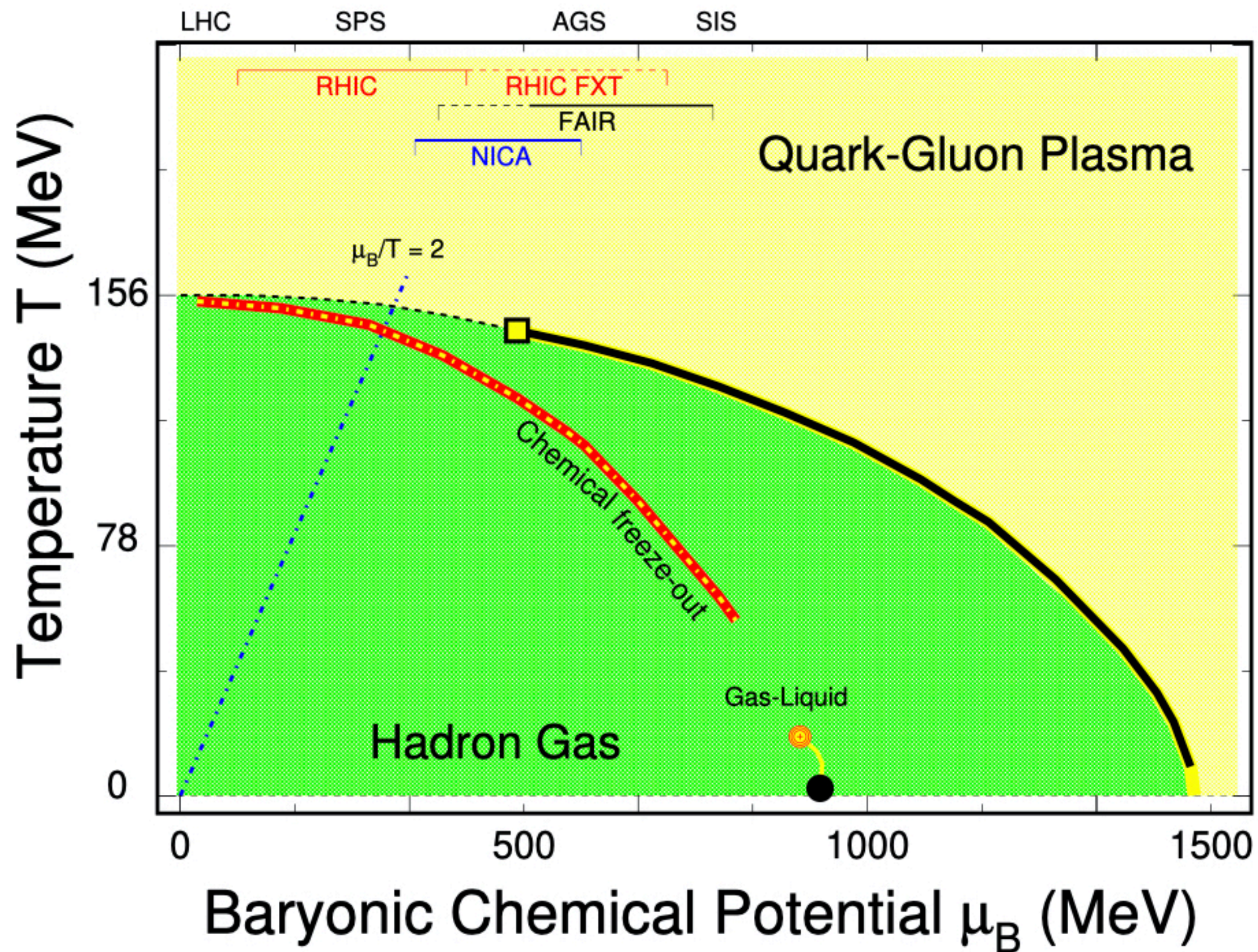
# INTRODUCTION: QCD CRITICAL POINT



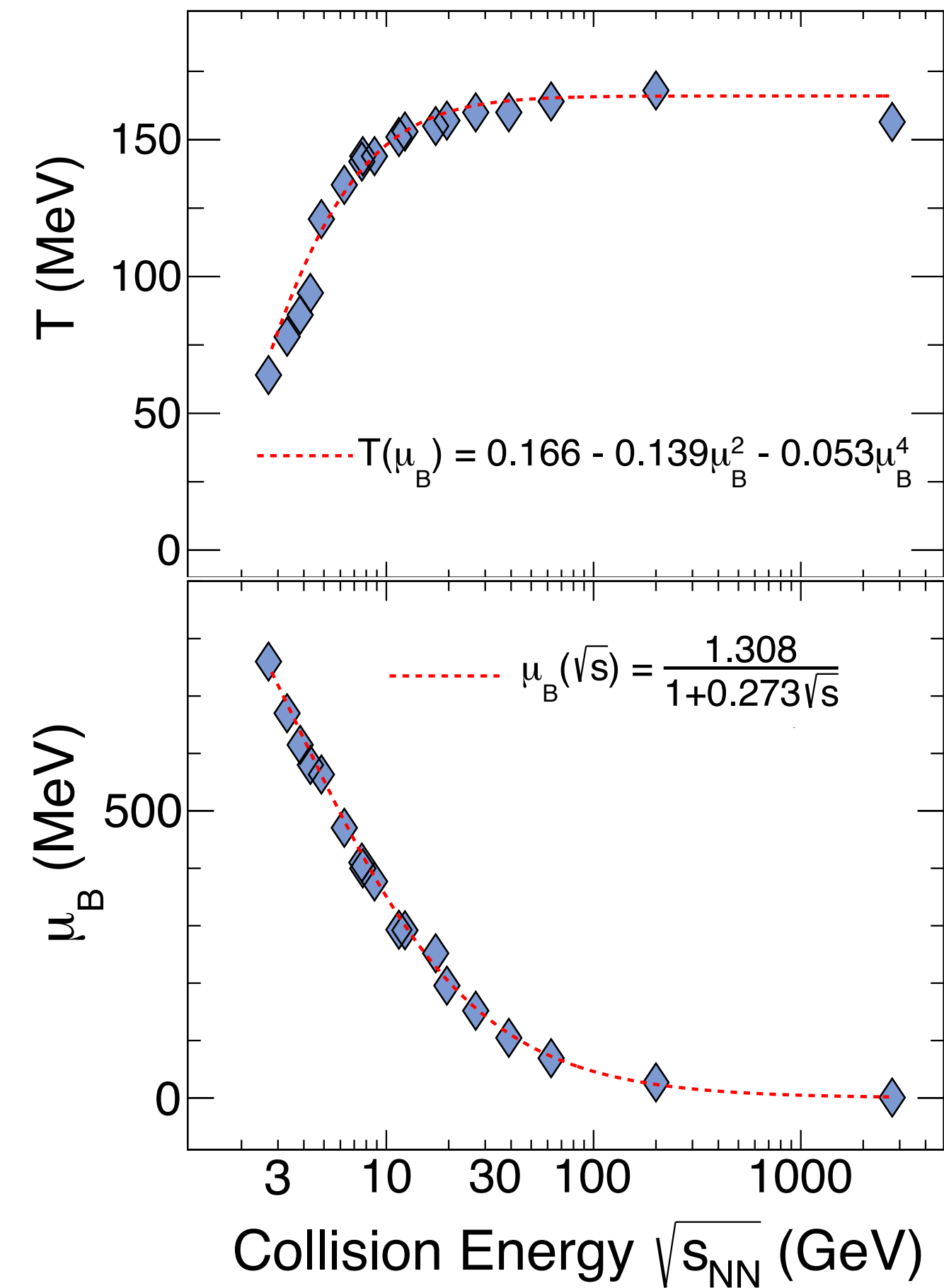
- Lattice calculations at high  $\mu_B$  suffer from sign problem
- Effective models have several underlying assumptions/ approximations

□ Theory predictions in  $\mu_B - T$  plane. **Experimental search very important.**

# INTRODUCTION: EXPERIMENTALLY ACCESSING PHASE DIAGRAM



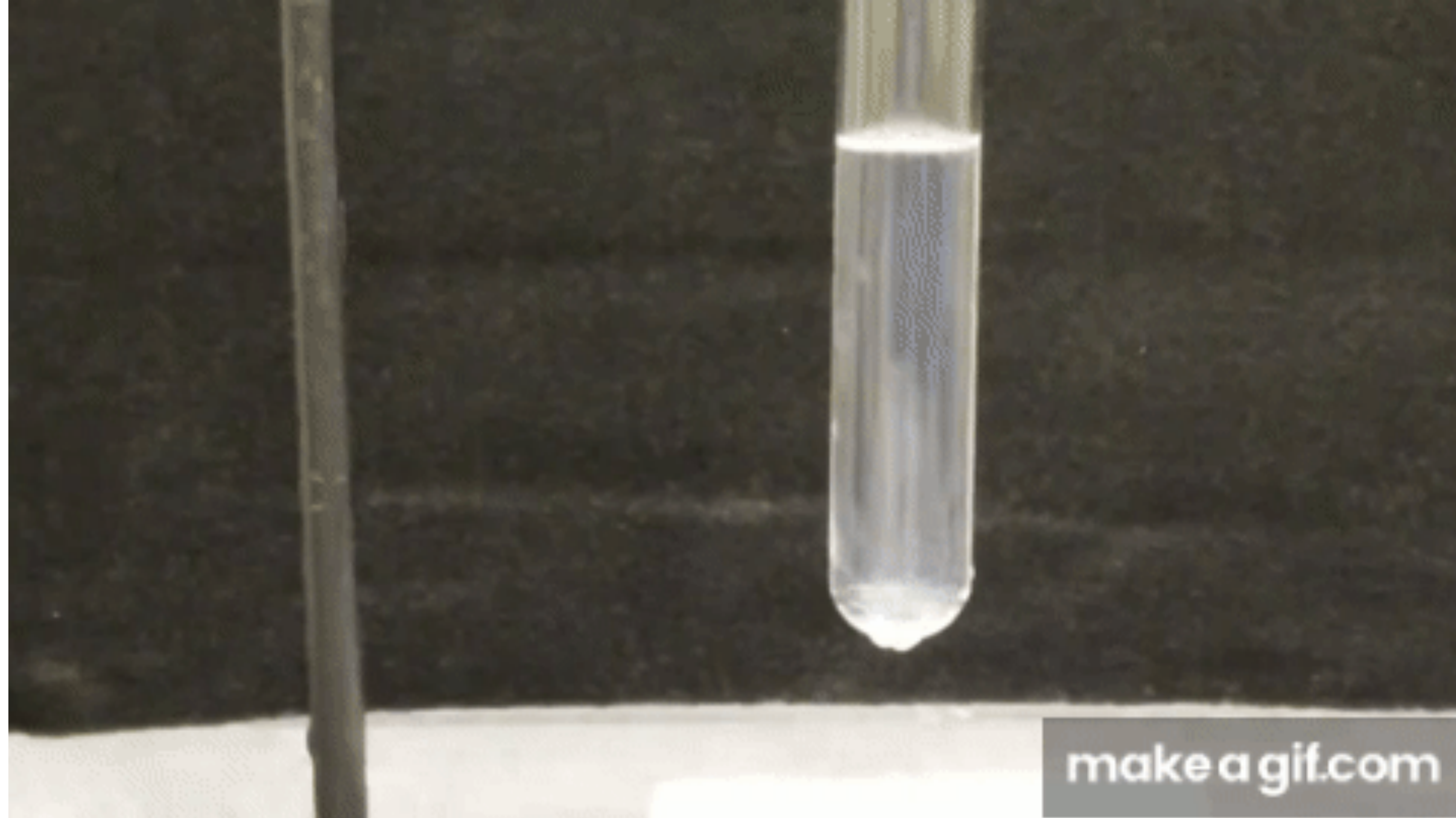
*B. Mohanty, N. Xu, arXiv:2101.09210, A. Pandav, D. Mallick, B. Mohanty, PPNP. 125, 103960 (2022)*



*P. Braun-Munzinger, J. Stachel, Nature 448 (2007) 302*

- Varying collision energy, impact parameter, rapidity acceptance, collision species, varies  $T$  and  $\mu_B$  of system created
- Study energy/centrality/rapidity/species dependence of CP sensitive observables

# FLUCTUATIONS NEAR CP:



Development of long range density fluctuations,

Divergence of correlation length, thermodynamic response functions (susceptibility, compressibility etc)

**Enhanced fluctuations expected near CP.**

How to quantify fluctuations? measure cumulants.

**Critical Opalescence:  $CO_2$  appears milky white**

# CUMULANTS:

● Cumulants:  $n = \text{net-proton multiplicity in an event}$

$$C_1 = \langle n \rangle$$

$$C_2 = \langle \delta n^2 \rangle \quad * \delta n = n - \langle n \rangle$$

$$C_3 = \langle \delta n^3 \rangle$$

$$C_4 = \langle \delta n^4 \rangle - 3 \langle \delta n^2 \rangle^2$$

$$C_5 = \langle \delta n^5 \rangle - 10 \langle \delta n^3 \rangle \langle \delta n^2 \rangle$$

$$C_6 = \langle \delta n^6 \rangle - 15 \langle \delta n^4 \rangle \langle \delta n^2 \rangle - 10 \langle \delta n^3 \rangle^2 + 30 \langle \delta n^2 \rangle^3$$

● Factorial cumulants (irreducible correlation function):

$$\kappa_1 = C_1$$

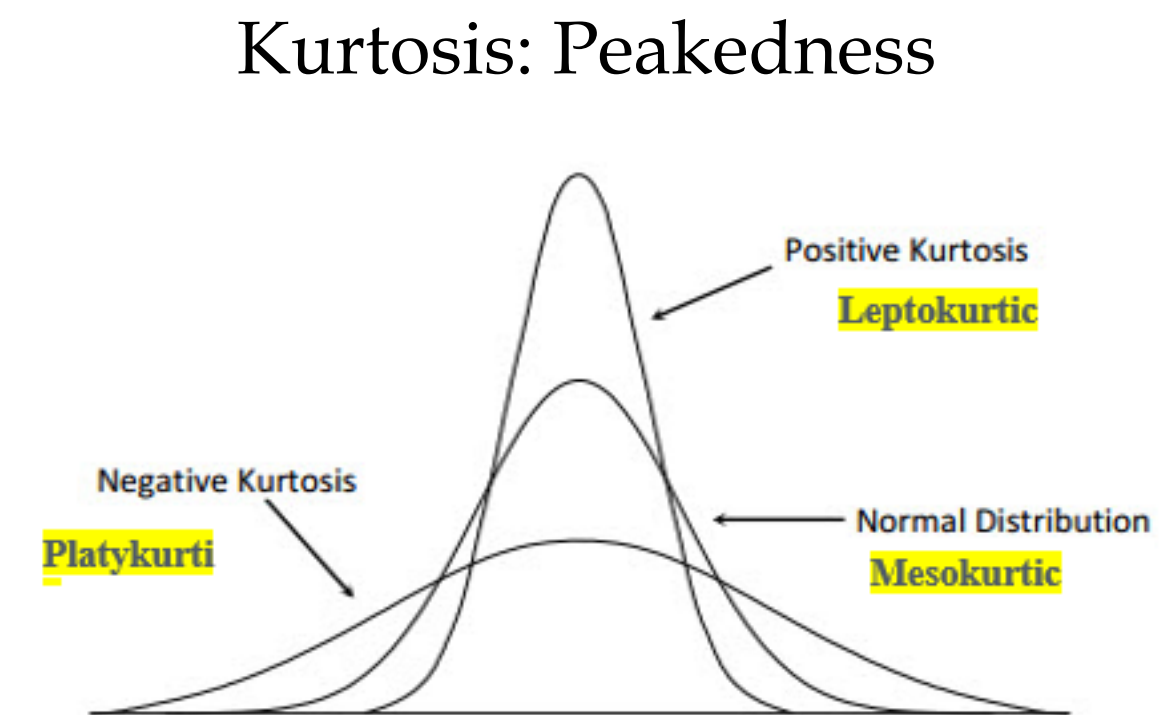
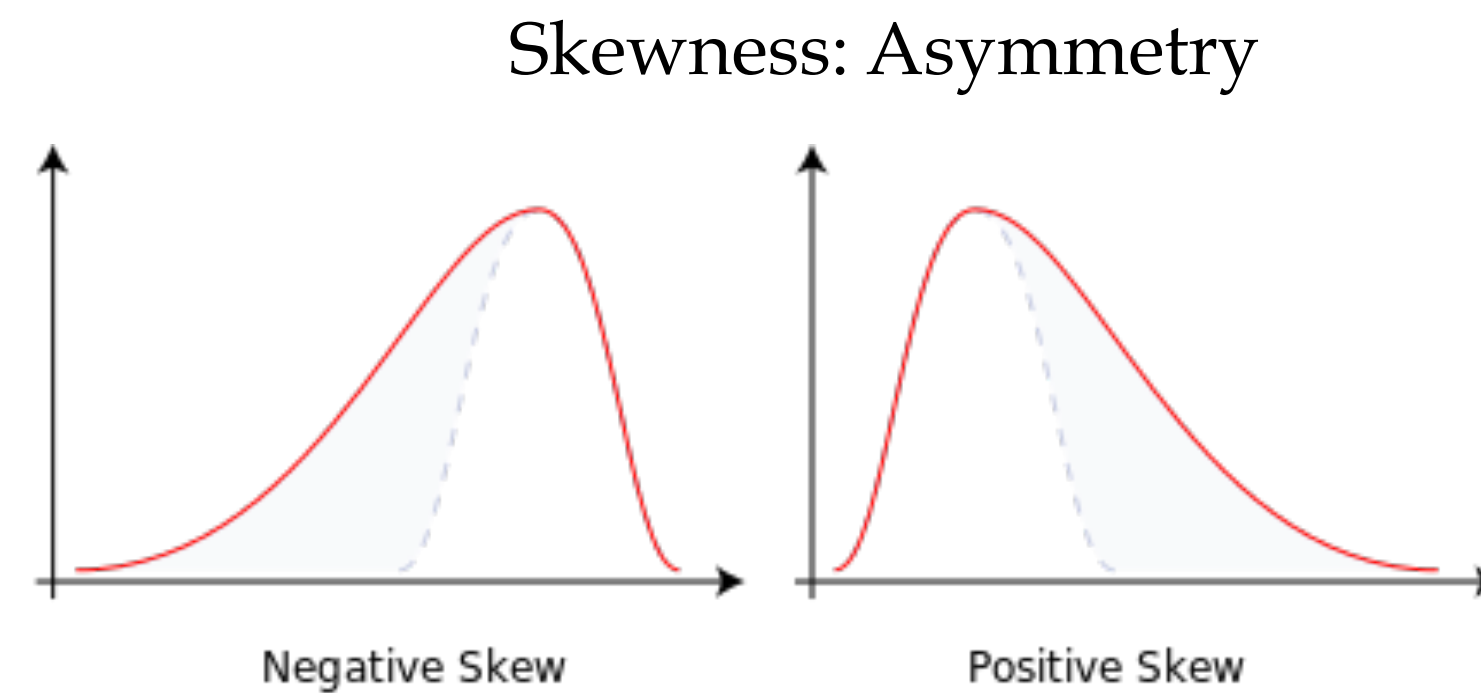
$$\kappa_2 = -C_1 + C_2$$

$$\kappa_3 = 2C_1 - 3C_2 + C_3$$

$$\kappa_4 = -6C_1 + 11C_2 - 6C_3 + C_4$$

$$\kappa_5 = 24C_1 - 50C_2 + 35C_3 - 10C_4 + C_5$$

$$\kappa_6 = -120C_1 + 274C_2 - 225C_3 + 85C_4 - 15C_5 + C_6$$



# CUMULANTS AND CP SEARCH:

**Related to correlation length:**  $C_2 \sim \xi^2$ ,  $C_4 \sim \xi^7$

Finite size/time effects reduces  $\xi$

Higher order  $\rightarrow$  more sensitivity

**Related to susceptibilities:**  $\frac{C_{4q}}{C_{2q}} = \frac{\chi_4^q}{\chi_2^q}$ ,  $\frac{C_{6q}}{C_{2q}} = \frac{\chi_6^q}{\chi_2^q}$   $q = B, Q, S$

