



U.S. DEPARTMENT OF
ENERGY

***Azimuthal-angle dependence of pion femtoscopy
relative to the first-order event plane
in $\sqrt{s_{NN}} = 200 \text{ GeV}$ Au+Au and Cu+Au collisions at
STAR***

Yota Kawamura
for the STAR Collaboration
January 10th, 2019
WWND2019 @ Beaver Creek

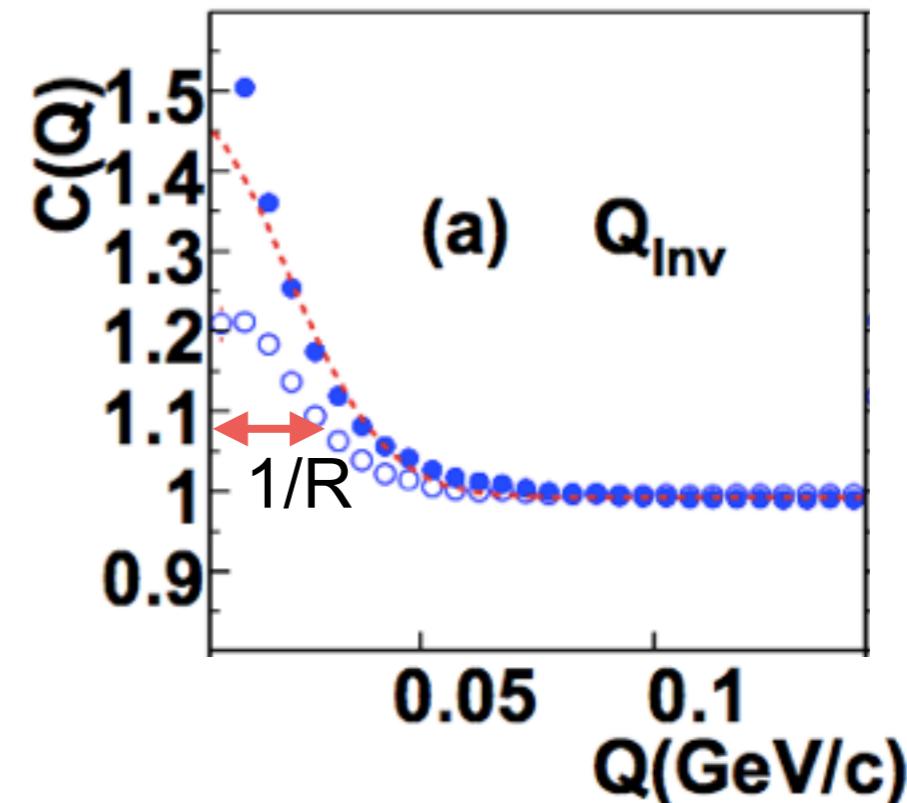
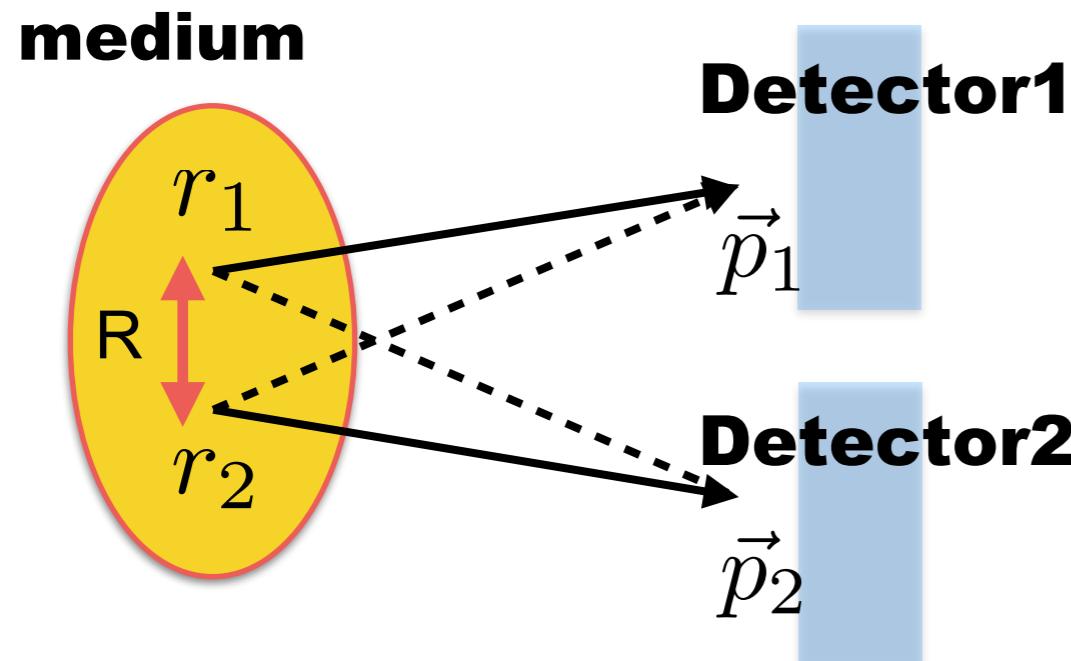


筑波大学
University of Tsukuba



Tomonaga Center
for the History of the Universe

- HBT can scope the source size at kinetic freeze-out
 - ✓ Measure quantum interference between two identical particles



STAR Collaboration, Phys. Rev. Lett. 87 (2001) 82301

- Theory

$$C_2 = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} \approx 1 + \exp(-R^2 Q_{inv}^2)$$

$$\vec{q} = \vec{p}_2 - \vec{p}_1 \quad Q_{inv} = \sqrt{q_x^2 + q_y^2 + q_z^2 - q_0^2}$$

- Experimentally

$$C(q) = \frac{N(q)}{D(q)}$$

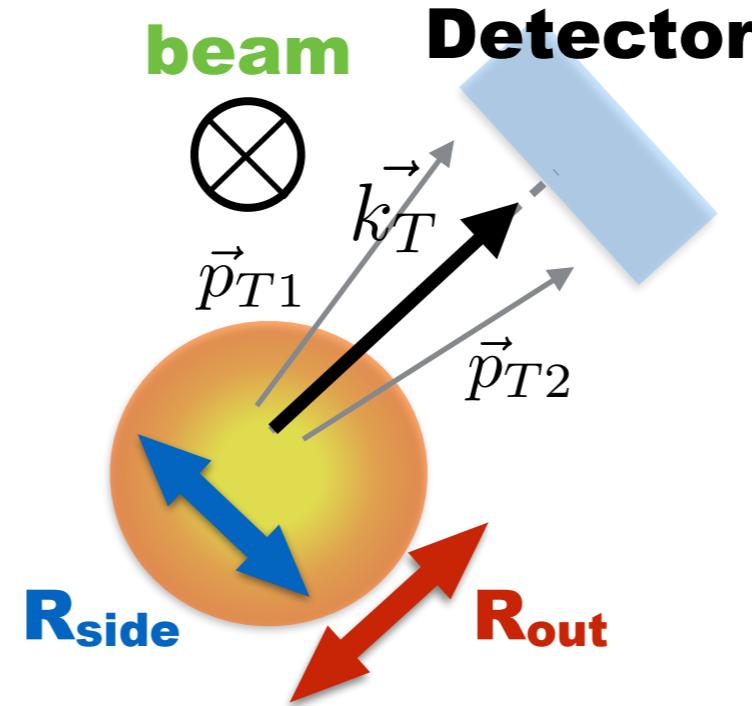
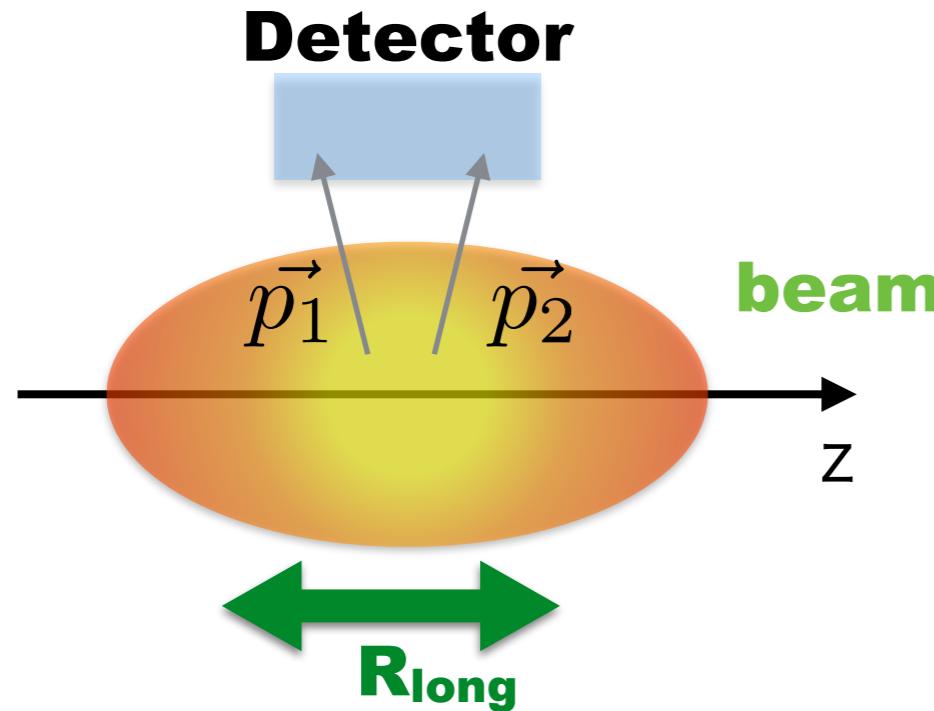
- Event mixing

N: pair distribution in same event (real)

D: pair distribution from different events (mixed)

- Make correlation function as a function of relative momentum (q)
- One can extract the source radius by fitting with theoretical formula

- **Bertsch-Pratt Parameterization** (S. Pratt, Phys. Rev. D 33, (1986) 72 , G. Bertsch et al., Phys. Rev. C 37, (1988) 1896)



$$\vec{k}_T = \frac{1}{2}(\vec{p}_{T1} + \vec{p}_{T2})$$

$$\vec{q}_{\text{out}} \parallel \vec{k}_T$$

$$\vec{q}_{\text{side}} \perp \vec{k}_T$$

- 3-dimensional radii use
 - ✓ R_{long} : Source size parallel to the beam direction
 - ✓ R_{out} : Source size parallel to the pair transverse momentum (k_T) + emission duration
 - ✓ R_{side} : Source size perpendicular to R_{out} and R_{long}

✓ Fit function:

$$C(\vec{q}) = N[(1 - \lambda) + \lambda K(\vec{q})(1 + G(\vec{q}))]$$

$$G(\vec{q}) = \exp(-R_{\text{out}}^2 q_{\text{out}}^2 - R_{\text{side}}^2 q_{\text{side}}^2 - R_{\text{long}}^2 q_{\text{long}}^2)$$

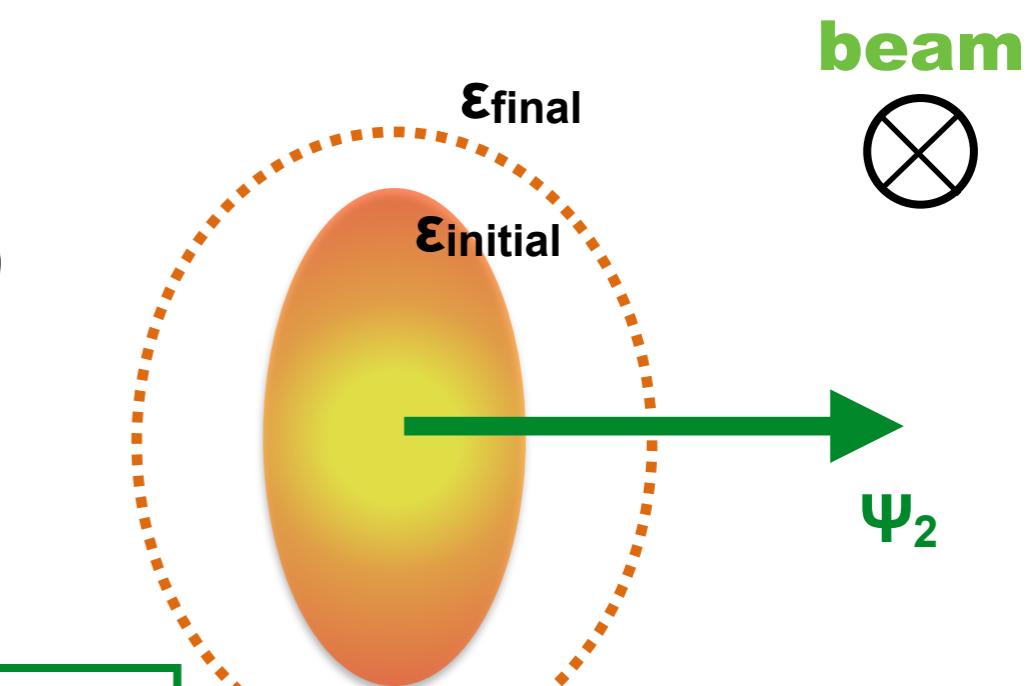
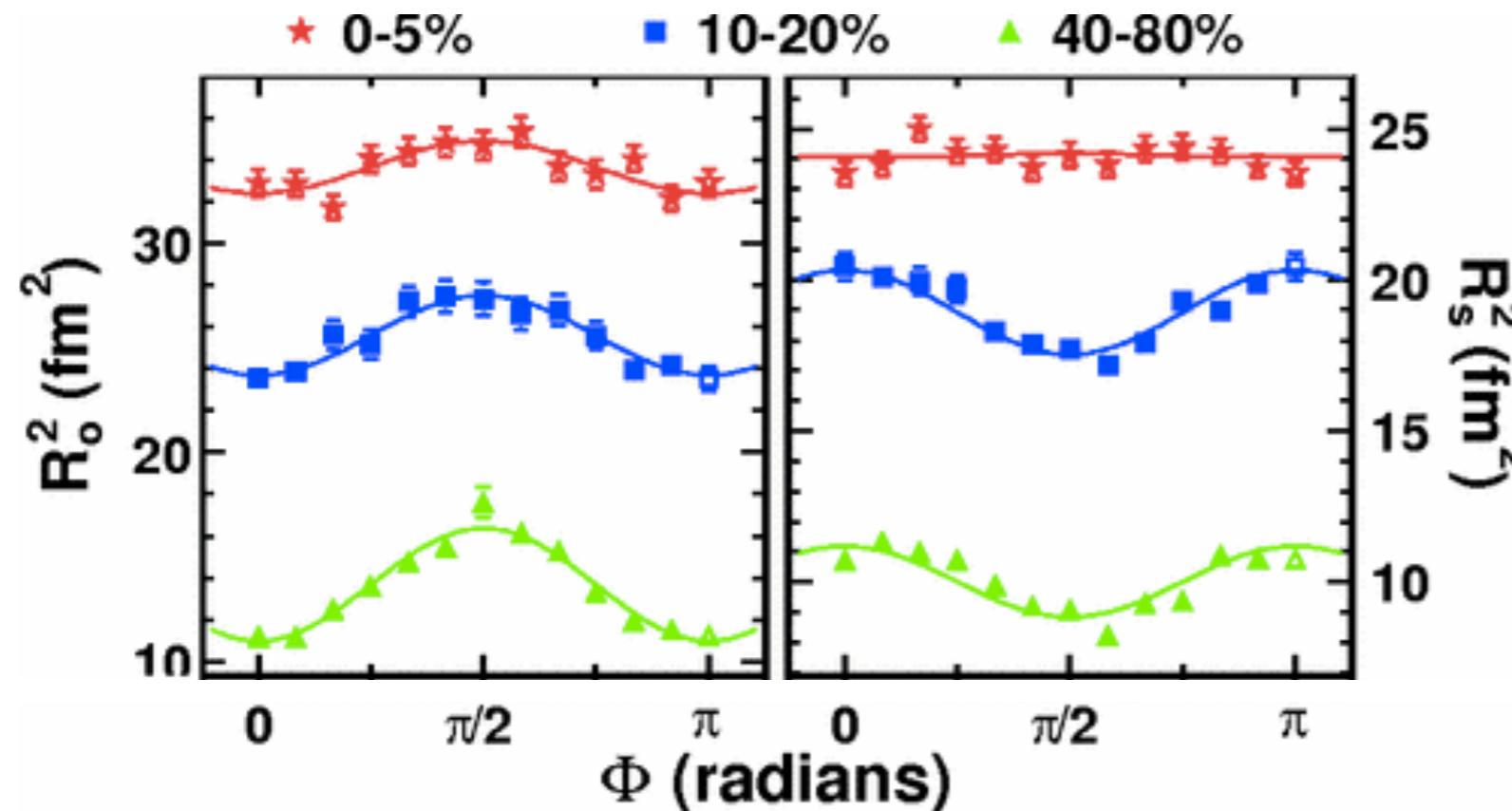
N : Normalization , $K(q)$: Coulomb correction, λ : Correlation strength

- ✓ Pair relative momentum \vec{q} is decomposed into three projection
 $\{\mathbf{q}_{\text{out}}, \mathbf{q}_{\text{side}} \text{ and } \mathbf{q}_{\text{long}}\}$

- ✓ Extract radii from fit of correlation function

Azimuthal sensitive HBT

STAR collaboration, Phys. Rev. Lett. 93, (2004) 012301



$$\varepsilon_{\text{final}} \approx 2 \frac{R_{s,2}^2}{R_{s,0}^2}$$

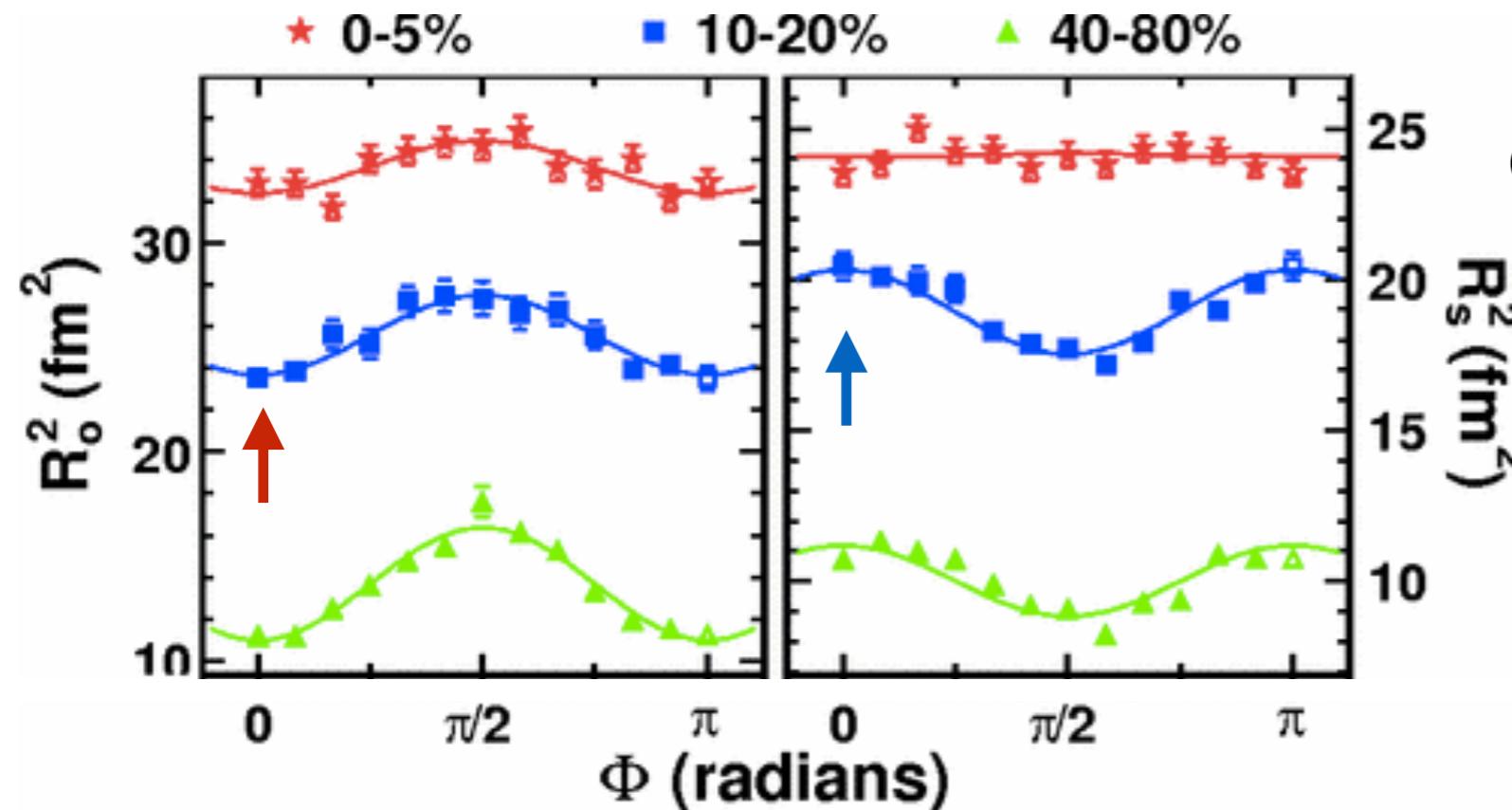
$R_{\mu,0}^2$: Average radius
 $R_{\mu,2}^2$: 2nd-order oscillation magnitude

- Fit function:
 $R_{\mu,0}^2 + 2 R_{\mu,2}^2 \cos(2(\phi - \Psi_{RP}))$, ($\mu = \text{out, side}$)

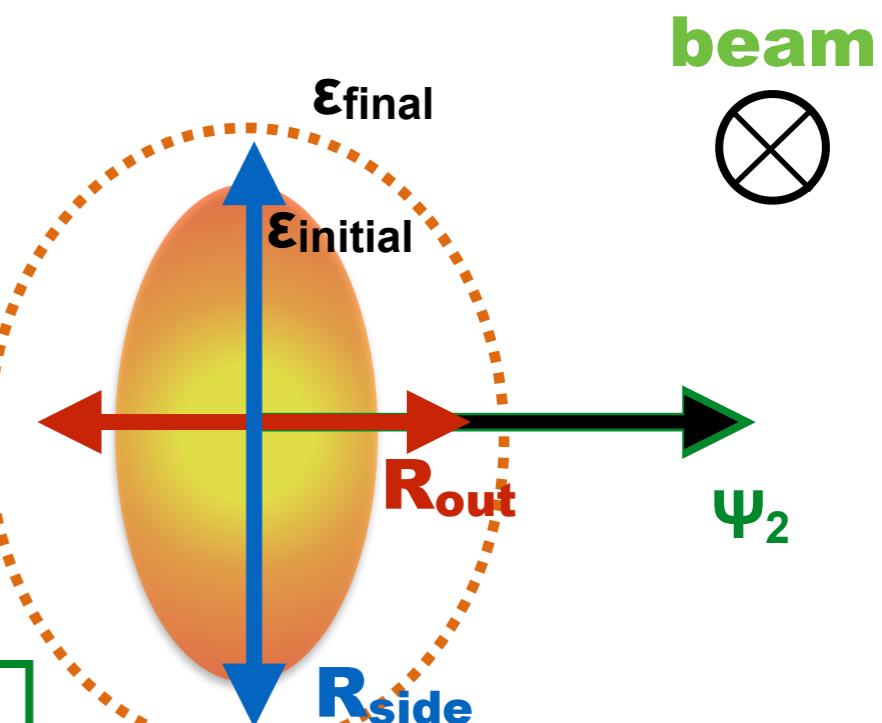
- Final eccentricity can be measured by HBT radii w.r.t. Ψ_2
- $\phi = 0^\circ$ R_{out} : short axis of ellipse, R_{side} : long axis of ellipse
- $\phi = 90^\circ$ R_{out} : long axis of ellipse, R_{side} : short axis of ellipse
- Out-of-plane expanded final source ($\varepsilon_{\text{final}} > 0$) can be measured
- It depends on initial eccentricity, source evolution, etc

