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Application of profile likelihood method for extraction of $A_L(\eta)$ from multiple datasets

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Abstract

The **profile likelihood method** (PLM) allows for simultaneous extraction of pairs of physics observables $A_L(+\eta), A_L(-\eta)$ from quads of yields, N_s, depending on beams polarization directions 's' and the STAR detector η-slice. We used Poisson statistics and multiplied the global likelihoods to combine the data from multiple years. For the most complex case we used 16 yields to extract 2 parameters of interest while marginalizing over 9 nuisance parameters. The RooStats package from ROOT/CERN has been extended to account for the constrain on the support of the likelihood function while computing the confidence interval. The ready to use code examples are included.
This STAR note does NOT contain any physics results for W-boson A_L.

Keywords
spin asymmetry, profile likelihood method, Poisson statistics

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Contents

Introduction

This work has been motivated by the desire to combine the 2011 and 2102 STAR data sets to extract the W-boson spin asymmetry $A_L(\eta)$. Due to low statistics in the 2011 data set we could not justify use of the Gaussian error propagation. Instead we used multi-dimensional likelihood, constructed from many Poisson distributions.

The essence of problem we want to solve is finding the most probable value (PMV) and the confidence interval (CI) of the unknown parameter A given a pair of measured yields N_+, N_- obeying Poisson distribution, and knowing that the true relation between those quantities are

$$N_\pm = N^0(\ 1\ \pm\ A \cdot P) \tag{1}$$

where $N^0 > 0$ is a free parameter of no interest to us and P is a known constant , $|P| < 1$.

At the large statistics limit the p.d.f. of measured yields $N_\pm$ are well approximated by the Gauss distribution. In such case can solve eq. 1 for A

$$A = \frac{1}{P}\frac{N_+ - N_-}{N_+ + N_-} \tag{2}$$

and propagate the statistical errors of $N_\pm$ in to 1 standard deviation of A, $\sigma(A)$

$$\sigma(A) = \frac{2}{P}\sqrt{\frac{N_+N_-}{(N_+ + N_-)^3}} \tag{3}$$

In such case the CI $[A-\sigma, A+\sigma]$ corresponds to CL=0.678.

Let further assume we repeated the experiment K-times, measured K pairs of yields $N_{\pm,k}$, however each time the constant P_k was different. We want again extract a single, common value of A based on the combined data from this K experiments.

At the large statistics limit we can compute A_k, σ_k for each dataset k, then compute weighted average

$$\hat{A} = \sum_{k=1}^{K} A_k w_k, \tag{4}$$

$$w_k = \sigma_k^{-2} / \sum_{k=1}^{K} \sigma_k^{-2} \tag{5}$$

The procedure described above fails at the low statistic limit, say for $N_\pm$ of few, when Poisson p.d.f. is not symmetric around the central value any more. Consequently, the 1-σ CI for A_k is not centered around MPV. For given CL we have CI $[A_k^{lo}, A_k^{hi}]$ such that $A_k - A_k^{lo} \neq A_k^{hi} - A_k$. This means eq. 5 can't be applied to compute the relative weights needed in eq. 4.

The alternative approach to find MPV of A from many, low statistics experiments is to apply the likelihood method (LM). Knowing the p.d.f. for each measurement,$N_{\pm,k}$, obeys the Poisson distribution, we construct the likelihood of measuring each pair $N_{\pm,k}$ given A assuming physics justified model. Next, multiply the likelihoods from all experiments. Finally, we find global maximum and CI of A by marginalization of nuisance parameters.

Section 1 describes application of the profile likelihood method (PLM) for such simple 2-yield experiment. Section 2 will expand PLM for more realistic case of a series 8-yield measurements and simultaneous extraction of multiple parameters of interest.

1. PLM for 2 spin states and one-observable

Lets start with a very simple case of the **profile likelihood method** applied to extract the single-spin asymmetry (SSA) from an experiment using a polarized beam hitting an unpolarized target and involving one detector.

1.1 Model

Let $N_\pm$ be the measured yields of W-boson events in our experiment for 2 opposite polarizations of the beam ($\pm$). Let $\mu_\pm$ be the expected values of the yields from our model of the experiment, discussed below.