Direct comparison with lattice QCD,  
HRG, QCD-based model calculations

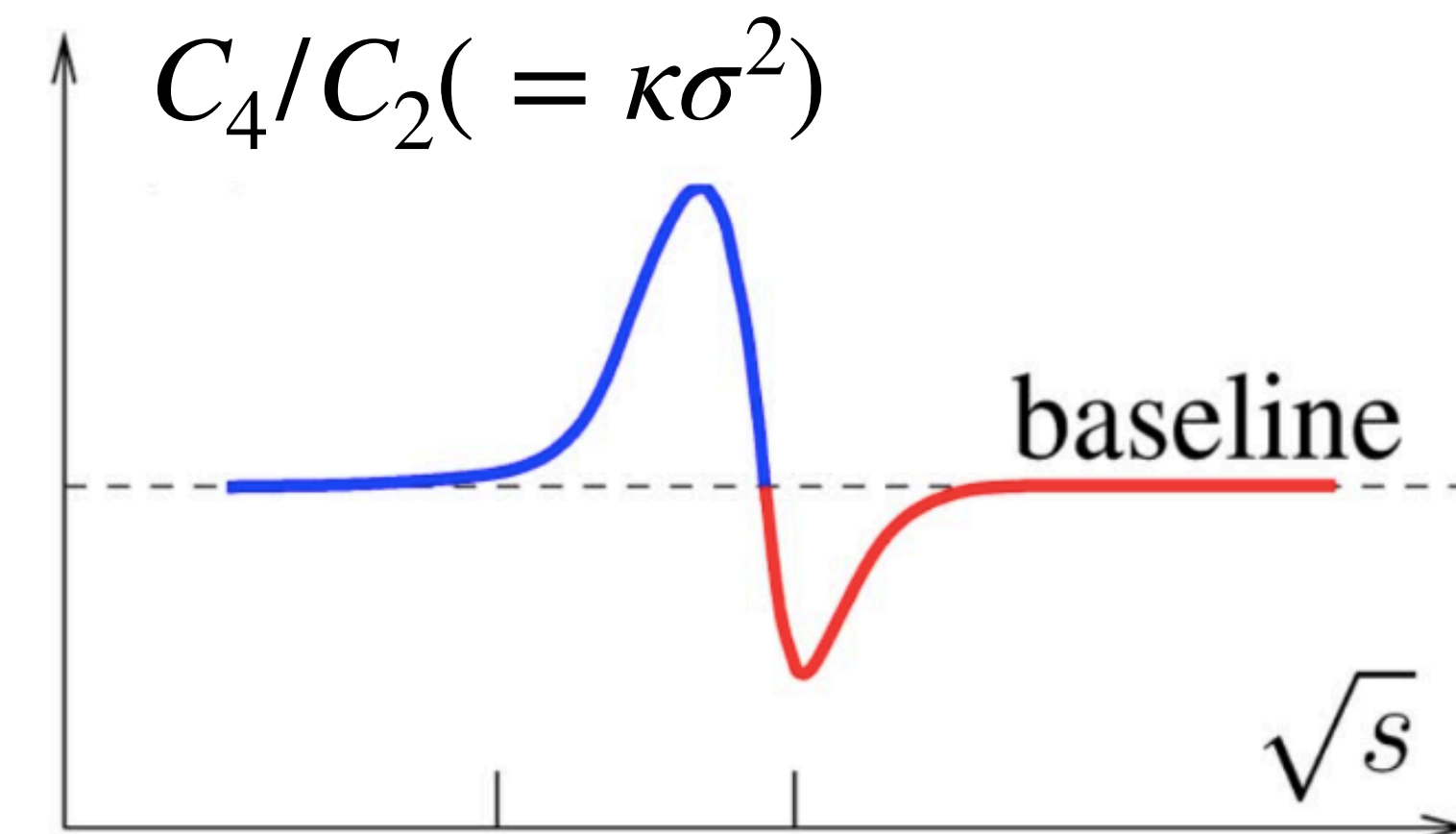
*R.V. Gavai and S. Gupta, PLB696, 459(11)*

*S. Ejiri, F. Karsch, K. Redlich, PLB633, 275(06)*

*A. Bazavov et al., PRL109, 192302(12)*

*S. Borsanyi et al., PRL111, 062005(13)*

## CP search



*M. A. Stephanov, PRL 107 (2011) 052301*

Assumption: Thermodynamic equilibrium

**Non-monotonic  $\sqrt{s_{NN}}$  dependence of  $C_4/C_2$  of conserved quantity - existence of a critical region**



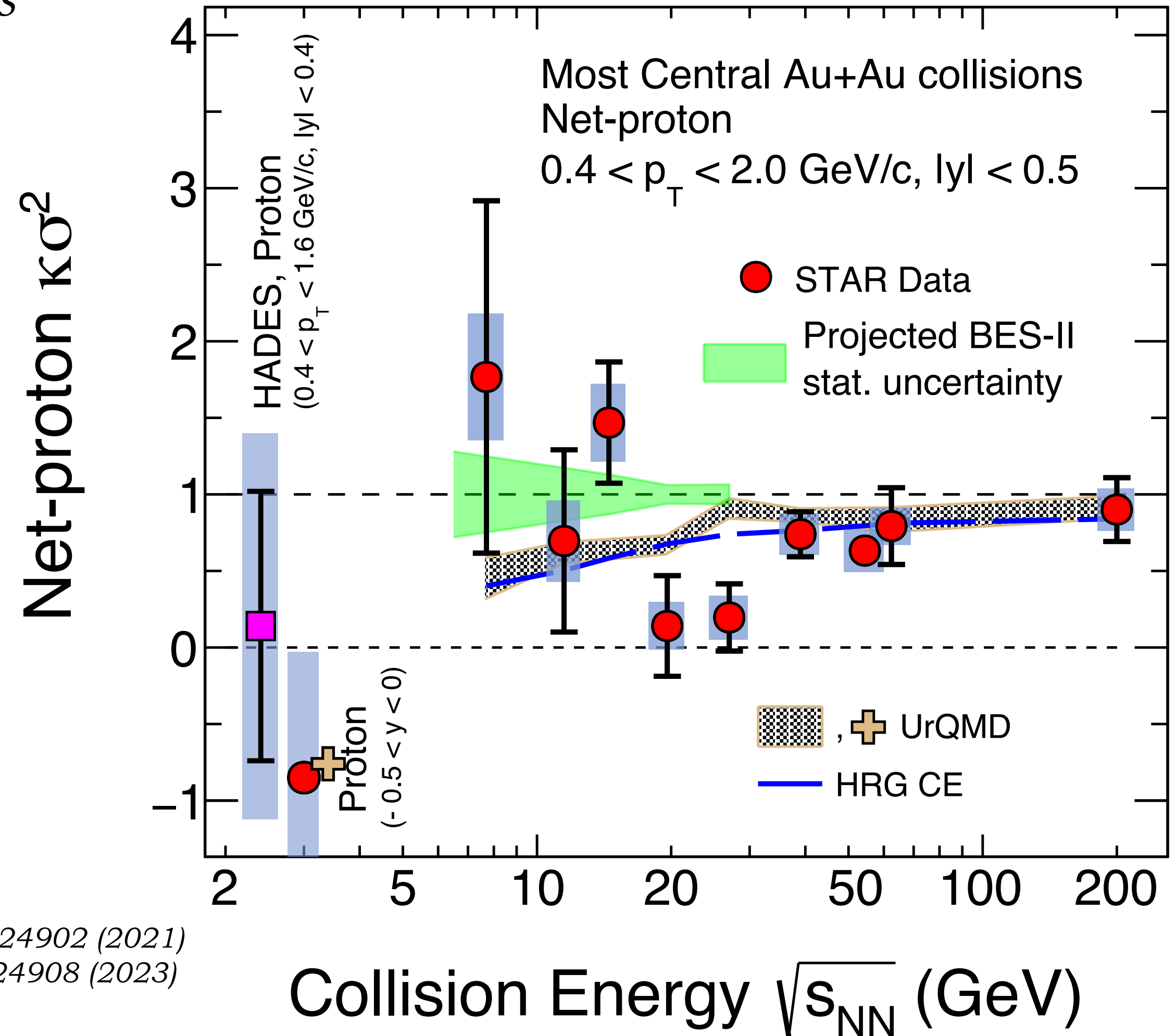
# EXPERIMENTAL SEARCH FOR CP: BES SCAN AT STAR-RHIC

## Phase I of BES program (BES-I): Au+Au collisions

*J. Cleymans, et. al, PRC. 73, 034905 (2006)*

$\sqrt{s_{NN}}$ (GeV)	Events ( $10^6$ )	$\mu_B$ (MeV)
200	220	25
62.4	43	75
54.4	550	85
39	92	112
27	31	156
19.6	14	206
14.5	14	262
11.5	7	316
7.7	2.2	420
3.0	140	750

STAR : PRL 127, 262301 (2021), PRC 104, 24902 (2021)  
 : PRL 128, 202302 (2022), PRC 107, 24908 (2023)  
 HADES: PRC 102, 024914 (2020)



Observed hint of non-monotonic trend in BES-I ( $3 \sigma$ ): consistent with model expectation with a CP  
 Robust conclusion require confirmation from precision measurement from BES-II.  
 Extend reach to even lower collision energies with FXT energies

# STAR BES-II PROGRAM: PRECISION MEASUREMENTS

## Au+Au Collisions at RHIC

### Collider Runs

### Fixed-Target Runs

Sl. no.	$\sqrt{s_{NN}}$ (GeV)	No. of collected events (millions)	$\mu_B$ (MeV)	Sl. no.	$\sqrt{s_{NN}}$ (GeV)	No. of collected events (millions)	$\mu_B$ (MeV)
1	<b>200</b>	<b>380</b>	<b>25</b>	1	13.7 (100)	50	280
2	62.4	46	75	2	11.5 (70)	50	316
3	54.4	1200	85	3	9.2 (44.5)	50	372
4	39	86	112	4	7.7 (31.2)	260	420
5	27	585	156	5	7.2 (26.5)	470	440
6	19.6	595	206	6	6.2 (19.5)	120	490
7	17.3	256	230	7	5.2 (13.5)	100	540
8	14.6	340	262	8	4.5 (9.8)	110	590
9	11.5	257	316	9	3.9 (7.3)	120	633
10	9.2	160	372	10	3.5 (5.75)	120	670
11	7.7	104	420	11	3.2 (4.59)	200	699
				12	<b>3.0 (3.85)</b>	<b>260 + 2000</b>	<b>750</b>

$$3 \leq \sqrt{s_{NN}} \text{ (GeV)} \leq 200 \rightarrow 750 \geq \mu_B \text{ (MeV)} \geq 25$$

High precision, widest  $\mu_B$  coverage to date

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# STAR BES-II PROGRAM: PRECISION MEASUREMENTS

## Au+Au Collisions at RHIC

### Collider Runs

### Fixed-Target Runs

Events used for net-proton fluctuation studies (Collider runs)

### BES-II vs BES-I

$\sqrt{s_{NN}}$ (GeV)	Events BES-I ( $10^6$ )	Events BES-II ( $10^6$ )
7.7	3	<b>45</b>
<b>9.2</b>	-	<b>78</b>
11.5	7	<b>110</b>
14.5	20	<b>178</b>
<b>17.3</b>	-	<b>116</b>
19.6	15	<b>270</b>
27	30	<b>220</b>

**~10-18 fold improvement in statistics**  
**9.2 and 17.3 GeV added to energy scan**

Sl. no.	$\sqrt{s_{NN}}$ (GeV)	No. of collected events (millions)	$\mu_B$ (MeV)	Sl. no.	$\sqrt{s_{NN}}$ (GeV)	No. of collected events (millions)	$\mu_B$ (MeV)
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# STAR DETECTOR: BES-II UPGRADE

endcap **T**ime-**O**f-**F**light

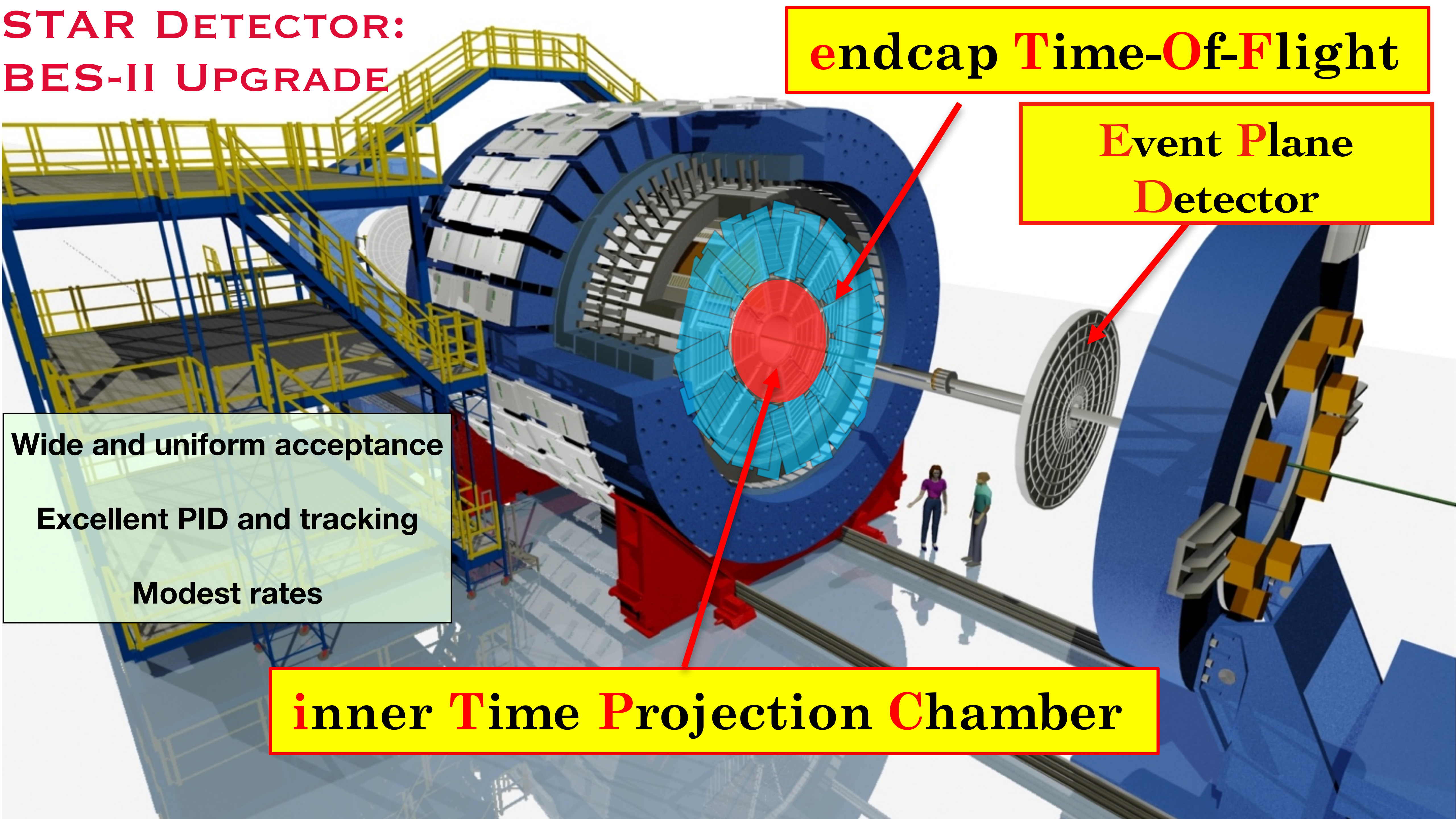
**E**vent **P**lane  
**D**etector

Wide and uniform acceptance

Excellent PID and tracking

Modest rates

inner **T**ime **P**rojection **C**hamber



# STAR Major Upgrades for BES-II



## iTPC:

- Improves  $dE/dx$
- Extends  $\eta$  coverage from 1.0 to 1.6
- Lowers  $p_T$  cut-in from 125 to 60 MeV/c
- Ready in 2019



## eTOF:

- Forward rapidity coverage
- PID at  $\eta = 1.05$  to 1.5
- Borrowed from CBM-FAIR
- Ready in 2019



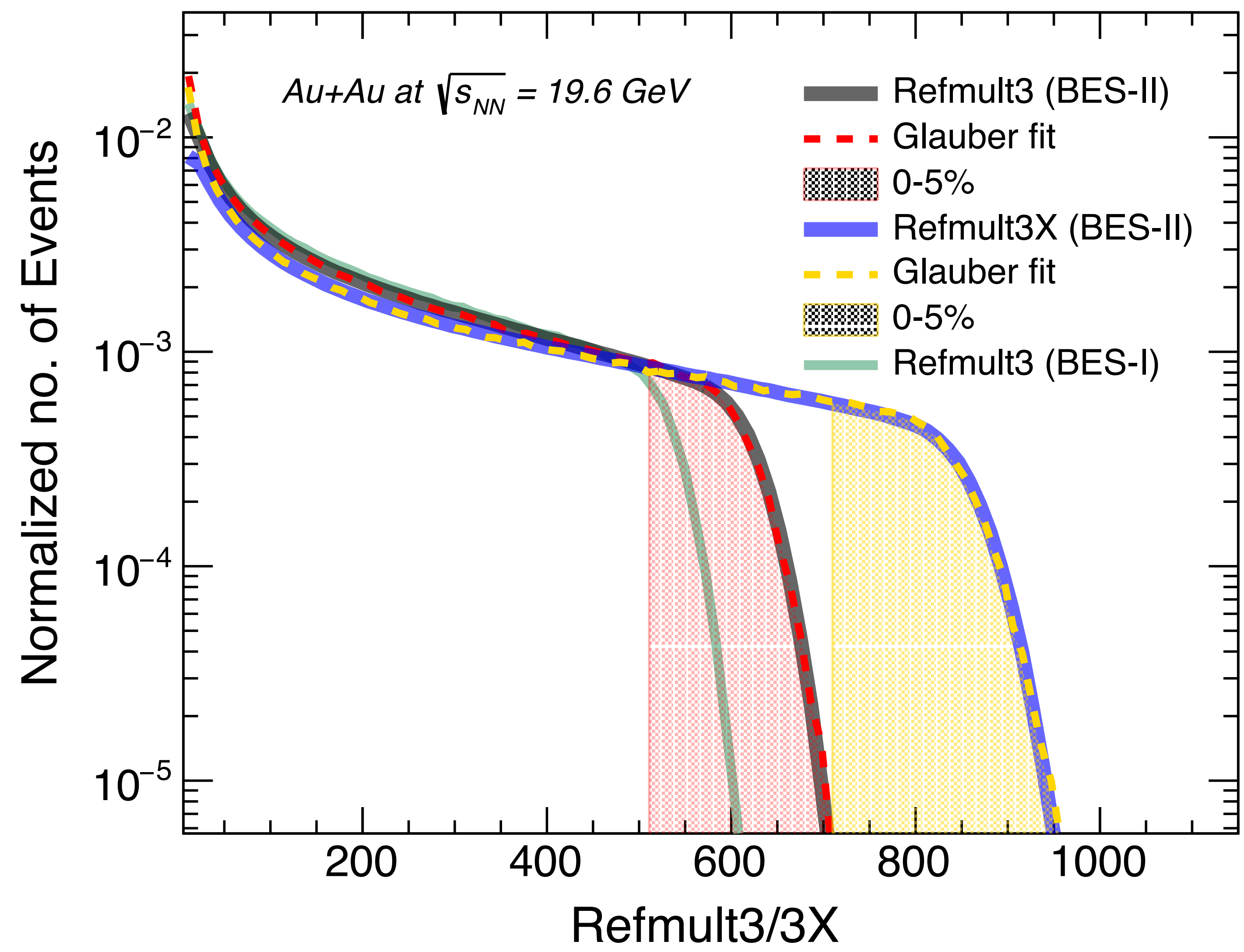
## EPD:

- Improves trigger
- Better centrality & event plane measurements
- Ready in 2018

- 1) Enlarge rapidity acceptance:  $|\eta| \leq 1.0 \rightarrow |\eta| \leq 1.6$
- 2) Improve particle identification:  $p_T \geq 125 \text{ MeV}/c \rightarrow p_T \geq 60 \text{ MeV}/c$
- 3) Enhance centrality/event plane resolution, suppress auto correlations
- 4) Enable the fixed-target program:  $\mu_B \leq 420 \text{ MeV} \rightarrow \mu_B \leq 750 \text{ MeV}$

# CENTRALITY:

- Defined using charged particle multiplicity measured by STAR
- Exclude protons and antiprotons to avoid self correlation



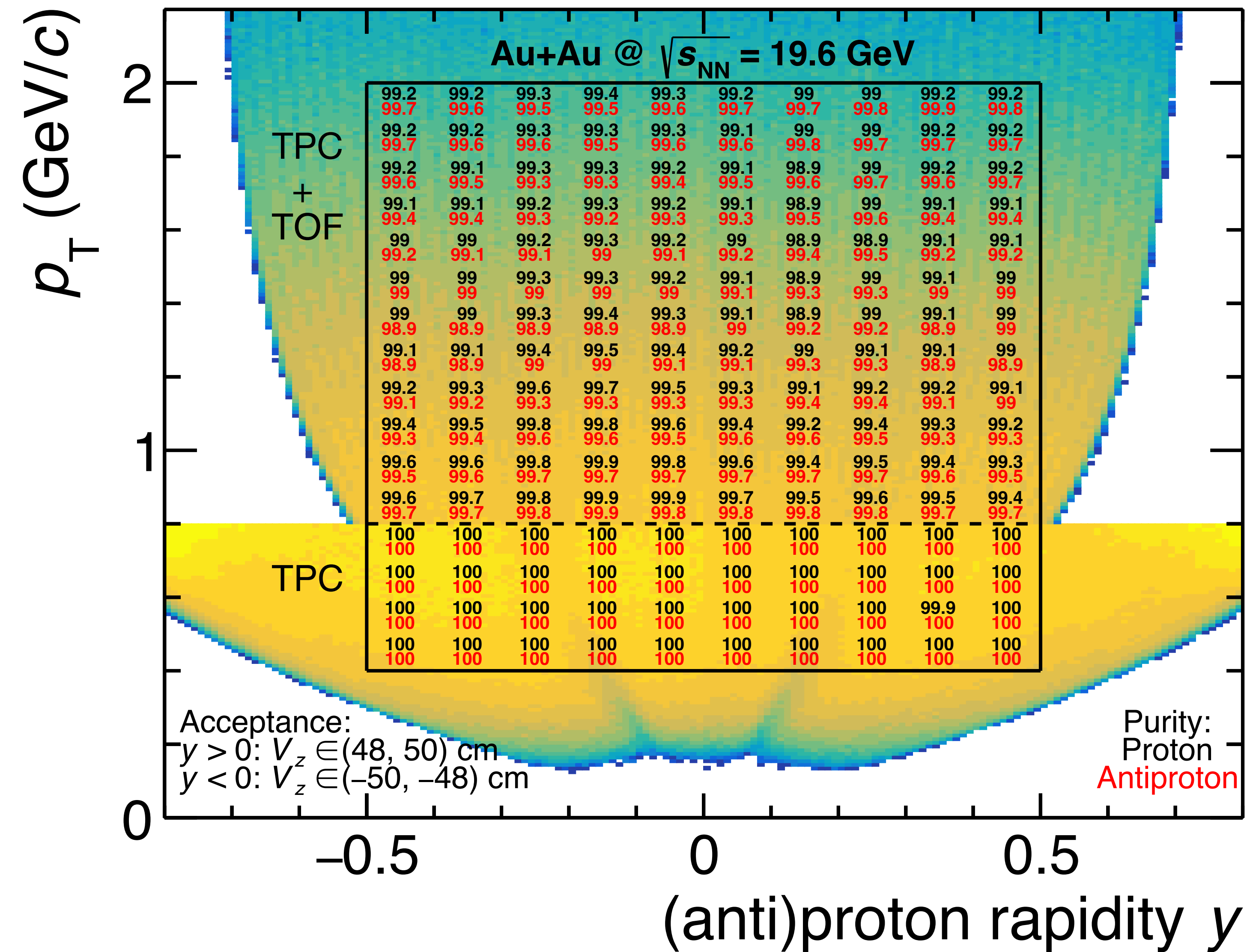
Two choices of centrality

**Refmult3:** Charged particle multiplicity excluding protons measured within  $|\eta| < 1.0$

**Refmult3X:** Charged particle multiplicity excluding protons measured within  $|\eta| < 1.6$   
*Possible due to iTPC upgrade*

Larger multiplicity leads to better centrality resolution:  
 $\text{Refmult3X (BES-II)} > \text{Refmult3 (BES-II)} > \text{Refmult3 (BES-I)}$

# PROTON SELECTION:



TPC and TOF detector used for identifying protons over  
 $p_T = 0.4 - 2.0$  GeV/c, and  $|y| < 0.5$

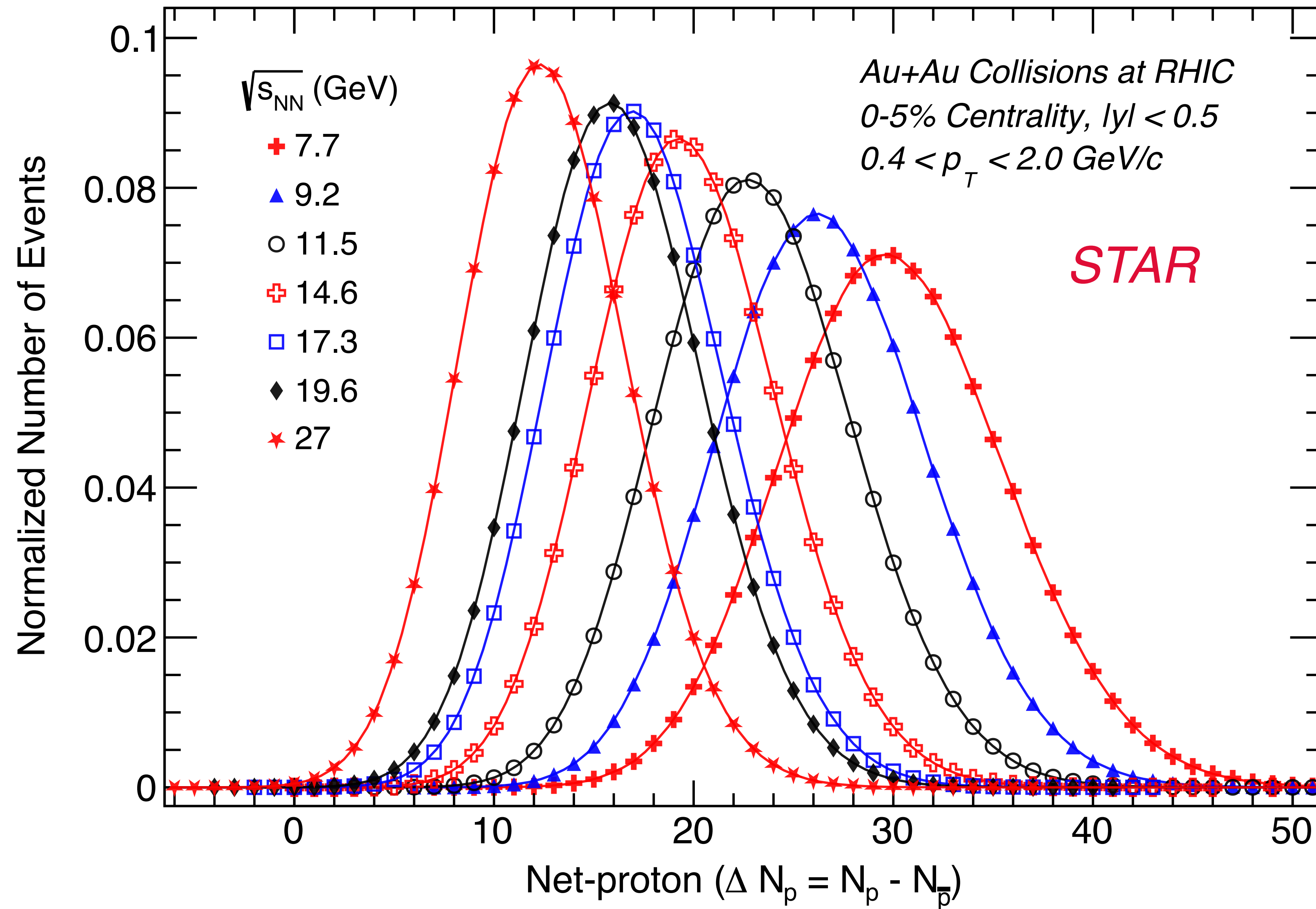
$p_T = 0.4 - 0.8$  GeV/c (PID using TPC)  
 Using dE/dx measurements  $\rightarrow |n\sigma| < 2$

$p_T = 0.8 - 2.0$  GeV/c (using TPC+TOF)  
 In addition to TPC,  $mass^2$  from TOF used  
 $\rightarrow 0.6 < mass^2 < 1.2$  GeV<sup>2</sup>/c<sup>4</sup>

Bin-by-bin  
**proton/antiproton purity > 99%**



# EVENT-BY-EVENT NET-PROTON DISTRIBUTION:



Raw net-proton distributions from BES-II: Uncorrected for detector efficiency

Mean increases with decreasing collision energy: Effect of baryon stopping

Larger width leads to larger Stat. uncertainties

$$\text{Stat. error } C_r \propto \frac{\sigma^r}{\sqrt{N}}$$

# CORRECTIONS:

- Corrected for detector efficiency

Binomial detector efficiency response considered  
 ~10% higher proton efficiency compared to BES-I  
 Better control on uncertainty on efficiency: 2% compared to 5% in BES-I

- Corrected for PID cut efficiency

$PID_{eff} = S_a / S_{tot}$   
 $S_a = \text{area under } |n\sigma_p| < 2.0$   
 $S_{tot} = \text{total area under } n\sigma_p$   
 distribution( $|n\sigma_p| < 5.0$ )

- Corrected for finite centrality bin width

$C_n = \sum_r w_r C_{n,r}$   
 where  $w_r = n_r / \sum_r n_r$ ,  $n=1,2,3,4\dots$   
 Here,  $n_r$  is no. of events in  $r^{th}$  multiplicity bin

X. Luo, T Nonaka, PRC 99 (2019)  
 X. Luo et al, J.Phys. G 40, 105104 (2013),

## Percentage stat. and sys. error in net-proton cumulants at 0-5% centrality

$\sqrt{s_{NN}}$	7.7 GeV		19.6 GeV	
	% stat. error	% sys. error	% stat. error	% sys. error
$C_2/C_1$	0.1%	0.3%	0.06%	0.3%
$C_3/C_2$	2.1%	1.3%	0.7%	1%
$C_4/C_2$	61%	29%	22%	11%

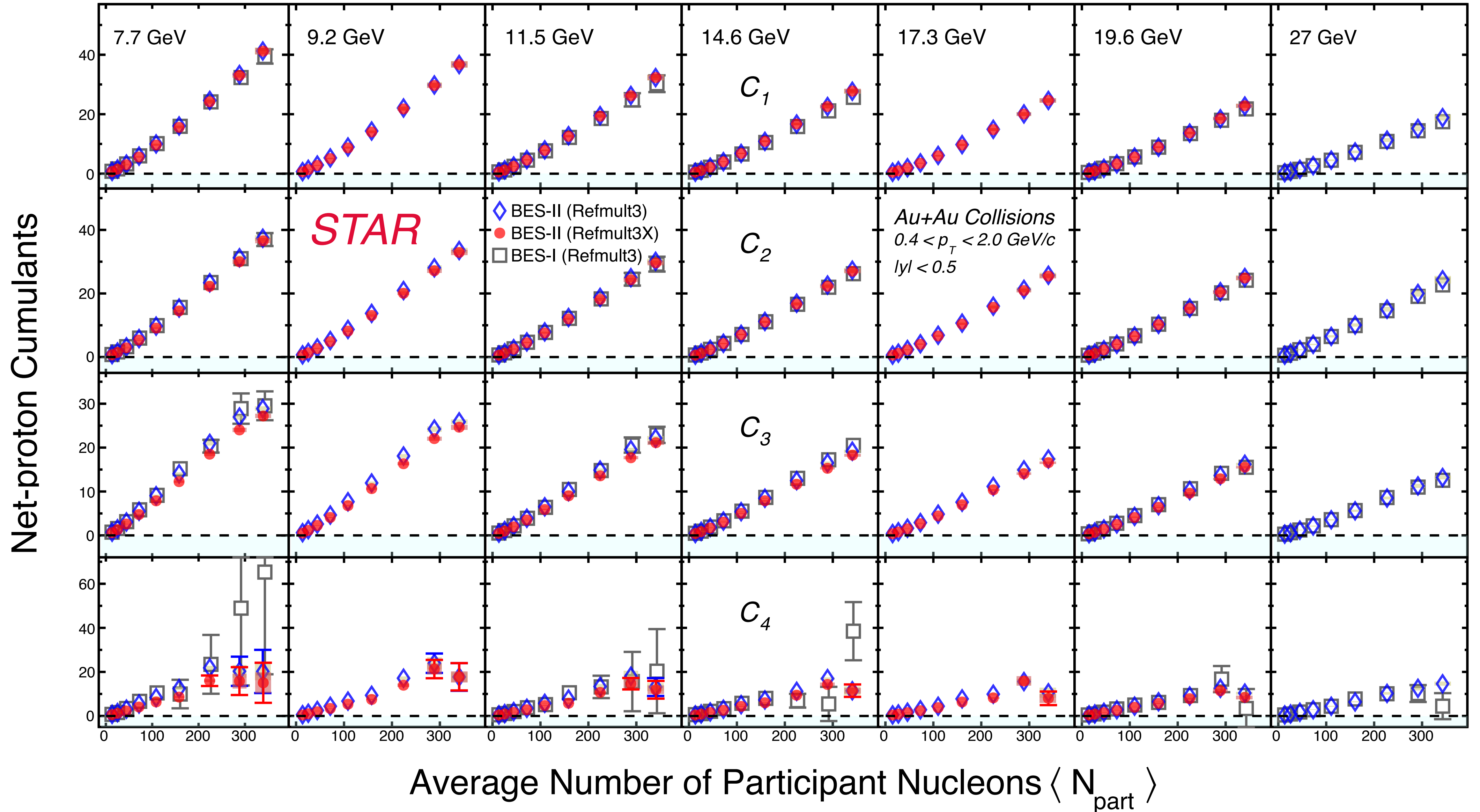
## Reduction factor in uncertainties on 0-5% $C_4/C_2$ : BES-II vs BES-I