Azimuthal sensitive HBT

STAR collaboration, Phys. Rev. Lett. 93, (2004) 012301



$\varphi_{\text{pair}} = 0^\circ$



$$\varepsilon_{\text{final}} \approx 2 \frac{R_{s,2}^2}{R_{s,0}^2}$$

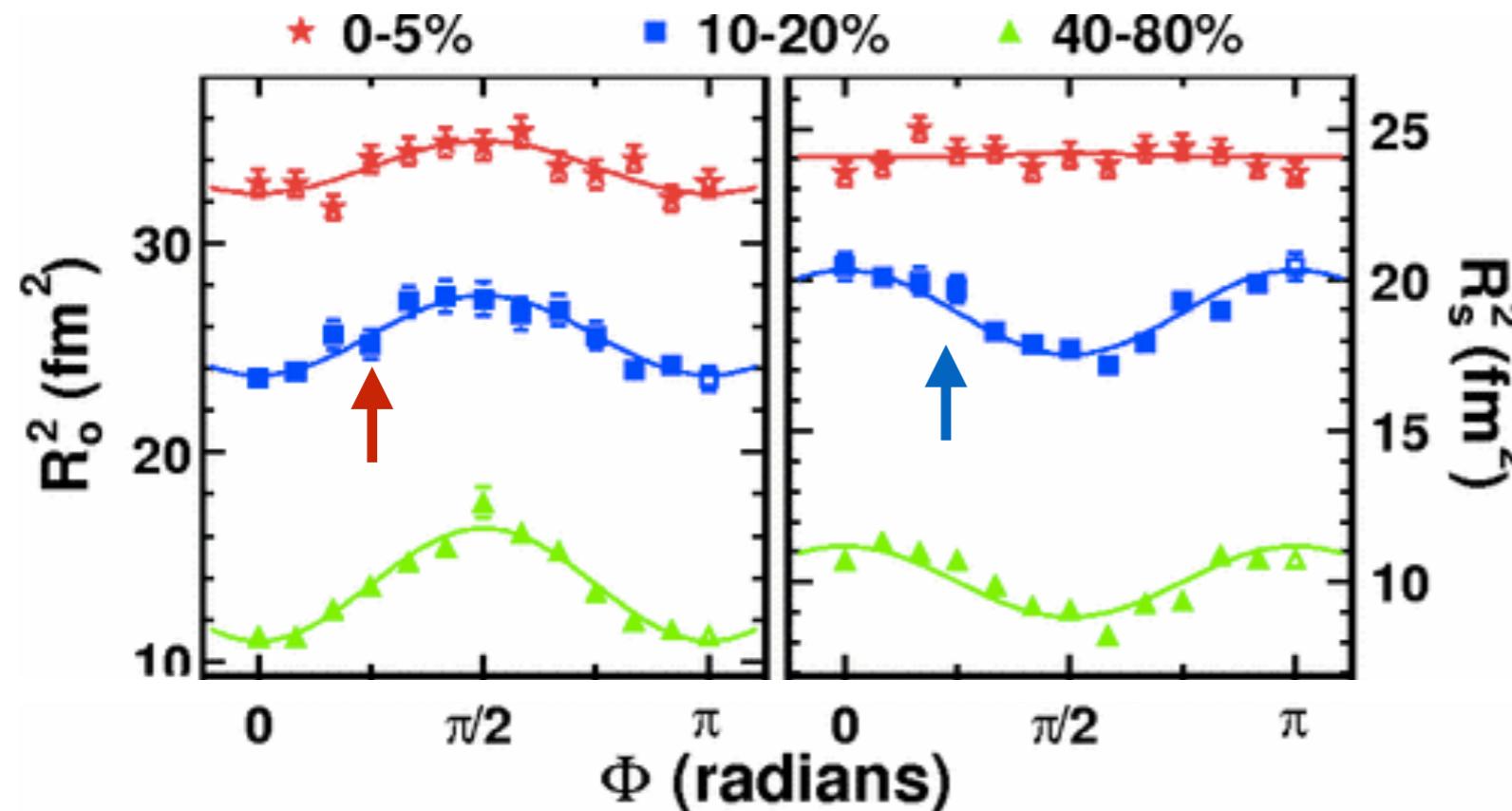
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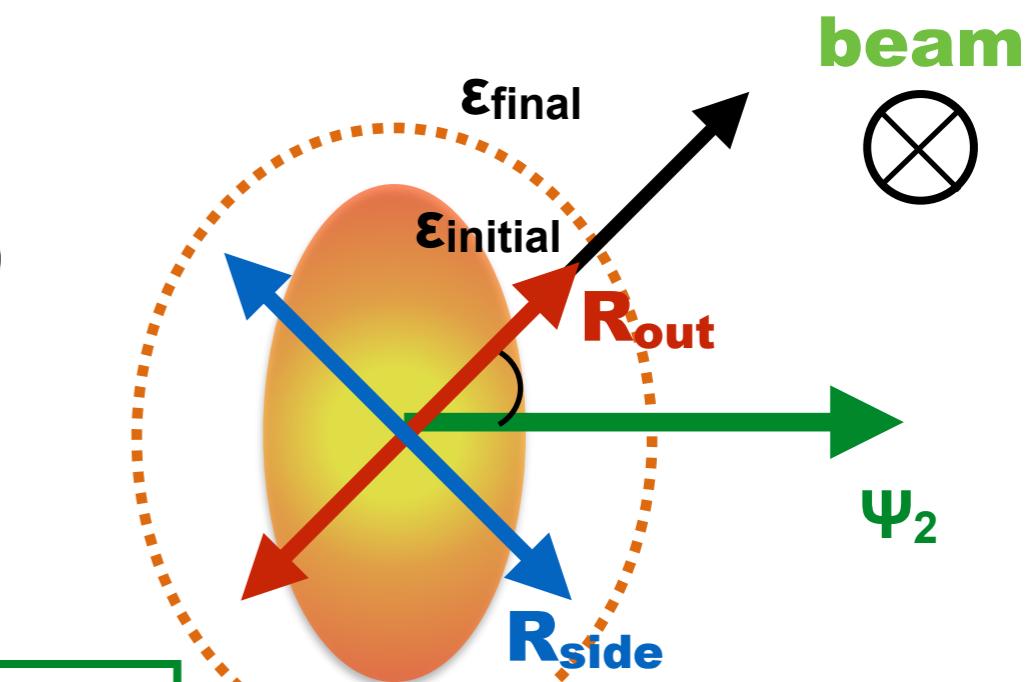
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Azimuthal sensitive HBT

STAR collaboration, Phys. Rev. Lett. 93, (2004) 012301



$$\varphi_{\text{pair}} = 45^\circ$$



$$\varepsilon_{\text{final}} \approx 2 \frac{R_{s,2}^2}{R_{s,0}^2}$$

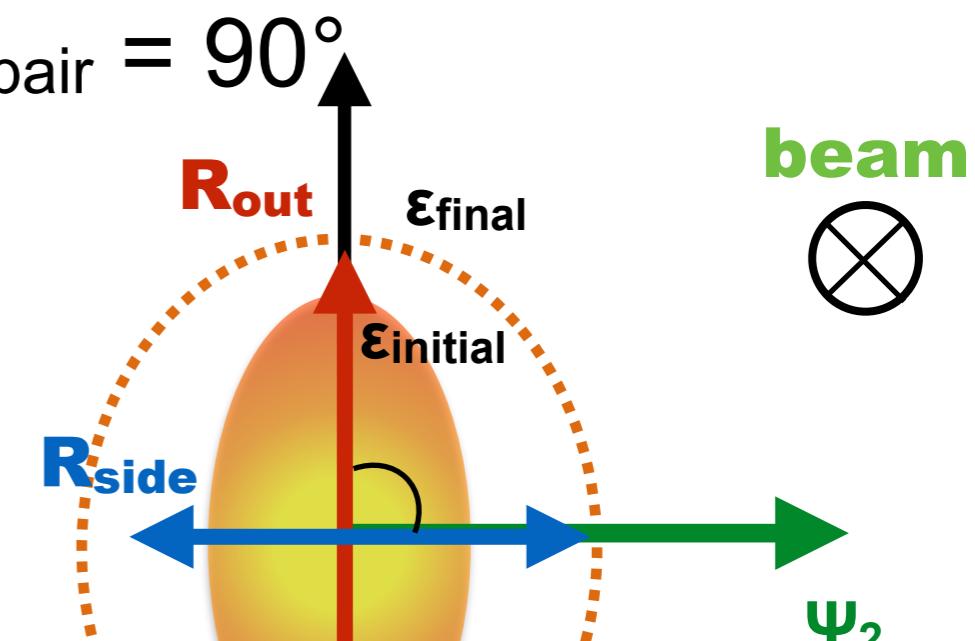
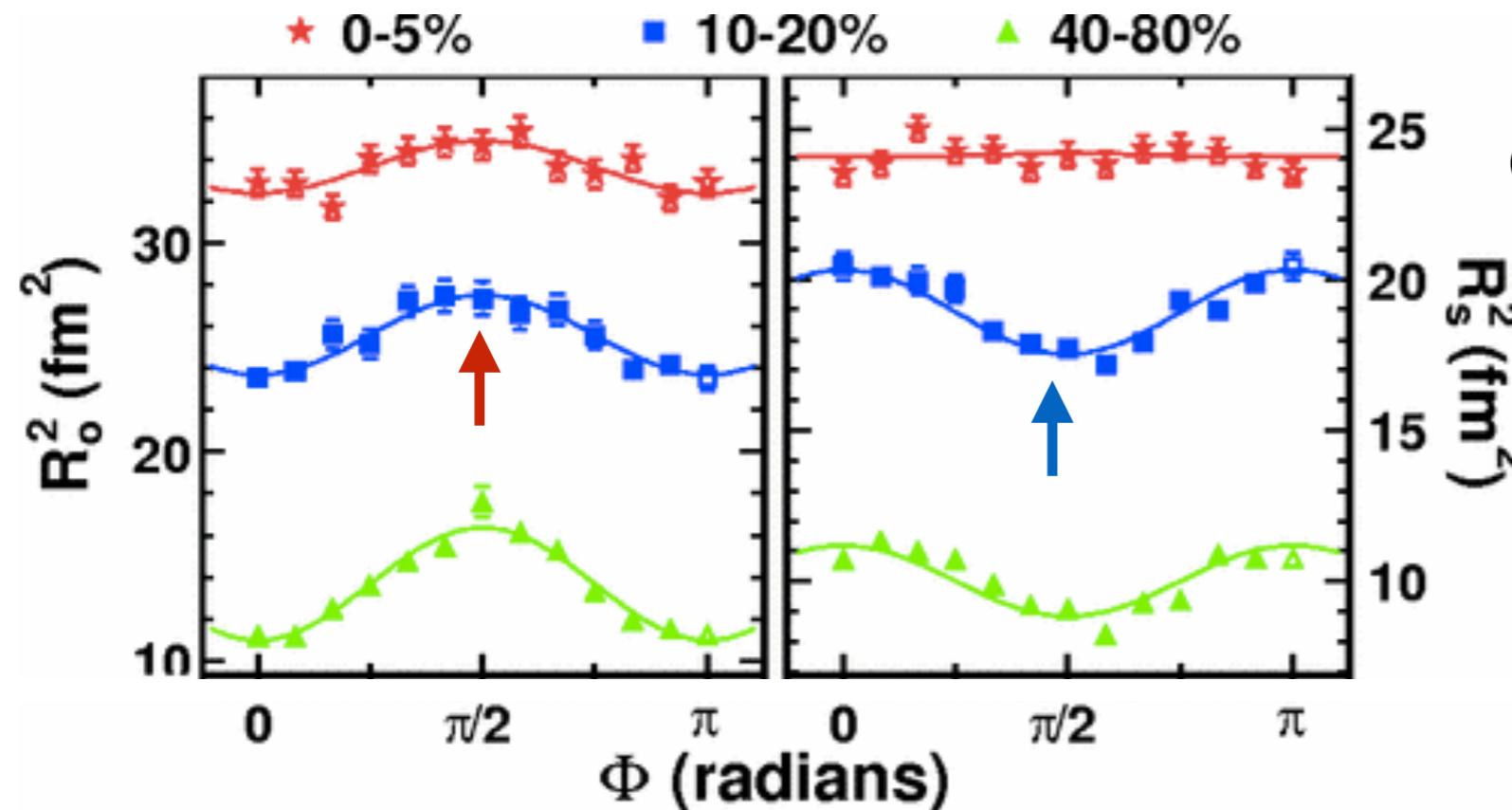
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Azimuthal sensitive HBT

STAR collaboration, Phys. Rev. Lett. 93, (2004) 012301



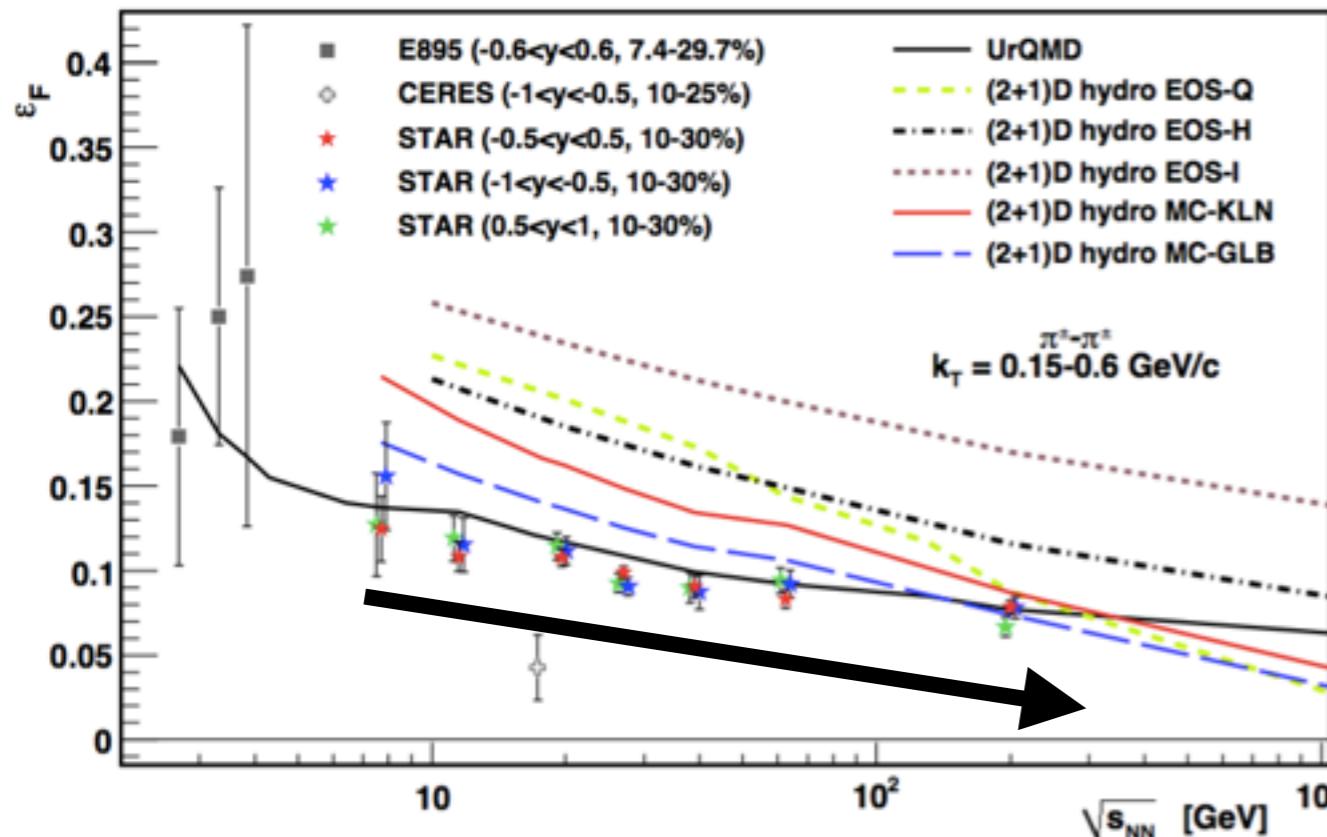
$$\varepsilon_{\text{final}} \approx 2 \frac{R_{s,2}^2}{R_{s,0}^2}$$

$R_{\mu,0}^2$: Average radius
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- Fit function:
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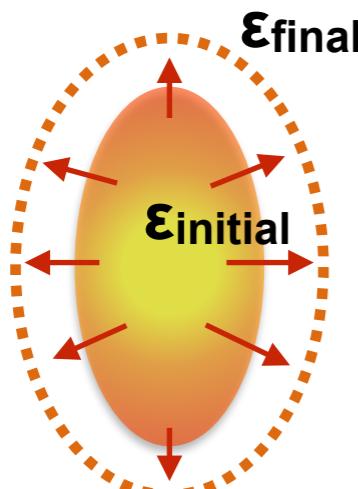
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- Out-of-plane expanded final source ($\varepsilon_{\text{final}} > 0$) can be measured
- It depends on initial eccentricity, source evolution, etc

- Final eccentricity via HBT

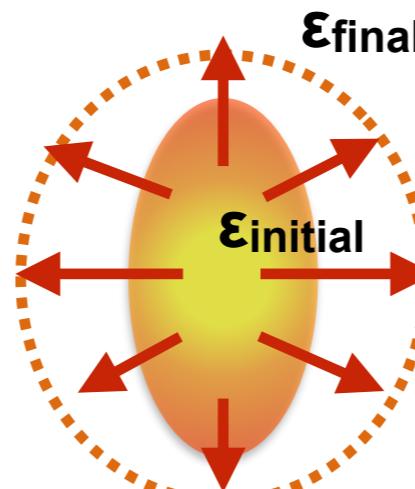


STAR Collaboration, Phys. Rev. C 92 (2015) 014904

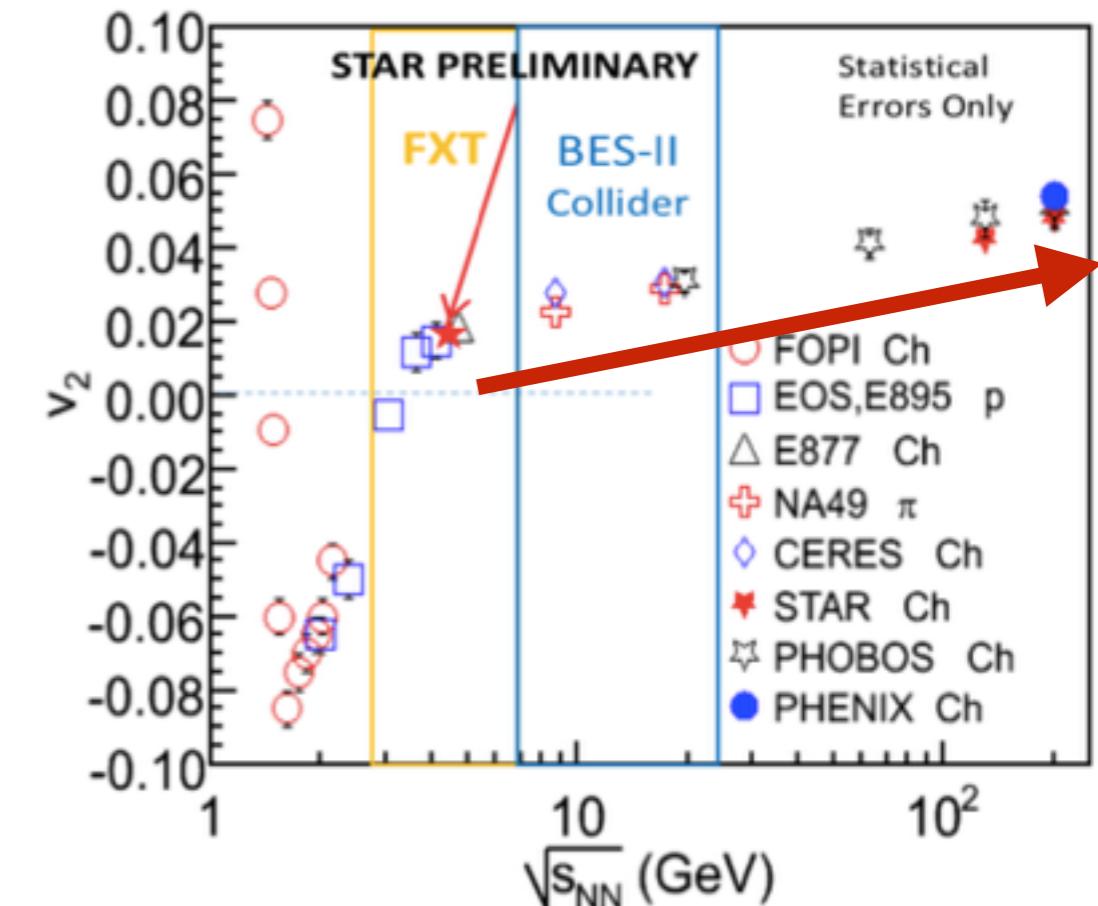
✓ Low energy



✓ High energy



- Momentum space anisotropy



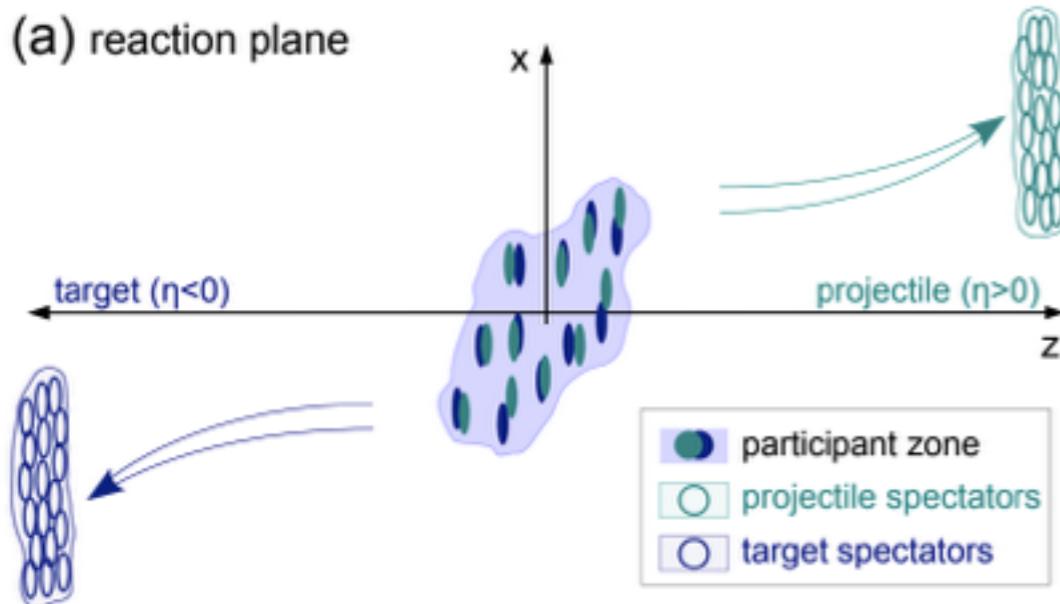
STAR, Quark Matter 2018

- Final eccentricity decreases (more round shape) with increasing collision energy due to longer lifetime and stronger pressure gradients

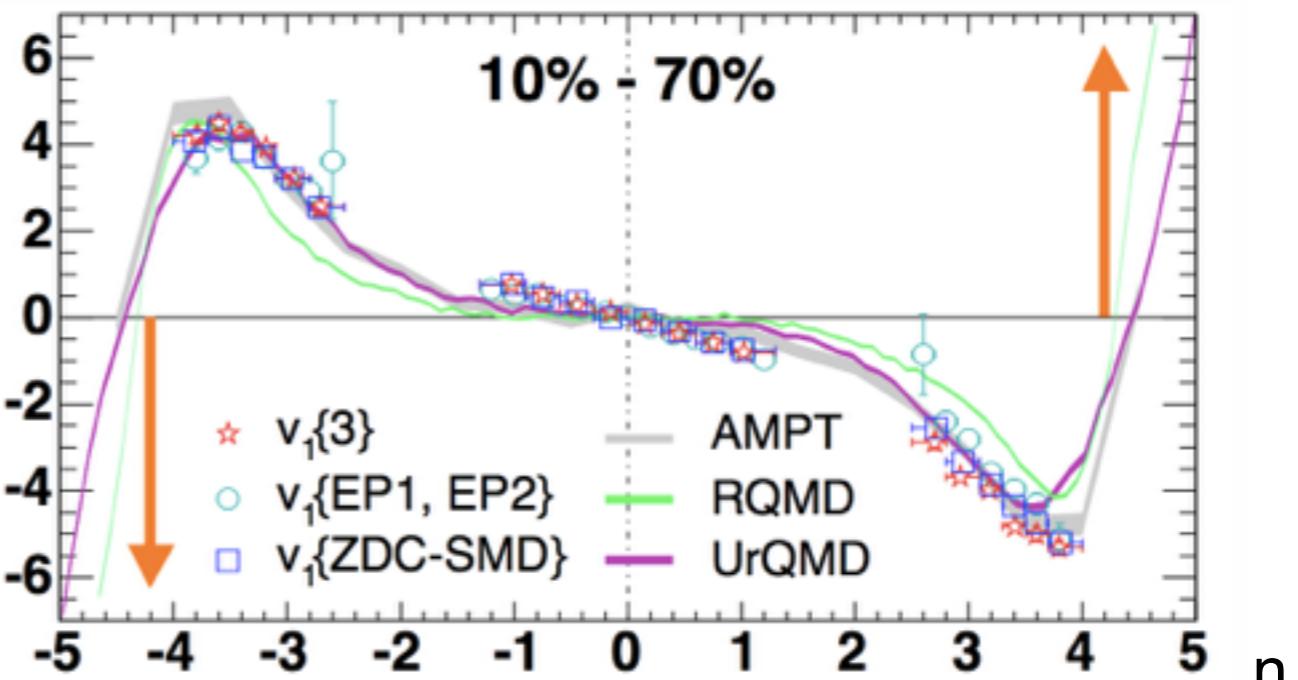
Directed flow

ALICE Collaboration, Phys. Rev. Lett. 111 (2013) 232302

(a) reaction plane



STAR Collaboration, Phys. Rev. C 73 (2006) 34903



Au+Au 62.4 GeV, Charged particle v_1

The direction of flow for spectator neutrons (measured in ZDC).

