Estimate of nonflow baseline for the chiral magnetic effect in isobar collisions at RHIC

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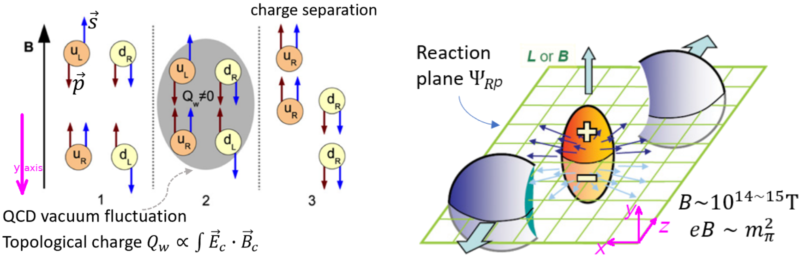
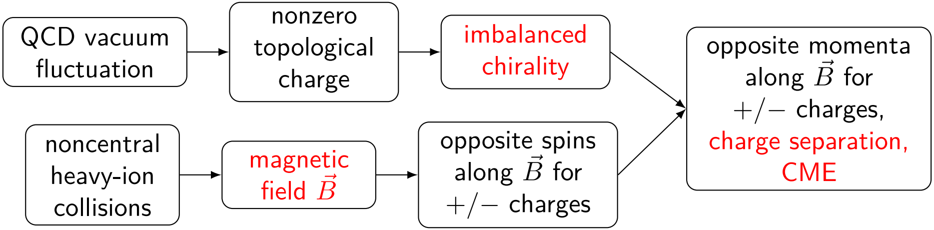
Outline

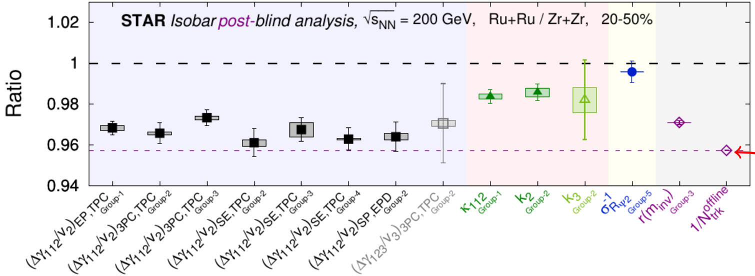


CME schematics. [Kharzeev, McLerran and Warringa, NPA **803**, 227 (2008)]



Outline

Isobar Results



Post-blind results from STAR isobar analysis [STAR, PRC **105**, 014901 (2022)].

- Isobar expectation: $\Delta\gamma/v_2$ in ${}^{96}_{44}$Ru + ${}^{96}_{44}$Ru is larger than in ${}^{96}_{40}$Zr + ${}^{96}_{40}$Zr.
- The main reason that the observed isobar ratio is less than unity is the multiplicity difference.
- The better quantity is $N\Delta\gamma/v_2$. Its naive background baseline is unity.
- Isobar data are all above this naive baseline. Investigate nonflow effects.

$\Delta\gamma$ Observable

- The CME-sensitive observable $\Delta\gamma \equiv C_3/v_2^*$:

$$
\begin{aligned}
C_{3,\mathrm{os}} =& \langle \cos(\phi_\alpha^\pm + \phi_\beta^\mp - 2\phi_c) \rangle, \\
C_{3,\mathrm{ss}} =& \langle \cos(\phi_\alpha^\pm + \phi_\beta^\pm - 2\phi_c) \rangle, \\
C_3 =& C_{3,\mathrm{os}} - C_{3,\mathrm{ss}}
\end{aligned}
$$

OS: opposite-sign pair
SS: same-sign pair

- The asterisk (*) on v_2 indicates it is the measured v_2 containing nonflow
- $\Delta\gamma$ contains CME and a major background proportional to v_2 (true v_2 flow)

Nonflow Contribution to Isobar Baseline

- The naive baseline of unity would be correct if there was no nonflow. Nonflow correlations will cause the baseline to deviate from unity.
- Nonflow in v_2^*: $v_2^{*2} = v_2^2 + v_{2,\mathrm{nf}}^2, \quad \epsilon_{\mathrm{nf}} \equiv v_{2,\mathrm{nf}}^2 / v_2^2$
- C_3 is composed of flow-induced background (major), 3p nonflow correlations (minor), and possible CME (not written out) [Y. Feng, et al., PRC **105**, 024913 (2022)]:

Note:
ϵ is not eccentricity

$$C_3 = \frac{N_{2\mathrm{p}}}{N^2} C_{2\mathrm{p}} v_{2,2\mathrm{p}} v_2 + \frac{N_{3\mathrm{p}}}{2N^3} C_{3\mathrm{p}} = \frac{v_2^2 \epsilon_2}{N} + \frac{\epsilon_3}{N^2},$$

$$\frac{N\Delta\gamma}{v_2^*} = \frac{NC_3}{v_2^{*2}} = \frac{\epsilon_2}{1+\epsilon_{\mathrm{nf}}} + \frac{\epsilon_3}{Nv_2^2(1+\epsilon_{\mathrm{nf}})} = \frac{\epsilon_2}{1+\epsilon_{\mathrm{nf}}}\left(1 + \frac{\epsilon_3/\epsilon_2}{Nv_2^2}\right)$$

- 2-particle (2p) nonflow (e.g., resonance): $C_{2\mathrm{p}} \equiv \langle\cos(\phi_\alpha + \phi_\beta - 2\phi_{2\mathrm{p}})\rangle$, $\epsilon_2 \equiv \frac{N_{2\mathrm{p}} v_{2,2\mathrm{p}}}{N v_2} C_{2\mathrm{p}}$
- 3-particle (3p) nonflow (e.g., jets): $C_{3\mathrm{p}} \equiv \langle\cos(\phi_\alpha + \phi_\beta - 2\phi_c)\rangle_{3\mathrm{p}}$, $\epsilon_3 \equiv \frac{N_{3\mathrm{p}}}{2N} C_{3\mathrm{p}}$
- $N \approx N_+ \approx N_-$ is POI (particle of interest) mult. $N_{2\mathrm{p}}$ ($N_{3\mathrm{p}}$) is 2p (3p) nonflow pair (triplet) mult.

Nonflow Contribution to Isobar Baseline

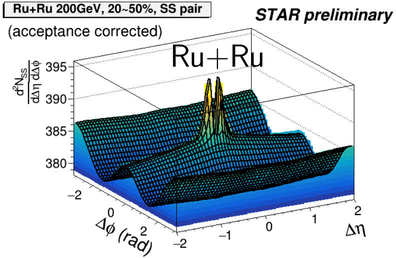
- Isobar ratio:

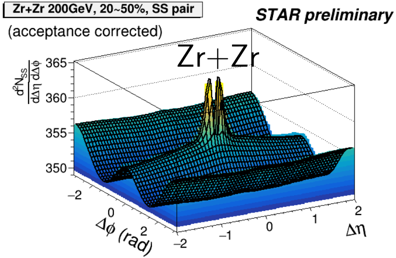
$$\frac{(N\Delta\gamma/v_2^*)^{\text{Ru}}}{(N\Delta\gamma/v_2^*)^{\text{Zr}}} \equiv \frac{(NC_3/v_2^{*2})^{\text{Ru}}}{(NC_3/v_2^{*2})^{\text{Zr}}} = \frac{\epsilon_2^{\text{Ru}}}{\epsilon_2^{\text{Zr}}} \cdot \frac{(1+\epsilon_{\text{nf}})^{\text{Zr}}}{(1+\epsilon_{\text{nf}})^{\text{Ru}}} \cdot \frac{\left[1+\epsilon_3/\epsilon_2/(Nv_2^2)\right]^{\text{Ru}}}{\left[1+\epsilon_3/\epsilon_2/(Nv_2^2)\right]^{\text{Zr}}}$$

$$\approx 1 + \frac{\Delta\epsilon_2}{\epsilon_2} - \frac{\Delta\epsilon_{\text{nf}}}{1+\epsilon_{\text{nf}}} + \frac{\epsilon_3/\epsilon_2/(Nv_2^2)}{1+\epsilon_3/\epsilon_2/(Nv_2^2)}\left(\frac{\Delta\epsilon_3}{\epsilon_3} - \frac{\Delta\epsilon_2}{\epsilon_2} - \frac{\Delta N}{N} - \frac{\Delta v_2^2}{v_2^2}\right)$$