The W-boson reconstruction algorithm accepts a small fraction of non-W events (*i.e.* background) - this impacts the value of measured SSA and it needs to be corrected for. We have identified 3 dominant background sources indexed by the subscript i=Z,E,Q, and W:
'Z' labels $Z \to e^+e^-$ events accepted if one of leptons misses the BEMC or EEMC,
'E' labels QCD $\to$ jet-jet events for which one jet heads toward non-existent East EMC endcap,
'Q' labels other subset of QCD events which sometimes hadronize in such a way that they pass the W reconstruction algorithm. For completeness, 'W' labels W-boson events of interest.

Let f_i denote fraction of reconstructed event yield, n_i, of a given type :

$$f_i = \frac{n_i}{\sum n_i} \quad ; \quad \sum_i f_i = 1 \tag{6}$$

In general, the SSA for each background process, A_i, may be different and non-zero which leads to the following **model of spin dependent yields**, $\mu_\pm$, for all events accepted by the W algorithm:

$$N_\pm \quad \to \quad \mu_\pm = \mu_{W\pm} + \mu_{Z\pm} + \mu_{E\pm} + \mu_{Q\pm} \tag{7}$$

$$\begin{aligned} N_{W\pm} &\to \mu_{W\pm} = l_\pm N_0 f_W (1 \pm A^W P) \\ N_{Z\pm} &\to \mu_{Z\pm} = l_\pm N_0 f_Z (1 \pm A^Z P) \\ N_{E\pm} &\to \mu_{E\pm} = l_\pm N_0 f_E (1 \pm A^E P) \\ N_{Q\pm} &\to \mu_{Q\pm} = l_\pm N_0 f_Q (1 \pm A^Q P) \end{aligned} \tag{8}$$

Eq. 7 implies the full model for the spin dependent yields, $\mu_\pm$, is the sum of all possible processes

$$\begin{aligned} \mu_\pm &= l_\pm N^0 [f_W + f_Z + f_E + f_Q \pm \\ &\quad P(f_W A^W + f_Z A^Z + f_E A^E + f_Q A^Q) \quad (9) \\ &= l_\pm N^0 \left[1 \pm P(\beta A^W + \alpha)\right] \quad (10) \end{aligned}$$

where

$$\beta = \frac{f_W}{f_W + f_Z + f_E + f_Q} \tag{11}$$

$$\alpha = \frac{f_Z A^Z + f_E A^E + f_Q A^Q}{f_W + f_Z + f_E + f_Q} \tag{12}$$

In practice the α-term is much smaller than the statistical uncertainty of the experiment so we will ignore it in this paper. The final model of spin dependent yields is

$$\mu_\pm = l_\pm N^0 (1 \pm P\beta A^W) \tag{13}$$

The beam polarization, P, is a **constant**. The relative luminosities $l_\pm$ are assumed also to be **constants**. $l_\pm$ depend on (very large) number of events recorded by the luminosity monitor for both spin states $N_{LUM\pm}$.

$$l_\pm = \frac{2N_{LUM\pm}}{N_{LUM+} + N_{LUM-}}; \quad \frac{1}{2}(l_+ + l_-) = 1 \tag{14}$$

$$\sigma_{l_\pm} = 1/\sqrt{N_{LUM+} + N_{LUM-}} \quad \text{are small} \tag{15}$$

1.2 Total likelihood function L_Ω

The total likelihood $L_\Omega(A^W, N^0, \beta)$ is constructed as the joint probability using all information we gather from various sources:

$$L_\Omega(A^W, N^0, \beta) \equiv L_{PHY}(A^W) \cdot L_{SPIN}(A^W, N^0, \beta) \cdot L_{BCK}(\beta) \tag{16}$$

where:

- A^W is the SSA we want to extract from the experiment (*i.e.* variable of interest),

- $L_{PHY}(A^W) = H(1-|A^W|)$: restricts the range of the physically allowed values of SSA, where $H(x)$ is the step function,
- N_0 and β are nuisance parameters describing the unpolarized expected yield and unpolarized background, respectively,
- $L_{SPIN}(A^W, N_0, \beta) = \prod_i^2 f(N_i|\mu_i(A^W, N^0, \beta)$ is product of Poisson's functions $f(N|\mu)$, describing probability of measured yields N given the expected value was μ from eq. 13,
- $L_{BCK}(\beta) = g(\beta - \hat{\beta}, \sigma_\beta)$ is the probability distribution function for the unpolarized background magnitude, here parametrized as a gaussian with the mean $\hat{\beta}$ and standard deviation σ_β.