- ✓ Directed flow is generated by the interaction between spectator and participant particles
- ✓ Quantified by the 1st harmonic in the Fourier expansion as v_1

$$v_1 = \langle \cos(\phi - \Psi_1) \rangle$$

- ✓ $v_1(\eta)$ is crossing zero 3 times at around midrapidity, forward and backward rapidities
-> “wiggle structure”

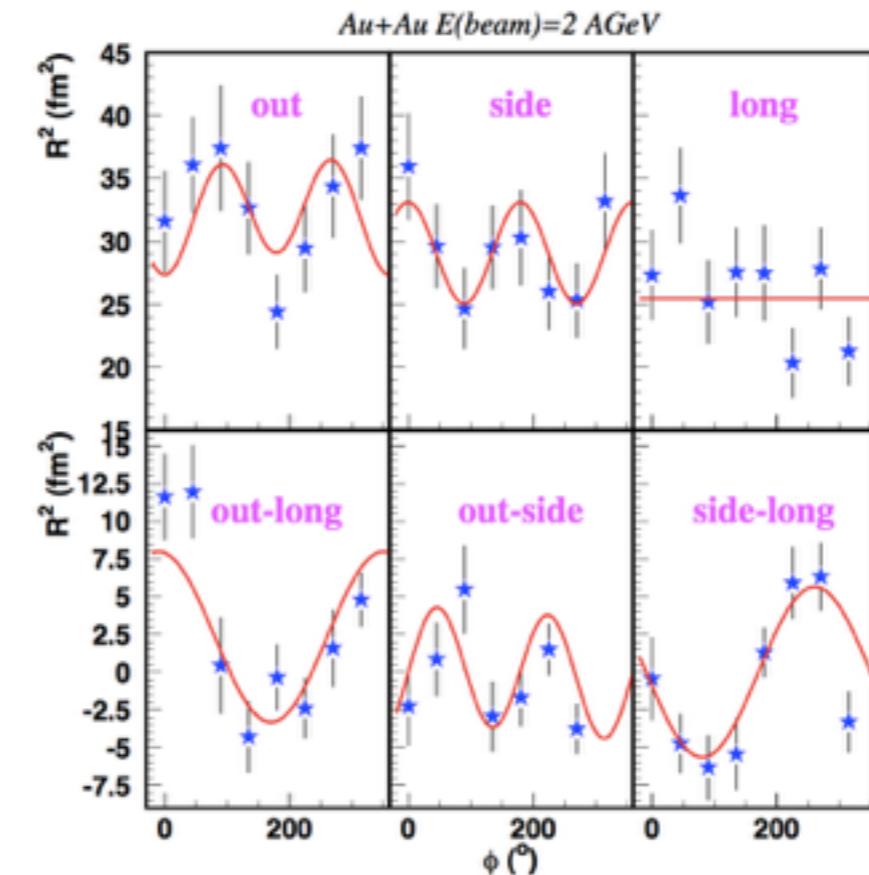
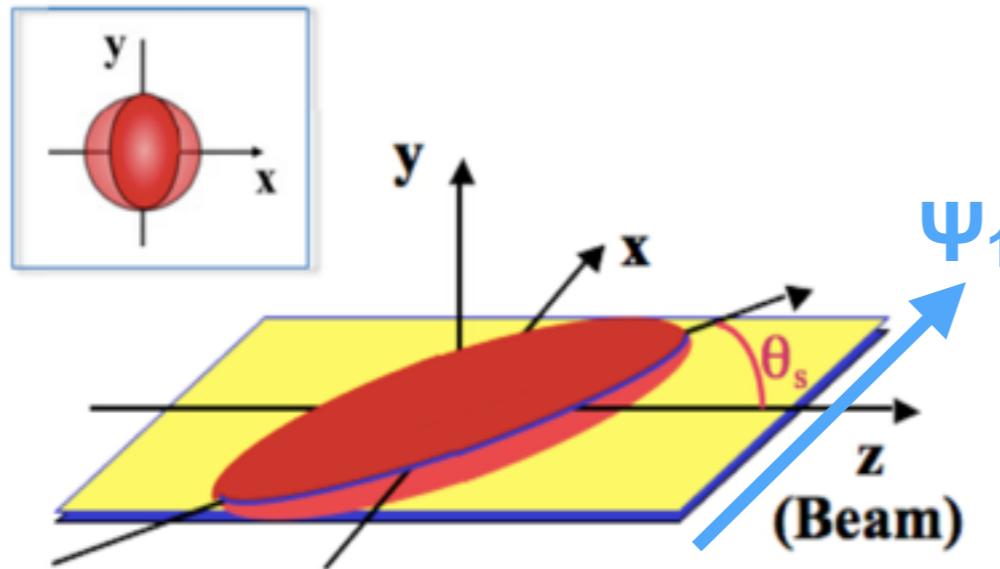
- ✓ Possible signature of phase transition

J. Brachmann et al. Phys. Rev. C 61 (2000) 024909

- ✓ Hydrodynamic models cannot explain only v_1 (unlike to v_2 or v_3)

HBT radii w.r.t. Ψ_1

M. A. Lisa et al. New J. Phys. 13 (2011) 065006



- v_1 signal can be generated from assuming the “tilted” source initial conditions
- HBT measurement w.r.t. Ψ_1 can scope source tilt at freeze-out by including cross terms in the fit function

✓ Fit function with cross terms:

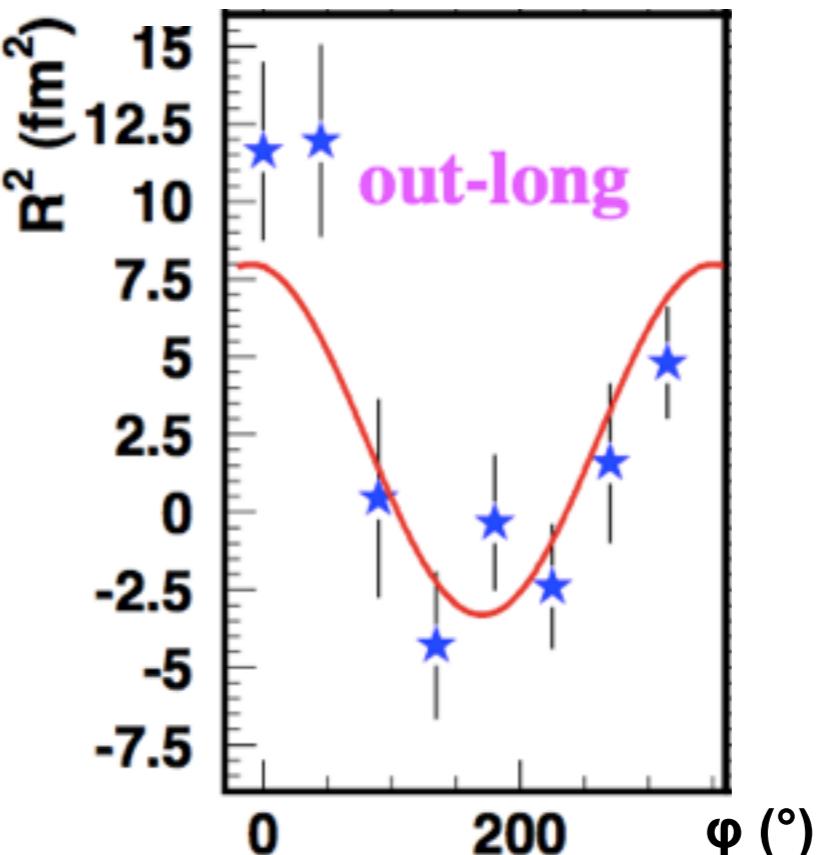
$$C(\vec{q}) = N[(1 - \lambda) + \lambda K(\vec{q})(1 + G(\vec{q}))]$$

$$G(\vec{q}) = \exp(-R_{out}^2 q_{out}^2 - R_{side}^2 q_{side}^2 - R_{long}^2 q_{long}^2 - 2R_{os}^2 q_{out} q_{side} - 2R_{ol}^2 q_{out} q_{long} - 2R_{sl}^2 q_{side} q_{long})$$

- Important parameters: R_{ol} , R_{sl}
- If final source is tilted, R_{ol} and R_{sl} cross terms will have oscillation w.r.t. Ψ_1

- 3D

- :Out - Long plane
- Projection Out - Long plane



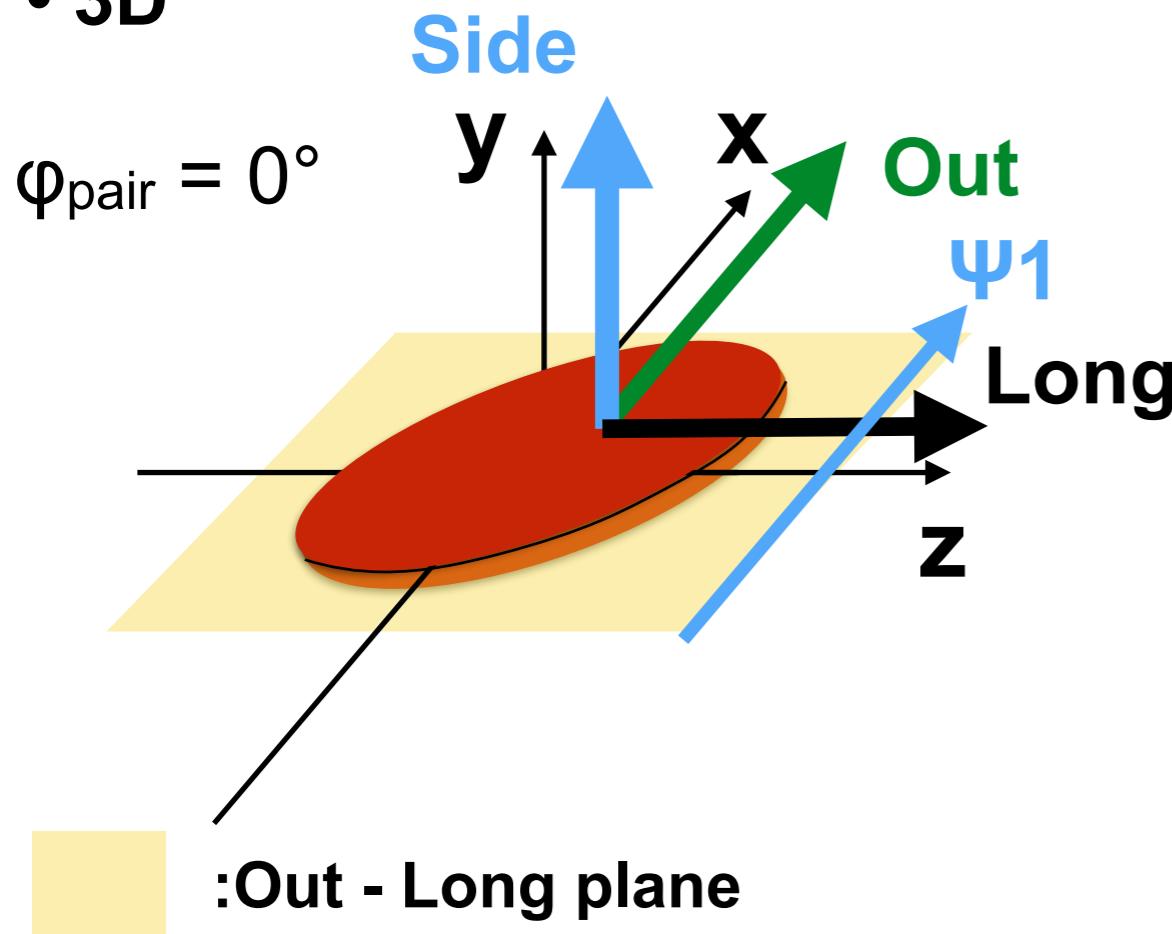
$$R_{ol}^2 \propto \cos(\phi)$$

$$R_{sl}^2 \propto \sin(\phi)$$

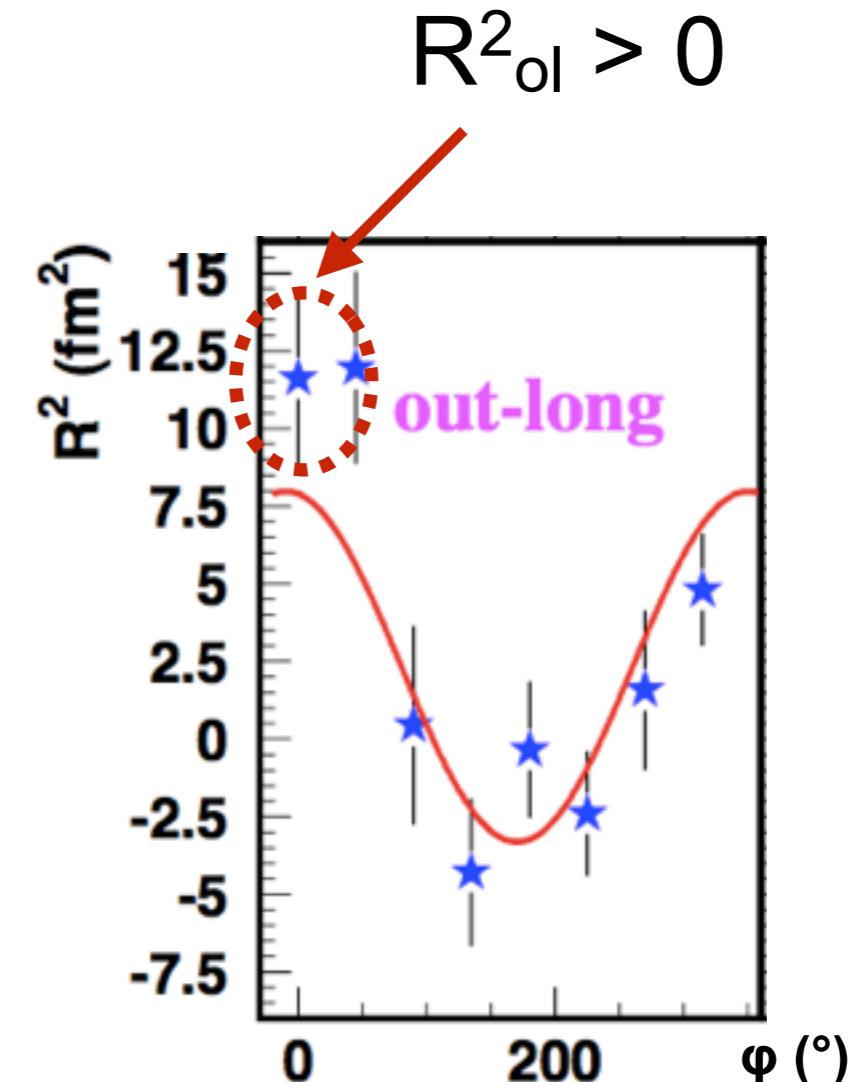
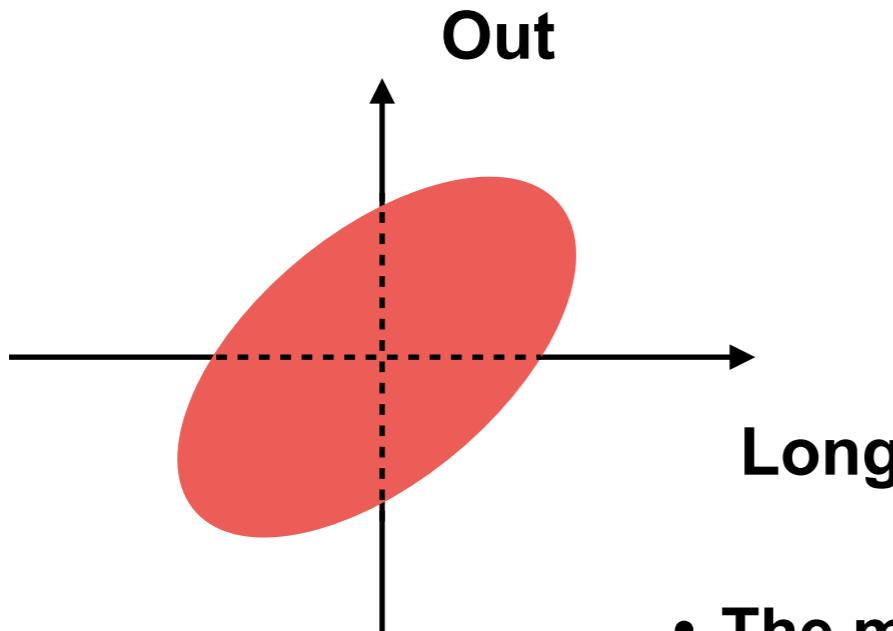
- R^2_{sl} has its $+\pi/2$ oscillation
- The magnitude of oscillation corresponds to the tilt angle

HBT radii w.r.t. Ψ_1

- 3D



- Projection Out - Long plane



$$R_{ol}^2 \propto \cos(\phi)$$

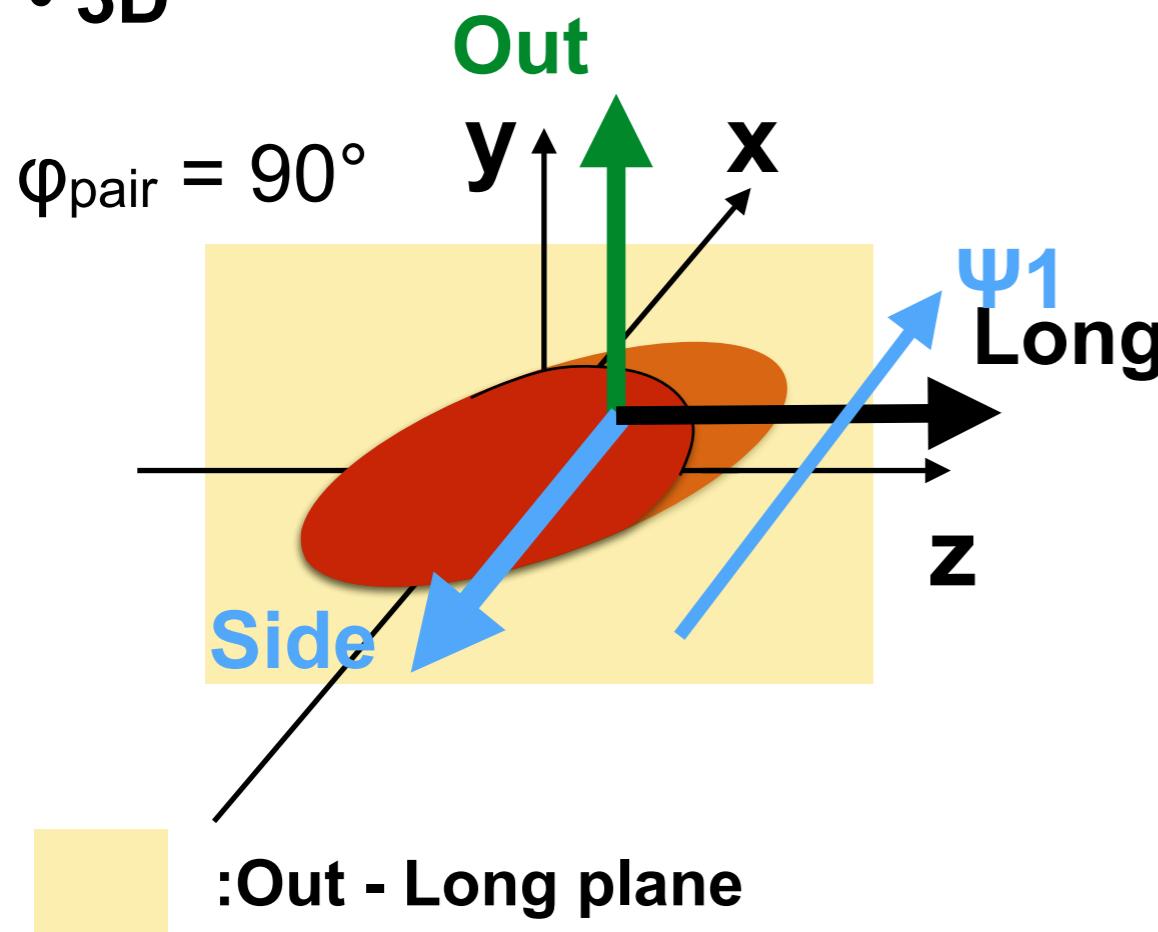
$$R_{sl}^2 \propto \sin(\phi)$$

- R^2_{sl} has its $+\pi/2$ oscillation

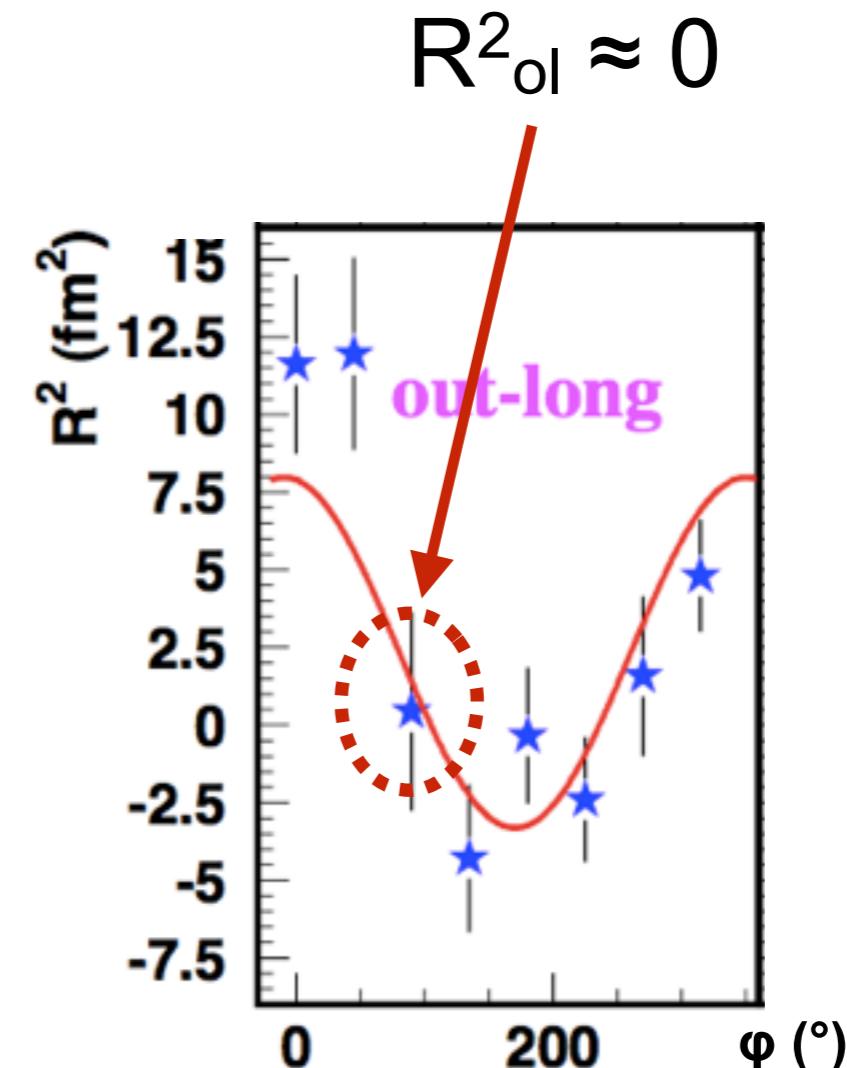
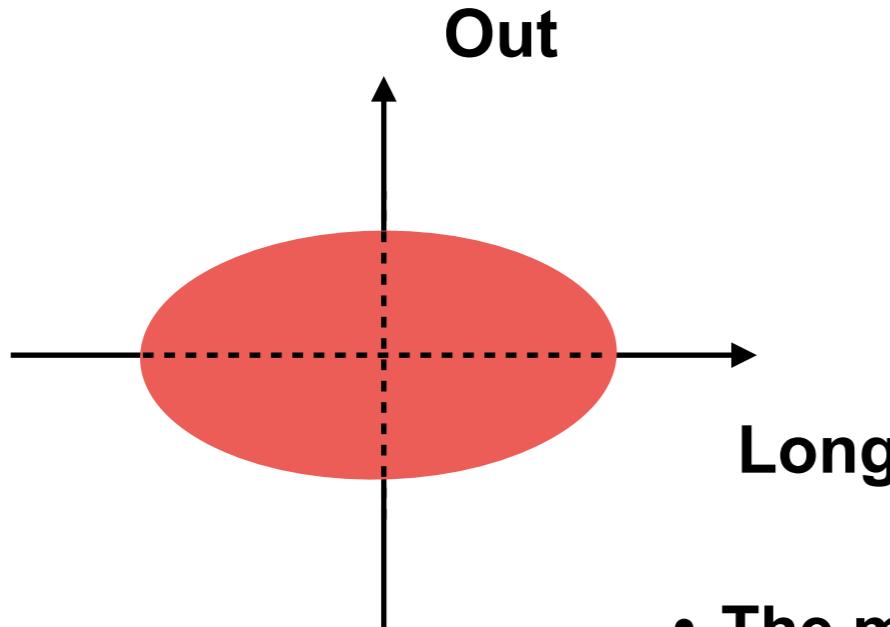
- The magnitude of oscillation corresponds to the tilt angle