$$\Delta X = X^{\text{Ru}} - X^{\text{Zr}}$$

- Need ϵ_{nf}, ϵ_2, ϵ_3 for background estimate
  - $\epsilon_{\text{nf}} \equiv \frac{v_{2,\text{nf}}^2}{v_2^2} = \frac{v_2^{*2} - v_2^2}{v_2^2}$
  - $\epsilon_2 \equiv \frac{C_{2\text{p}} N_{2\text{p}} v_{2,2\text{p}}}{N v_2} = \frac{N_{2\text{p}} v_{2,2\text{p}}}{N v_2} \langle \cos(\phi_\alpha + \phi_\beta - 2\phi_{2\text{p}}) \rangle$
  - $\epsilon_3 \equiv \frac{C_{3\text{p}} N_{3\text{p}}}{2N} = \frac{N_{3\text{p}}}{2N} \langle \cos(\phi_\alpha + \phi_\beta - 2\phi_c) \rangle_{3\text{p}}$





- $0.2 < p_T < 2.0$ GeV
- $|\eta| < 1$
- Centrality $20-50\%$ defined by POI multiplicity
- Mixed-event acceptance corrected

Fit function:

Nearside

$$f(\Delta\eta,\Delta\phi) = A_1 G_{\rm NS,W}(\Delta\eta)G_{\rm NS,W}(\Delta\phi) + A_2 G_{\rm NS,N}(\Delta\eta)G_{\rm NS,N}(\Delta\phi) + A_3 G_{\rm NS,D}(\Delta\eta)G_{\rm NS,D}(\Delta\phi)$$

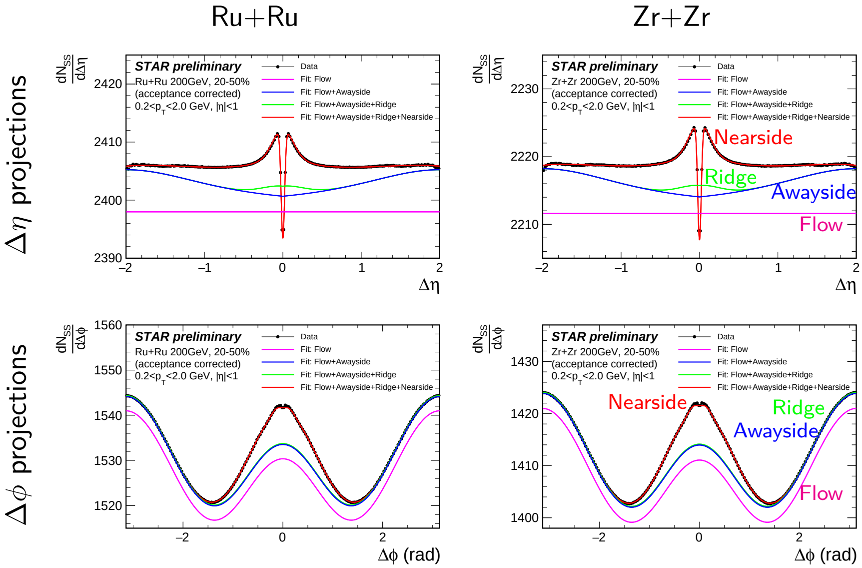
$$+ \frac{B}{2-|\Delta\eta|}\mathrm{erf}\left(\frac{2-|\Delta\eta|}{\sqrt{2}\sigma_{\Delta\eta,\rm AS}}\right)G_{\rm AS}(\Delta\phi\pm\pi) + DG_{\rm RG}(\Delta\eta) + C\left[1+2V_1\cos(\Delta\phi)+2V_2\cos(2\Delta\phi)+2V_3\cos(3\Delta\phi)\right]$$

Awayside Ridge Flow

$G_s(x)$ Gaussian function, $V_n = v_n^2$ assumed η-independent.

NS–nearside, AS–awayside, RG–ridge; W–wide, N–narrow, D–dip.

(a) Nonflow to v_2^*: measurement of ϵ_{nf}



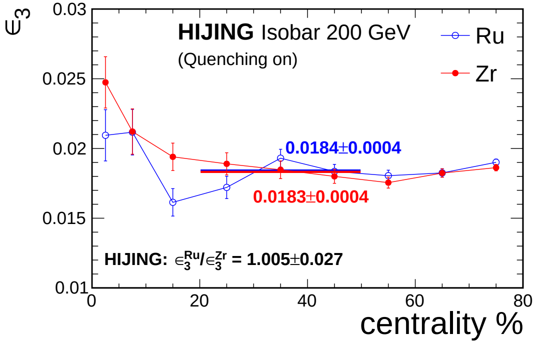
- Data: markers in black.
- Flow: flow component in fit.
- Flow+Awayside
- Flow+Awayside+Ridge: the $\Delta\phi$ ridge is a 1D Gaussian centering at $\Delta\eta = 0$, which is small and independent from $\Delta\phi$.
- Flow+Awayside+Ridge+Nearside: the total fit function, where the nearside includes 3 2D Gaussians centering at $\Delta\eta = 0$ $\Delta\phi = 0$, which are the wide, the narrow, and the dip.