The following additional parameters: $l_\pm, P$, needed to compute the numerical values of μ (see eq. 13), are assumed to be constant.
Note, for the practical reasons the ranges of all nuisance parameters are bracketed to $\pm 10\sigma$ around the respective central values.

1.3 Extracting parameter of interest from L_Ω

The following inputs are required to extract the asymmetry of interest A^W

- $N_\pm$: spin sorted yields from STAR experiment,
- $l_\pm$: relative luminosities from STAR experiment,
- $\hat{\beta}, \sigma_\beta$: describe background p.d.f., based on embedding, simulations, and theory,
- P : beam polarization from RHIC.

With these inputs we can build the 3-dimensional total likelihood function $L_\Omega(A^W, N^0, \beta)$. To find the central value of the A^W we need to remove the nuisance parameters (N^0 and β) from the problem. One method to accomplish this is to marginalize L_Ω (or integrate over) the nuisance parameters to produce the 1-dimensional likelihood vs. the variable of interest, A^W:

$$L_{\text{marg}}(A^W) = \int dN^0 \int d\beta \; L_\Omega(A^W, N^0, \beta). \tag{17}$$

Another method to treat the nuisance parameters, described in the PDG statistics review [1] (specifically Sec. 36.3.2.3) and a longer review from Cowan [2], is the profile likelihood method which we will use and should yield the same central value and confidence intervals as the marginalization method.

The profile likelihood method consists of two steps: (i) construction of the profile likelihood $L_{\text{prof}}(A^W)$ and (ii) extraction of central value and confidence interval for A^W.

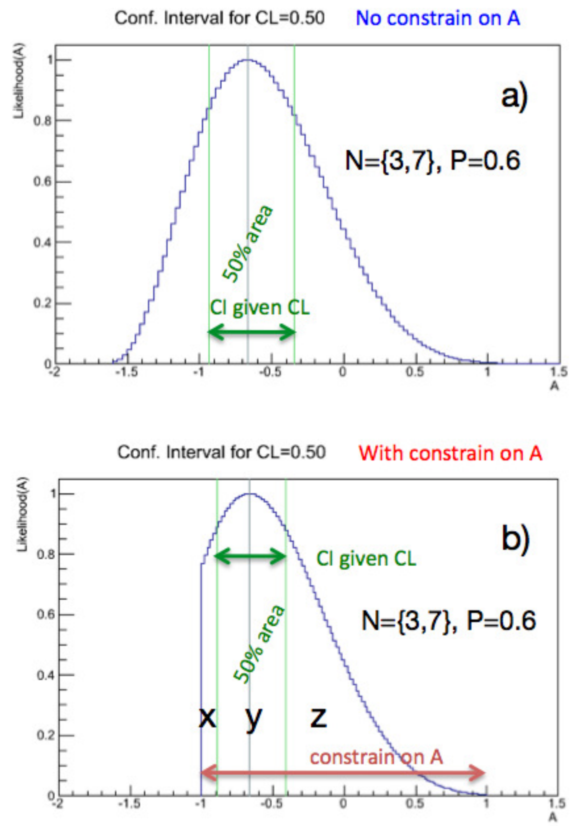

Figure 1. Definition of confidence interval for the case w/o constrain on support (a) and with additional constrain (b).

1.3.1 Profile likelihood

Let's group all nuisance parameters as a vector $\nu \equiv (N^0, \beta)$. For each value of A^W, there exist $\hat{\nu}(A^W)$ which maximizes $L_\Omega(A^W, \hat{\nu})$ defined by eq. 16. The **profile likelihood** $L_{\text{prof}}(A^W)$ is defined as

$$L_{\text{prof}}(A^W) = L_\Omega(A^W, \hat{\nu}) \tag{18}$$

It is a 1-dimensional likelihood, depending only on A^W. Often one conveniently normalizes the profile likelihood by constructing the **profile likelihood ratio** defined as

$$\lambda_{\text{prof}}(A^W) \equiv \frac{L_{\text{prof}}(A^W)}{L_{\Omega 0}}, \tag{19}$$

where $L_{\Omega 0}$ is the global maximum of the 3D likelihood in the $[A^W, N^0, \beta]$ parameter space.