7.7 GeV		19.6 GeV	
stat. error	sys. error	stat. error	sys. error
4.7	3.2	4.5	4

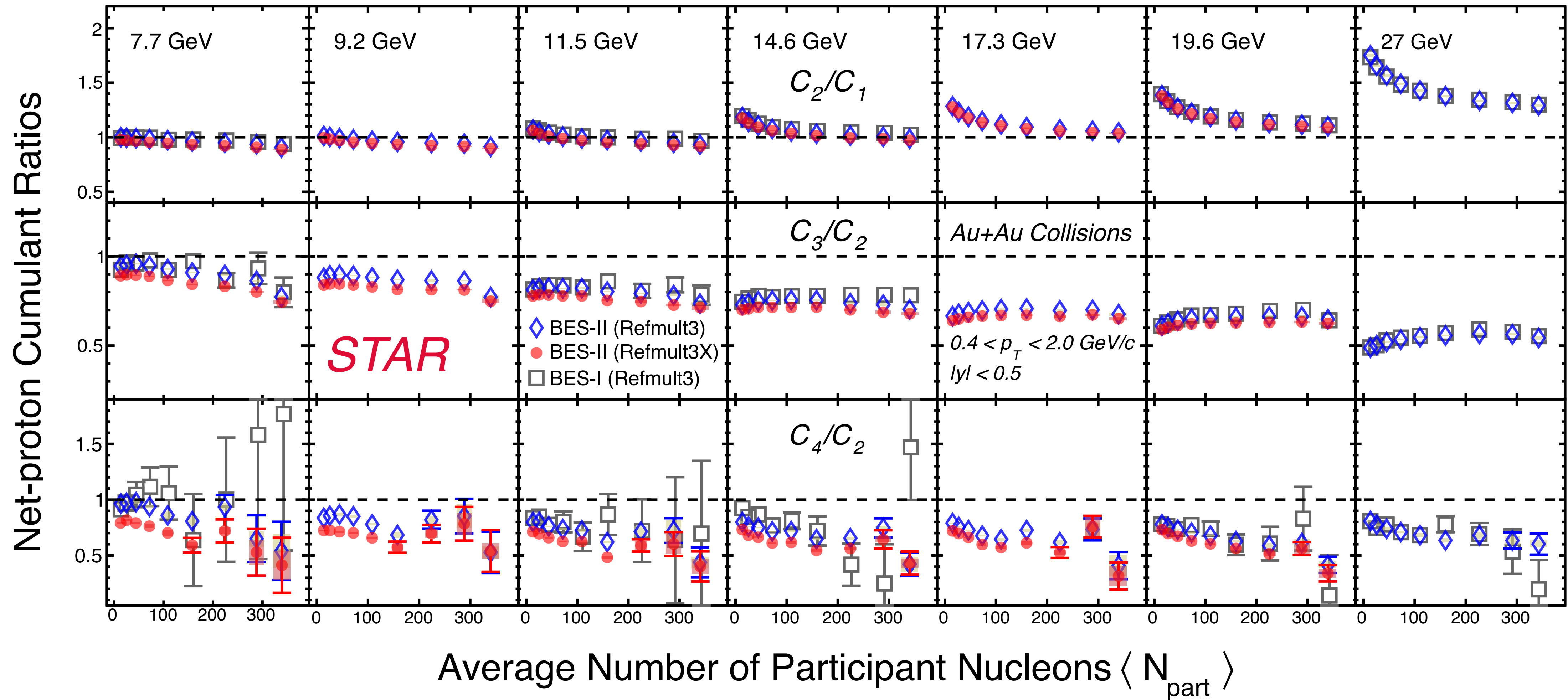
# **LATEST NET-PROTON FLUCTUATION**

## **RESULTS FROM BES-II:**

# CENTRALITY DEPENDENCE AND COMPARISON WITH BES-I

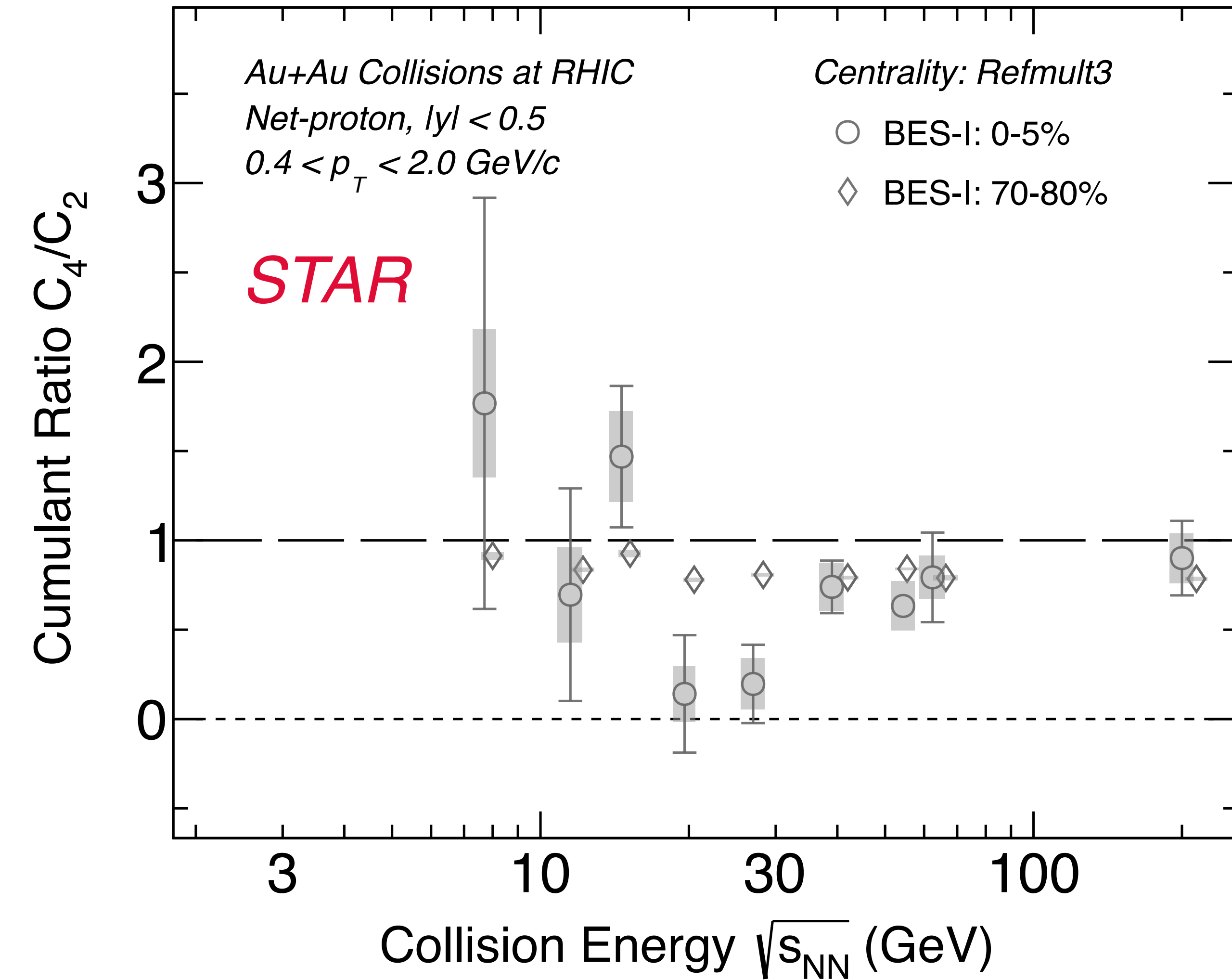


# CENTRALITY DEPENDENCE AND COMPARISON WITH BES-I

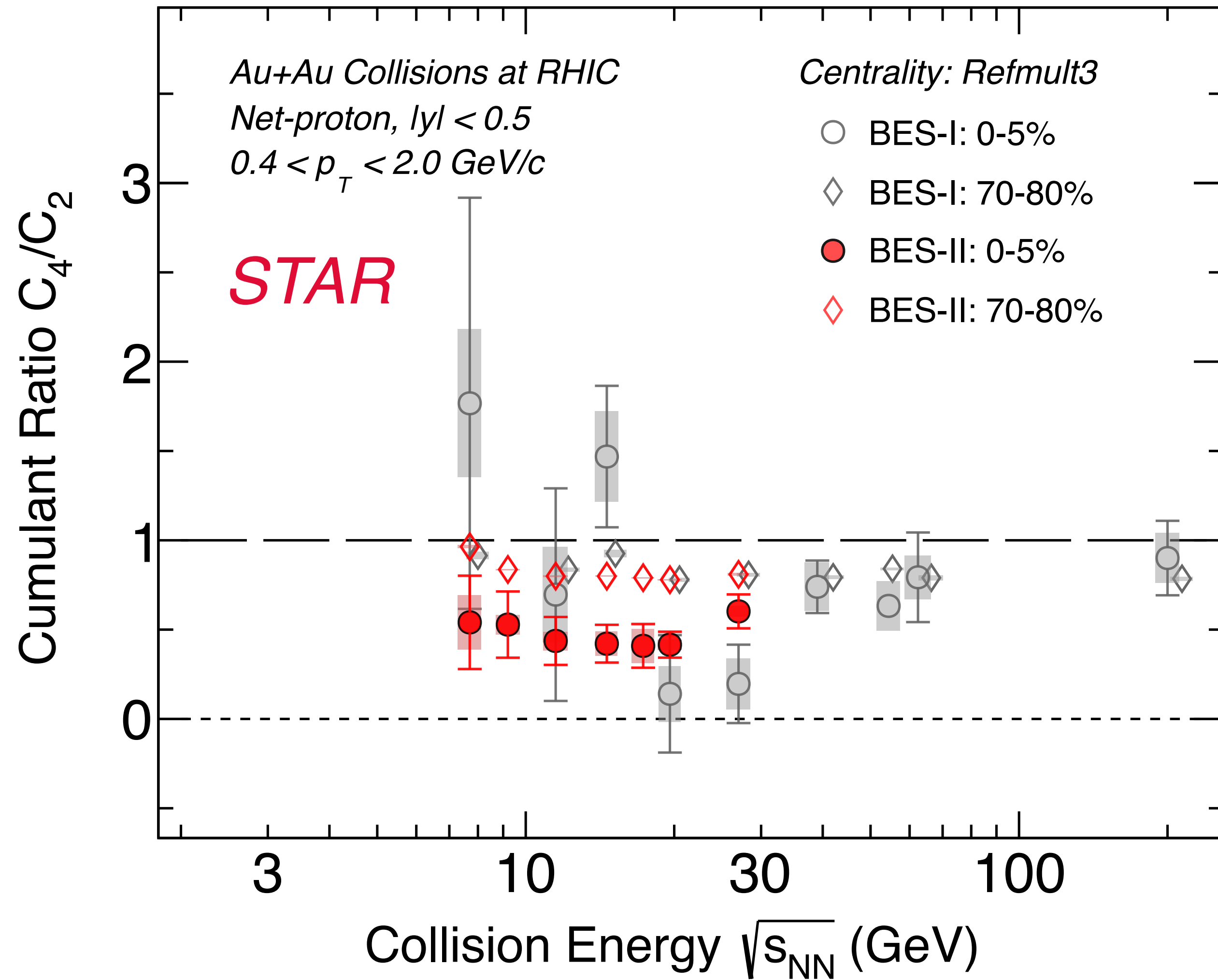


1. Precision measurements: smooth variation across centrality and collision energy observed.
2. Higher centrality resolution leads to lower ratios (especially in mid central collisions):  
**Results from Refmult3X (BES-II) < Refmult3 (BES-II) < Refmult3 (BES-I)**
3. For 0-5%  $C_4/C_2$ , weak effect of centrality resolution seen.

# ENERGY DEPENDENCE OF $C_4/C_2$ : COMPARISON WITH BES-I



# ENERGY DEPENDENCE OF $C_4/C_2$ : COMPARISON WITH BES-I

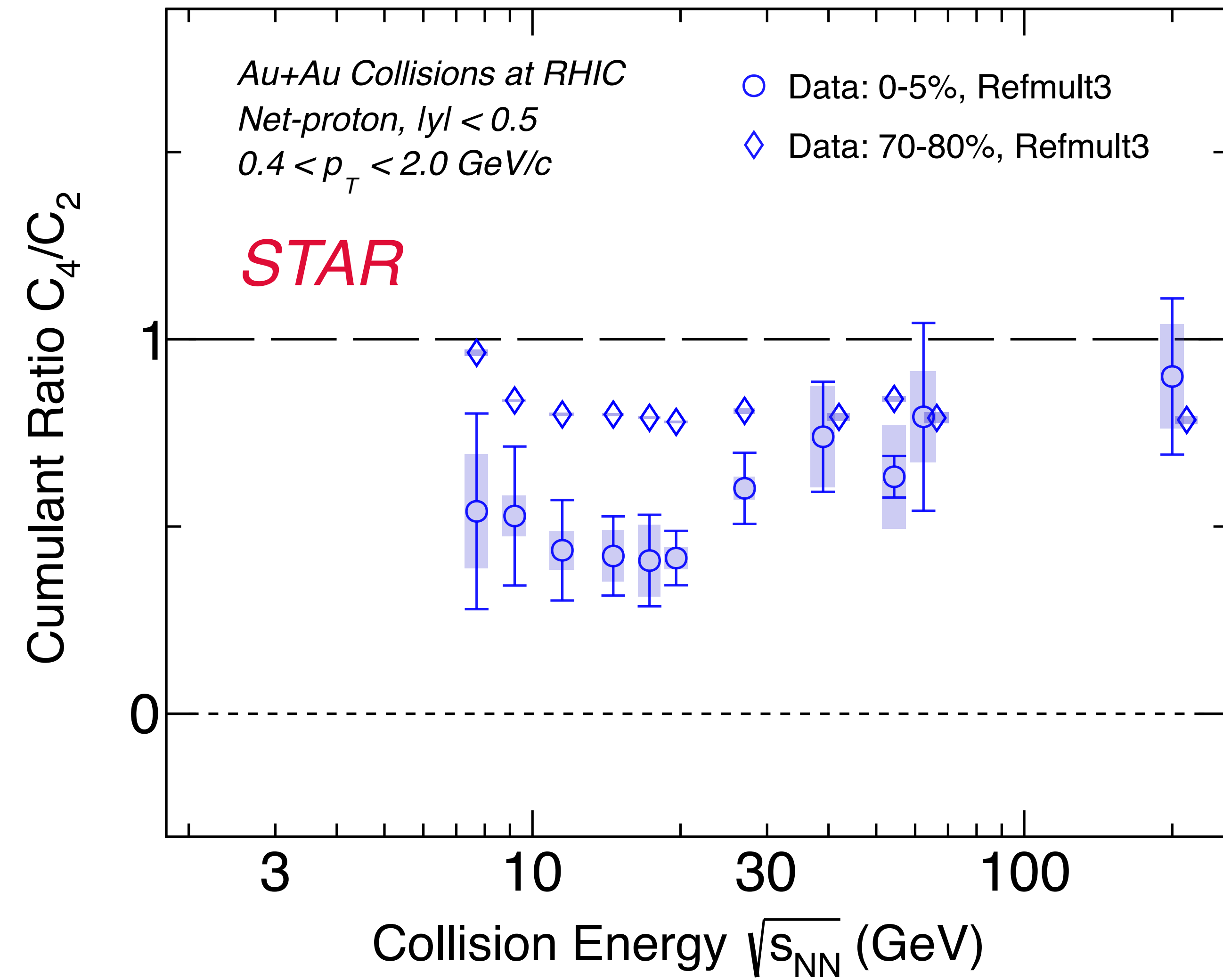


Deviation between BES-II and BES-I data

$\sqrt{s_{NN}}$ (GeV)	0-5%	70-80%
7.7	$1.0\sigma$	$0.9\sigma$
11.5	$0.4\sigma$	$1.3\sigma$
14.6	$2.2\sigma$	$2.5\sigma$
19.6	$0.7\sigma$	$0.0\sigma$
27	$1.4\sigma$	$0.2\sigma$

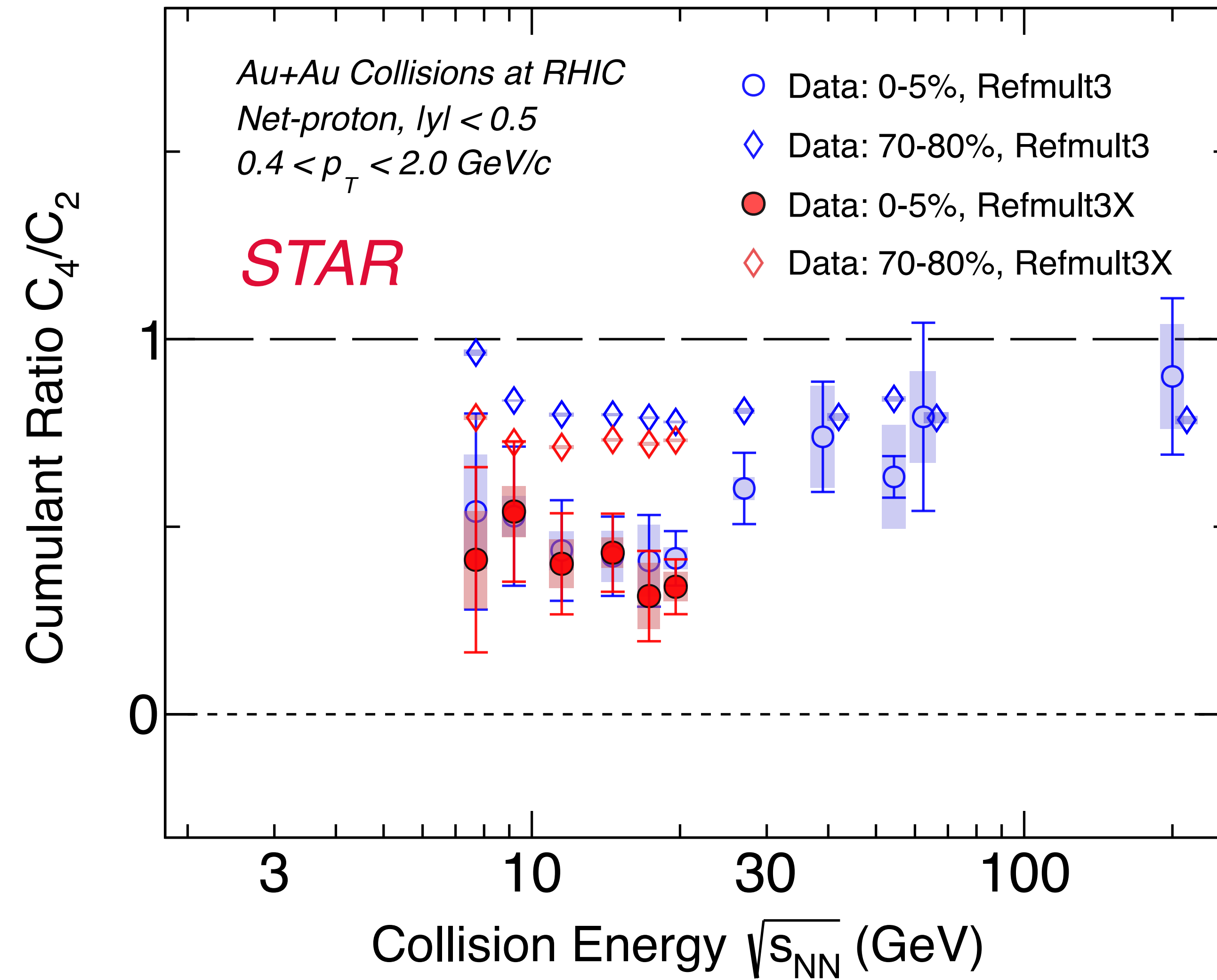
- BES-II results consistent with BES-I within uncertainties.

