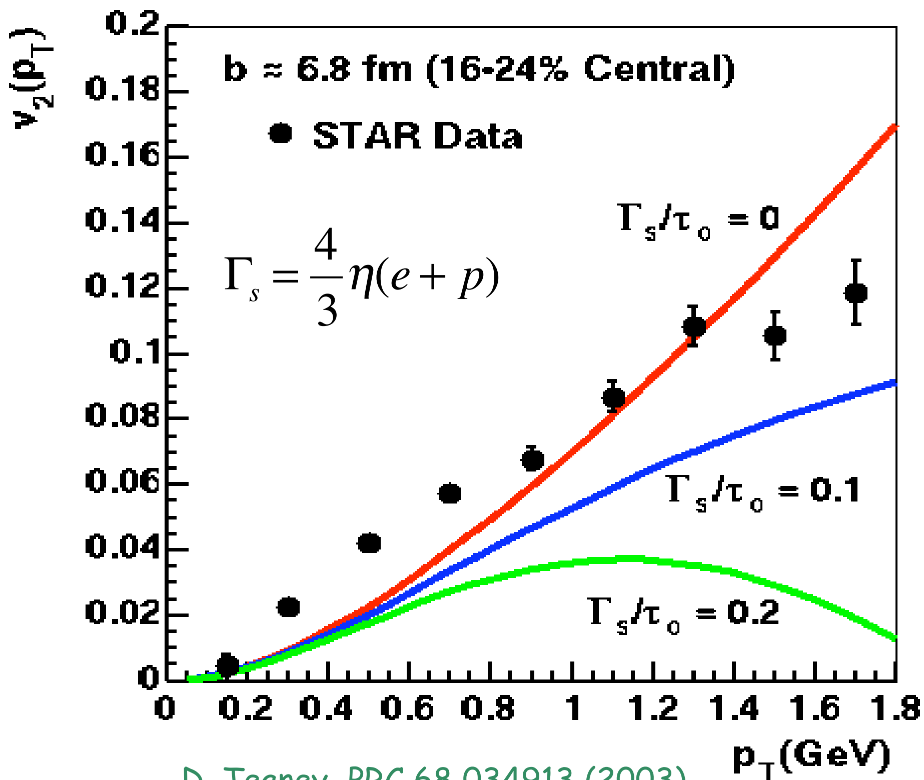
BROOKHAVEN NATIONAL LABORATORY

Flow Results and Hints of Incomplete Thermalization

Aihong Tang for the STAR Collaboration

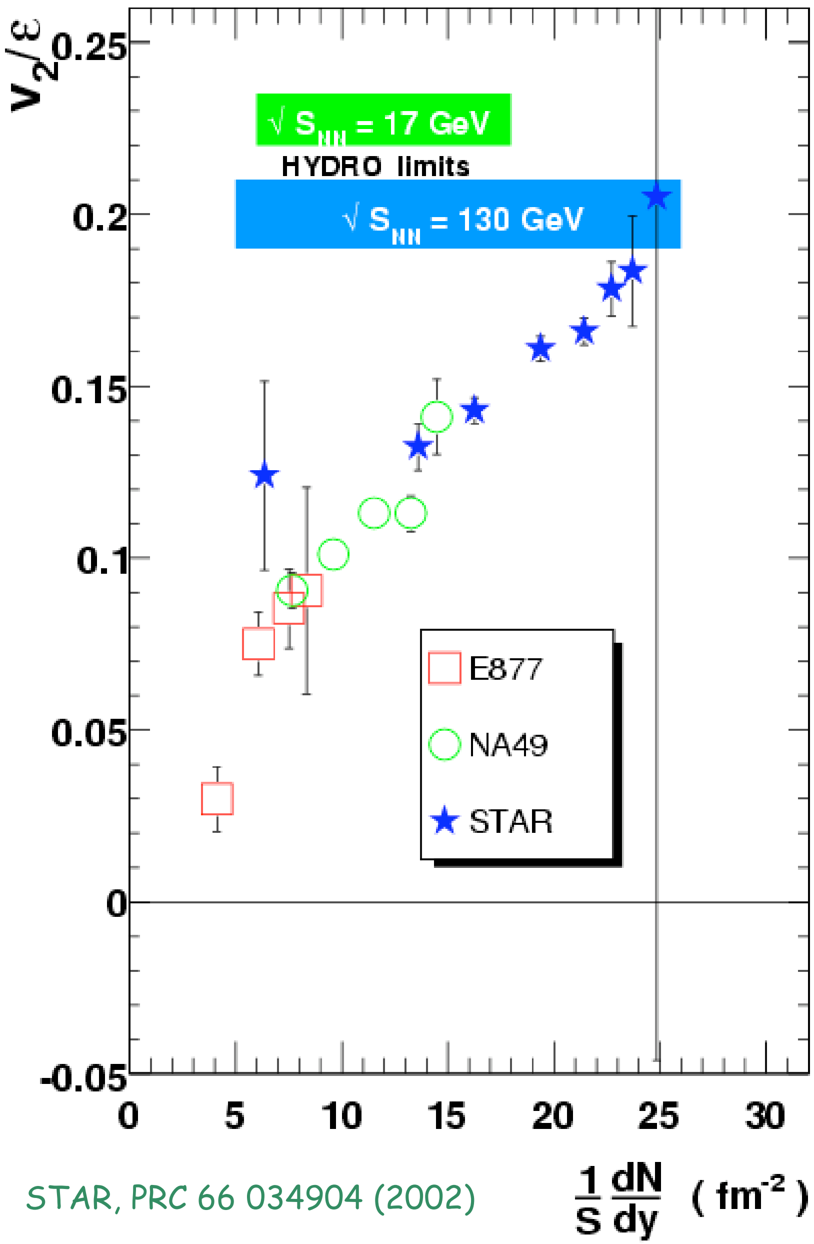