The central value of A^W is the one which maximizes the profile likelihood $L_{\text{prof}}(A^W)$, or equivalently minimizes the negative log-likelihood, $-\ln L_{\text{prof}}$.

1.3.2 Confidence interval

For a given confidence level (CL) the confidence interval of A^W is computed as the pair $[A^W_{lo}, A^W_{hi}]$ satisfying the integral

$$\int_{A^W_{lo}}^{A^W_{hi}} L_{\text{prof}}(A)\, dA = CL \cdot \int_{support} L_{\text{prof}}(A)\, dA \tag{20}$$

In the absence of constrain on A and for a non-symmetric p.d.f., as show in fig. 1a), we need to impose additionally

$$L_{\text{prof}}(A^W_{lo}) = L_{\text{prof}}(A^W_{hi}) \tag{21}$$

for unambiguous definition of CI.

Presence of the constrain on the support $L_{PHY}(A^W)$, see fig. 1b), complicates this picture slightly. For clarity, assume that the lower bound is closer to the maximum (as in the figure). In general 2 CI values divide the whole area on 3 parts, labeled x,y,z.

$$\int_x + \int_y + \int_z = \int_{-1}^{+1} L_{\text{prof}}(A)\, dA \tag{22}$$

In particular, $\int_x$ may be zero if chosen CL is too large. In such case we set $A^W_{lo} = -1$ (*i.e.* the lower boundary of the constrain) and A^W_{hi} is defined by the modified relation

$$\int_{-1}^{A^W_{hi}} L_{\text{prof}}(A)\, dA = CL \cdot \int_{-1}^{+1} L_{\text{prof}}(A)\, dA \tag{23}$$

1.4 Numerical example

For numerical computation we will use the RooStats [3] (extension of CERN root). For educational purposes we have prepared several ready to use macros placed in the MIT disc at BNL [4].

The code used to produce fig. 1 is named `spin2Asy_constrain.C` . Since the original RooStast did not handled properly the constrain on support, we developed our own after-burner macro `getIntervGivenConstrSimple.C`, used in all sections of this paper.

2. PLM for 4 spin states and two-observables

In section 1 we have applied the profile likelihood method to a simplified case of extraction of one parameter of interest (A^W) out of a pair of measurements (N_+, N_-), using a model (eq. 13) with additional 2 nuisance params (N^0, β) and 3 fixed params ($l_\pm, P$). The complexity of real-life problem discussed in this section, extraction of $A_L(\eta)$ for W-boson measured at STAR, is much higher.

2.1 Model

In the following we will retain the naming convection of physical quantities, however we will add additional indexing. The indexes s, η, k denote the following:
s spin state of colliding beams, $s = [++, +-, -+, --]$,
η pseudo-rapidity of 2 detectors, $\eta = [\eta_1, \eta_2]$,
k labels the datasets, k=[1,2,..,K].

Let review previously defined quantities with extended indexing:

$N_{s\eta k}$ are yields **measured** by STAR for spin state (s), detector (η), dataset (k),

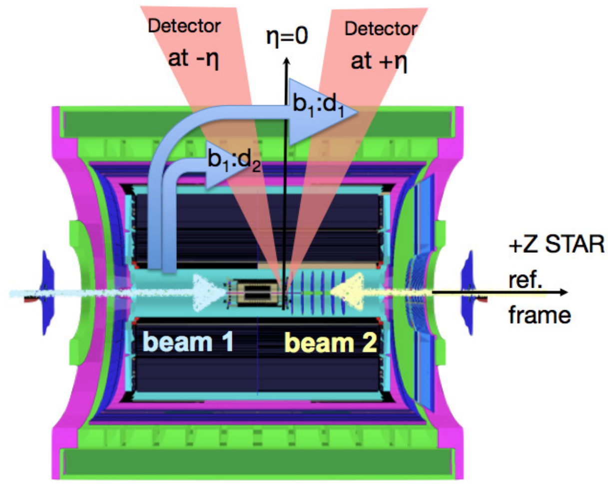

Figure 2. Definition directions of the beams and signs of the angles of the detector with respect to polarized beam needed to define dependence of polarized yields on SSA & DSA in eqs. 25.