# EFFECT OF CENTRALITY RESOLUTION ON $C_4/C_2$





# EFFECT OF CENTRALITY RESOLUTION ON $C_4/C_2$

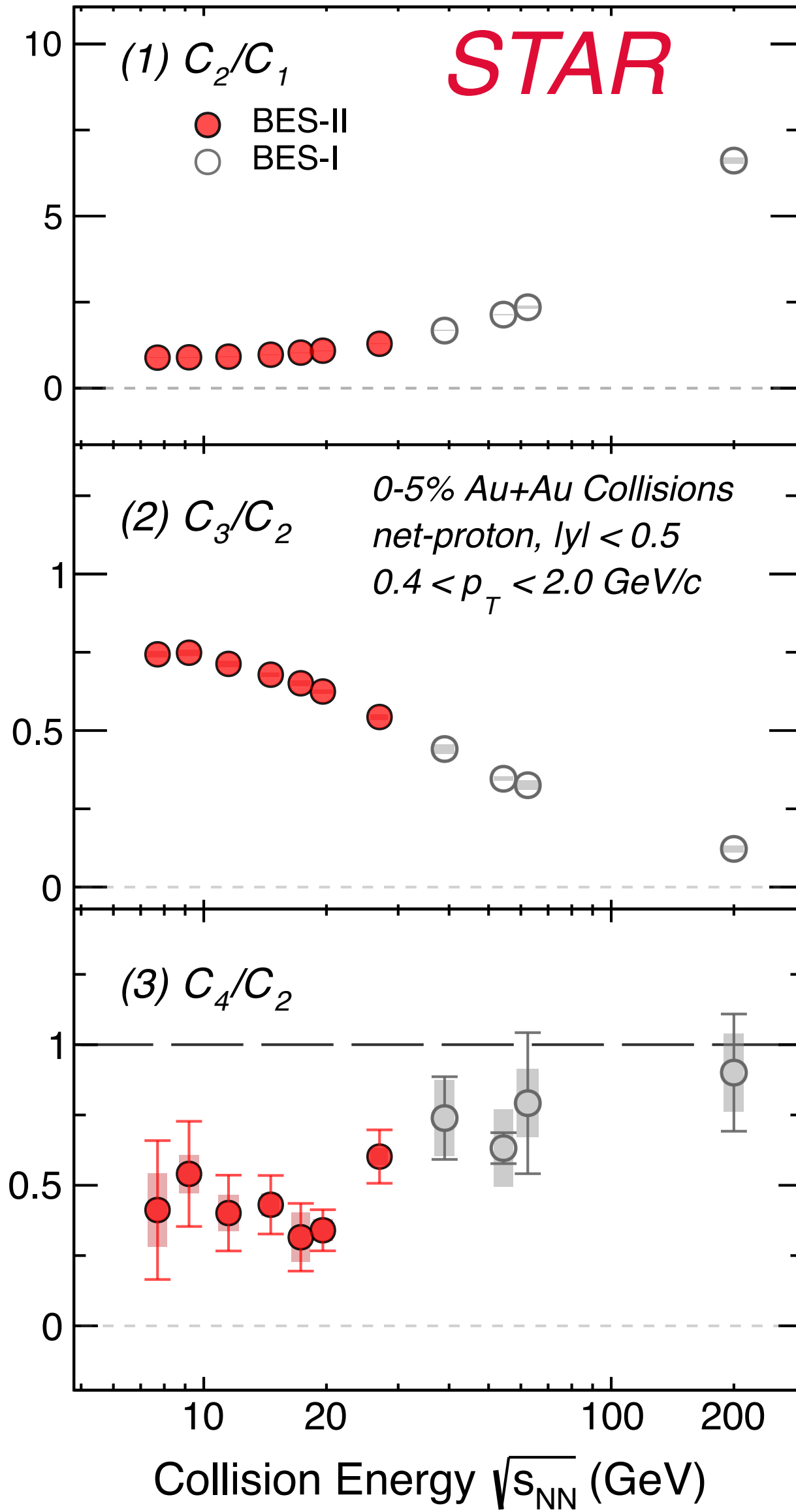


1. **0-5% centrality  $C_4/C_2$  results show good agreement between Refmult3 and Refmult3X: weak effect of centrality resolution.**

2. BES-II results shown hereafter are with Refmult3X

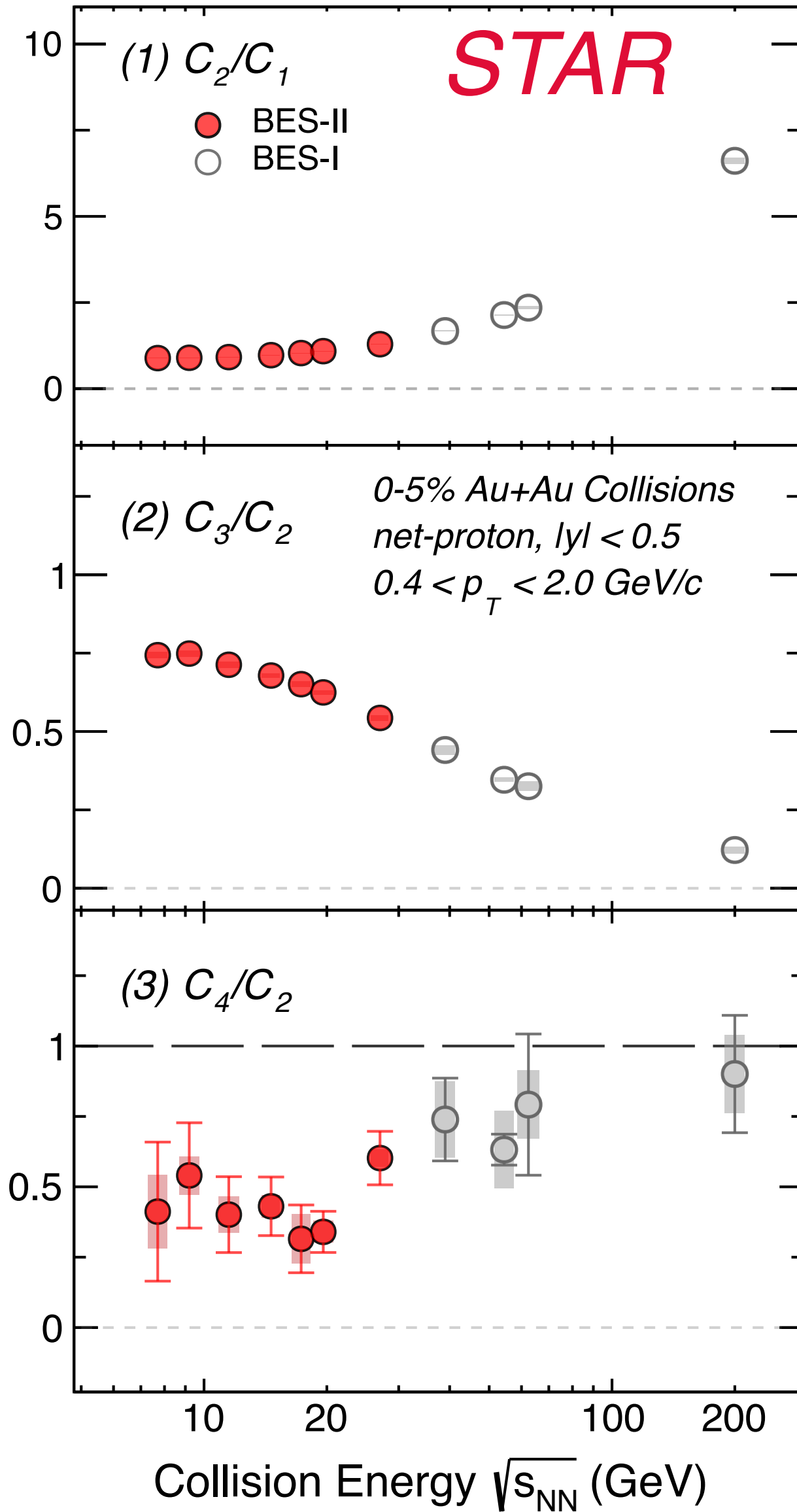
# ENERGY DEPENDENCE: MODEL COMPARISON

## Net-proton cumulant ratios



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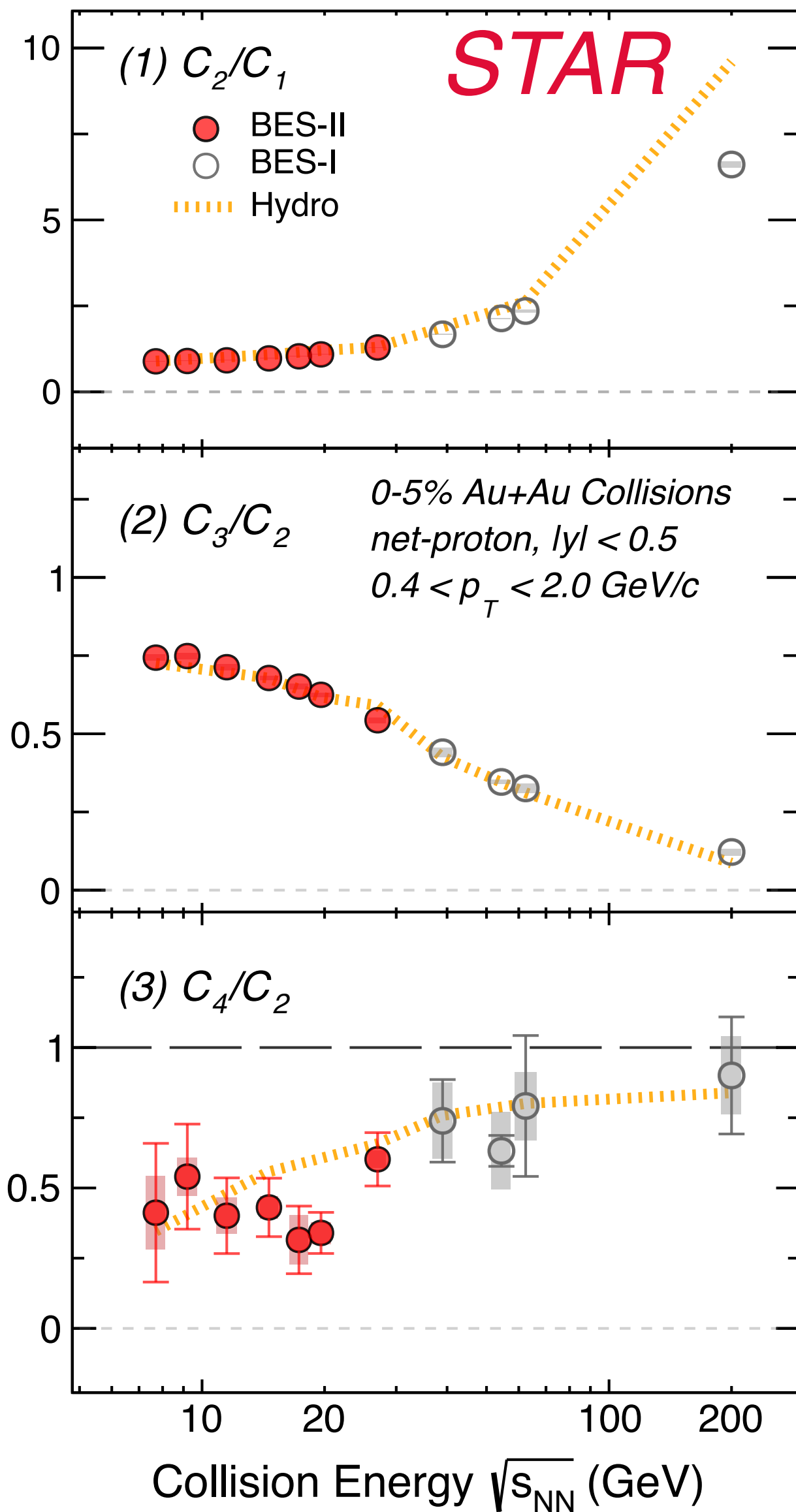
## Net-proton cumulant ratios



1. Smooth variation vs  $\sqrt{s_{NN}}$  in  $C_2/C_1$  and  $C_3/C_2$  observed.  $C_4/C_2$  decreases with decreasing  $\sqrt{s_{NN}}$ .

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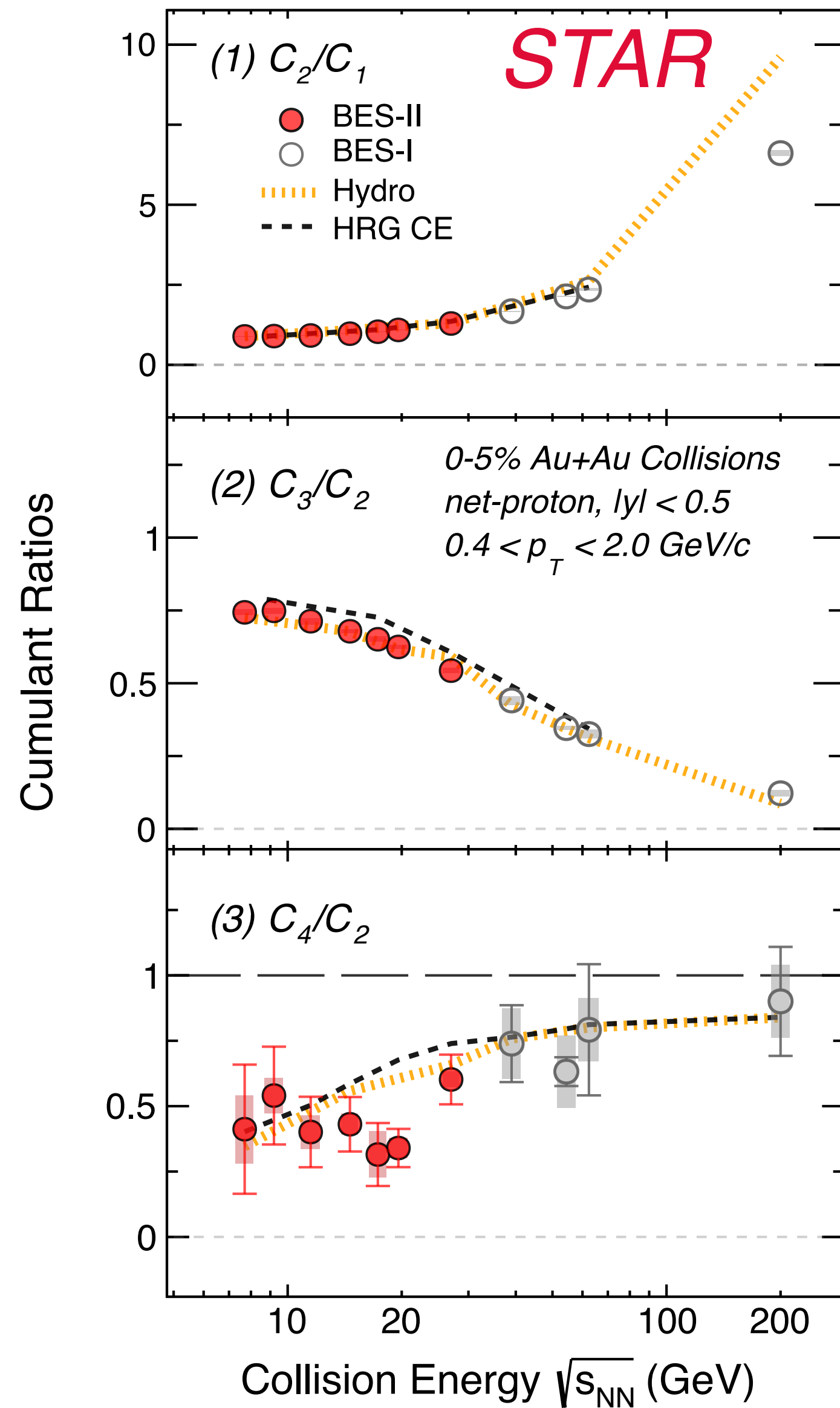
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2. Non-CP models used for comparison:  
A. Hydro: Hydrodynamical model

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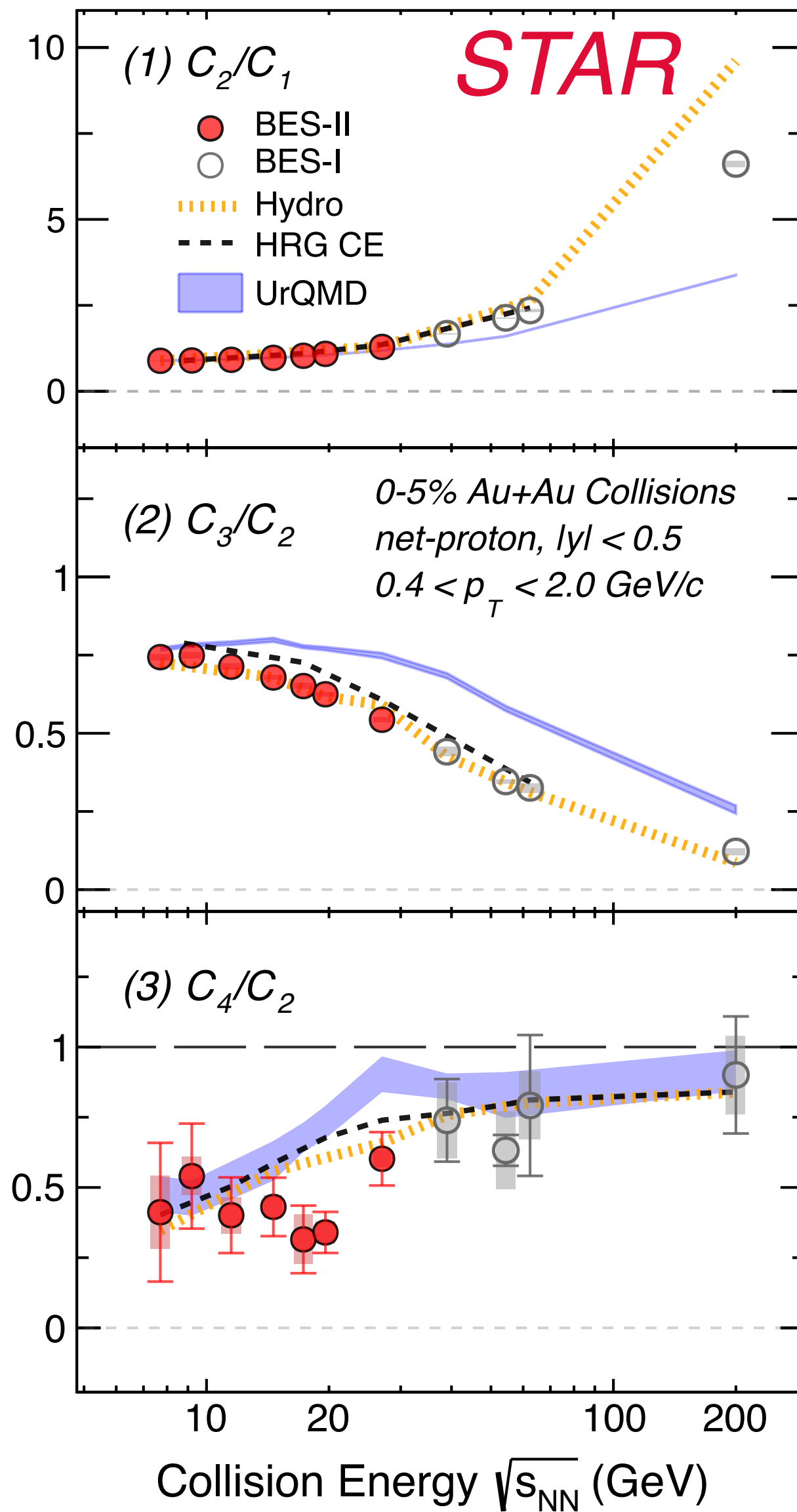
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A. Hydro: Hydrodynamical model

B. HRG CE: Thermal model with canonical treatment of baryon charge

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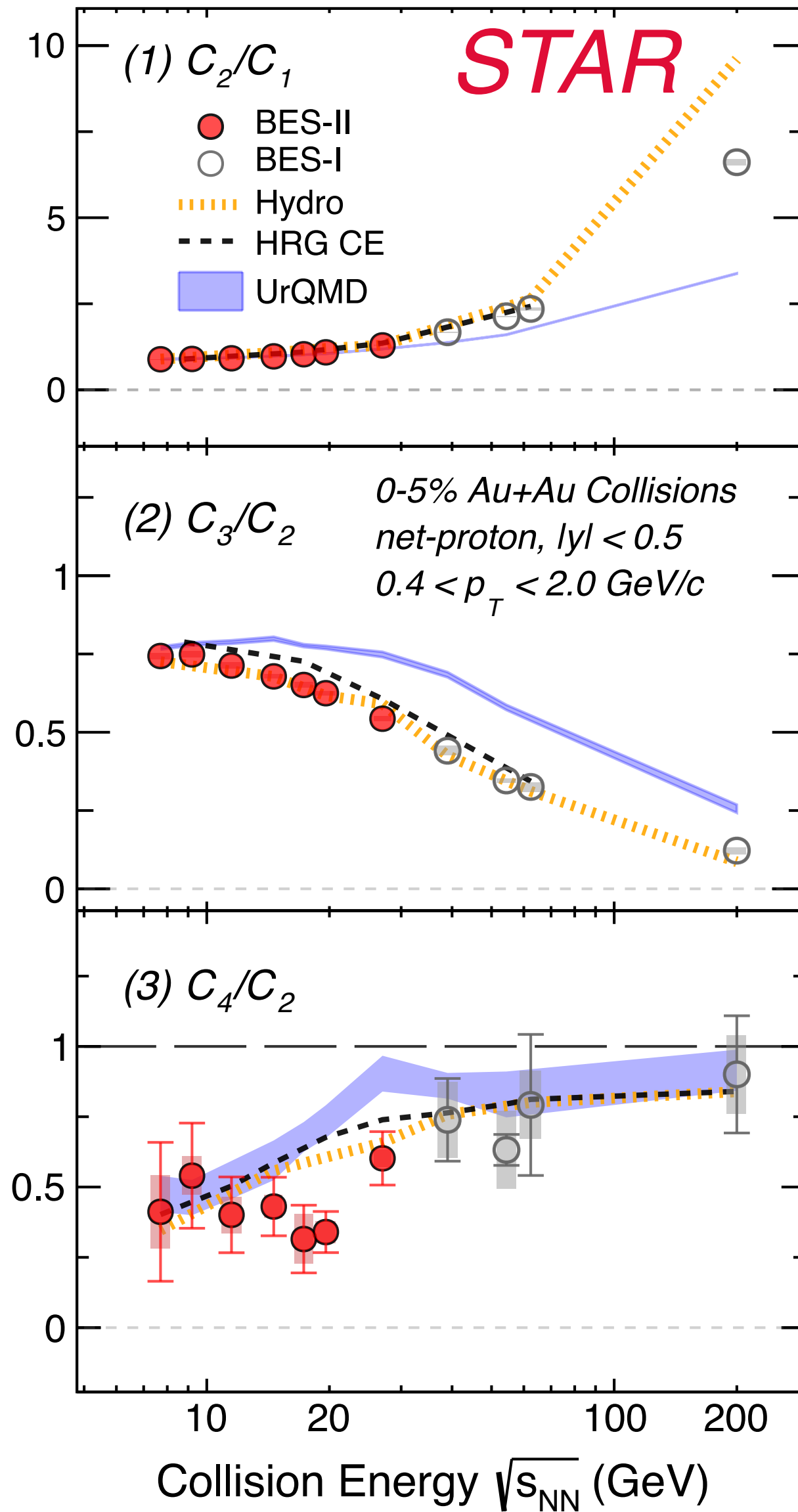
B. HRG CE: Thermal model with canonical treatment of baryon charge