STAR preliminary		Ru+Ru	Zr+Zr
SS	fit parameter C	381.651 ± 0.011	351.988 ± 0.009
	fit parameter $V_2 = v_2^2$	0.0029716 ± 0.0000029	0.0028668 ± 0.0000025
	$\langle\cos(2\Delta\phi)\rangle_{\mathrm{SS}}$ $(\lvert\Delta\eta\rvert > 0.05)$	0.0035968 ± 0.0000010	0.0034930 ± 0.0000010
inclusive	$\langle\cos(2\Delta\phi)\rangle = {v_2^*}^2$ $(\lvert\Delta\eta\rvert > 0.05)$	0.0037161 ± 0.0000007	0.0036088 ± 0.0000007
	nonflow $U = \langle\cos(2\Delta\phi)\rangle - V_2$	0.0007446 ± 0.0000030	0.0007420 ± 0.0000026
	$\epsilon_{\mathsf{nf}} = U/V_2$	$(25.06 \pm 0.10)\%$	$(25.88 \pm 0.09)\%$

- Nonflow in ${v_2^*}^2$ is $\sim 25\%/(1+25\%) = 20\%$.
- The nearside wide Gaussian (A_1 term) is dominant.
- We take half of it as systematics:
  $\Delta\epsilon_{\mathsf{nf}} = (-0.82 \pm 0.13 \mp 0.30)\%$, $-\Delta\epsilon_{\mathsf{nf}}/(1+\epsilon_{\mathsf{nf}}) = (0.65 \pm 0.11 \pm 0.22)\%$.
  $\Delta v_2^2/v_2^2 = \Delta V_2/V_2 = (3.7 \pm 0.1 \mp 0.3)\%$.

- ϵ_2 can be obtained from ZDC measurement (no nonflow, assuming negligible CME) [STAR, PRC **105**, 014901 (2022)]
  $\epsilon_2 = \frac{N\Delta\gamma\{\text{ZDC}\}}{v_2\{\text{ZDC}\}} \approx 0.57 \pm 0.04 \pm 0.02$ (tracking efficiency $\sim 80\%$)
- The $\Delta\epsilon_2$ precision from ZDC is too poor: $\Delta\epsilon_2/\epsilon_2 \approx (2.3 \pm 9.2)\%$, but we can estimate it as follows:
  - Assuming $C_{\text{2p}}^{\text{Ru}} = C_{\text{2p}}^{\text{Zr}}$, then $\epsilon_2 \propto Nr$, where the pair multiplicity difference $r \equiv \frac{N_{\text{OS}}-N_{\text{SS}}}{N_{\text{OS}}}$ is precisely measured [STAR, PRC **105**, 014901 (2022)]
    $\Delta\epsilon_2/\epsilon_2 = \Delta r/r + \Delta N/N = (-2.95 \pm 0.08)\% + 4.4\% = (1.45 \pm 0.08)\%$
  - For a point of reference, AMPT simulation w.r.t. RP gives $\Delta\epsilon_2/\epsilon_2 \approx (3.5 \pm 1.4)\%$

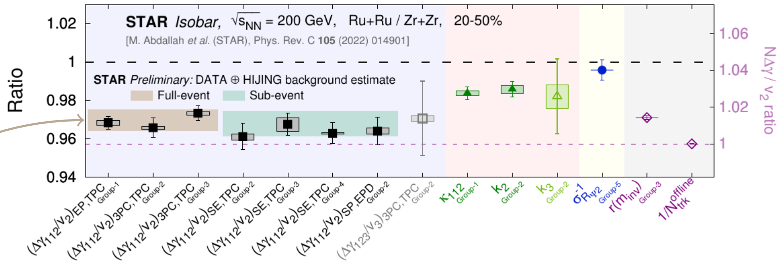
- 3p nonflow study in real data is difficult (work ongoing)
- We use HIJING simulation (which has no flow) to obtain $\epsilon_3 \approx (1.84 \pm 0.04)\%$, and $\Delta\epsilon_3/\epsilon_3 = (0.5 \pm 2.7)\%$ ($\sim 8.6 \times 10^8$ events for each isobar).
- HIJING without jet quenching gives $\epsilon_3 = (2.24 \pm 0.05)\%$, differing by 22%.
- We assign 50% systematic uncertainty for ϵ_3 ($\pm 0.92\%$), and assume $\Delta\epsilon_3/\epsilon_3$ is presently dominated by statistics.