$\mu_{s\eta k}$ are yields **predicted** by the model defined below.

For clarity lets ignore for the moment the dataset index k. The generic formula for model $\mu_{s\eta}$ depends on similar parameters to eq. 13:

$$\mu_{s\eta} = l_s N^0_\eta \left[\, 1 \oplus_s P_1 \beta_\eta A^W_{\eta'} \oplus_s P_2 \beta_\eta A^W_{\eta''} \oplus_s P_1 P_2 \beta_\eta A^{LL} \right] \tag{24}$$

where $\oplus_s$ means the sign switch depending on the spin state 's'.The index of A^W depend on the angle between polarized beam and detector η-bin. The definition of parameters used in eq. 24 is below :

L_s are relative luminosity corrections, normalization: $\sum_s l_s = 4$, do not depend on the detector,

N^0_η are predicted spin-average yields, nuisance params, change with the detector

P_1, P_2 are beam polarization magnitudes,

$A^W_{\eta'}, A^W_{\eta''}$ are SSAs for a pair of symmetric η-bins with respect to the polarized beam; those 2 are parameters of interest,

β_η are unpolarized corrections to A^W_η, depend on detector angle, nuisance parameter,

A^{LL} is DSA nuisance param, it has no $\pm\eta$ detector dependence.

2.1.1 Model for 8 yields

In total there are 8 different possibilities for the index $s\eta$ defined in the full model (eq. 24). Below we will write them

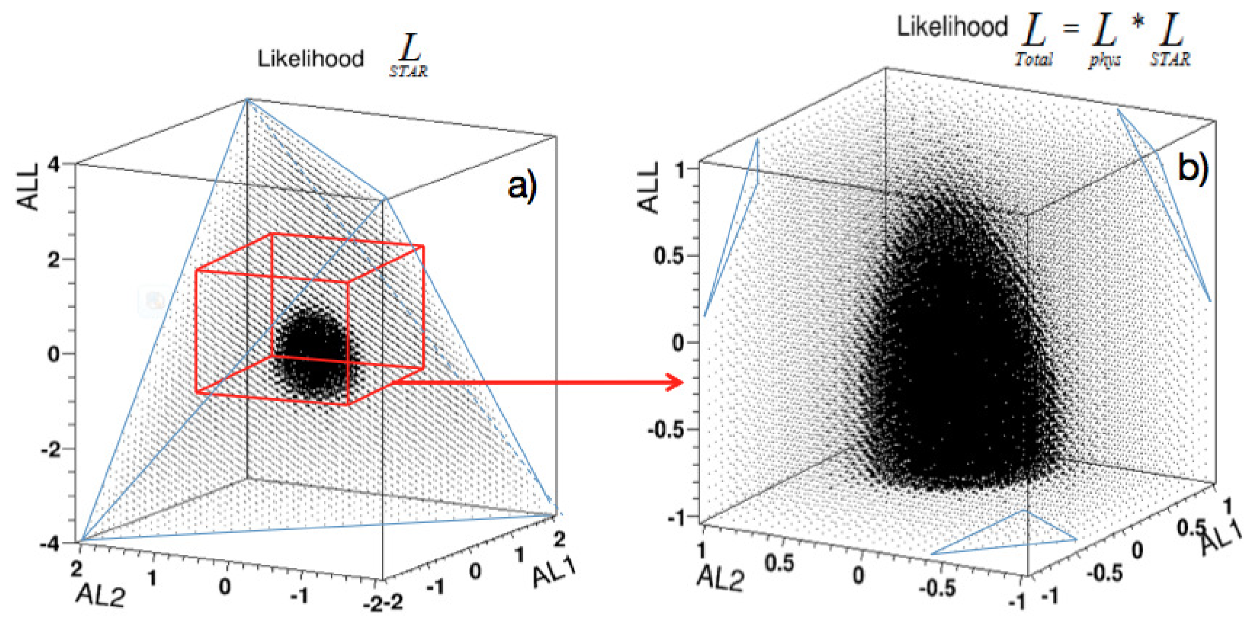

Figure 3. Illustration of applying the physics constrain, L_{PHY}, on the support of the total likelihood for eq. 16 . a) no constrains, b) after constrains are applied the support has been reduced.