HBT radii w.r.t. Ψ_1

- 3D



- Projection Out - Long plane



$$R_{\text{ol}}^2 \propto \cos(\phi)$$

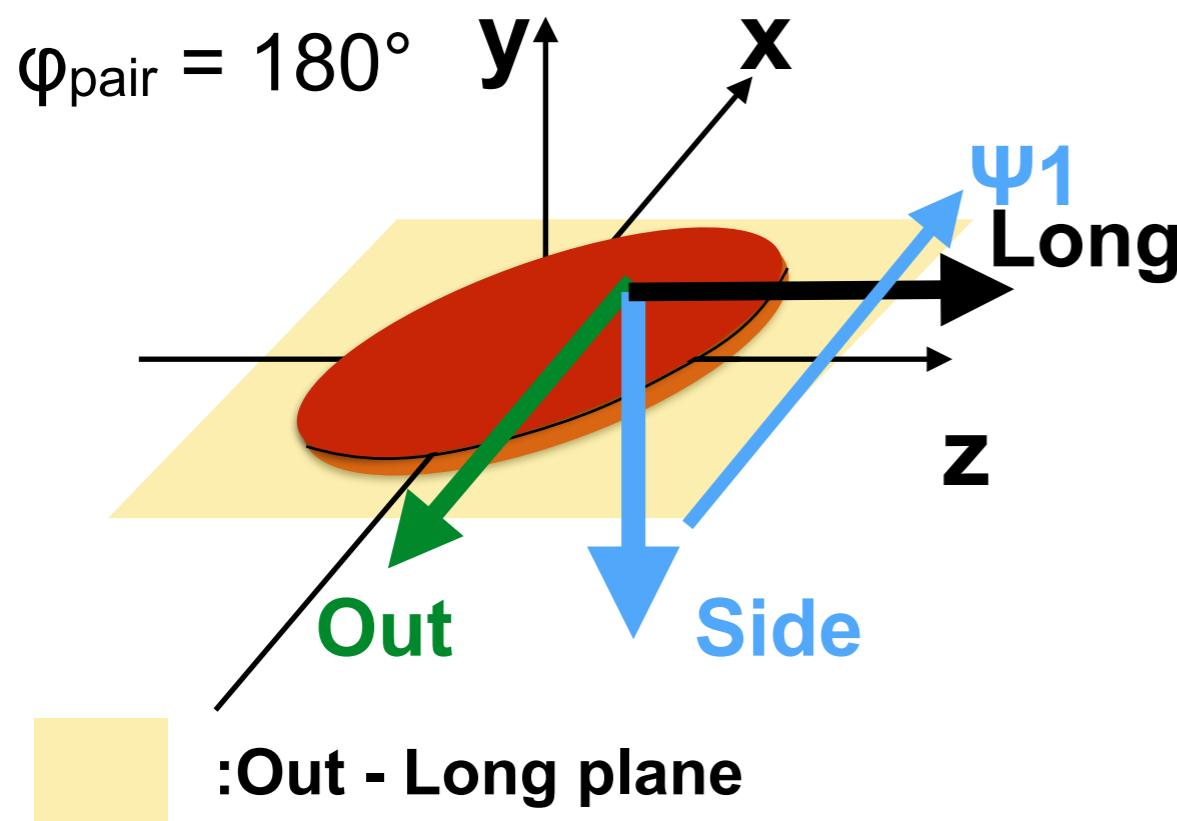
$$R_{\text{sl}}^2 \propto \sin(\phi)$$

- R^2_{sl} has its $+\pi/2$ oscillation

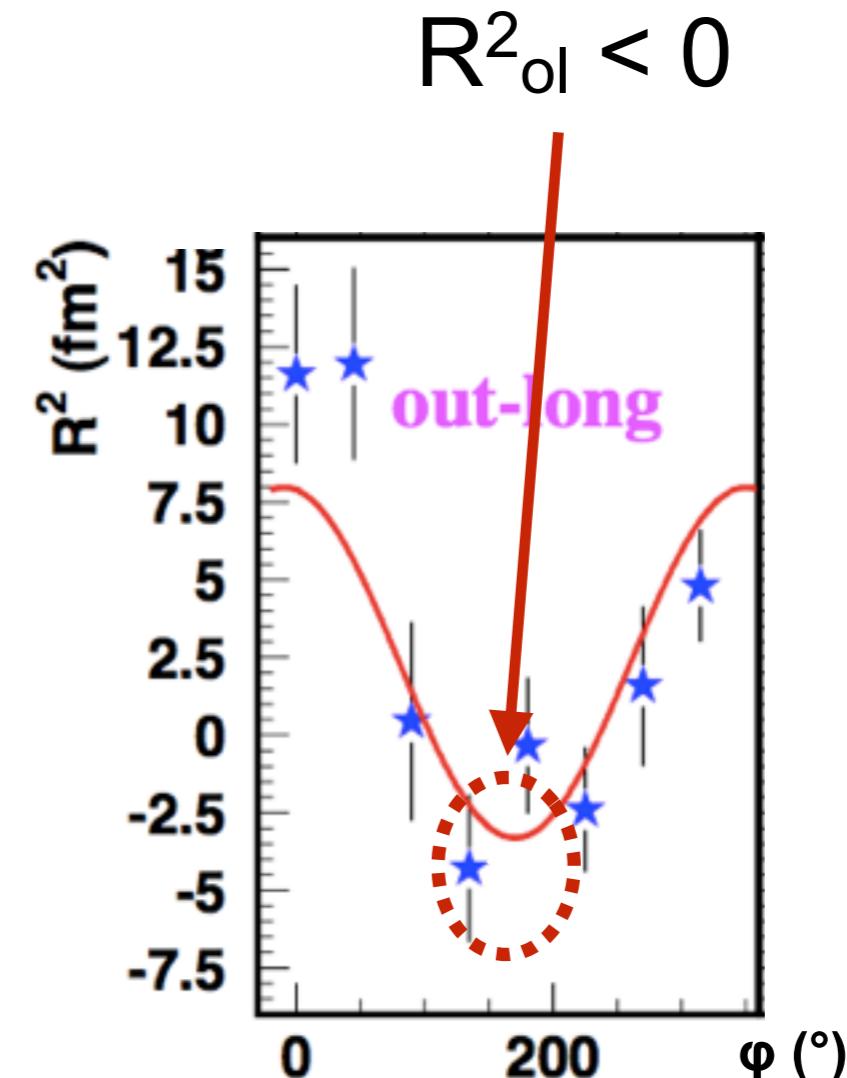
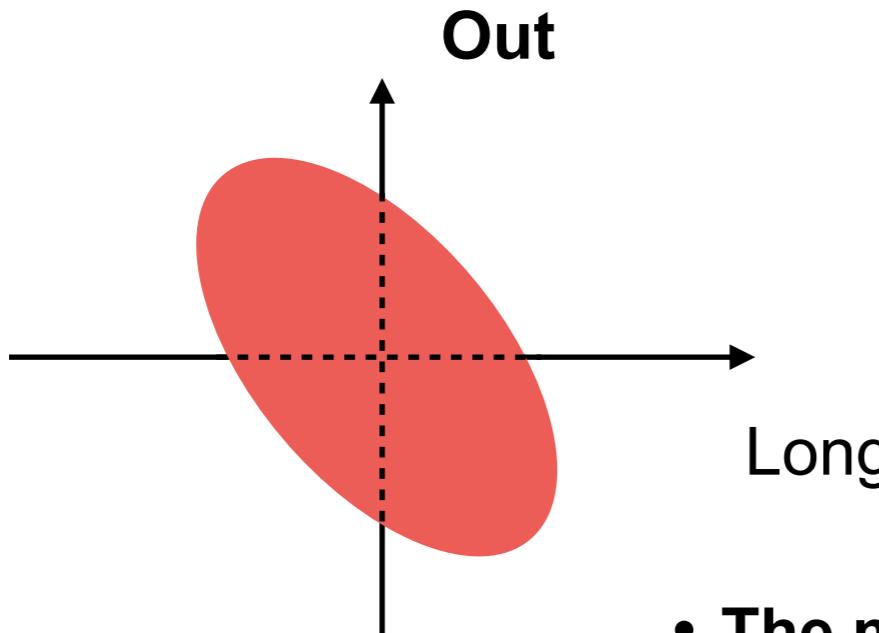
- The magnitude of oscillation corresponds to the tilt angle

HBT radii w.r.t. Ψ_1

- 3D



- Projection Out - Long plane



$$R^2_{ol} \propto \cos(\phi)$$

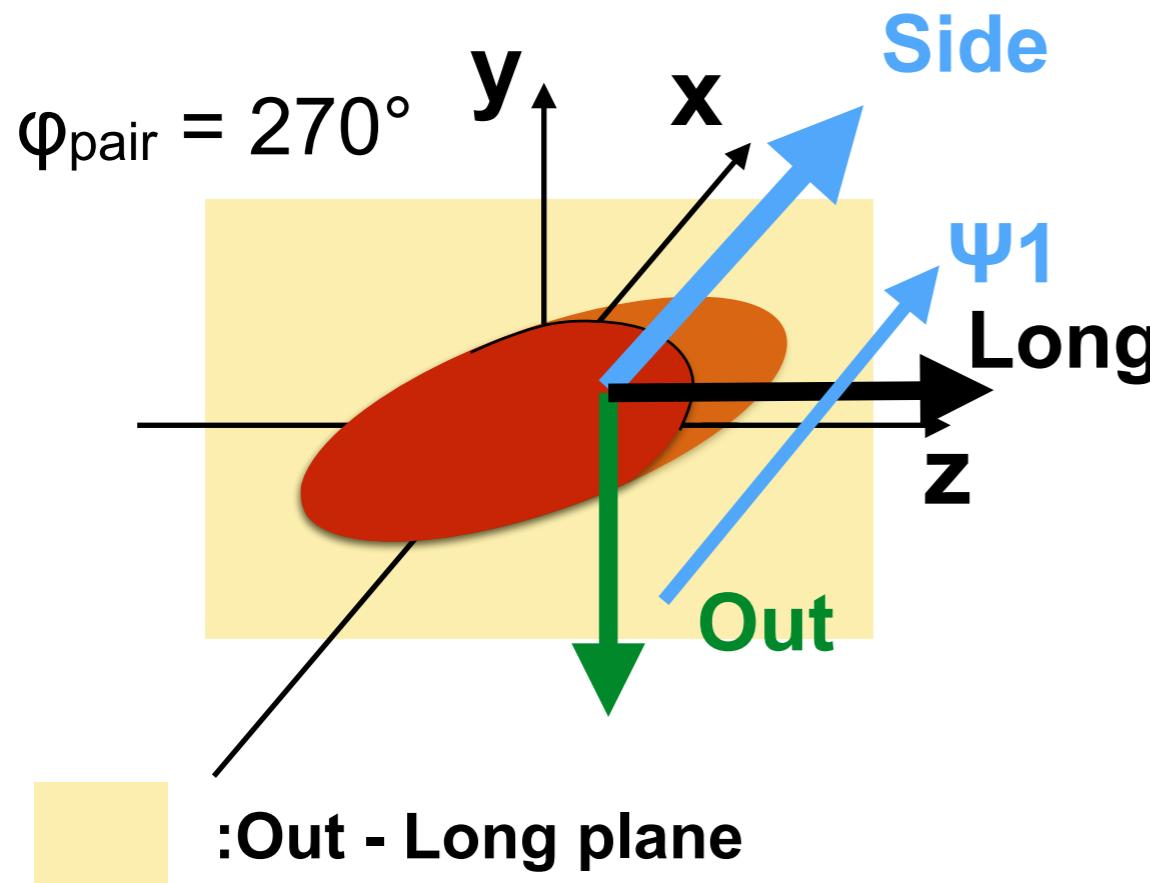
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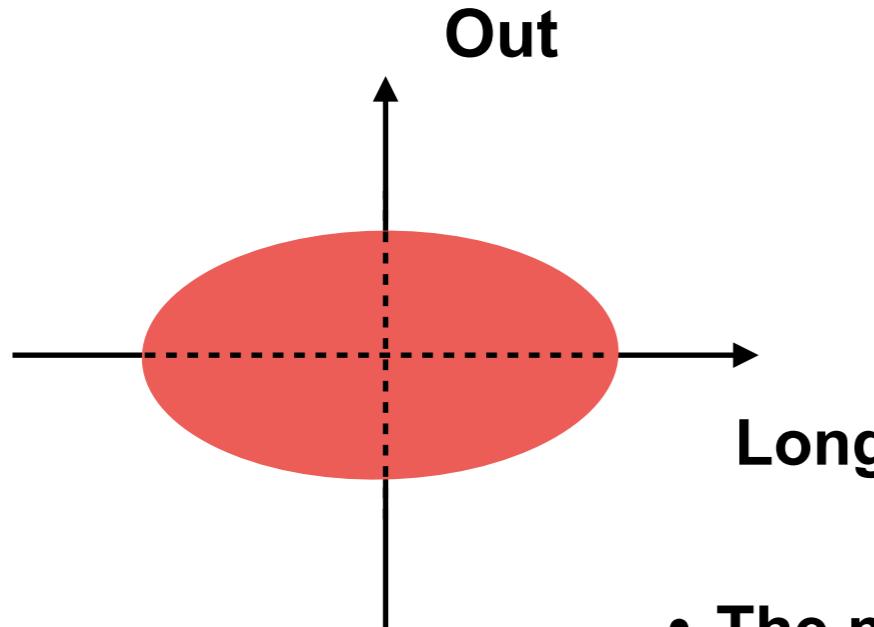
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HBT radii w.r.t. Ψ_1

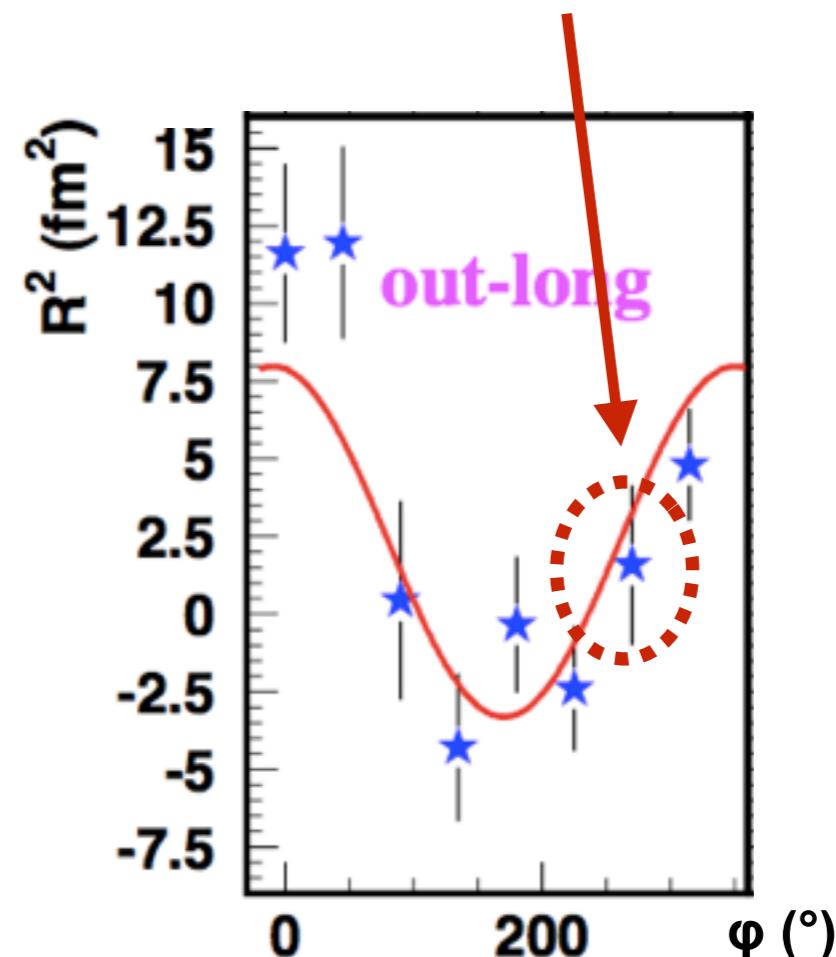
- 3D



- Projection Out - Long plane



$$R^2_{ol} \approx 0$$



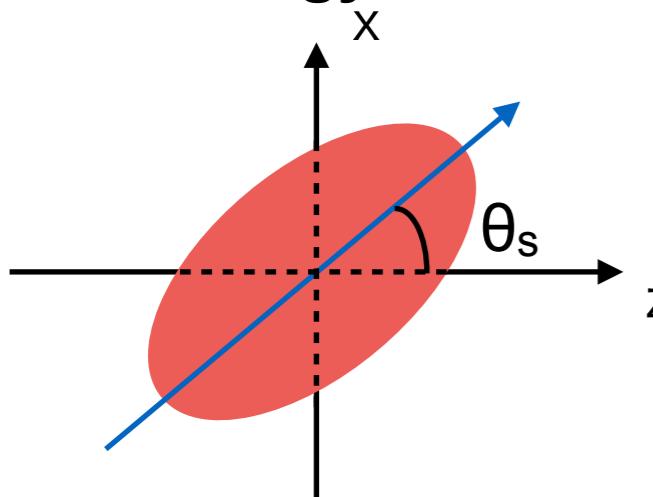
$$R^2_{ol} \propto \cos(\phi)$$

$$R^2_{sl} \propto \sin(\phi)$$

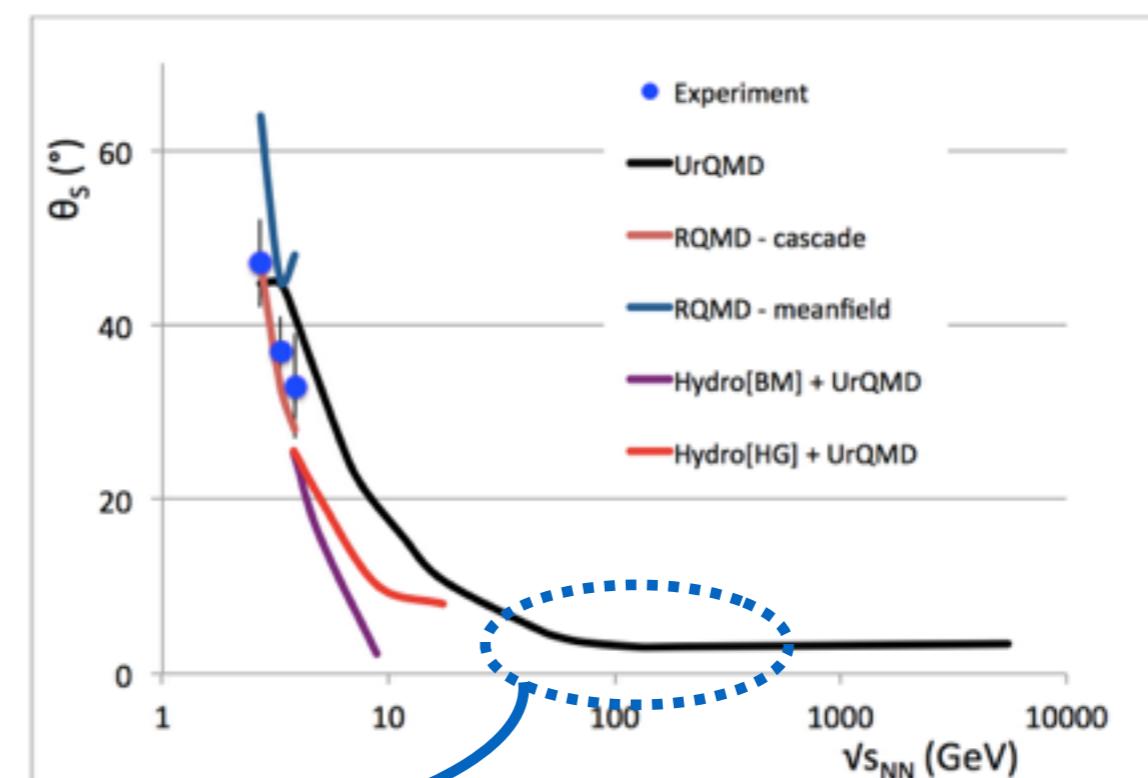
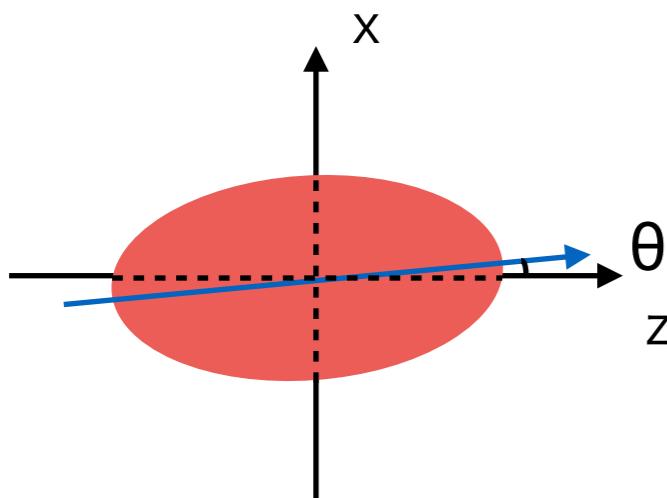
- R^2_{sl} has its $+\pi/2$ oscillation

- The magnitude of oscillation corresponds to the tilt angle

✓ Low energy



✓ High energy expectation



Is there a signal ?

M. A. Lisa et al. New J. Phys. 13 (2011) 065006

Tilt angle

$$\theta_s = \frac{1}{2} \tan^{-1} \left(\frac{-4R_{sl,1}^2}{R_{l,0}^2 - R_{s,0}^2 + 2R_{s,2}^2} \right)$$

- Fit function:

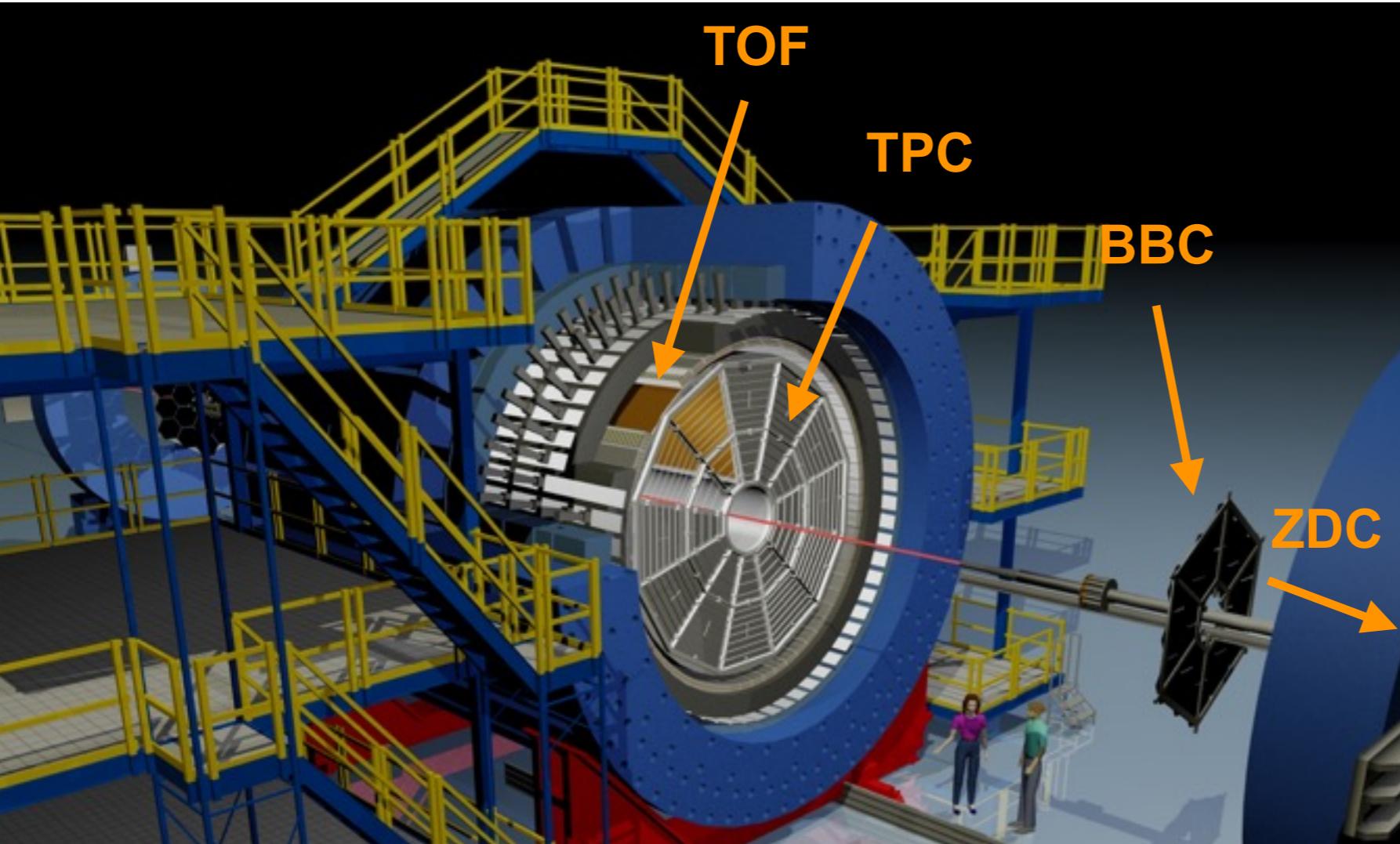
$$R_{\mu,0}^2 + 2 R_{\mu,1}^2 \cos(\varphi - \Psi_1) + 2 R_{\mu,2}^2 \cos(2(\varphi - \Psi_1)) , (\mu = o, s, ol)$$

$$R_{\mu,0}^2 + 2 R_{\mu,1}^2 \sin(\varphi - \Psi_1) + 2 R_{\mu,2}^2 \sin(2(\varphi - \Psi_1)) , (\mu = os, sl)$$

- Experimentally, source tilt has been only measured at low energies
- Tilt angle is inversely proportional to the beam energy
- At RHIC energy (200 GeV), source tilt value is expected to be nearly 0 or signal is very small

✓ Perform HBT measurement w.r.t Ψ_1 and scope tilt signal
using both Au+Au and Cu+Au in 200 GeV
✓ Cu+Au have initial density asymmetry...
-> How does it affect HBT measurement?