C. UrQMD: Hadronic transport model

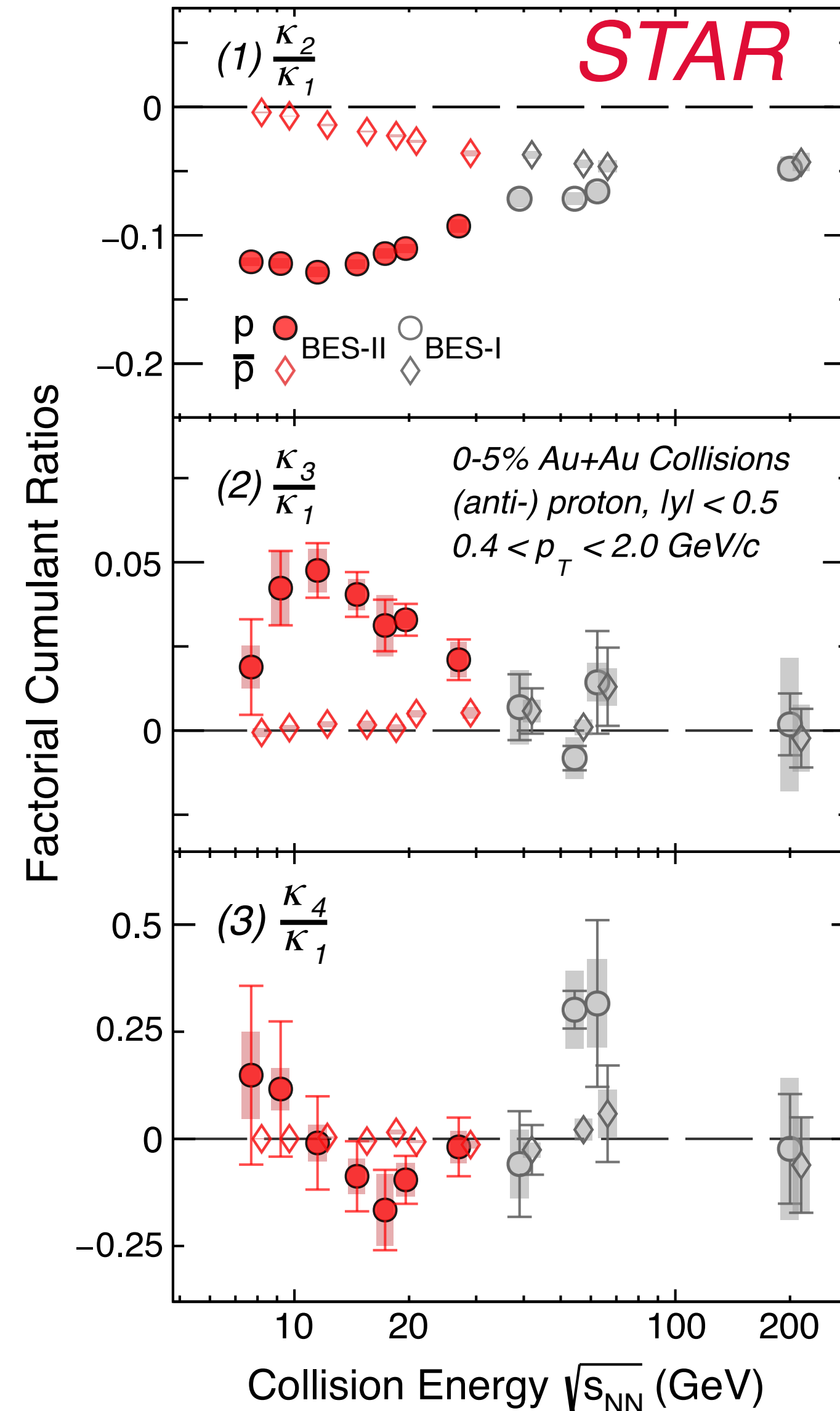
(All models include baryon number conservation)

# ENERGY DEPENDENCE: MODEL COMPARISON

## Net-proton cumulant ratios



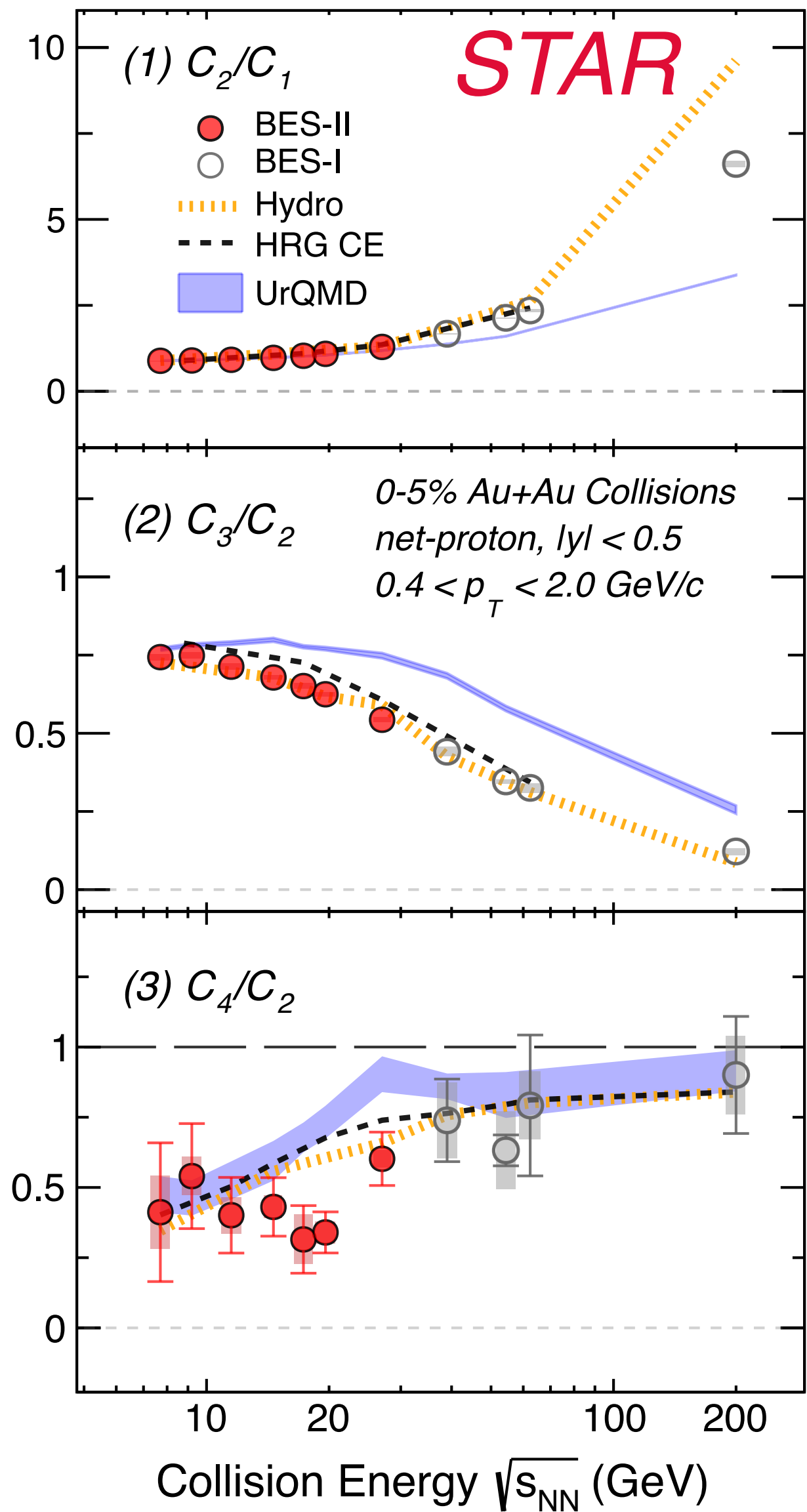
## Proton/antiproton factorial cumulant ratios



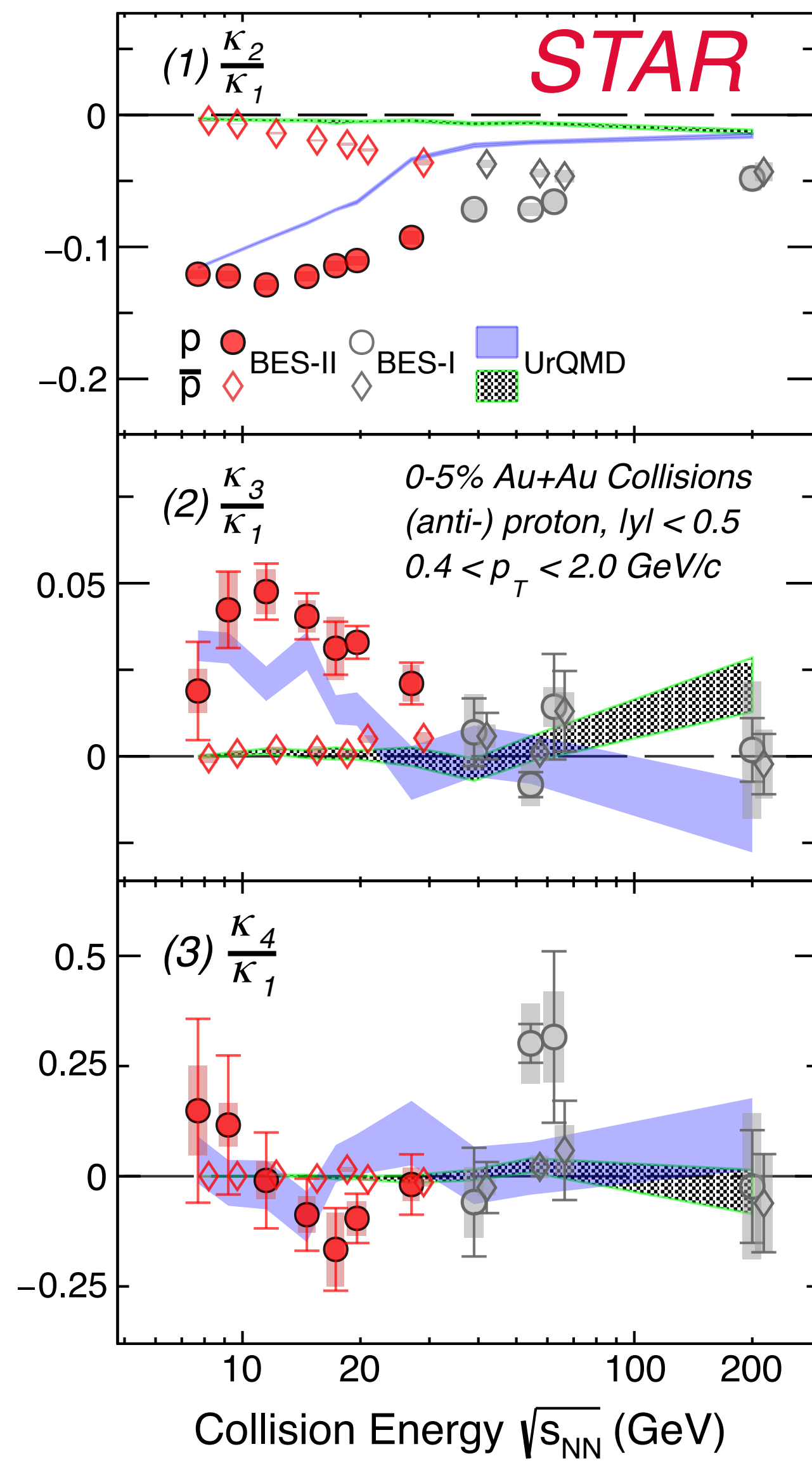
- Smooth variation vs  $\sqrt{s_{NN}}$  in  $C_2/C_1$  and  $C_3/C_2$  observed.  $C_4/C_2$  decreases with decreasing  $\sqrt{s_{NN}}$ .
- Non-CP models used for comparison:
  - Hydro: Hydrodynamical model
  - HRG CE: Thermal model with canonical treatment of baryon charge
  - UrQMD: Hadronic transport model
 (All models include baryon number conservation)
- Proton factorial cumulant ratios deviates from poisson baseline at 0. Antiproton  $\kappa_3/\kappa_1, \kappa_4/\kappa_1$  closer to 0.

# ENERGY DEPENDENCE: MODEL COMPARISON

## Net-proton cumulant ratios



## Proton/antiproton factorial cumulant ratios



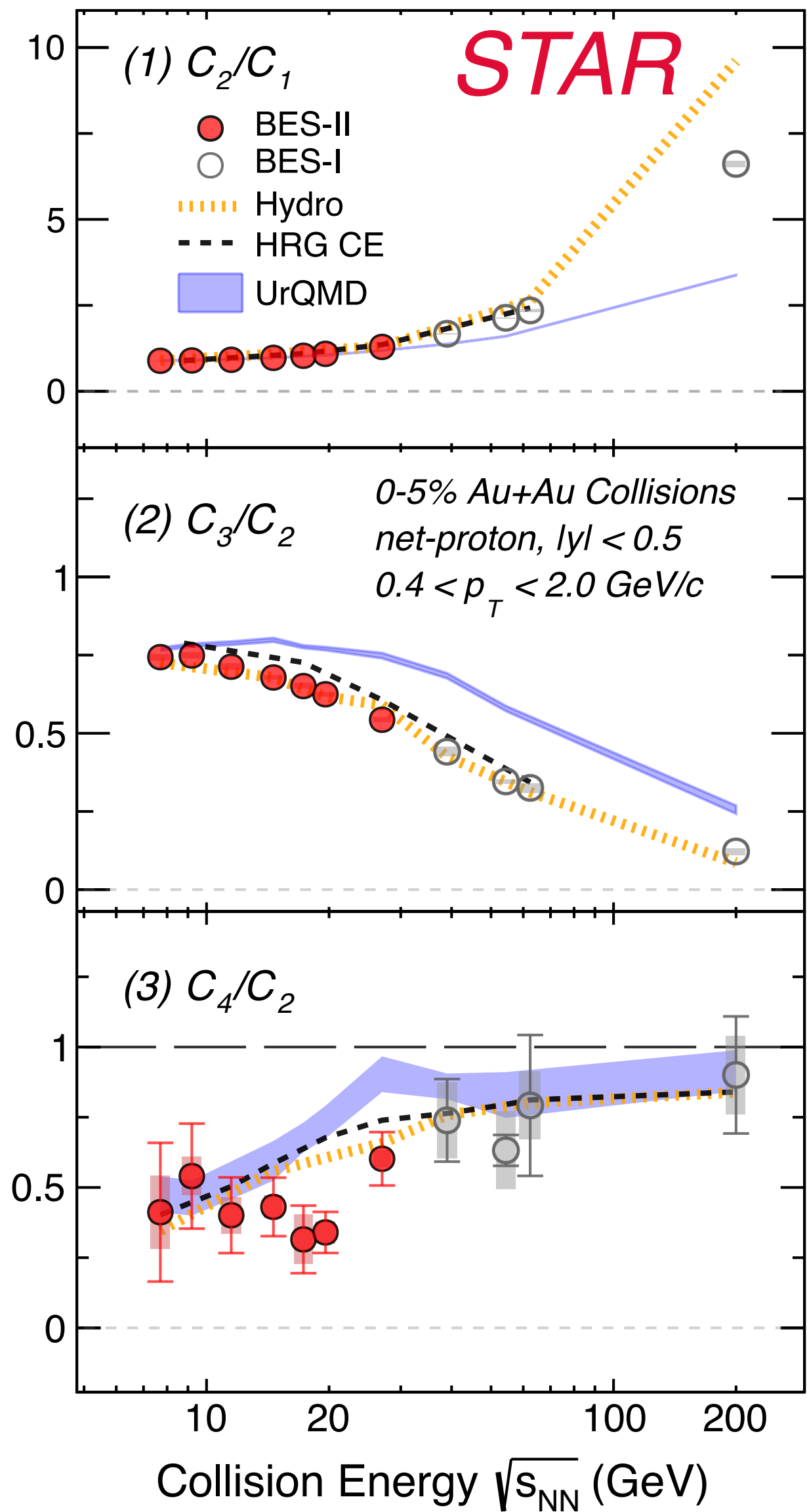
- Smooth variation vs  $\sqrt{s_{NN}}$  in  $C_2/C_1$  and  $C_3/C_2$  observed.  $C_4/C_2$  decreases with decreasing  $\sqrt{s_{NN}}$ .
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  - C. UrQMD: Hadronic transport model
 (All models include baryon number conservation)
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HRG CE: P. B Munzinger et al, NPA 1008, 122141 (2021)  
Hydro: V. Vovchenko et al, PRC 105, 014904 (2022)