explicitly

$$
\begin{aligned}
\mu_{++,1} &= l_{++}N_1^0[\,1\ +P_1\beta_1A_1^W\ +P_2\beta_1A_2^W\ +P_1P_2\beta_1A^{LL}\,] \quad (25)\\
\mu_{+-,1} &= l_{+-}N_1^0[\,1\ +P_1\beta_1A_1^W\ -P_2\beta_1A_2^W\ -P_1P_2\beta_1A^{LL}\,]\\
\mu_{-+,1} &= l_{++}N_1^0[\,1\ -P_1\beta_1A_1^W\ +P_2\beta_1A_2^W\ -P_1P_2\beta_1A^{LL}\,]\\
\mu_{--,1} &= l_{+-}N_1^0[\,1\ -P_1\beta_1A_1^W\ -P_2\beta_1A_2^W\ +P_1P_2\beta_1A^{LL}\,]\\
\mu_{++,2} &= l_{++}N_1^0[\,1\ +P_1\beta_2A_2^W\ +P_2\beta_2A_1^W\ +P_1P_2\beta_2A^{LL}\,]\\
\mu_{+-,2} &= l_{+-}N_1^0[\,1\ +P_1\beta_2A_2^W\ -P_2\beta_2A_1^W\ -P_1P_2\beta_2A^{LL}\,]\\
\mu_{-+,2} &= l_{++}N_1^0[\,1\ -P_1\beta_2A_2^W\ +P_2\beta_2A_1^W\ -P_1P_2\beta_2A^{LL}\,]\\
\mu_{--,2} &= l_{+-}N_1^0[\,1\ -P_1\beta_2A_2^W\ -P_2\beta_2A_1^W\ +P_1P_2\beta_2A^{LL}\,]
\end{aligned}
$$

From the mathematical perspective, the model of $\mu_{s\eta}$ defined by eqs. 25 does not need any justification. However, if you are a curious physicist fig. 2, defining directions of both beams and signs of the angles of the detector with respect to polarized beam, may be helpful.

2.1.2 Model for 4 yields

In certain cases we only have a single detector η. Then, we need only 4 equations for the model:

$$\mu_s = l_sN^0\left[\,1\ \oplus_s\ P_1\beta A_{\eta'}^W\ \oplus_s\ P_2\beta A_{\eta''}^W\ \oplus_s\ P_1P_2\beta A^{LL}\right] \quad (26)$$

and,

$$
\begin{aligned}
\mu_{++} &= l_{++}N^0[\,1\ +P_1\beta A_1^W\ +P_2\beta A_2^W\ +P_1P_2\beta A^{LL}\,] \quad (27)\\
\mu_{+-} &= l_{+-}N^0[\,1\ +P_1\beta A_1^W\ -P_2\beta A_2^W\ -P_1P_2\beta A^{LL}\,]\\
\mu_{-+} &= l_{++}N^0[\,1\ -P_1\beta A_1^W\ +P_2\beta A_2^W\ -P_1P_2\beta A^{LL}\,]\\
\mu_{--} &= l_{+-}N^0[\,1\ -P_1\beta A_1^W\ -P_2\beta A_2^W\ +P_1P_2\beta A^{LL}\,]
\end{aligned}
$$

2.2 Total likelihood

The total likelihood for one dataset consisting of 8 measured yields $N_{s\eta}$ is a product of all p.d.f.s, in analogy to eq. 16,

$$L_{\Omega 8}(A_1^W, A_2^W, \nu_8) = \prod_{s,\eta}^{8} f(N_{s\eta}|\mu_{s\eta}) \prod_{\eta}^{2} g(\beta_\eta) \prod_{\eta'}^{2} H(1-|A_{\eta'}^W|)\ H(1-|A_{LL}|) \quad (28)$$

where ν_8 represent 5 nuisance parameters $\nu_8 = [N_\eta^0, \beta_\eta, A^{LL}]$. The functions $f(...), g(...), H(...)$ were previously defined in the section 1.2. Fig. 3 illustrates the impact of the constrains, H(x), on the allowed parameter space of the total likelihood function.