The STAR detector



Time Projection Chamber (TPC)

- Main tracking detector, $|\eta| < 1.0$, full azimuth

Zero Degree Calorimeter (ZDC)

- $|\eta| > 6.3$
- Measure spectator neutron
- Event plane reconstruction using spectator neutrons

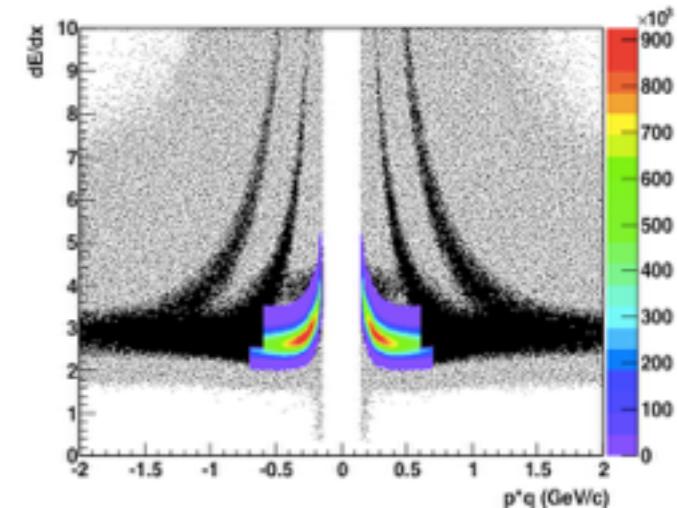
Beam-Beam Counters (BBC)

- $3.3 < |\eta| < 5$
- Event plane reconstruction using participants

TOF & TPC detector

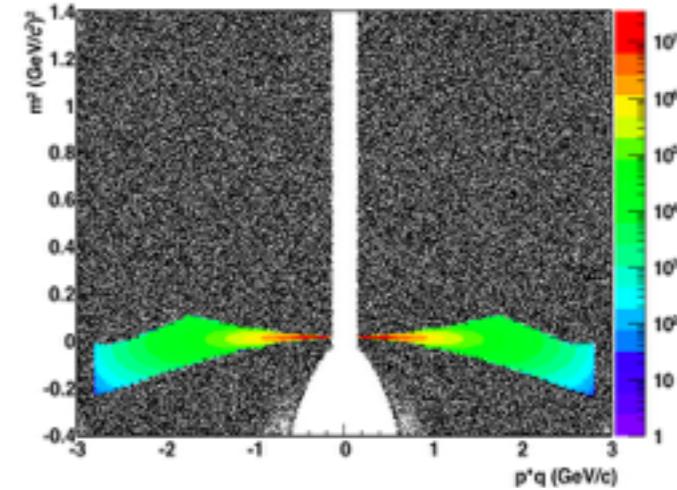
✓ Use PID (particle identification)

TPC (dE/dx) STAR Preliminary



TOF (time of flight)

STAR Preliminary



✓ Pion selected

Analysis

- Au+Au 200 GeV, Cu+Au 200 GeV
- Number of events: Au+Au ~ 430 M
Cu+Au ~ 45 M
- Correlation function

$$C(q) = \frac{N(q)}{D(q)}$$

N: pair distribution from the same event (real)
D: different event pair distribution
from the different events (mixed)

- Estimate Coulomb interaction correction factor
 $K(q)$: Coulomb correction factor
- Fit correlation function and extract radii parameters

$$C(\vec{q}) = N[(1 - \lambda) + \lambda K(q)(1 + G(\vec{q}))]$$

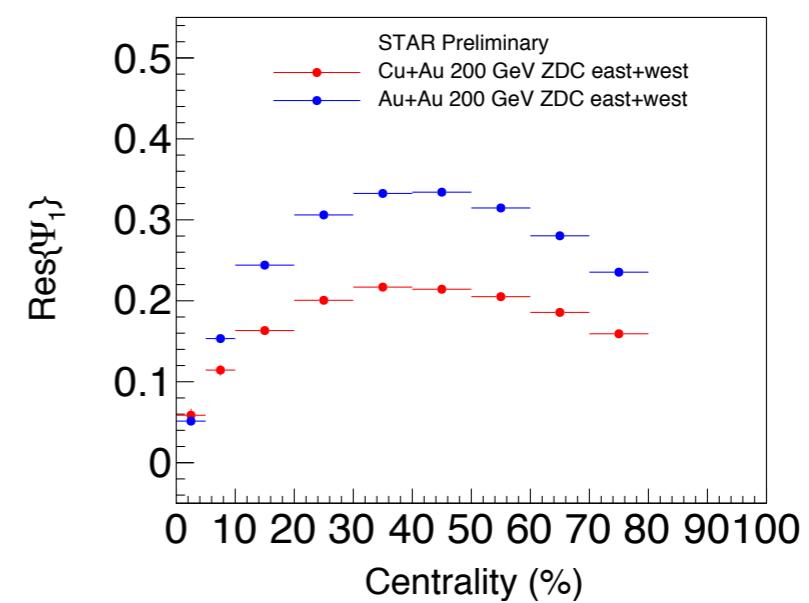
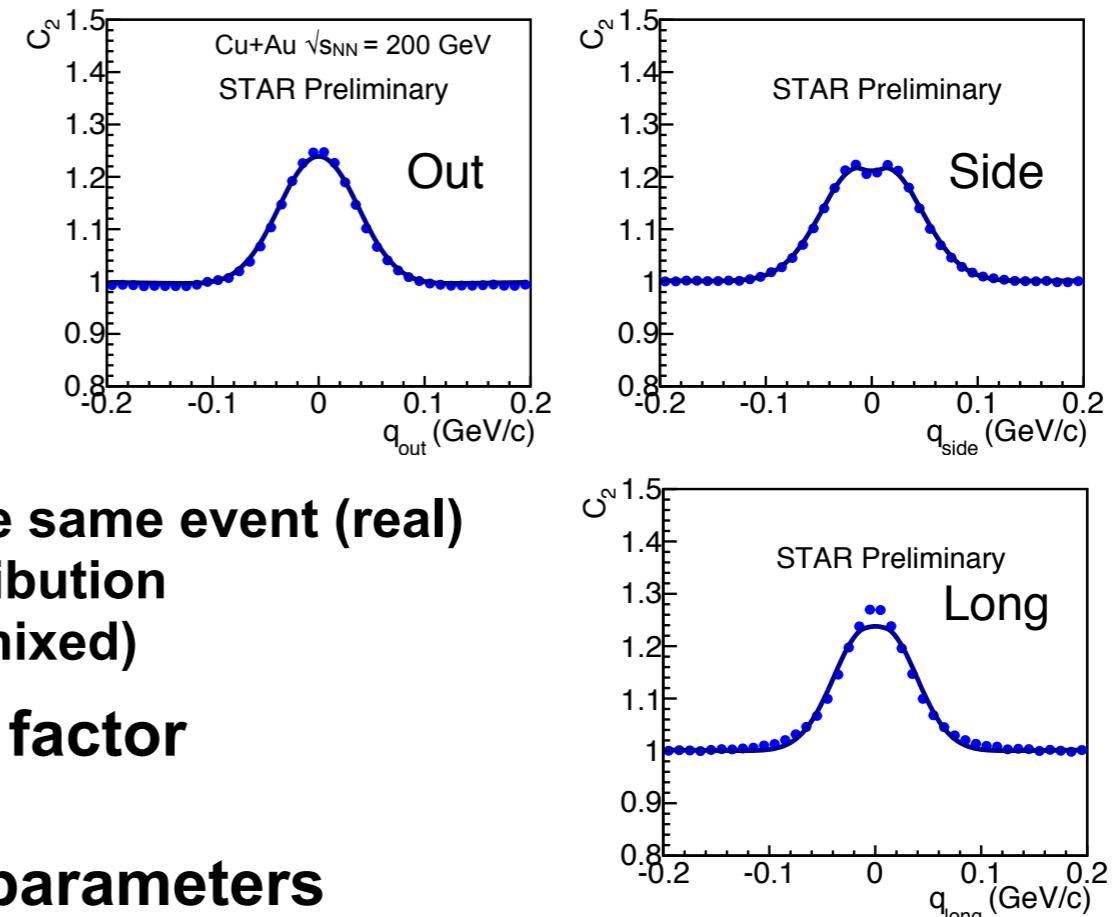
✓ Azimuthally-integrated analysis

$$G(\vec{q}) = \exp(-R_{out}^2 q_{out}^2 - R_{side}^2 q_{side}^2 - R_{long}^2 q_{long}^2)$$

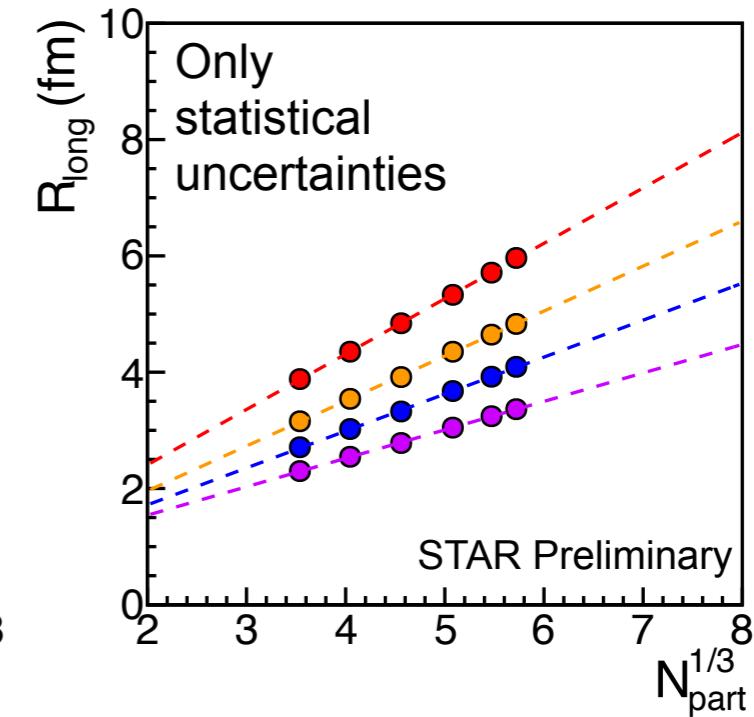
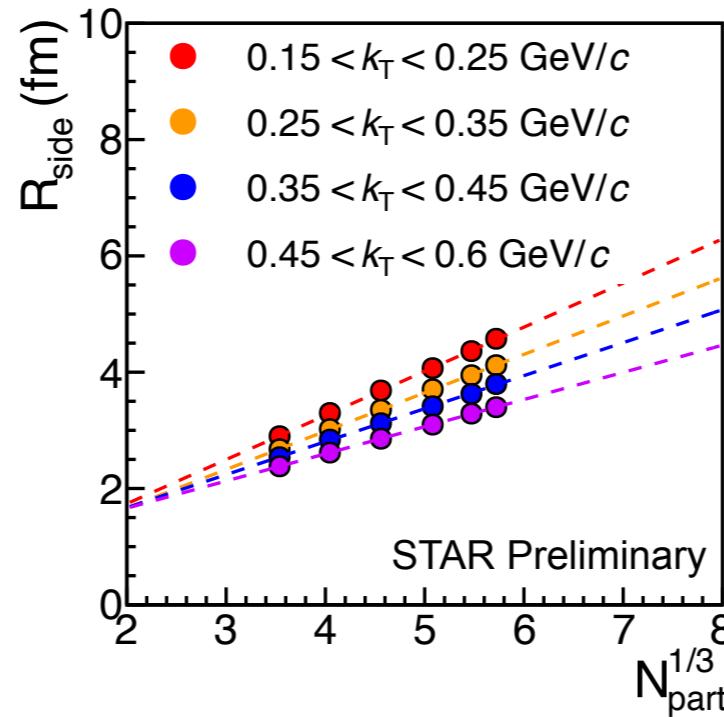
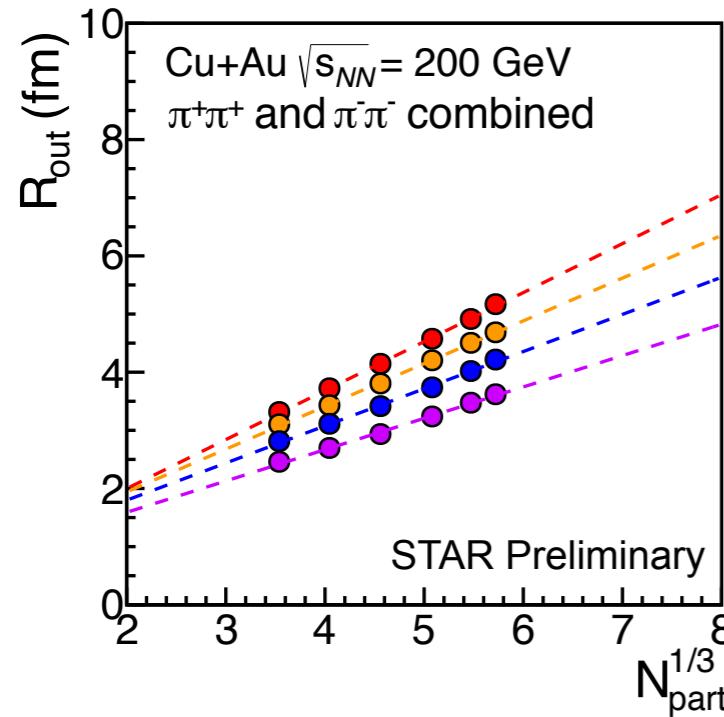
✓ Azimuthal-angle-dependent HBT analysis

$$G(\vec{q}) = \exp(-R_{out}^2 q_{out}^2 - R_{side}^2 q_{side}^2 - R_{long}^2 q_{long}^2 - 2R_{os}^2 q_{out} q_{side} - 2R_{ol}^2 q_{out} q_{long} - 2R_{sl}^2 q_{side} q_{long})$$

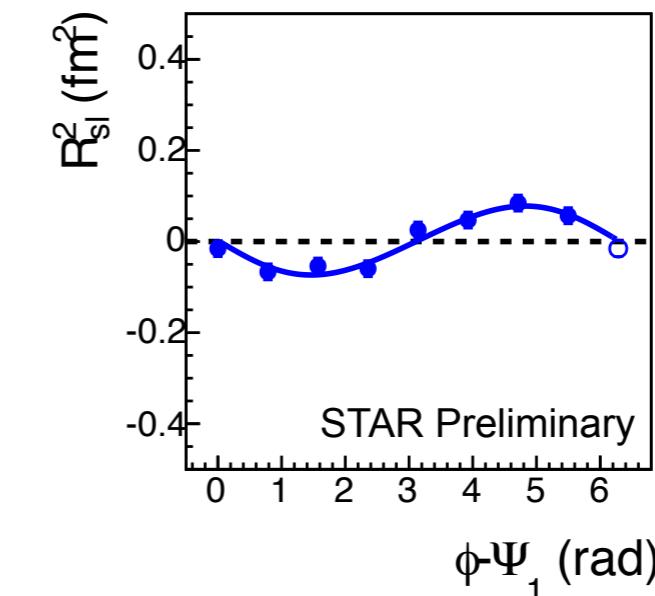
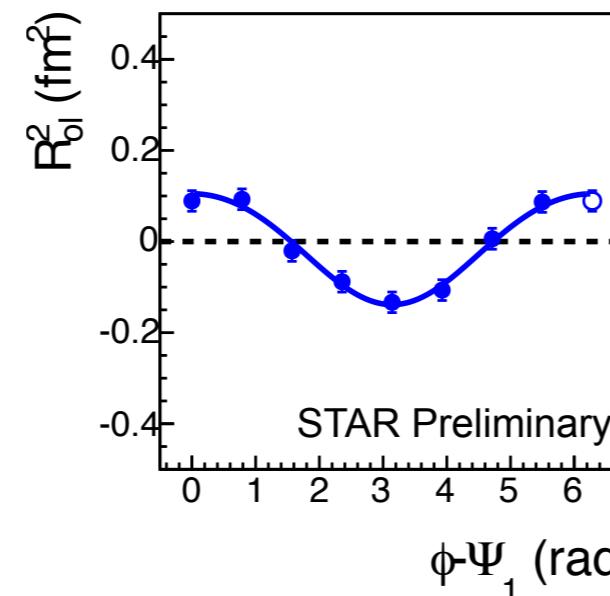
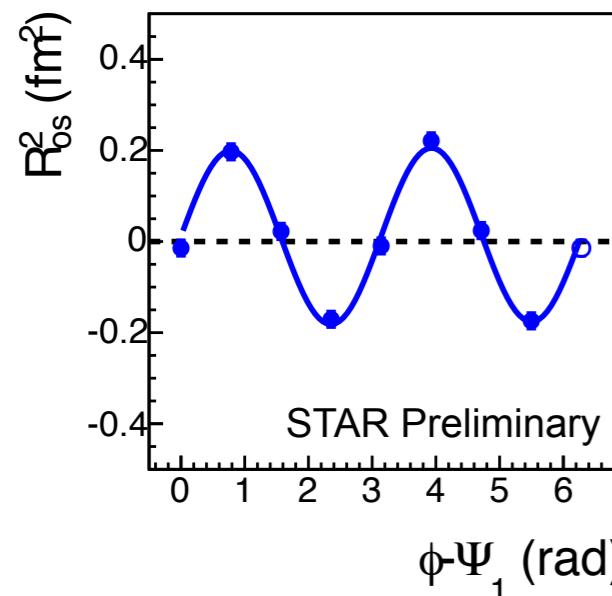
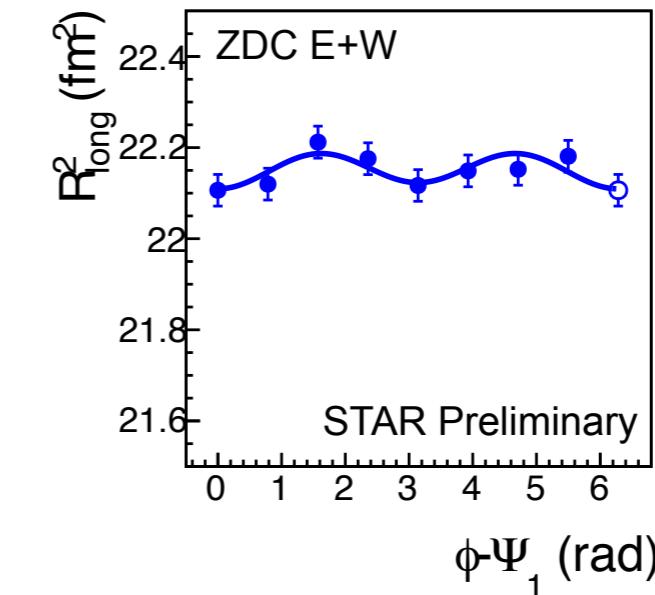
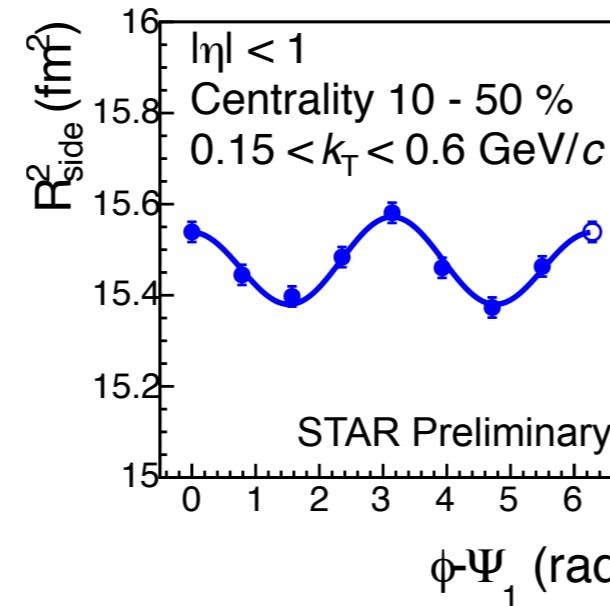
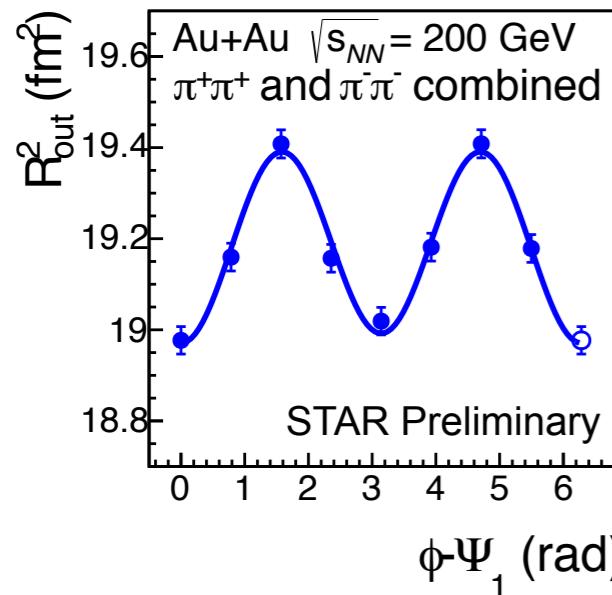
- Event plane reconstruction
 - ✓ ZDC east + west plane used
- Res Ψ_1 ~ 0.35 (Au+Au)
- Res Ψ_1 ~ 0.20 (Cu+Au)