Estimated Background Components for Isobar $N\Delta\gamma/v_2$ Ratio

Quantity		Method	Systematic uncertainty	Full-event value	Sub-event value
Multiplicity $\Delta N/N$	Measured		Negligible	4.4%	4.4%
Flow $\Delta v_2^2/v_2^2$	Measured	Nonflow subtracted as per below	From nonflow syst.	$\Delta v_2^2/v_2^2 = (3.7 \pm 0.1 \pm 0.3)\%$	$\Delta v_2^2/v_2^2 = (3.7 \pm 0.1 \pm 0.3)\%$
v_2 nonflow	Measured	$(\Delta\eta, \Delta\phi)$ correlations, experimentally measured	Nonflow$\sim 25\%$ (full event), dominated by NS wide Gaus; consider $\pm 1/2$ WG as syst. uncertainty	$-\Delta\epsilon_{\text{nf}} = (0.82 \pm 0.13 \pm 0.30)\%$ $\frac{-\Delta\epsilon_{\text{nf}}}{1+\epsilon_{\text{nf}}} = (0.65 \pm 0.11 \pm 0.22)\%$	$-\Delta\epsilon_{\text{nf}} = (0.59 \pm 0.15 \pm 0.27)\%$ $\frac{-\Delta\epsilon_{\text{nf}}}{1+\epsilon_{\text{nf}}} = (0.48 \pm 0.12 \pm 0.22)\%$
v_2-induced bkgd: $\epsilon_2 = N\Delta\gamma/v_2$	Measured	Measured by ZDC (assume negligible CME)	Small	$\epsilon_2 = (0.57 \pm 0.04 \pm 0.02)\%$	$\epsilon_2 = (0.79 \pm 0.05 \pm 0.01)\%$
v_2-induced bkgd difference: $\frac{\Delta\epsilon_2}{\epsilon_2} \sim \frac{\Delta(N_{\text{2p}}/N)}{(N_{\text{2p}}/N)} = \frac{\Delta(rN)}{rN}$	Measured	$r = (N_{\text{OS}} - N_{\text{SS}})/N_{\text{OS}}$ experimentally measured	Negligible	$\frac{\Delta\epsilon_2}{\epsilon_2} = (1.45 \pm 0.08)\%$	$\frac{\Delta\epsilon_2}{\epsilon_2} = (1.45 \pm 0.08)\%$
3p contribution to C_3: $\epsilon_3 = C_{3\text{p}}N_{3\text{p}}/(2N)$	Model estimate	HIJING simulations quenching-on	Quenching-on and off difference$\sim 20\%$. Take $\pm 50\%$ as syst. uncertainty	$\epsilon_3 = (1.84 \pm 0.04 \pm 0.92)\%$	$\epsilon_3 = (1.91 \pm 0.09 \pm 0.95)\%$
3p contribution difference: $\Delta\epsilon_3/\epsilon_3$	Model estimate	HIJING simulation quenching-on	Assumed negligible relative to the large stat. uncertainty	$\frac{\Delta\epsilon_3}{\epsilon_3} = (0.5 \pm 2.7)\%$ $\frac{\epsilon_3/\epsilon_2}{Nv_2^2} = 0.104 \pm 0.008 \pm 0.053$	$\frac{\Delta\epsilon_3}{\epsilon_3} = (-1.8 \pm 6.3)\%$ $\frac{\epsilon_3/\epsilon_2}{Nv_2^2} = 0.079 \pm 0.006 \pm 0.040$
background estimate				$1.013 \pm 0.003 \pm 0.005$	$1.011 \pm 0.005 \pm 0.005$