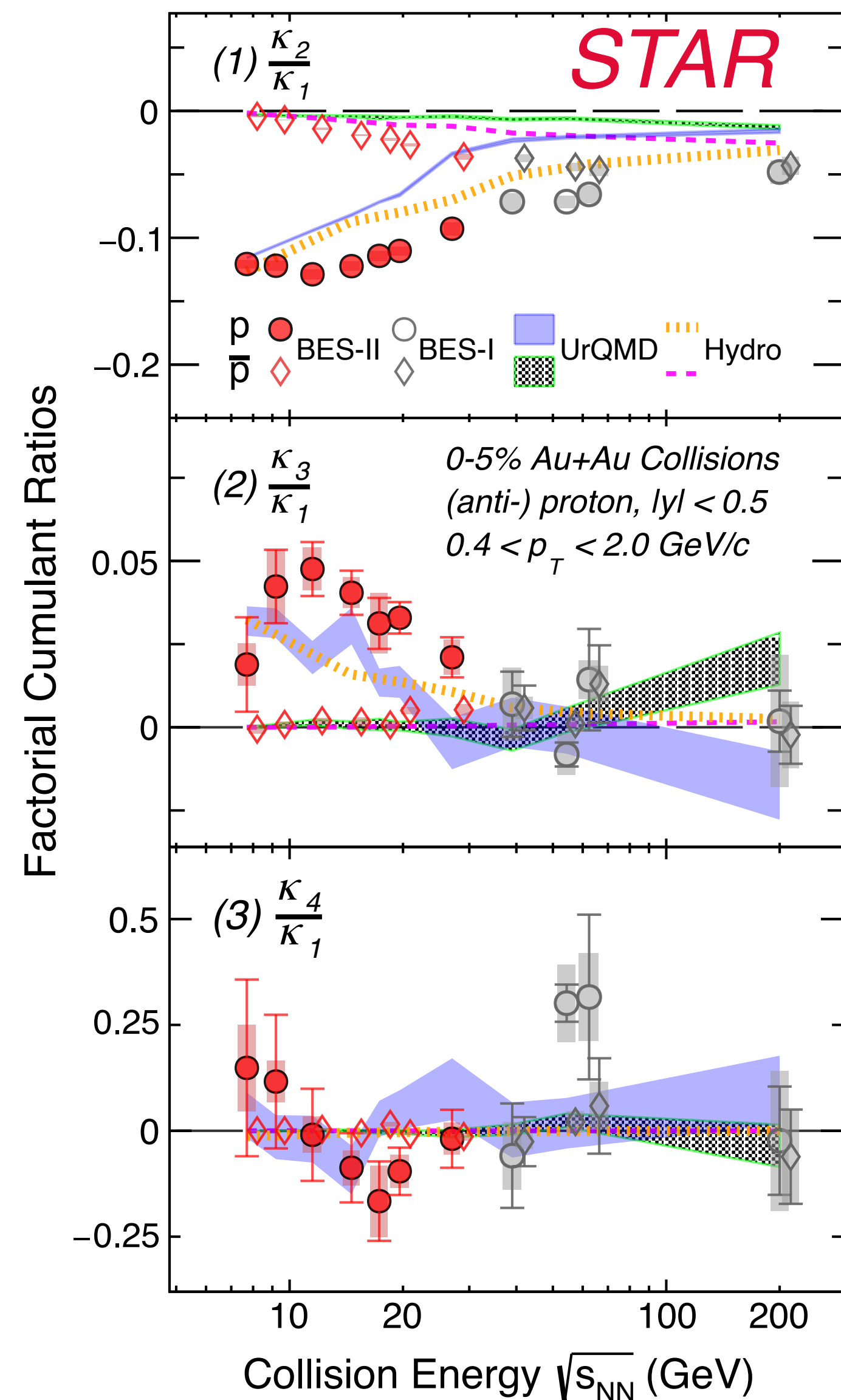


# ENERGY DEPENDENCE: MODEL COMPARISON

## Net-proton cumulant ratios



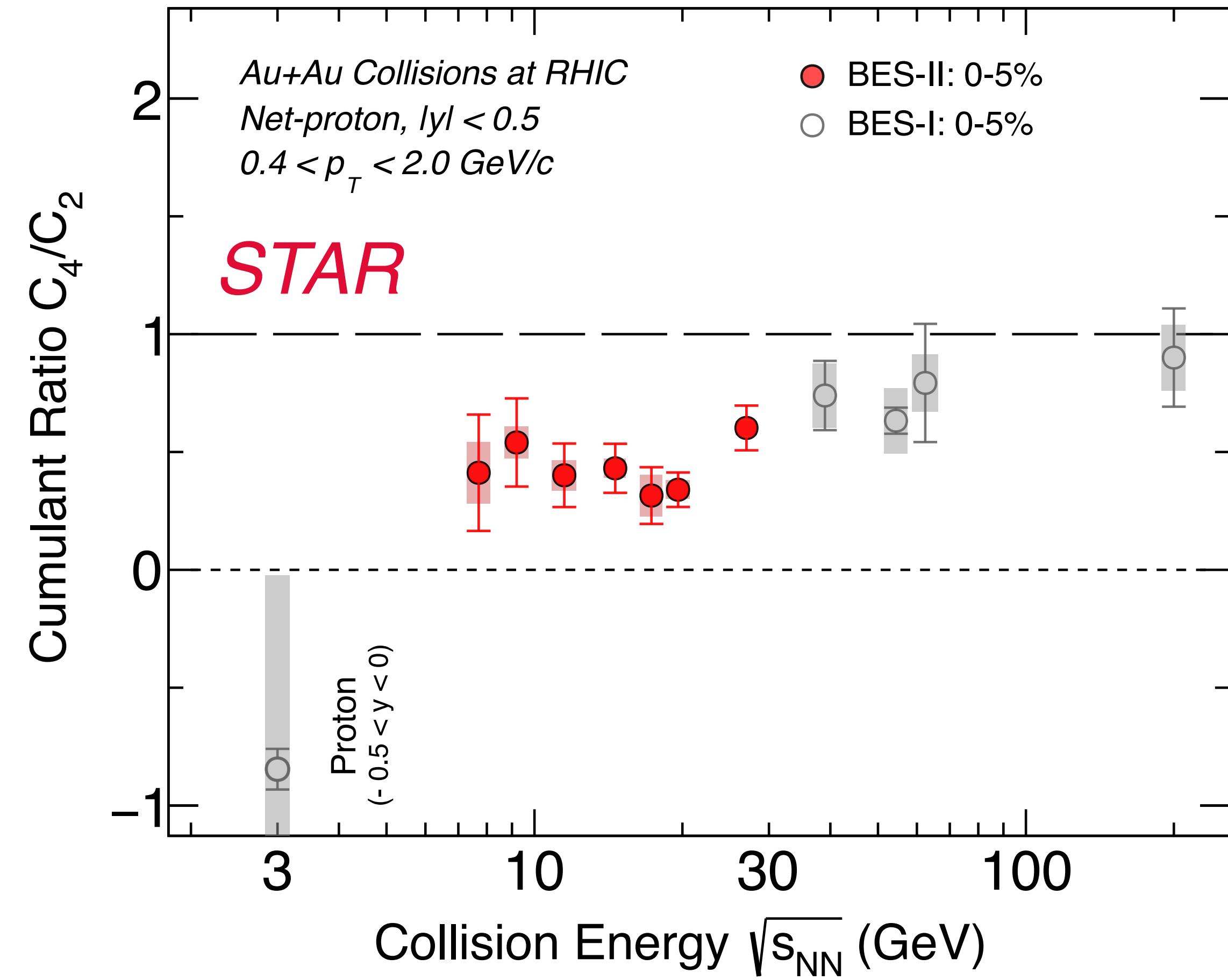
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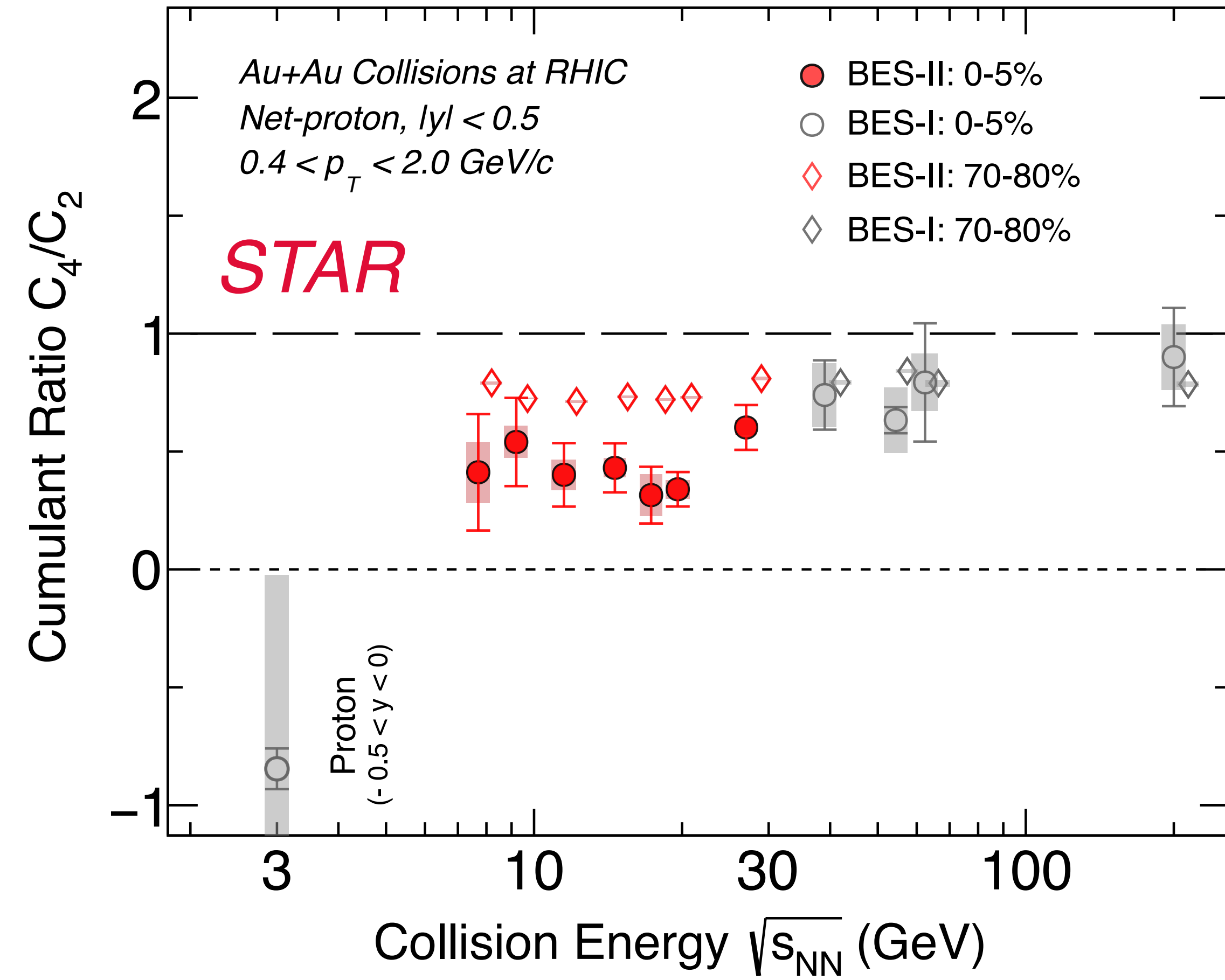
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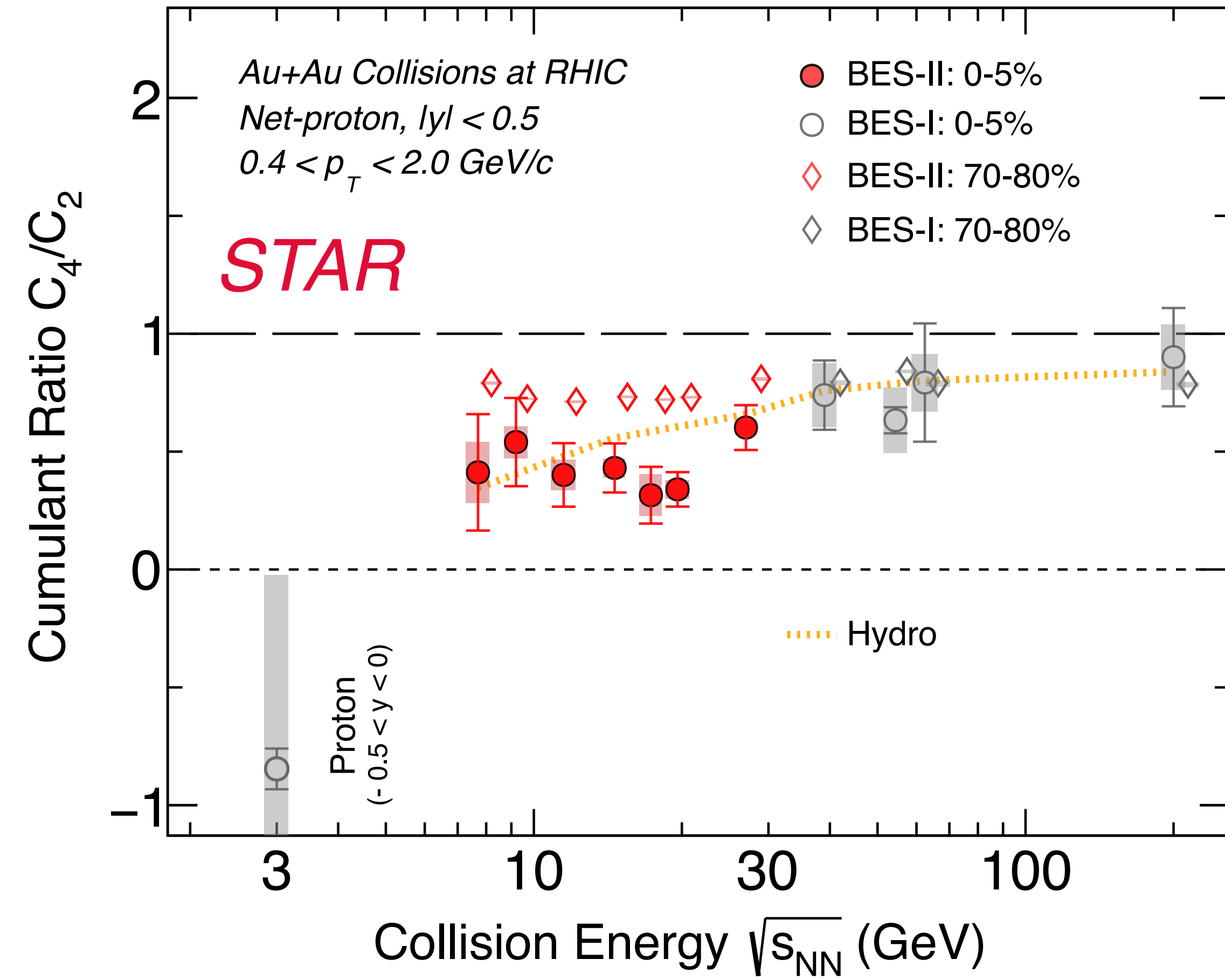
# ENERGY DEPENDENCE OF $C_4/C_2$ : QUANTIFYING DEVIATION