N_{part} dependence

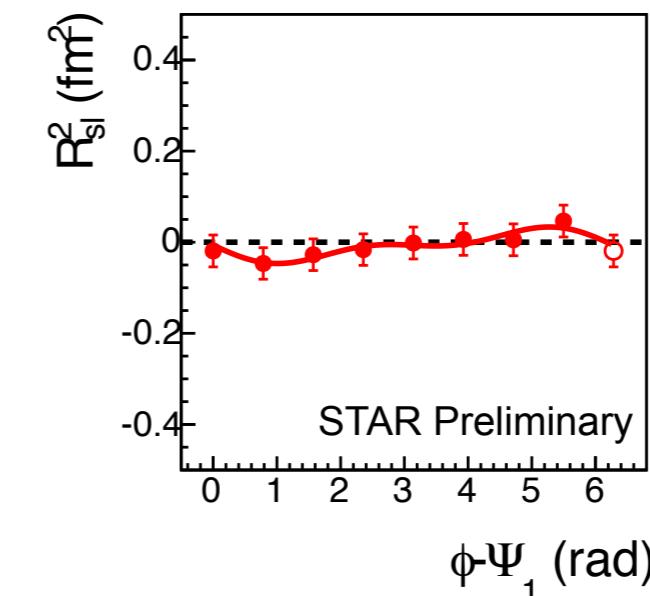
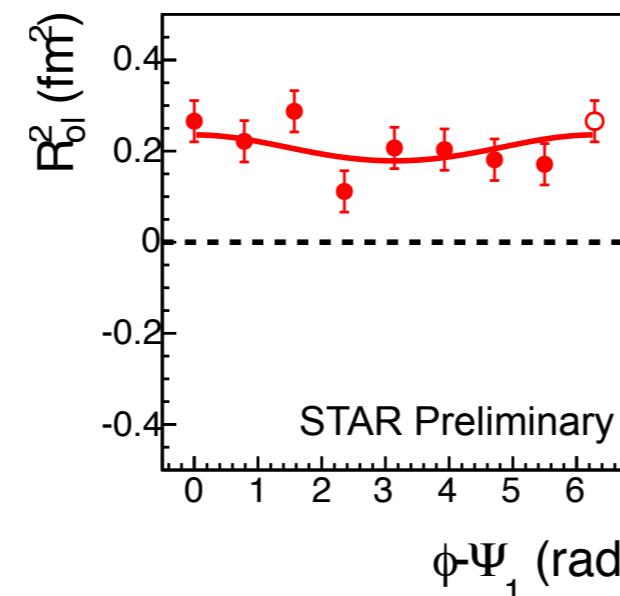
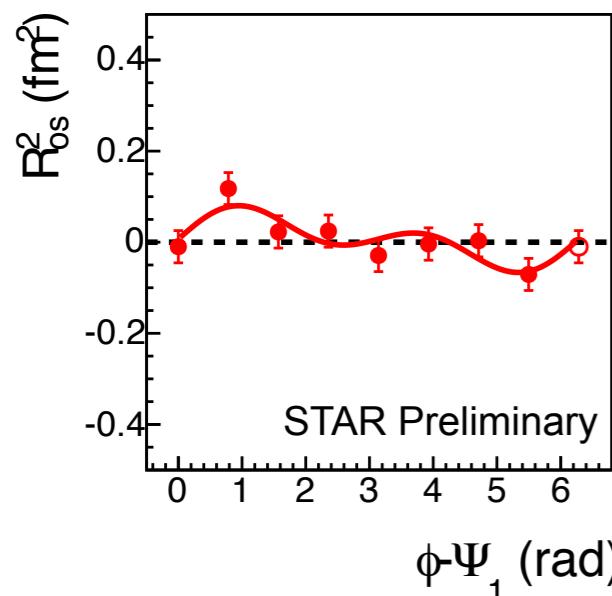
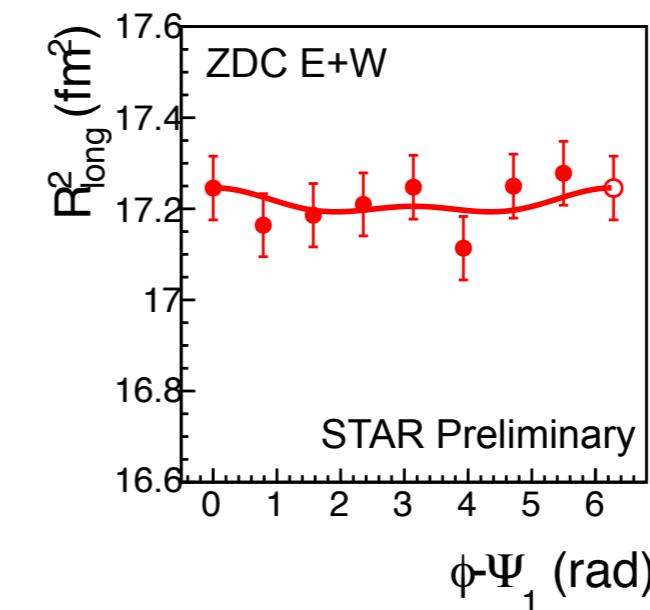
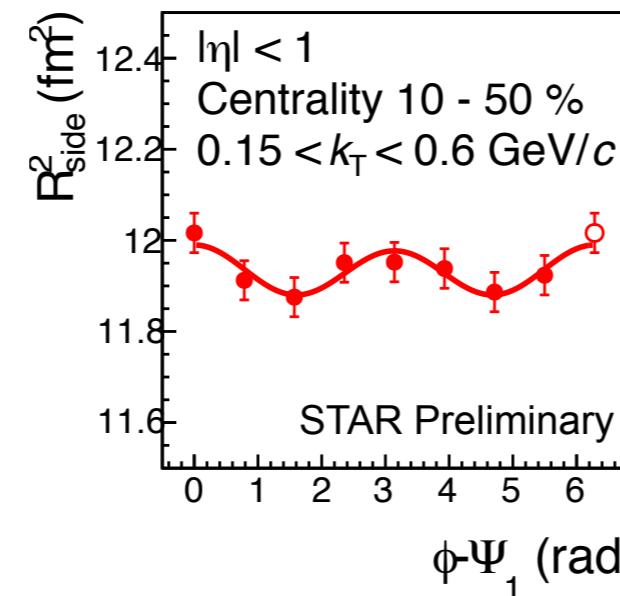
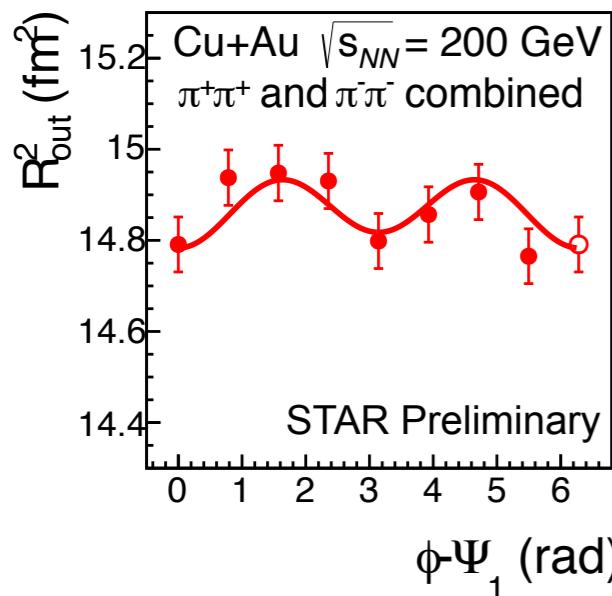


- $N_{part}^{1/3}$ corresponds to the source radius at the collision time
- Checked HBT radii $\propto N_{part}^{1/3}$



- EP resolution correction is not applied

- R_{out} , R_{side} and R_{os} have a 2nd-order oscillation due to the elliptic source shape with respect to Ψ_1
- Small (but $\neq 0$) 1st-order oscillation can be found in R_{ol} and R_{si} due to the source tilt signal
- These results indicate that the source shape at freeze-out is tilted even at the top RHIC energy
 - Note that $\phi - \Psi_1 = 0$ point is replotted at $\phi - \Psi_1 = 2\pi$



- EP resolution correction is not applied

- In R_{ol} , average magnitude is shifted from 0 because center-of-mass rapidity is not 0 (shift to Au-going side ($\eta < 0$))
- Oscillation sign is similar trend with Au+Au
- Oscillation is distorted -> simply due to the poor EP resolution? or density asymmetry affects and distorts oscillation ?
-> More statistics may reveal where this trend comes from

- Fit function:

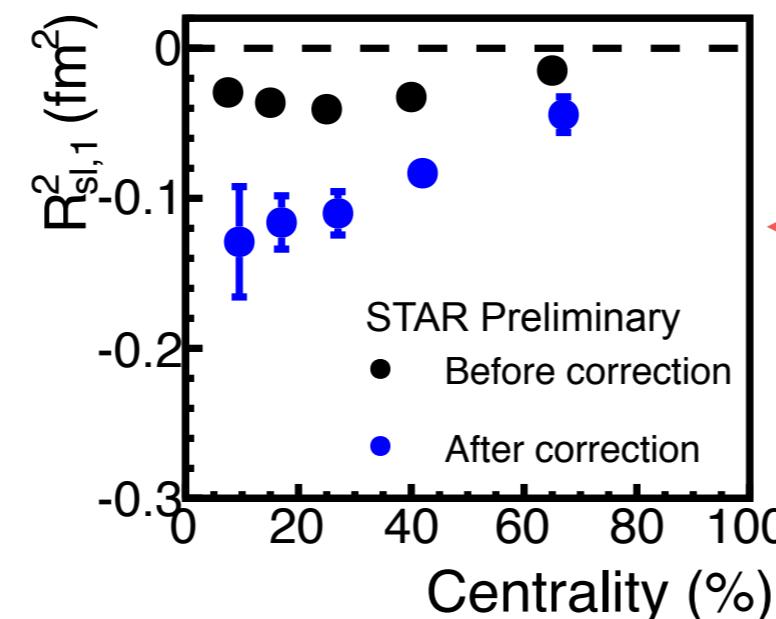
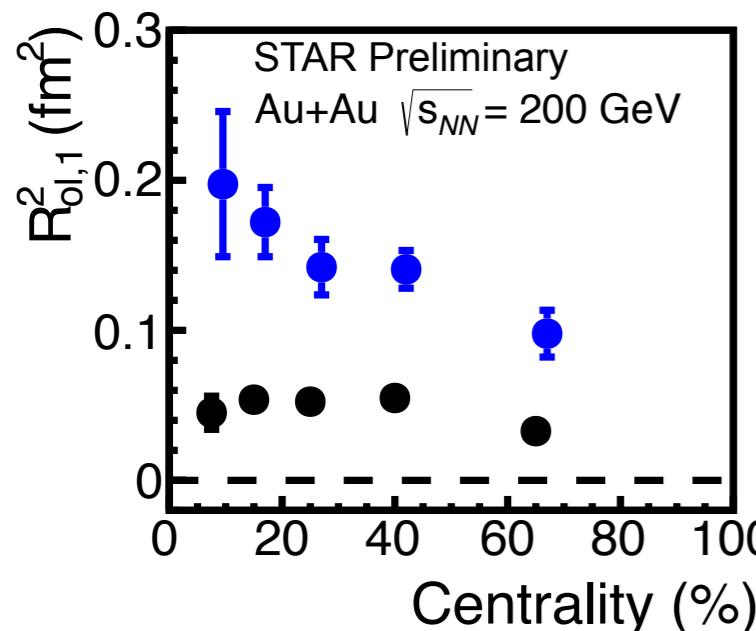
$$R_{\mu,0}^2 + 2 R_{\mu,1}^2 \cos(\varphi - \Psi_1) + 2 R_{\mu,2}^2 \cos(2(\varphi - \Psi_1)), (\mu = o, s, ol)$$

$$R_{\mu,0}^2 + 2 R_{\mu,1}^2 \sin(\varphi - \Psi_1) + 2 R_{\mu,2}^2 \sin(2(\varphi - \Psi_1)), (\mu = os, sl)$$

$$R_{\mu,n}^{2true} = R_{\mu,n}^{2obs} \times \frac{n\Delta/2}{\sin(n\Delta/2)\langle \cos(n(\Psi_1 - \Psi_R)) \rangle}$$

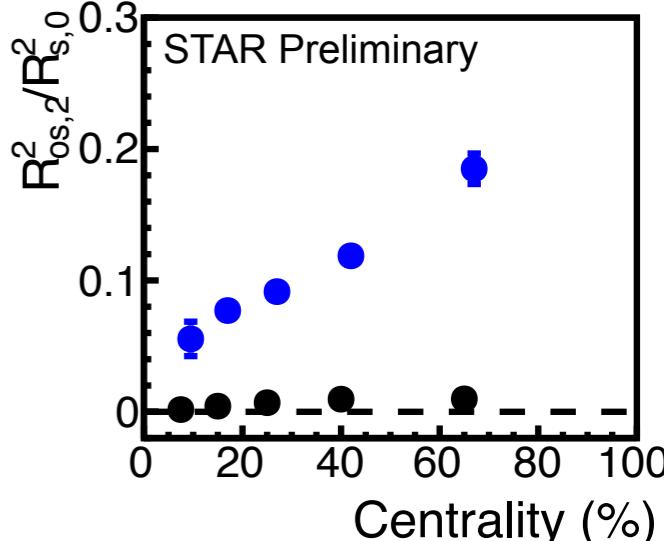
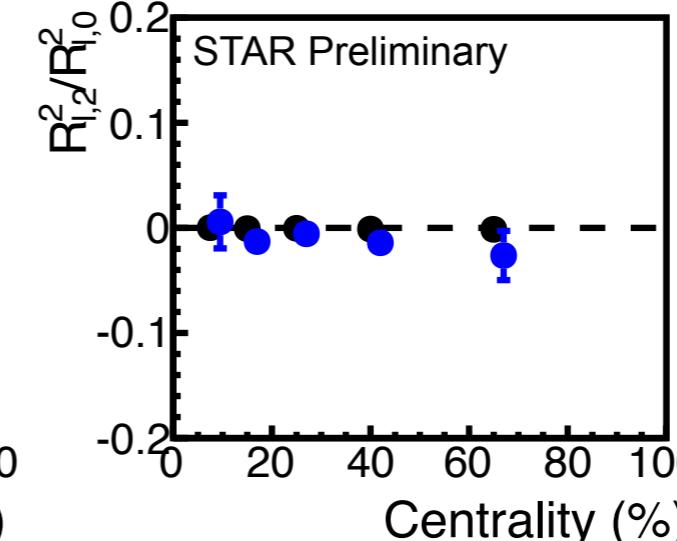
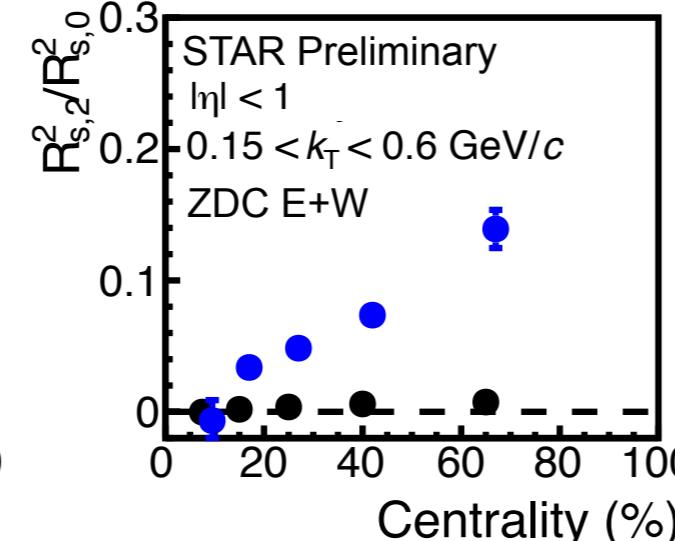
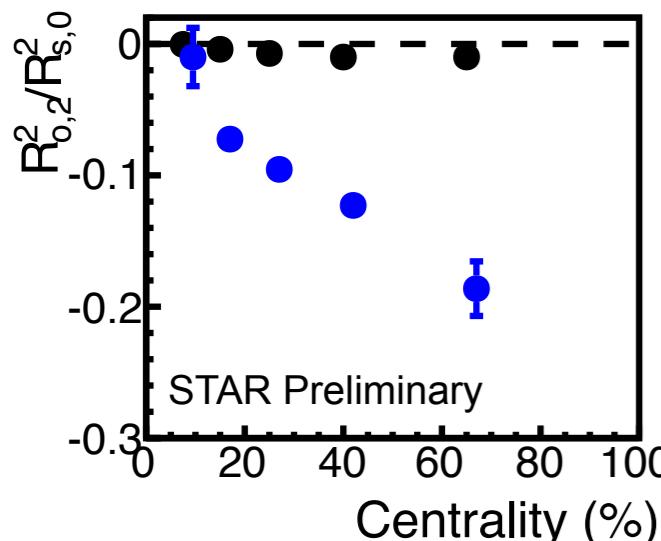
Correction is applied by dividing EP resolution
(for 1st- and 2nd-order resolution)
together with finite azimuthal angle bin width correction

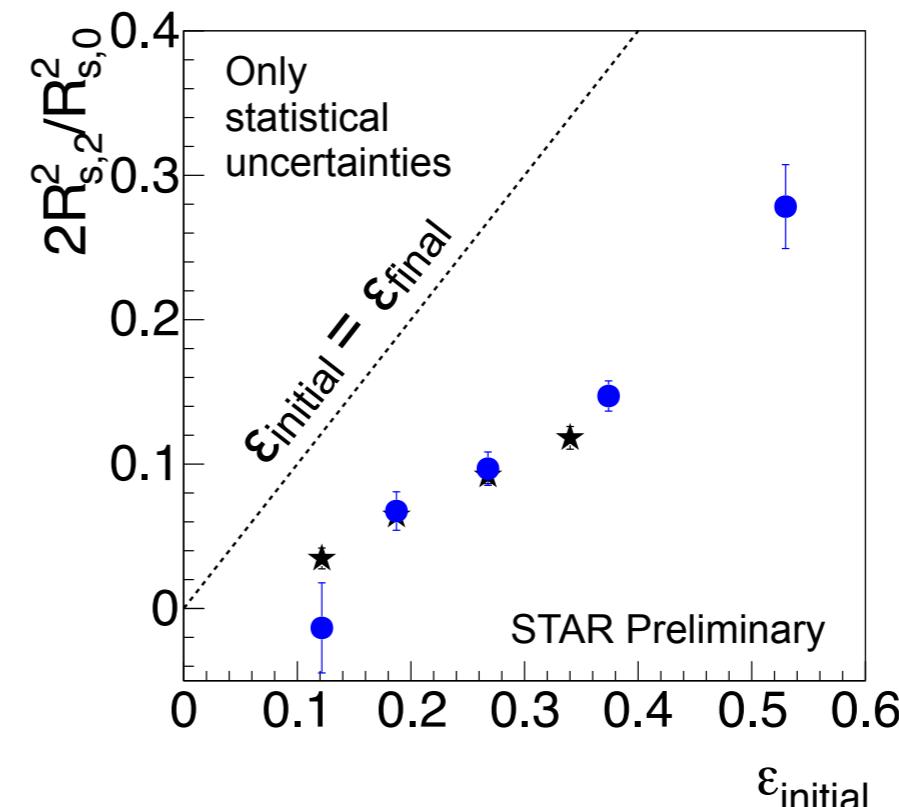
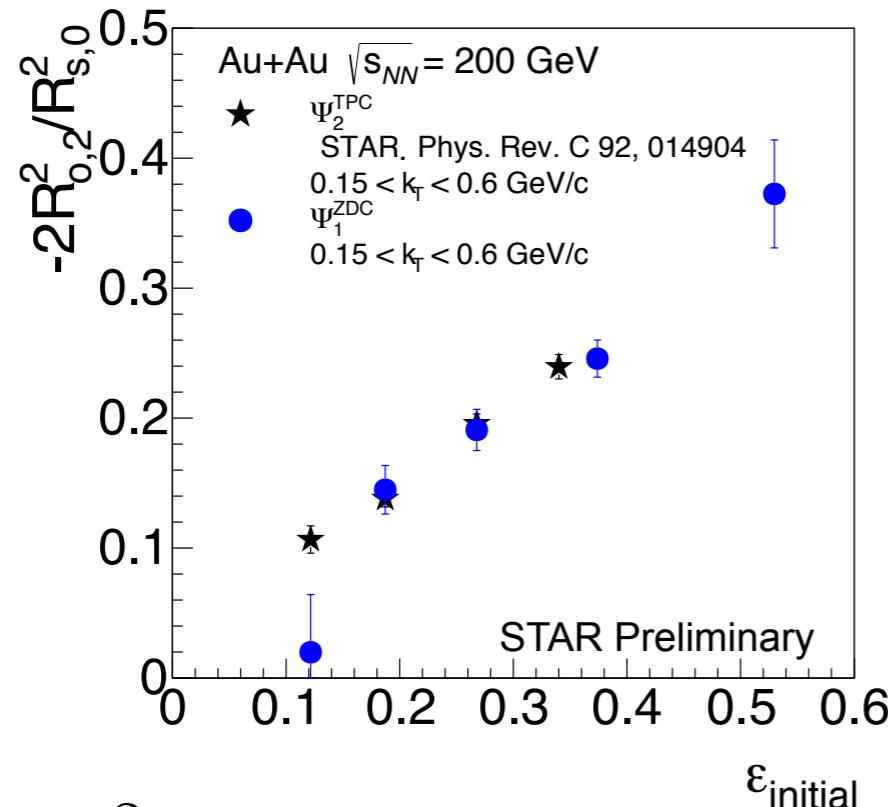
- 1st-order oscillations



- The 1st-order oscillations decrease with increasing centrality
- The 2nd-order oscillations increase with increasing centrality

- The 2nd-order oscillations relative to the average radii

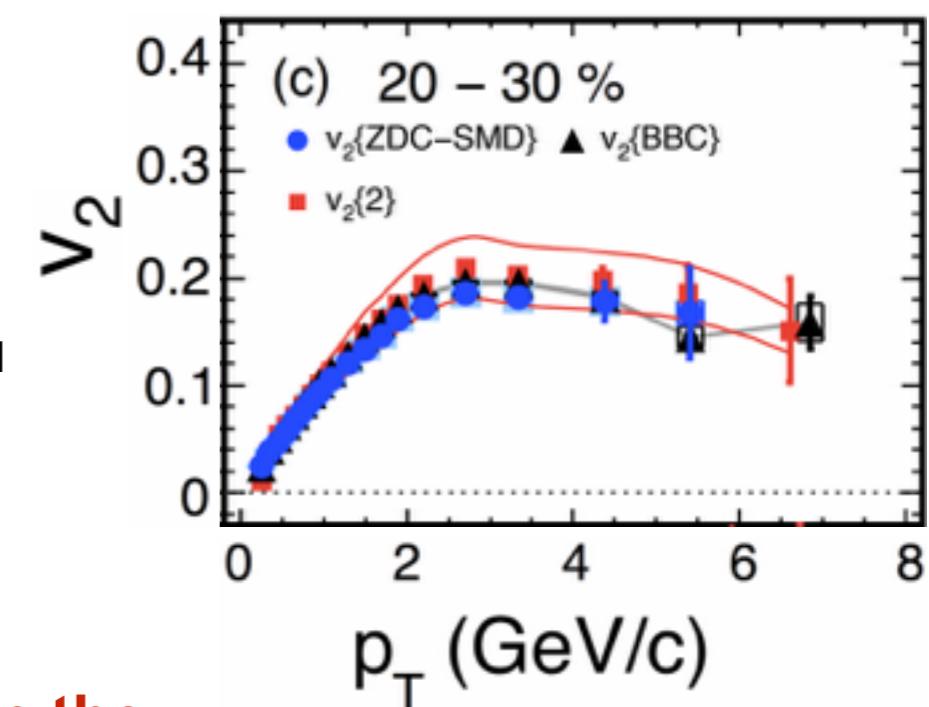


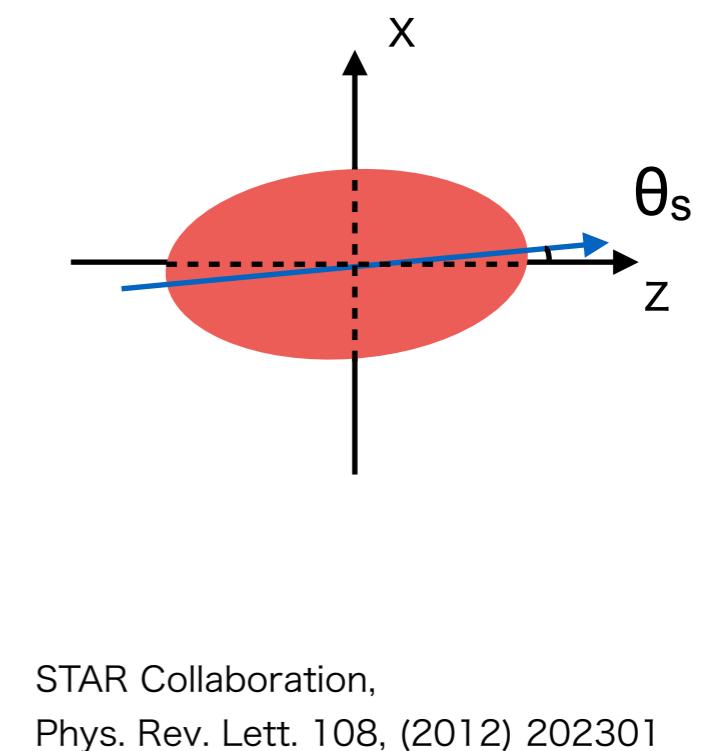
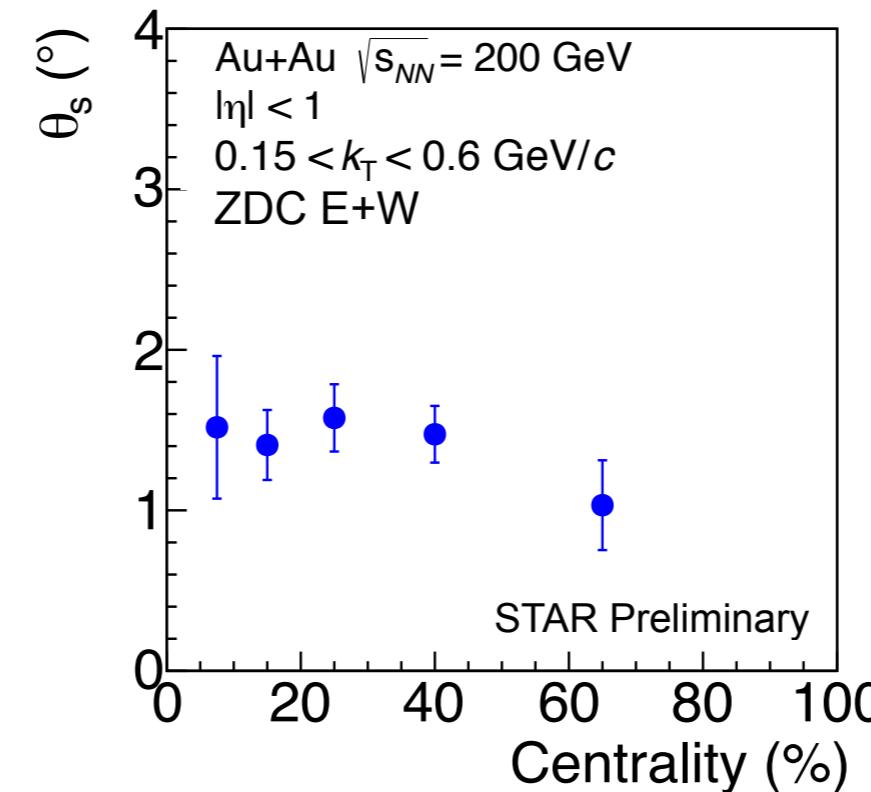
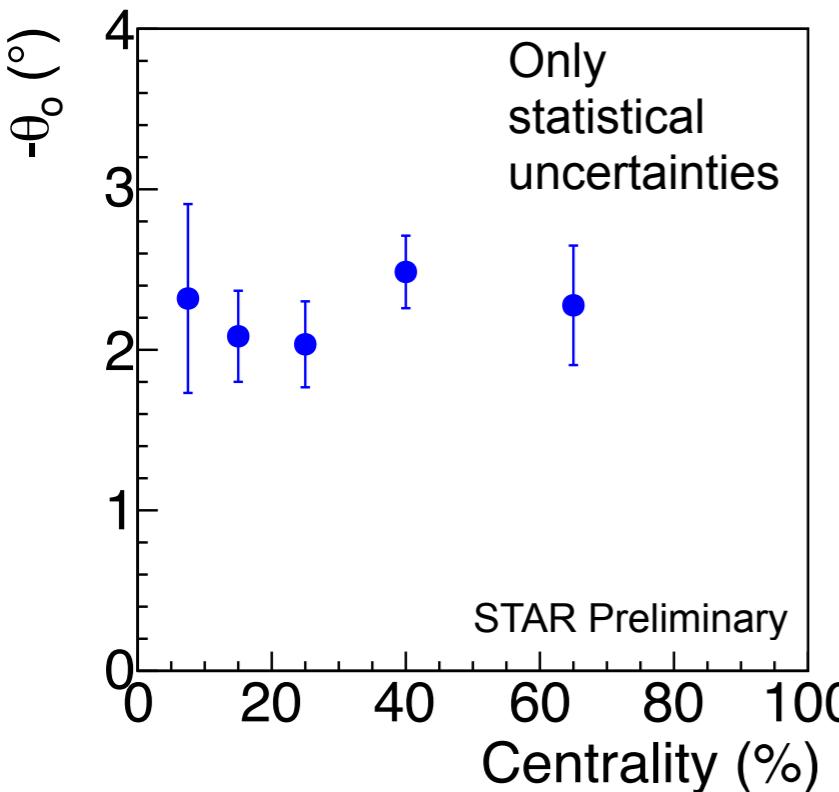