Estimated Background Level for Isobar $N\Delta\gamma/v_2$ Ratio



$$\frac{(N\Delta\gamma/v_2^*)^{\text{Ru}}}{(N\Delta\gamma/v_2^*)^{\text{Zr}}} \approx 1 + \frac{\Delta\epsilon_2}{\epsilon_2} - \frac{\Delta\epsilon_{\text{nf}}}{1+\epsilon_{\text{nf}}} + \frac{\epsilon_3/\epsilon_2/(Nv_2^2)}{1+\epsilon_3/\epsilon_2/(Nv_2^2)}\left(\frac{\Delta\epsilon_3}{\epsilon_3} - \frac{\Delta\epsilon_2}{\epsilon_2} - \frac{\Delta N}{N} - \frac{\Delta v_2^2}{v_2^2}\right)$$

$$= 1 + (1.45 \pm 0.08)\% + (0.65 \pm 0.11 \pm 0.22)\%$$

$$+ (0.094 \pm 0.007 \pm 0.048)\left[(0.5 \pm 2.7)\% - (1.45 \pm 0.08)\% - 4.4\% - (3.7 \pm 0.1 \pm 0.3)\%\right]$$

$$= 1 + (1.45 \pm 0.08)\% + (0.65 \pm 0.11 \pm 0.22)\% - (0.85 \pm 0.26 \pm 0.44)\%$$

$$= \mathbf{1.013 \pm 0.003 \pm 0.005} \quad \text{(full-event)}$$

Outline

The R variable and its baseline

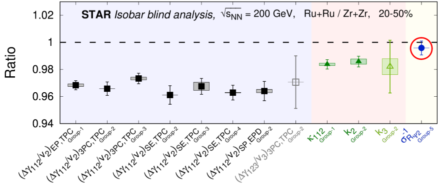
$\Delta S = \langle \sin(\phi^{+} - \Psi_2) \rangle - \langle \sin(\phi^{-} - \Psi_2) \rangle$

ΔS distribution $N_{\text{real}}(\Delta S) \xrightarrow{\text{charge shuffling}} N_{\text{shuffled}}(\Delta S)$

$C_{\Psi_2}(\Delta S) = N_{\text{real}}(\Delta S)/N_{\text{shuffled}}(\Delta S) \xrightarrow{\Psi_2 \to \Psi_2 + \pi/2} C^{\perp}_{\Psi_2}(\Delta S)$

$R_{\Psi_2}(\Delta S) = C_{\Psi_2}(\Delta S)/C^{\perp}_{\Psi_2}(\Delta S)$

$\Delta S \xrightarrow[\text{normalization}]{\text{shuffled width}} \Delta S' \xrightarrow[\text{correction}]{\text{resolution}} \Delta S''$; The final observable is $R_{\Psi_2}(\Delta S'')$.



[Choudhury et al., CPC 46, 014101 (2022)]:

$$\frac{S_{\text{concavity}}}{\sigma^2_{R2'}} = \frac{S_{\text{concavity}}}{\sigma^2_{R2}} \langle (\Delta S_{2,\text{shuffled}})^2 \rangle \approx -\frac{M}{2}(M-1)\Delta\gamma_{112} \times \frac{2}{M} \approx -M\Delta\gamma_{112}. \quad (23)$$

$\propto \frac{v_2}{M}$ $\propto v_2$

Normalized by shuffled width, so the N_{ch} is already scaled out.

That's why the isobar ratio of $1/\sigma^2_{R_{\Psi_2}}$ is closer to unity than $\frac{\Delta\gamma}{v_2}$.

STAR has concluded that $1/\sigma^2_{R_{\Psi_2}}$ is approximately proportional to v_2. After v_2 scaling, the isobar ratio is even further below unity.

[STAR, PRC **105**, 014901 (2022)]:

> The scaling relations extracted in Ref. [81] indicate an approximate relation between $1/\sigma^2_{R_{\Psi_2}}$, multiplicity N and $\Delta\gamma$, which would imply for this analysis $1/\sigma^2_{R_{\Psi_2}} \approx N\Delta\gamma$; an estimate based on the measurements from this analysis indicates this ratio for Ru + Ru over Zr + Zr to be approximately 1.02

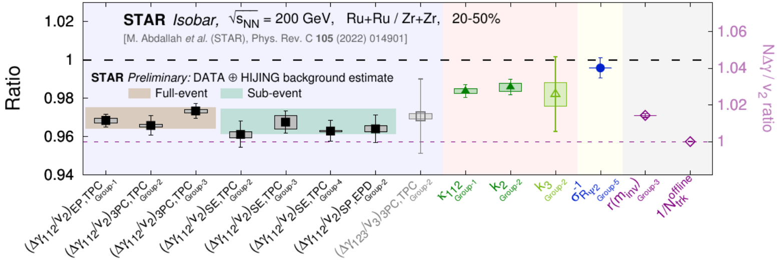
Recent AVFD simulations indicate a linear dependence between $1/\sigma^2_{R_{\Psi_2}}$ and $1/N_{\text{ch}}$. An additional $1/N_{\text{ch}}$ scaling is applied to $1/\sigma^2_{R_{\Psi_2}}$ making a CME claim [R. Lacey et al., arXiv:2203.10029]. This contradicts the STAR isobar conclusion.