Finally, lets allow for multiple datasets and restore the index 'k'. For 2 datasets, k=1,2, we measure total of 16 yields $N_{s\eta k}$ and need to almost double the number of nuisance parameters for the total likelihood. This is the final formula:

$$L_{\Omega 16}(A_1^W, A_2^W, \nu_{16}) = \prod_{s,\eta,k}^{16} f(N_{s\eta k}|\mu_{s\eta k}) \prod_{\eta,k}^{4} g(\beta_{\eta k}) \prod_{\eta'}^{2} H(1-|A_{\eta'}^W|)\ H(1-|A_{LL}|) \quad (29)$$

where ν_{16} represent 9 nuisance parameters $[N_{\eta k}^0, \beta_{\eta k}, A^{LL}]$.

Similarly, for the two dataset consisting of only 4 measured yields N_s for one pseudo-rapidity of the detector the total likelihood is:

$$L_{\Omega 8}(A_1^W, A_2^W, \nu_8) = \prod_{s,k}^{8} f(N_{sk}|\mu_{sk}) \prod_{k}^{2} g(\beta_k) \prod_{k}^{2} H(1-|A_{\eta'}^W|)\ H(1-|A_{LL}|) \quad (30)$$

2.3 RooStats implementation

In section 1.4 RooStats [3] was mentioned. It is a powerful tool to deal with the profile likelihood of multiple parameters. If we define all the models and likelihood PDFs, RooStats will do all the other works. To call RooStats package, 5.28.00 or higher version of ROOT is required. An example of solution of problem defined by eqs. 29 and eq. 30 are available at [4].

2.3.1 RooWorkspace

The RooWorkspace is a persistant container for RooFit projects. A workspace can contain and own variables, p.d.f.s, functions and datasets. All objects that live in the workspace are owned by the workspace. The import() method enforces consistency of objects upon insertion into the workspace (e.g. no duplicate object with the same name are allowed) and makes sure all objects in the workspace are connected to each other.
The code `creatPDF.C` is an example about how to define a RooWorkspace which contain all the variables, p.d.f.s and datasets will be used in our analysis.

```
> root -l creatPDF.C
```

run this code will create the RooWorkspace and print it out.

2.3.2 ProfileLikelihoodCalculator

In individual calculation, all the relative variables should be initialized with the experiment parameters. The function RooStats::ProfilelikelihoodCalculator will do the main computation of profile likelihood.

```
ProfileLikelihoodCalculator plC(*dataY,
*modelConfig);
```

where the dataY is RooDataSet which contains all the observables (namely spin sorted yields in W A_L analysis), and modelCofig is RooStats::ModelConfig which contain the full likelihood function (eg. eq. 29 and eq. 30) and the definition of parameters of interest(A^W) and the model (μ).

2.3.3 Confidence Interval

We can't use directly the output ProfilelikelihoodCalculator since it does not account correctly for the reduced support due to constraints. We call our "after-burner" code, discussed in section 1.3.2. Taking out the profile likelihood ratio from the ProfileLikelihoodCalculator, we can get the central value and the confidence interval with given confidence level. The code `getIntervalGivenConstrain.C` is an example to get the result from a profile likelihood ratio comes from ProfilelikelihoodCalculator.

2.4 Numerical results

To allow cross check by the reader we will report few results for synthetic data.

2.4.1 A^W for 2x8-yields

Lets assume the following 16 yields, $N_{s\eta d}$, for the pair of 2 detectors, for 2 years:
$N_{++,1,1}$=18, $N_{+-,1,1}$=24, $N_{-+,1,1}$=27, $N_{--,1,1}$=21
$N_{++,2,1}$=25, $N_{+-,2,1}$=14, $N_{-+,2,1}$=33, $N_{--,2,1}$=43
$N_{++,1,2}$=87, $N_{+-,1,2}$=184, $N_{-+,1,2}$=161, $N_{--,1,2}$=226
$N_{++,2,2}$=104, $N_{+-,2,2}$=186, $N_{-+,2,2}$=182, $N_{--,2,2}$=269
Other parameters are:
$L_{s,1}$= { 1.0180, 0.9891, 0.9926, 1.0002 }
$L_{s,2}$= { 0.9950, 1.0077, 0.9933, 1.0040 }
$P_{1,1}$=0.49, $P_{2,1}$=0.49; $P_{1,2}$=0.55, $P_{2,2}$=0.57
$\beta_{\eta,1}$ = { 0.976, 0.971 }; $\beta_{\eta,2}$ = { 0.967,0.962 }

A^W	Profile likelihood, CL=%68.27				Gauss method*	
	mpv A^W	A^W_{lo}	A^W_{hi}	δA^W	A^W	σA^W
A^W_1	-0.345	-0.391	-0.299	0.046	-0.345	0.046
A^W_2	-0.424	-0.470	-0.378	0.046	-0.425	0.045

Table 1. A^W_1,A^W_2 from RooStats and comparison with the Gxaussian method for 16-yields, input discussed in section. 2.4.1.
*Gauss method: calculate A^W for each detector eta bin of each year dataset and then average them with error wight.