$$\epsilon_{\text{final}} \approx 2 \frac{R_{s,2}^2}{R_{s,0}^2}$$

- $\epsilon_{\text{initial}}$ is from the Glauber simulation

- Final eccentricity ϵ_{final} w.r.t. Ψ_1^{ZDC} is measured
- $2R_{s,2}^2/R_{s,0}^2$ purely corresponds final eccentricity ϵ_{final}
- Momentum space anisotropy is same ($v_2\{2\} \approx v_2\{\text{ZDC}\}$) at low p_T
- Final eccentricity is smaller than initial eccentricity, but still remain out-of-plane extended ($\epsilon_{\text{final}} > 0$)
- Final eccentricity shows a rough agreement between the participant Ψ_2 and the spectator Ψ_1 planes

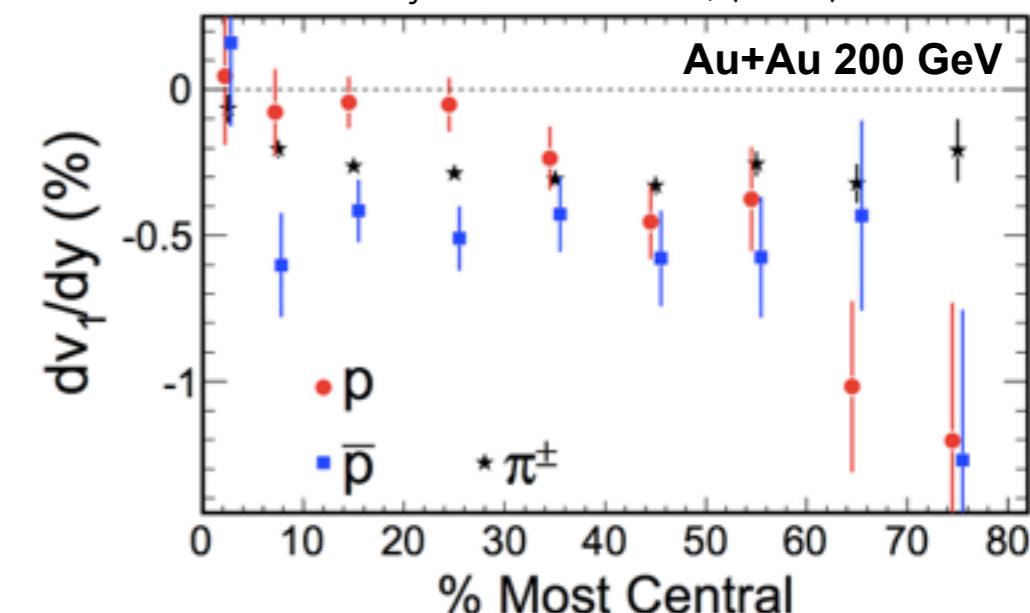




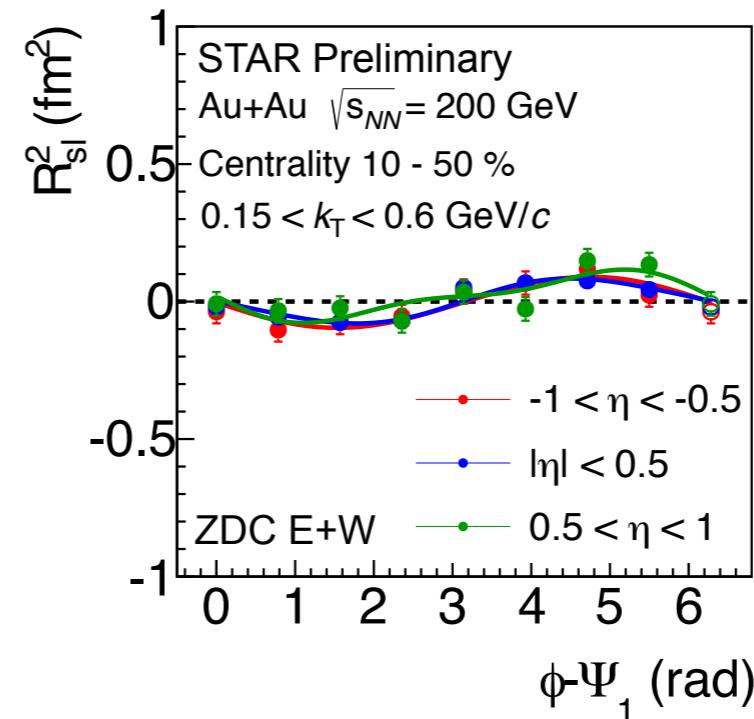
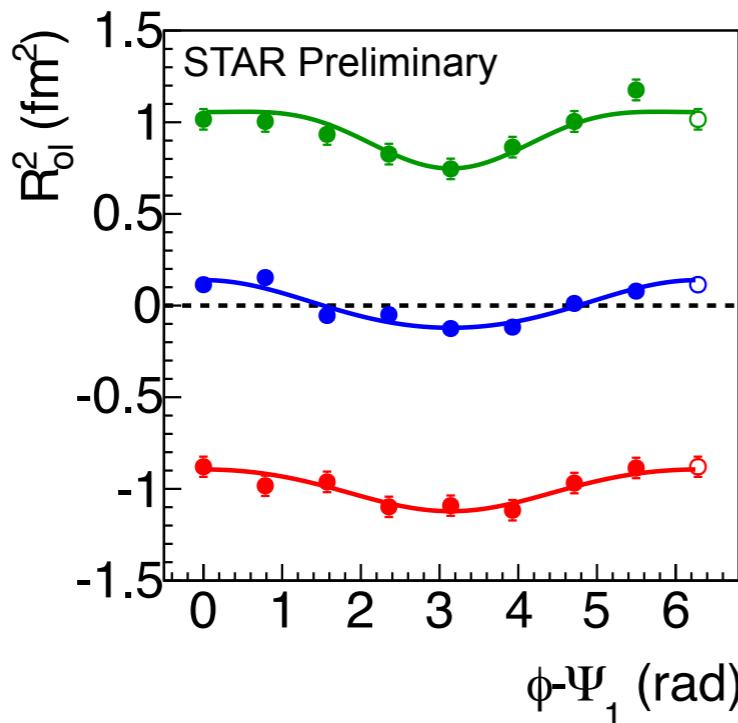
Tilt angle

$$\theta_s = \frac{1}{2} \tan^{-1} \left(\frac{-4R_{sl,1}^2}{R_{l,0}^2 - R_{s,0}^2 + 2R_{s,2}^2} \right)$$

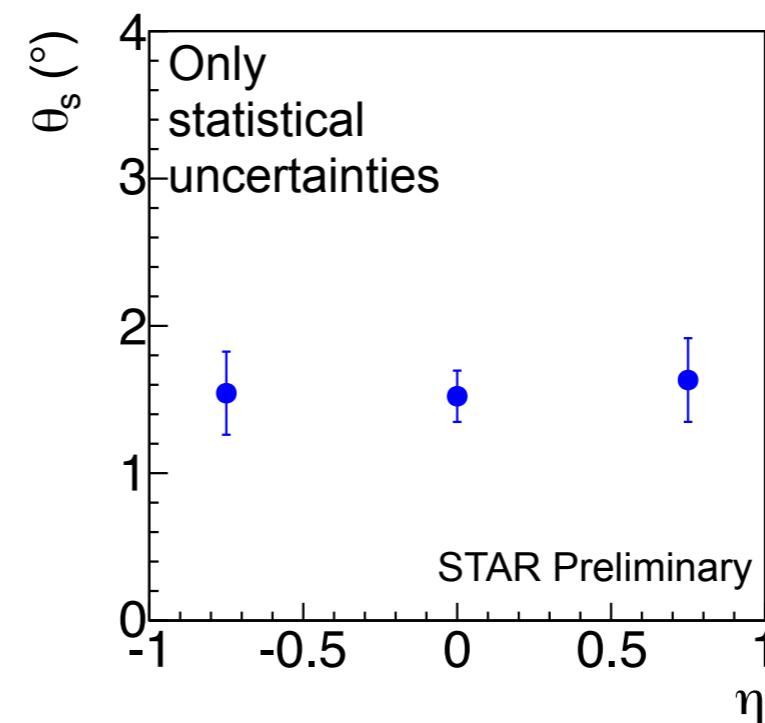
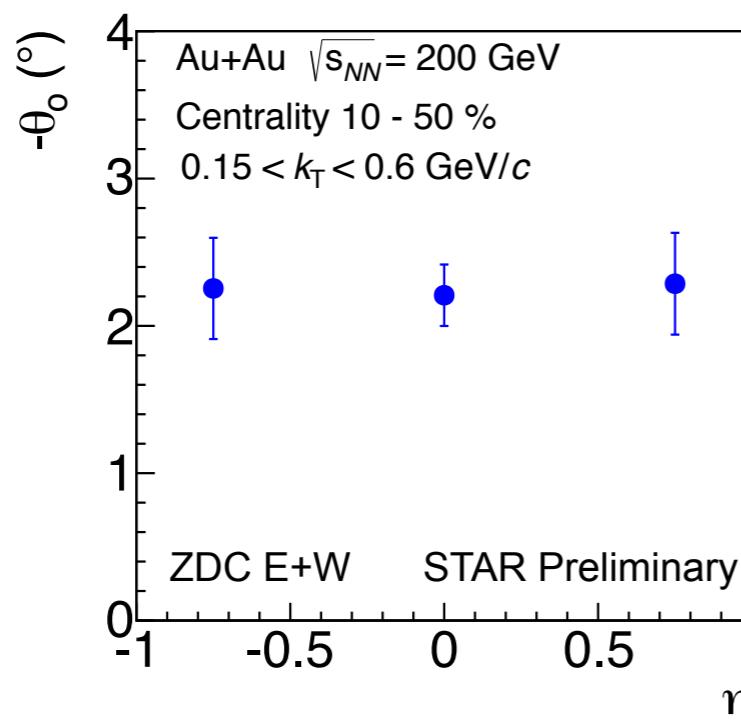
$$\theta_o = \frac{1}{2} \tan^{-1} \left(\frac{-4R_{ol,1}^2}{R_{l,0}^2 - R_{s,0}^2 + 2R_{s,2}^2} \right)$$



- θ_s purely corresponds to geometrical tilt (only side and long info. used)
- θ_o is that R_{ol,1} is used instead of R_{sl,1}
- Centrality dependence is very weak or absent
- Tilt angle shows similar trend to that of centrality dependence of v₁ slope



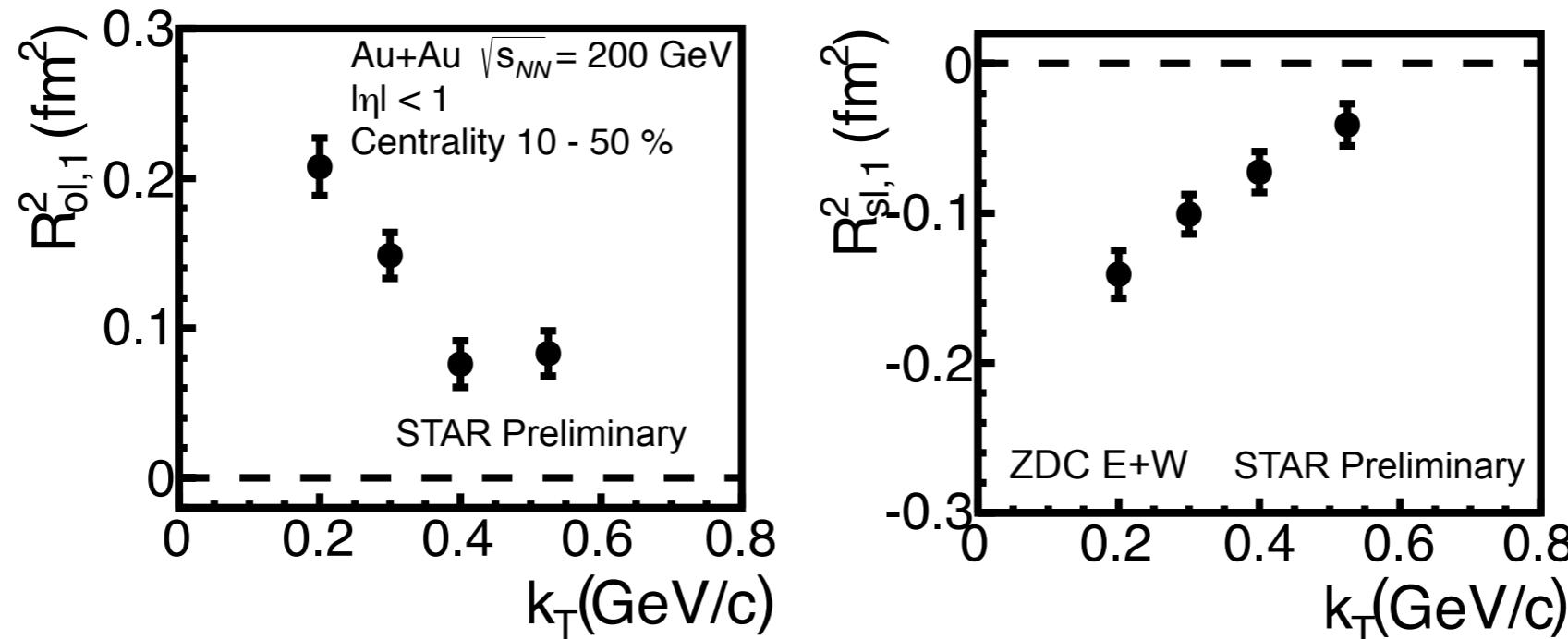
- EP resolution correction is not applied



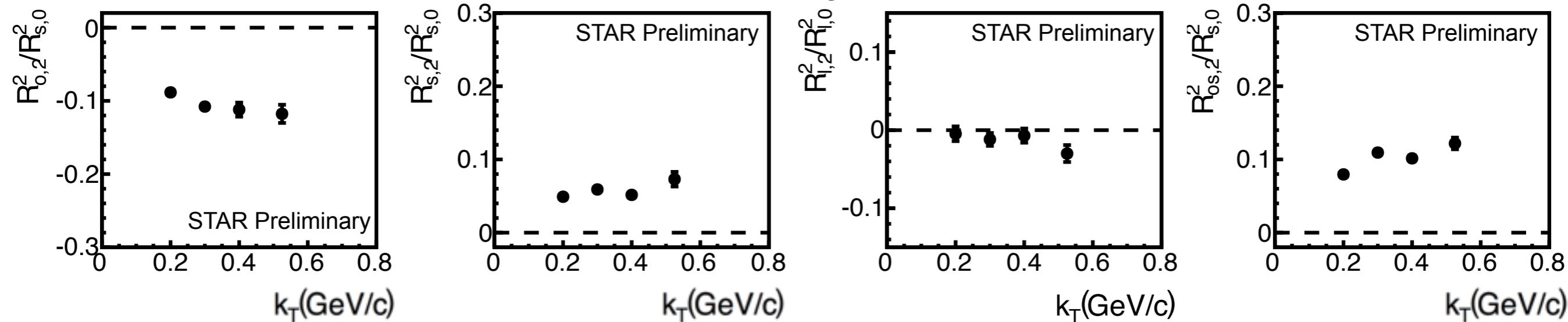
- The average R_{ol} value shifts when going away from center-of-mass rapidity
- The oscillation amplitude does not have significant dependence on η
- Same tilt angle can be seen in all η region
- These results are consistent with the linear dv_1/dy slope at midrapidity

k_T dependence of HBT radii

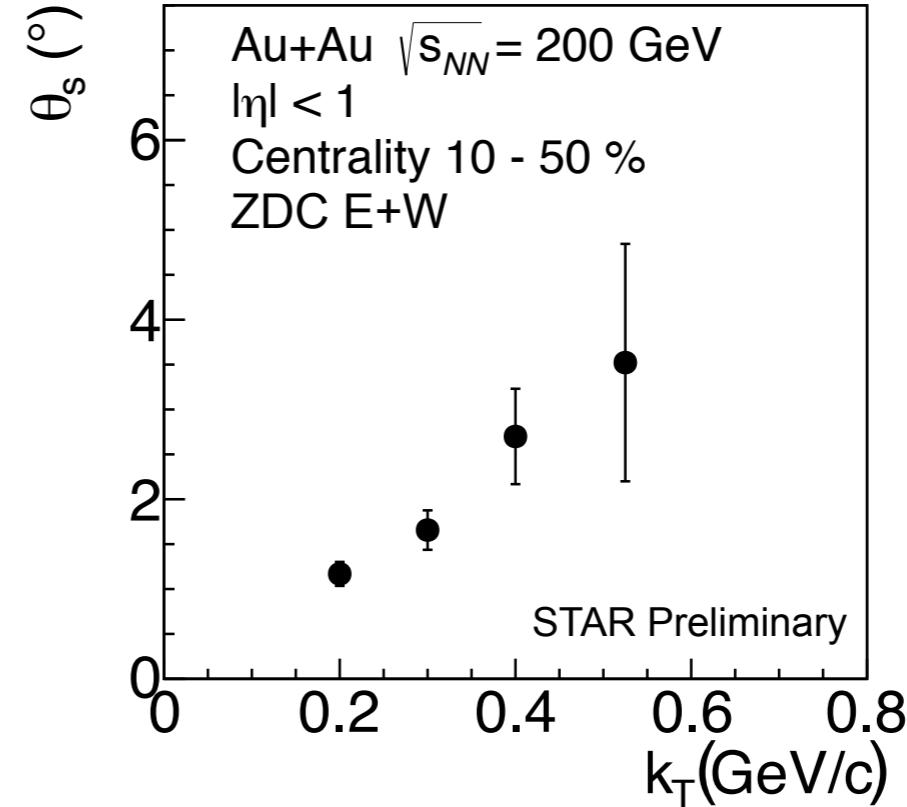
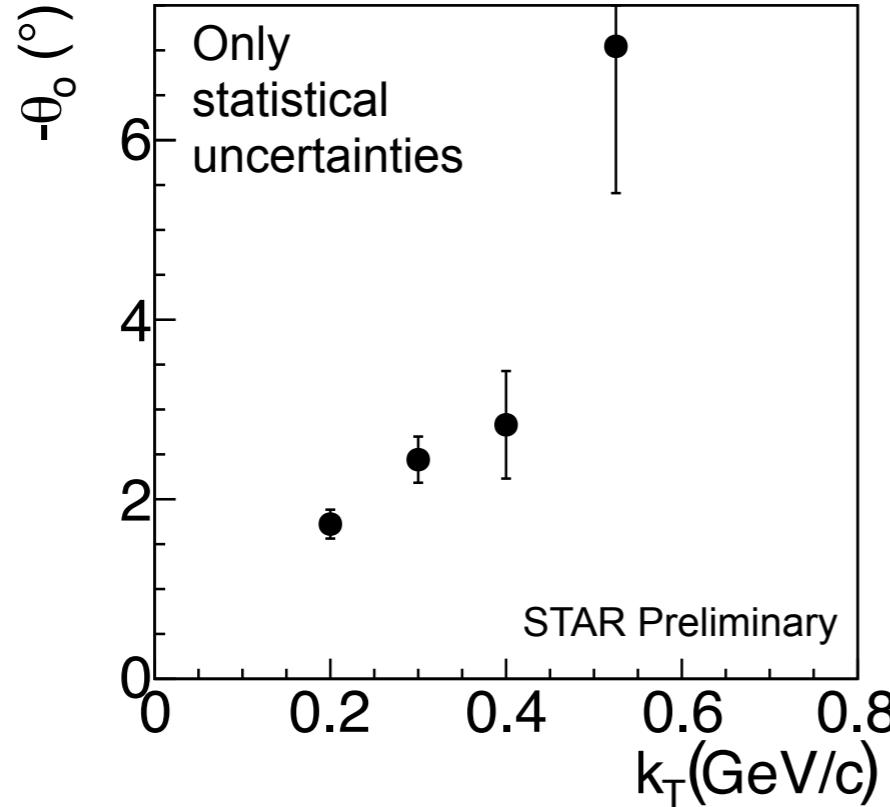
- The 1st-order oscillations



- The 2nd-order oscillations relative to the average radii



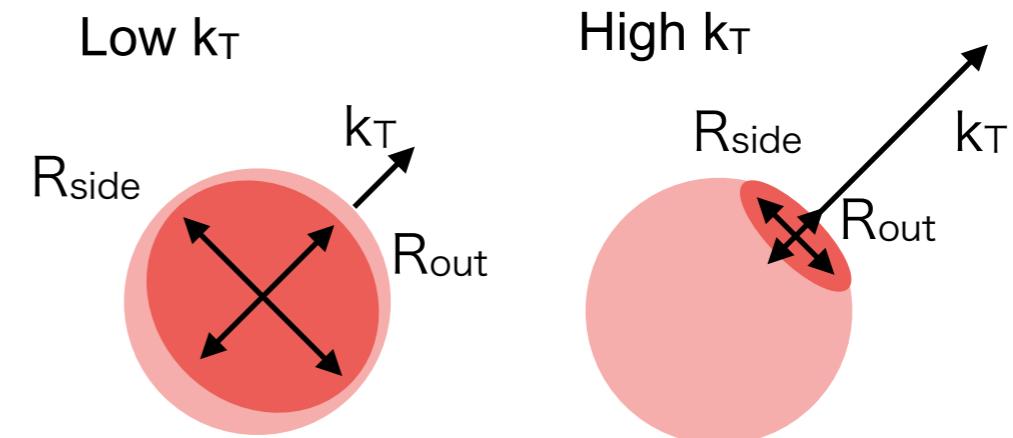
- The 1st-order oscillation magnitude $R_{sl,1}$ $R_{ol,1}$ seems to decrease with increasing k_T
 - > The same trend as the centrality dependence
- The 2nd-order oscillations $R_{o,2}/R_{s,0}$, $R_{s,2}/R_{s,0}$, $R_{os,2}/R_{s,0}$ have weak k_T dependence compared to the centrality dependence
 - > It means final eccentricity (ϵ_{final}) shows very weak dependence on k_T (in the measured k_T range)