$1/N_{\text{ch}}$ scaling is already incorporated in construction of R correlator. The R correlator explicitly depends on v_2, giving an apparent N_{ch} dependence. The $1/\sigma^2_{R_{\Psi_2}}$ should be scaled by v_2 [F. Wang, arXiv:2204.08450].

Outline

Summary

- v_2 nonflow and 2p nonflow in C_3 are measured. 3p nonflow in C_3 is estimated by HIJING. Large degree of cancellation between 2p and 3p nonflow.
- New preliminary isobar background estimate $\frac{(N\Delta\gamma/v_2^*)^{\mathrm{Ru}}}{(N\Delta\gamma/v_2^*)^{\mathrm{Zr}}} \approx (1.013 \pm 0.003 \pm 0.005)$ for full-event, $(1.011 \pm 0.005 \pm 0.005)$ for sub-event.



Backup

Table: fit results on slide 9

STAR preliminary	Ru+Ru	Zr+Zr
A_1	2.967 ± 0.009	2.801 ± 0.007
$\sigma_{\Delta\eta,\mathrm{NS},W}$	0.9878 ± 0.0030	0.9550 ± 0.0025
$\sigma_{\Delta\phi,\mathrm{NS},W}$	0.6329 ± 0.0009	0.6364 ± 0.0008
A_2	15.615 ± 0.011	14.515 ± 0.009
$\sigma_{\Delta\eta,\mathrm{NS},N}$	0.12668 ± 0.00008	0.12839 ± 0.00008
$\sigma_{\Delta\phi,\mathrm{NS},N}$	0.12889 ± 0.00006	0.12977 ± 0.00006
A_3	-72.522 ± 0.018	-66.943 ± 0.016
$\sigma_{\Delta\eta,\mathrm{NS},D}$	0.022288 ± 0.000006	0.022314 ± 0.000005
$\sigma_{\Delta\phi,\mathrm{NS},D}$	0.102971 ± 0.000029	0.102619 ± 0.000027
B	0.2140 ± 0.0037	0.1943 ± 0.0031
$\sigma_{\Delta\eta,\mathrm{AS}}$	0.591 ± 0.005	0.589 ± 0.005
$\sigma_{\Delta\phi,\mathrm{AS}}$	$1.1 \times 10^5 \pm 18.3 \times 10^5$	$1.4 \times 10^5 \pm 11.7 \times 10^5$
D	0.2759 ± 0.0032	0.2660 ± 0.0026
$\sigma_{\Delta\eta,\mathrm{RG}}$	0.2600 ± 0.0018	0.2524 ± 0.0015
C	381.651 ± 0.011	351.988 ± 0.009
V_1	-0.001916 ± 0.000006	-0.001943 ± 0.000005
V_2	0.0029716 ± 0.0000029	0.0028668 ± 0.0000025
V_3	0.0001766 ± 0.0000012	0.0001842 ± 0.0000011
χ^2/NDF	$1018458.1/159982 = 6.4$	$1136361.1/159982 = 7.1$

Fit function

$$\begin{aligned} & A_1 G_{\mathrm{NS},W}(\Delta\eta) G_{\mathrm{NS},W}(\Delta\phi) \\ & + A_2 G_{\mathrm{NS},N}(\Delta\eta) G_{\mathrm{NS},N}(\Delta\phi) \\ & + A_3 G_{\mathrm{NS},D}(\Delta\eta) G_{\mathrm{NS},D}(\Delta\phi) \\ & + \frac{B}{2-|\Delta\eta|} \mathrm{erf}\left(\frac{2-|\Delta\eta|}{\sqrt{2}\sigma_{\Delta\eta,\mathrm{AS}}}\right) \\ & \quad \times G_{\mathrm{AS}}(\Delta\phi \pm \pi) \\ & + D G_{\mathrm{RG}}(\Delta\eta) \\ & + C\big[1 + 2V_1 \cos(\Delta\phi) \\ & \quad + 2V_2 \cos(2\Delta\phi) + 2V_3 \cos(3\Delta\phi)\big] \end{aligned}$$

Large $\sigma_{\Delta\phi,\mathrm{AS}}$ turns G_{AS} into a flat line.