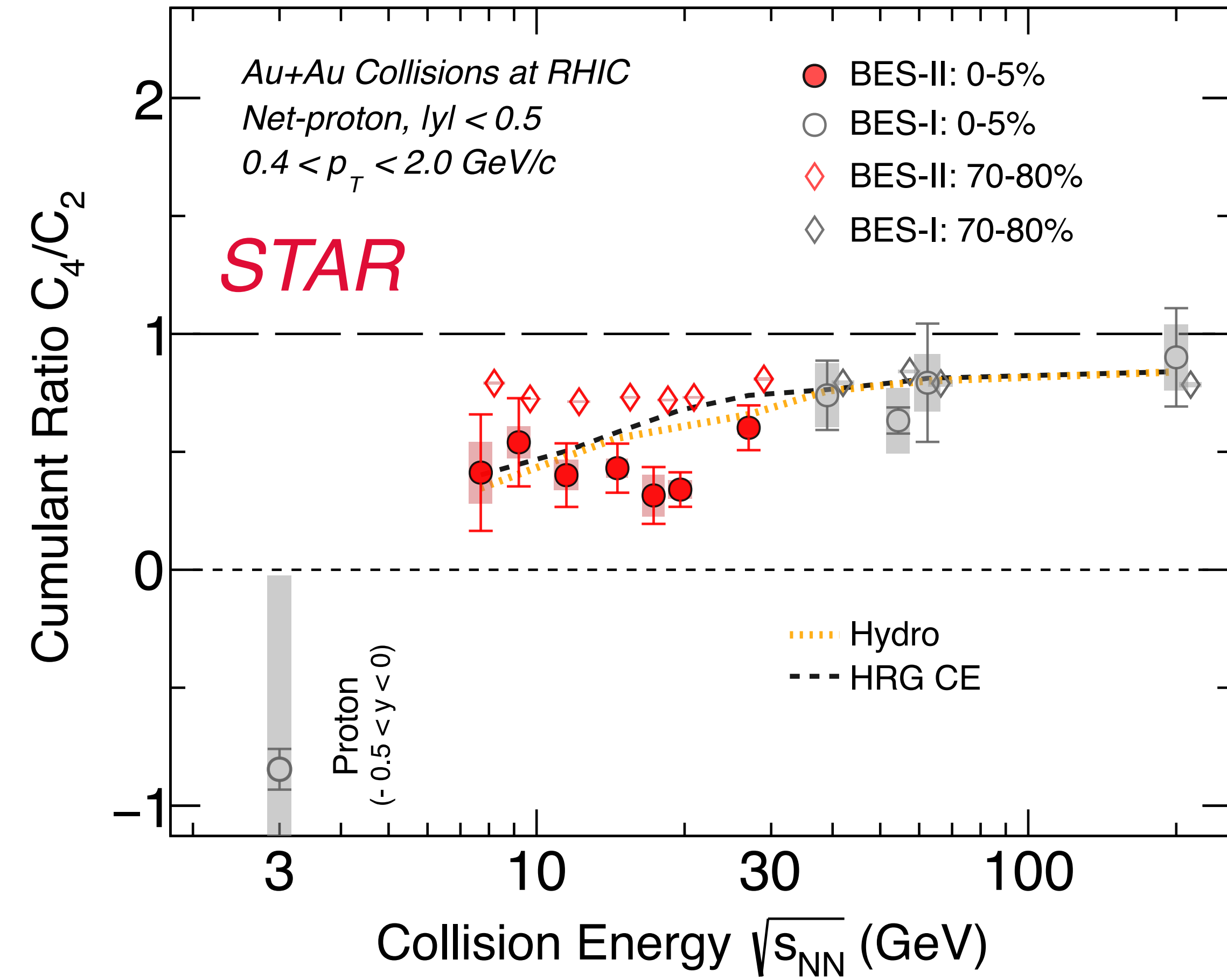
# ENERGY DEPENDENCE OF $C_4/C_2$ : QUANTIFYING DEVIATION



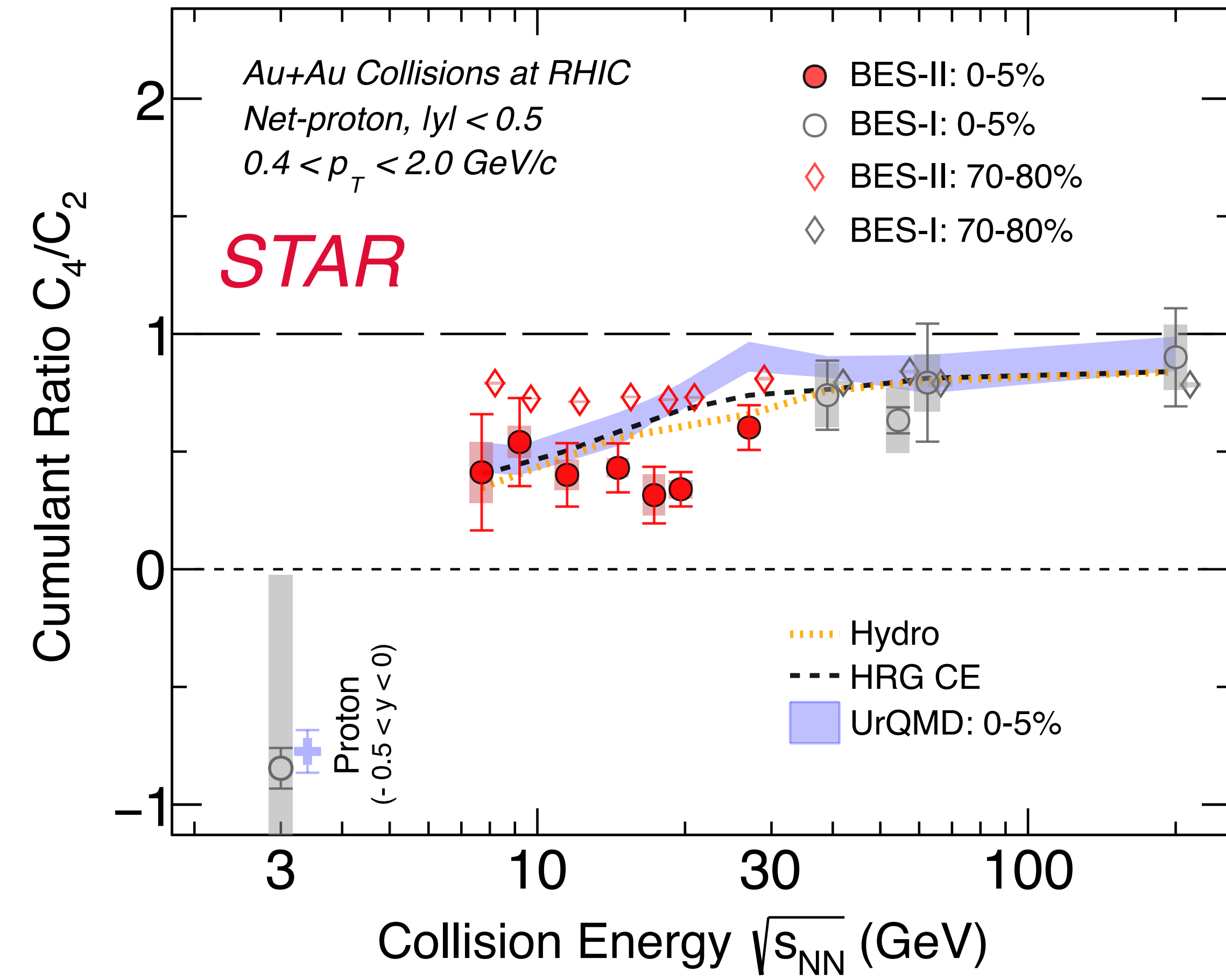
# ENERGY DEPENDENCE OF $C_4/C_2$ : QUANTIFYING DEVIATION



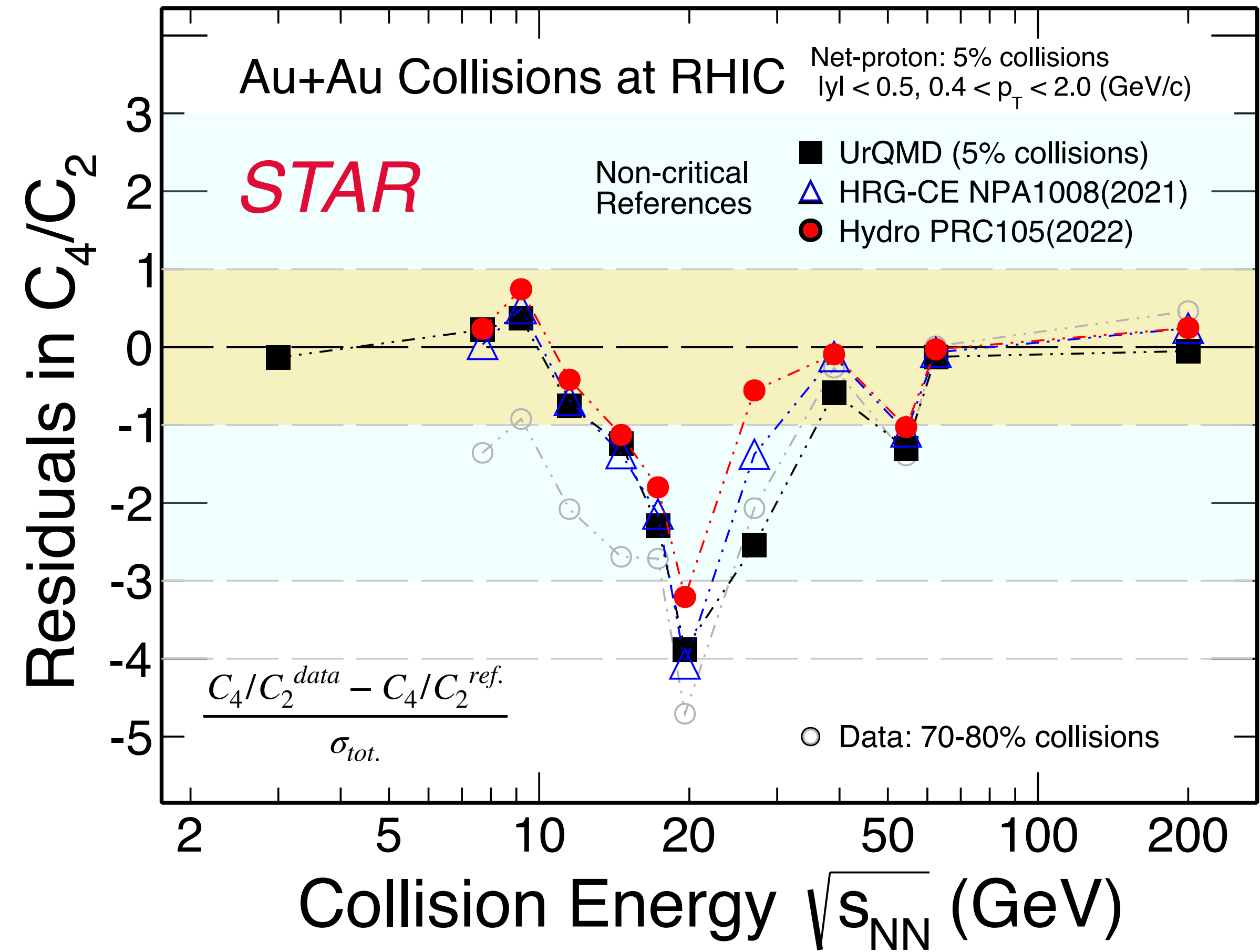
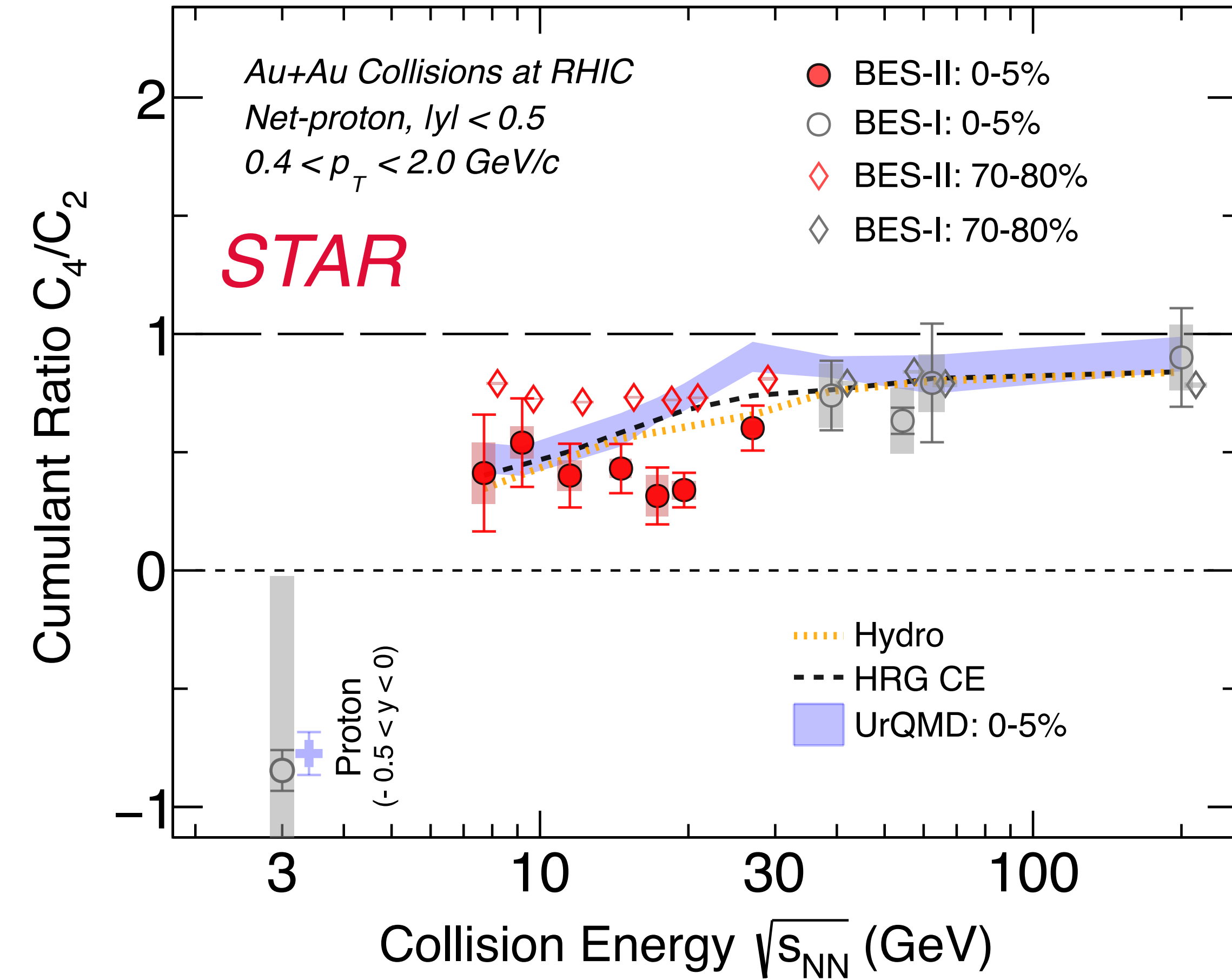
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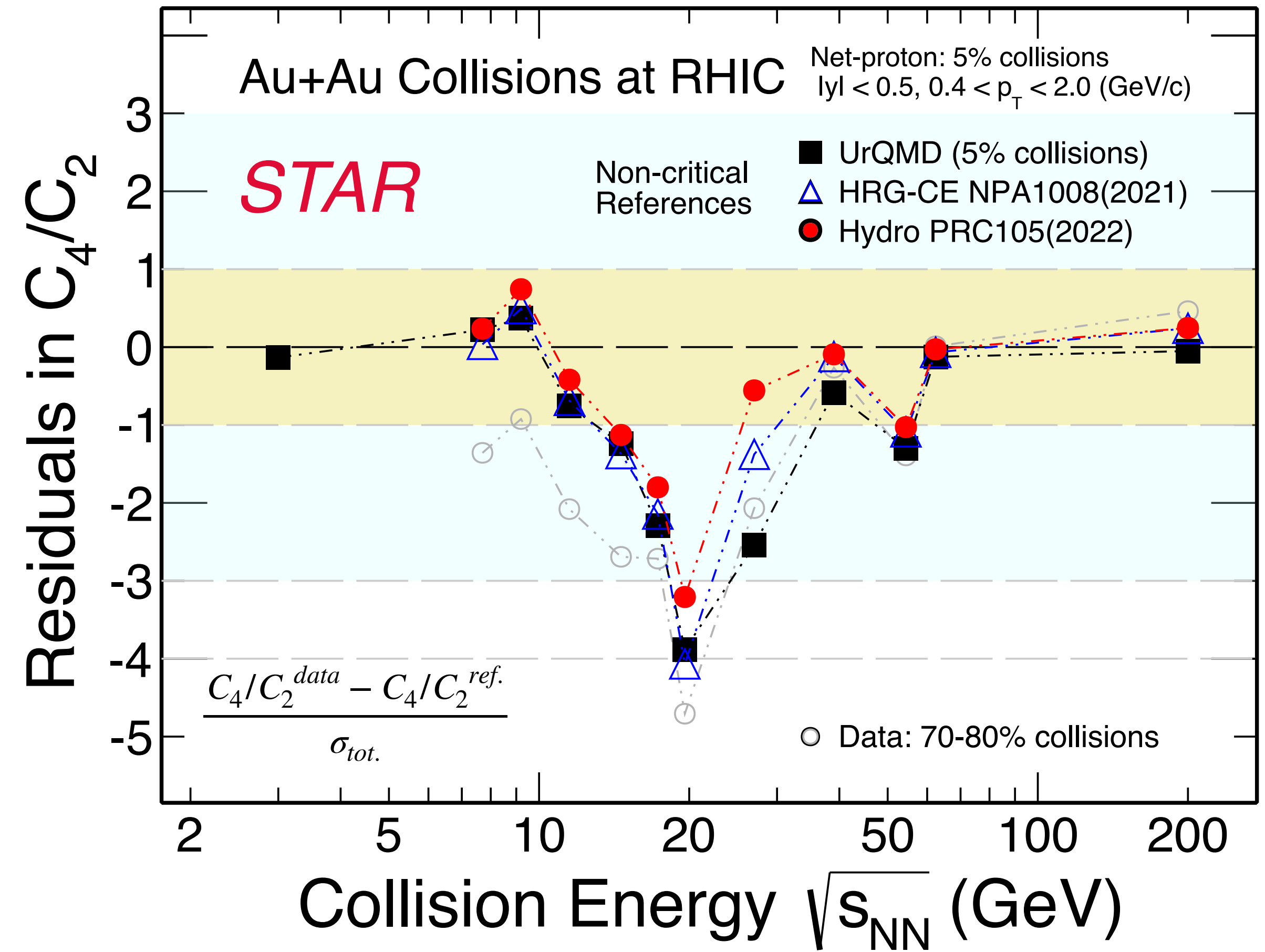
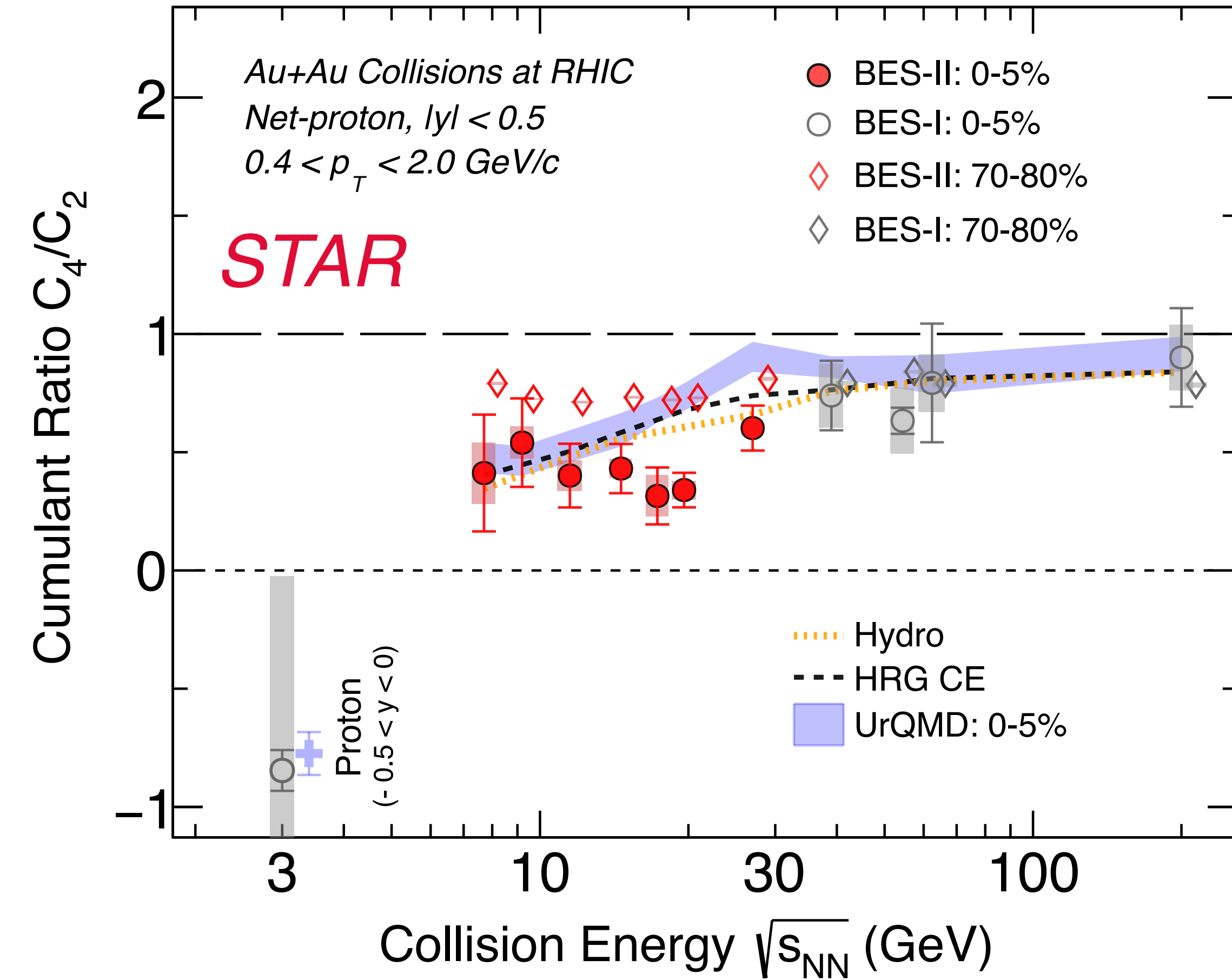


# ENERGY DEPENDENCE OF $C_4/C_2$ : QUANTIFYING DEVIATION



$C_4/C_2$  shows minimum around  $\sim 20$  GeV comparing to non-CP models, 70-80% data

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1. Maximum deviation:  $3.2 - 4.7\sigma$  at  $\sqrt{s_{NN}} = 20$  GeV ( $1.3 - 2\sigma$  at BES-I)

2. Overall deviation from  $\sqrt{s_{NN}} = 7.7 - 27$  GeV:  $1.9 - 5.4\sigma$  ( $1.4 - 2.2\sigma$  at BES-I)



# SUMMARY AND OUTLOOK:

## Summary:

1. Precision measurement of net-proton number fluctuations in Au+Au collisions from STAR BES-II reported. Centrality and energy dependence discussed. Compared to BES-I, we have **better statistical precision, better centrality resolution, better control on systematics!**
2. Measured net-proton  $C_4/C_2$  in 0-5% central collisions do not show deviation above non-CP model calculations. Maximum deviation in 0-5% data w.r.t. various non-CP model calculations and 70-80% data is observed at  $\sqrt{s_{NN}} = 20$  GeV with a significance level of  $3.2 - 4.7\sigma$ .

## Outlook:

1. Extend measurements to even higher orders of fluctuations:  $C_n, \kappa_n$  ( $n = 1 - 6$ ).
2. Examine transverse momentum dependence and rapidity dependence of fluctuations.
3. Complete the measurements in Au+Au collisions at fixed target (FXT) energies.

## **ACKNOWLEDGEMENTS:**

***RHIC operation for successfully completing collection of BES-II data,***

***Organizers for giving this opportunity.***

***Thank you all for listening.***

**BACK UP**