Tilt angle

$$\theta_s = \frac{1}{2} \tan^{-1} \left(\frac{-4R_{sl,1}^2}{R_{l,0}^2 - R_{s,0}^2 + 2R_{s,2}^2} \right)$$

$$\theta_o = \frac{1}{2} \tan^{-1} \left(\frac{-4R_{ol,1}^2}{R_{l,0}^2 - R_{s,0}^2 + 2R_{s,2}^2} \right)$$



- Unlike centrality dependence, tilt angle seems to increase with k_T
- As the k_T increases, HBT radii decrease because of collective radial flow
 -> $(R_{l,0}^2 - R_{s,0}^2 + 2R_{s,2}^2)$ decreases faster than $R_{sl,1}$, contributing to an increase in the θ_s

- **Azimuthal-angle dependence of HBT radii w.r.t. Ψ_1**
 - ✓ The 1st-order oscillations of R_{ol} and R_{sl} have been measured in both Au+Au and Cu+Au collisions at 200 GeV
- Final eccentricity (Au+Au 200 GeV)
 - ✓ Final eccentricity w.r.t. Ψ_1^{ZDC} is consistent with that measured by Ψ_2^{TPC}
 - ✓ Final eccentricity shows a centrality dependence and weakly depends on k_T
- Tilt angle (Au+Au 200 GeV)
 - ✓ Centrality dependence of tilt angle is very weak or absent and tilt angle seems to increase with increasing k_T
 - ✓ Tilt angle seems to be η -independent within the TPC acceptance ($|\eta| < 1$)

Outlook

- Estimate systematic uncertainties
- Examine beam-energy dependence in BES-II with high statistics and good event plane resolution due to the installation of Event Plane Detector (EPD)

Back up

Au+Au 200 GeV

Run11 minimum bias

- Events ~ 430 M

Event selection

- $|v_z| < 25 \text{ cm}$
- $|v_r| < 2 \text{ cm}$
- $|v_z - v_z^{\text{vpd}}| < 3 \text{ cm}$

Track selection

- $0.15 < p_T < 0.8 \text{ GeV}/c$
- $|\eta| < 1$
- $n\text{HitsFit} \geq 15$
- $n\text{HitsFit}/n\text{HitsPoss} \geq 0.52$
- $\text{DCA} < 3 \text{ cm}$

PID

- Tof Matched track
 - for $0.15 < p < 0.3 \text{ GeV}/c$
 $m_{\pi}^2 \pm 2\sigma, |n\sigma_{\pi}| < 3$
 - for $0.3 < p < 2.8 \text{ GeV}$
 $m_{\pi}^2 \pm 2\sigma, \text{veto } m_k^2 \pm 2\sigma, |n\sigma_{\pi}| < 3$
- TPC only
 - for $0.15 < p < 0.5 \text{ GeV}/c$
 $|n\sigma_{\pi}| < 2$
 - for $0.5 < p < 0.7 \text{ GeV}/c$
 $|n\sigma_{\pi}| < 2, |n\sigma_k| > 2$

Cu+Au 200 GeV

Run12 minimum bias

- Events: ~ 45 M

Event Selection

- $|v_z| < 30 \text{ cm}$
- $|v_r| < 2 \text{ cm}$
- $|v_z - v_z^{\text{vpd}}| < 3 \text{ cm}$

Track selection

- $0.15 < p_T < 0.8 \text{ GeV}/c$
- $|\eta| < 1$
- $n\text{HitsFit} \geq 15$
- $n\text{HitsdEdx} \geq 10$
- $n\text{HitsFit}/n\text{HitsPoss} \geq 0.52$
- $\text{DCA} < 3 \text{ cm}$

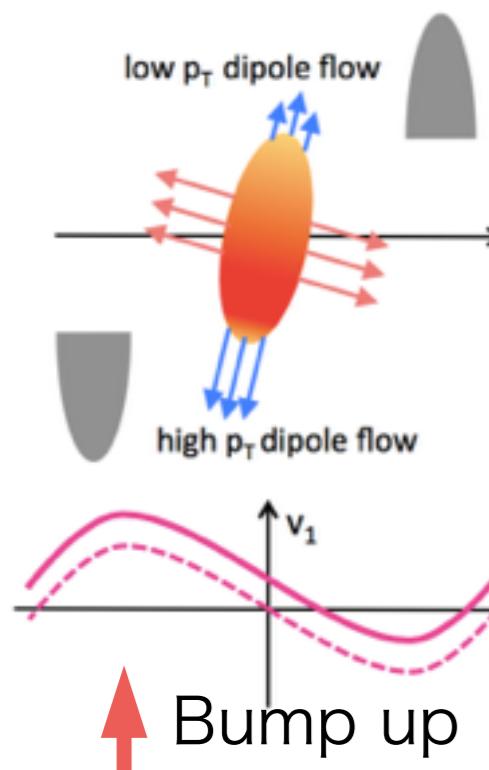
PID

- Tof Matched track
 - for $0.15 < p < 0.3 \text{ GeV}/c$
 $m_{\pi}^2 \pm 2\sigma, |n\sigma_{\pi}| < 3$
 - for $0.3 < p < 2.8 \text{ GeV}$
 $m_{\pi}^2 \pm 2\sigma, \text{veto } m_k^2 \pm 2\sigma, |n\sigma_{\pi}| < 3$
- TPC only
 - for $0.15 < p < 0.5 \text{ GeV}/c$
 $|n\sigma_{\pi}| < 2$
 - for $0.5 < p < 0.7 \text{ GeV}/c$
 $|n\sigma_{\pi}| < 2, |n\sigma_k| > 2$

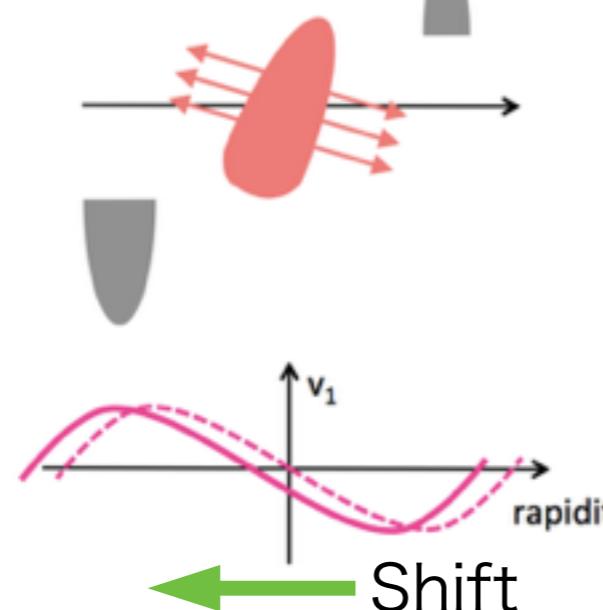
Cu+Au collisions

STAR Collaboration, Phys. Rev. C 98 (2018) 14915

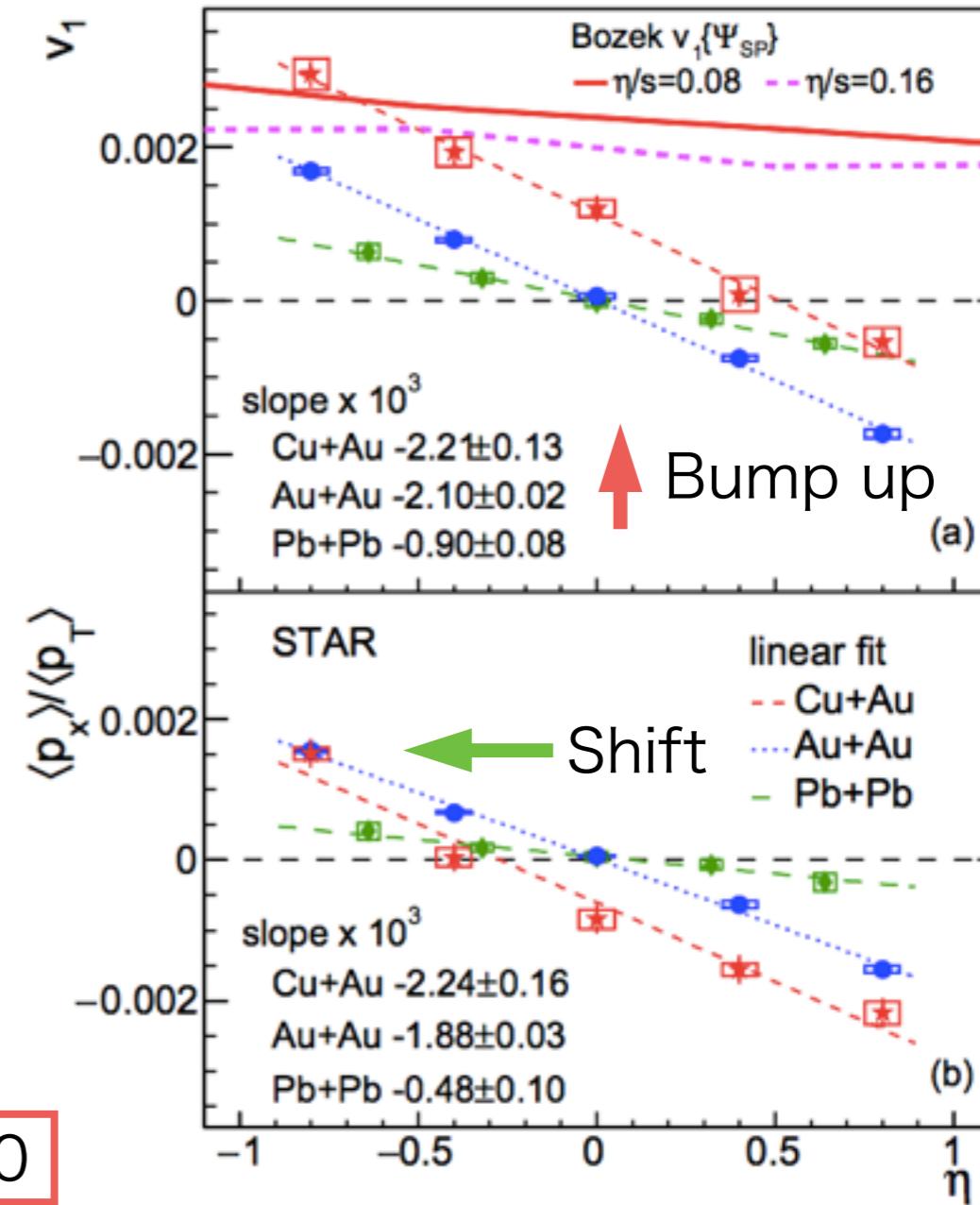
(b) tilted source
+ asymmetric density gradient



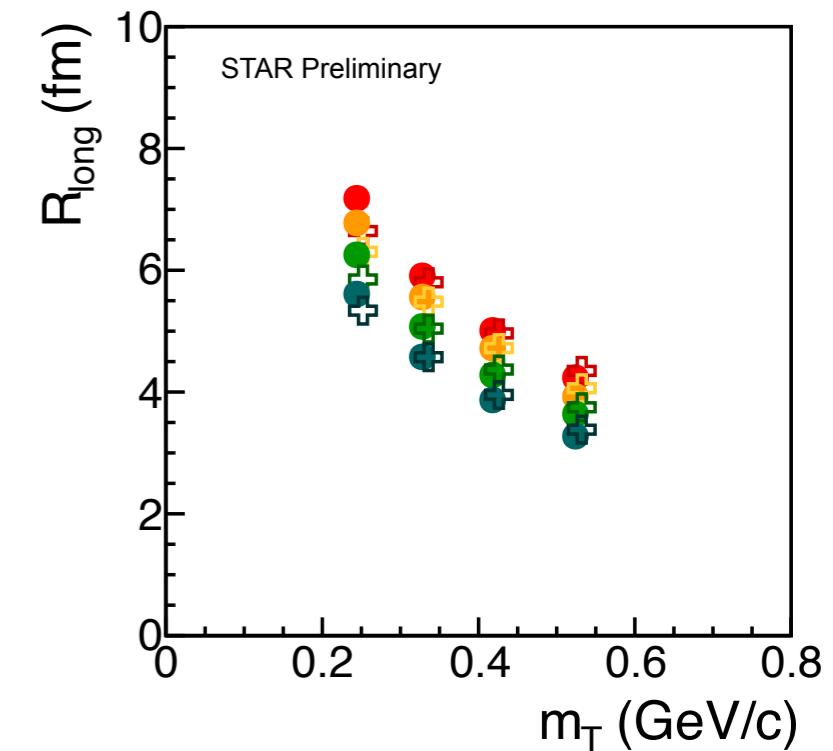
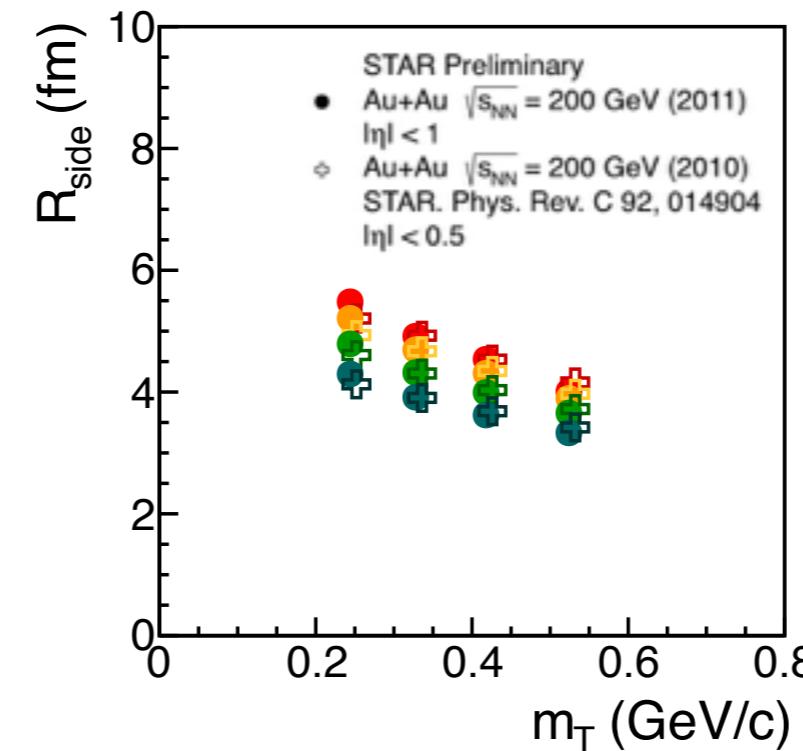
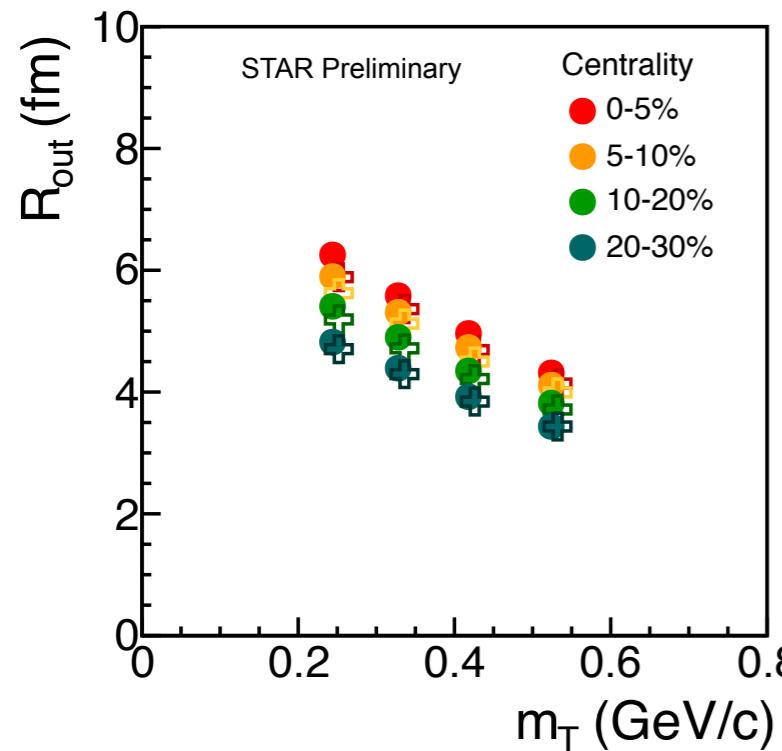
(c) tilted source
+ asymmetric participants



Note: $\langle p_x \rangle$ contribution from dipole flow ~ 0



- ✓ Cu+Au has asymmetric density gradient and it causes “dipole flow”.
 - > It bump up directed flow signals. (Fig. (b))
- ✓ In addition, Cu+Au collisions have a different number of participants between forward and backward directions.
 - > It shifts directed flow to the center-of-mass rapidity (Fig.(c))



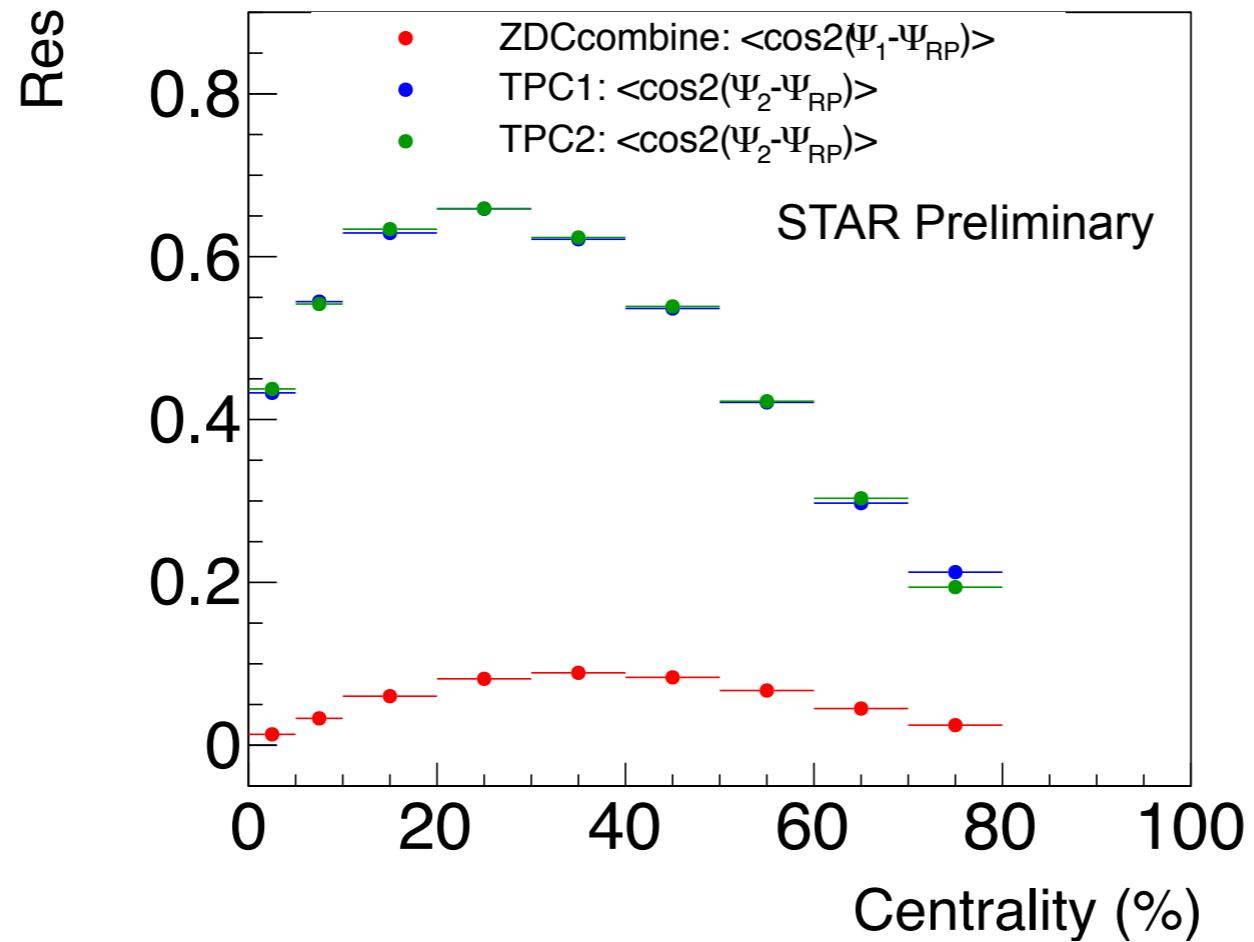
✓ Average radii are consistent within systematic uncertainties

Source	R_{out}	R_{side}	R_{long}	ϵ_F
Coulomb	4%	3%	4%	0.004
Fit Range	5%	5%	5%	0.002
FMH	7%	3%	3%	0.003
Total	9.5%	6.5%	7%	0.005

Systematic errors of average radii in
STAR Collaboration, Phys. Rev. C 92 (2015) 014904

- Au+Au 200 GeV

- $\langle \cos 2(\Psi_1 - \Psi_{RP}) \rangle$: 3 subevent method



There is a choice for EP resolution correction

- $\langle \cos(\Psi_1 - \Psi_{RP}) \rangle$ 2 subevent or 3 subevent method
- $\langle \cos 2(\Psi_1 - \Psi_{RP}) \rangle$ 2 subevent or 3 subevent method

S_{11} : source variance in x direction

S_{33} : source variance in z direction

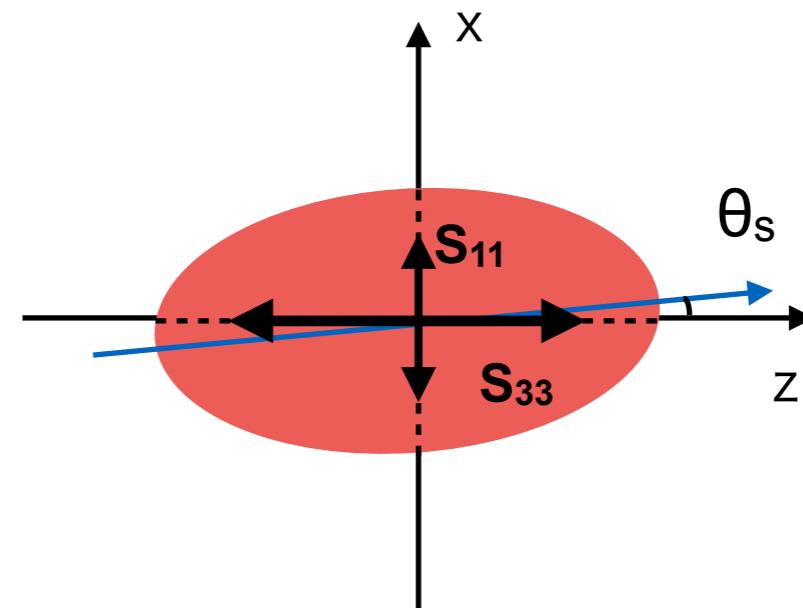
S_{13} : x-z covariance

$$S' = R_y^\dagger(\theta_s) \cdot S \cdot R_y(\theta_s)$$

$S_{\mu\nu}$: Spatial correlation tensor

$R_y(\theta_s)$: Rotation matrix

$$\theta_s = \frac{1}{2} \tan^{-1} \left(\frac{2S_{13}}{S_{33} - S_{11}} \right)$$



Rotating the spatial correlation tensor $S_{\mu\nu}$ by θ_s yields a purely diagonal tensor S'

$$R_{out}^2(\phi) = \frac{1}{2}(S_{11} + S_{22}) - \frac{1}{2}(S_{22} - S_{11})\cos(2\phi) + \beta_T^2 S_{00}$$

$$R_{os}^2(\phi) = \frac{1}{2}(S_{22} - S_{11})\sin(2\phi)$$

$$R_{side}^2(\phi) = \frac{1}{2}(S_{11} + S_{22}) + \frac{1}{2}(S_{22} - S_{11})\cos(2\phi)$$

$$R_{ol}^2(\phi) = S_{13}\cos(\phi)$$

$$R_{long}^2(\phi) = S_{33} + \beta_l^2 S_{00}$$

$$R_{sl}^2(\phi) = -S_{13}\sin(\phi)$$

Express in out-side-long coordinate

$$\theta_s = \frac{1}{2} \tan^{-1} \left(\frac{-4R_{sl,1}^2}{R_{l,0}^2 - R_{s,0}^2 + 2R_{s,2}^2} \right)$$