Awayside $\Delta\eta$ correlation

Suppose two particles (1, 2) correlated in η by momentum conservation or other nonflow effect. We let

$$\begin{cases} \Delta\eta = \eta_1 - \eta_2 \\ \delta = \eta_1 + \eta_2 \end{cases} \Rightarrow \begin{cases} \eta_1 = \dfrac{\delta + \Delta\eta}{2} \\ \eta_2 = \dfrac{\delta - \Delta\eta}{2} \end{cases} \quad (1)$$

where $\eta_1 = -\eta_2 + \delta$. For momentum conservation, the two particles tend to be back-to-back in η direction ($\eta_1 \sim -\eta_2$). δ serves as fluctuations, and the correlation could be a function of δ.

Since $|\eta_1| < 1$ and $|\eta_2| < 1$, the range of δ is

$$\begin{cases} \left|\dfrac{\delta + \Delta\eta}{2}\right| < 1 \\ \left|\dfrac{\delta - \Delta\eta}{2}\right| < 1 \end{cases} \Rightarrow |\delta| < 2 - |\Delta\eta| \quad (2)$$

Suppose the correlation function between two particles is $f(\eta_1, \eta_2) = g(\Delta\eta, \delta)$.

$$\begin{aligned} f(\eta_1, \eta_2)\mathrm{d}\eta_1\mathrm{d}\eta_2 &= g(\Delta\eta, \delta)\frac{\partial(\eta_1, \eta_2)}{\partial(\Delta\eta, \delta)}\mathrm{d}\Delta\eta\mathrm{d}\delta \\ &= g(\Delta\eta, \delta)\begin{vmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{vmatrix}\mathrm{d}\Delta\eta\mathrm{d}\delta = \frac{1}{2}g(\Delta\eta, \delta)\mathrm{d}\Delta\eta\mathrm{d}\delta \end{aligned} \quad (3)$$

Integral over δ to get the marginal distribution of $\Delta\eta$

$$h(\Delta\eta) = \int_{-2+|\Delta\eta|}^{2-|\Delta\eta|} \frac{1}{2}g(\Delta\eta, \delta)\mathrm{d}\delta \quad (4)$$

If there is no correlation, then $g(\Delta\eta, \delta) = f(\eta_1, \eta_2) = f(\eta_1)f(\eta_2) = \frac{1}{4}$, and the integral becomes $h(\Delta\eta) = \frac{1}{4}(2 - |\Delta\eta|)$, the acceptance triangle.

Awayside $\Delta\eta$ correlation

An intutitive assumption of the correlation from momentum conservation is δ obeys a Gaussian distribution centering at 0, which is $\delta \sim \mathcal{N}(0, \sigma)$.

$$g(\Delta\eta, \delta) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) \quad (5)$$

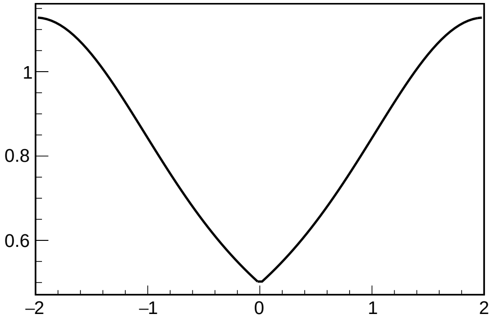
(may differ by a constant factor). And the marginal distribution becomes

$$h(\Delta\eta) = \frac{1}{2}\text{erf}\left(\frac{2 - |\Delta\eta|}{\sqrt{2}\sigma}\right) \quad (6)$$

After the acceptance correction, the function form should be

$$\frac{1}{2 - |\Delta\eta|}\text{erf}\left(\frac{2 - |\Delta\eta|}{\sqrt{2}\sigma}\right) \quad (7)$$

If we set $\sigma = 1$, then the function looks like below, which seems similar to the STAR data shape at awayside (large $|\Delta\phi|$).



Outlook

- ϵ_3 estimate in a data-driven way in future?
- Background estimates for each centrality bin separately.
- Improve the $(\Delta\eta, \Delta\phi)$ 2D fittings.