Implementation of the RooStats based code for the likelihood function of 16 yields (eq. 29) was applied on this numerical example. We got the most probable values and confidence intervals with %68 confidence level of A^W_1 and A^W_2 simultaneously.
A^W_1 = -0.345, with confidence interval [-0.391,-0.299]
A^W_2 = -0.424, with confidence interval [-0.469,-0.378]

Table 1 lists results of A^W_1 A^W_2 and comparison with Gaussian method. Results from both methods are consistent . To reproduce execute:

```
> root -l rdAprofF.C'(0,2,"AL")'
```

2.4.2 A^W for 2x4-yields

To test the case of one detector we assumed:
$N_{++,1}$=3, $N_{+-,1}$=2, $N_{-+,1}$=1, $N_{--,1}$=3
$N_{++,2}$=16, $N_{+-,2}$=15, $N_{-+,2}$=16, $N_{--,2}$=10

Other parameters as in section 2.4.1, except
$\beta_1 = 0.991$, $\beta_2 = 0.962$.

For CL of %68we got:
A^W_1 = 0.167, with confidence interval [-0.065,0.397]
A^W_2 = 0.181, with confidence interval [-0.045,0.403]

A^W	Profile likelihood , CL=%68.27				gauss method	
	mpv A^W	A^W_{lo}	A^W_{hi}	δA^W	A^W	σA^W
A^W_1	0.167	-0.065	0.399	0.232	0.169	0.234
A^W_2	0.181	-0.045	0.403	0.224	0.179	0.224

Table 2. A^W_1,A^W_2 from RooStats and comparison with the Gaussian method for 8-yields, input discussed in section. 2.4.2.

Table 2 lists results of A_1^W A_2^W and comparison with Gaussian method. As expected, result also looks reasonable. Due to the low statistics, the difference between two method is more significant. To reproduce execute:

```
> root -l rdAprofF.C'(1,7,"AL")'
```

2.5 A^{LL} extraction

Previous subsections are focused on A^W, the single spin asymmetry. For A^{LL}, the double spin asymmetry, the case is similar. By setting A^Ws as nuisance parameters and setting A^{LL} as the parameter of interest, we can extract the most probable value and confidence interval of A^{LL}.

We used the sample yields used in section 2.4.1 and 2.4.2 to get the numerical results of A^{LL}. We got the most probable value and confidence interval with 68% confidence level for 2x8-yields and 2x4-yields respectively.

```
> root -l rdAprofF.C'(0,2,"ALL")'
```

A^{LL} = -0.050, with confidence interval [-0.134, 0.035].

```
> root -l rdAprofF.C'(1,7,"ALL")'
```

A^{LL} = -0.155, with confidence interval [-0.556, 0.251]

2.6 Conclusion

Based on the knowledge from previous section, this method can be applied to extract W A_L from STAR data from multiple dataset. The Barrel part of STAR detector can be divided into pairs of symmetric pseudo-rapidity bins and eq 29 is aplicable. Since there is only one endcap at STAR the eq. 30 should be applied in this case. Applying the model and likelihood described in section 2.1 and 2.2, we can get a W AL dependence vs. pseudo-rapidity in full η range of STAR detector.

References

[1] G.Cowan, ”PDG Statistics Review”, http://pdg.lbl.gov/2013/reviews/rpp2012-rev-statistics.pdf

[2] G.Cowan, ”Statistics for Searches at the LHC”, http://arxiv.org/abs/1307.2487v1

[3] Statistical tool developed for LHC https://twiki.cern.ch/twiki/bin/view/RooStats

[4] Directory at MIT disc at BNL containing all example macros is /star/institutions/mit/balewski/freezer/2013-SN-590 . Note, we used `setup root 5.30.00` at rcas6nnn to execute the examples.

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