The Perfect Liquid



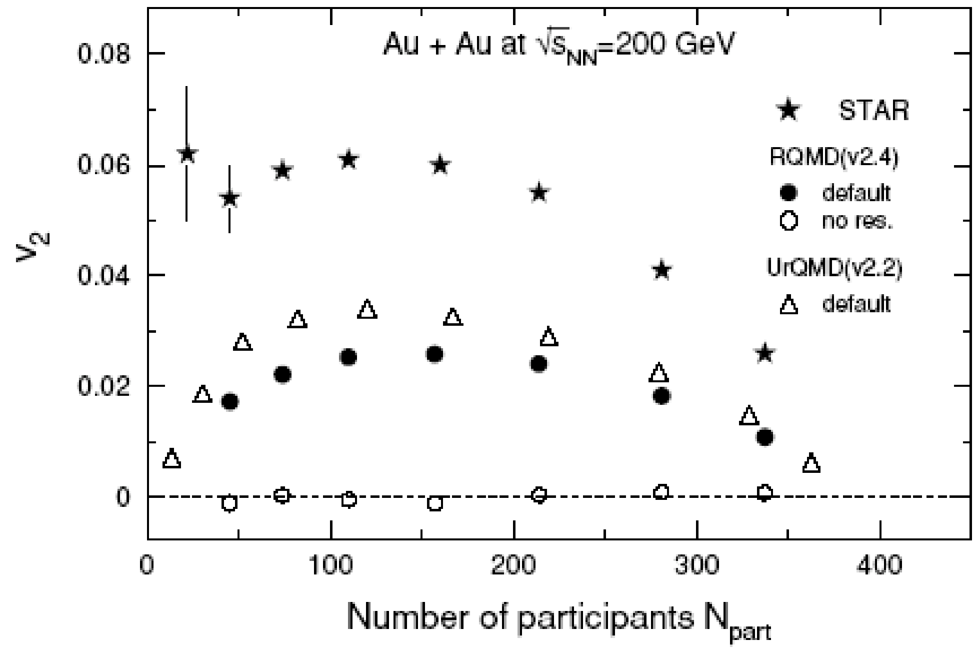
D. Teaney, PRC 68 034913 (2003)

Viscosity reduces v_2
Viscosity needs to be small in order to explain data

The Hydro Success



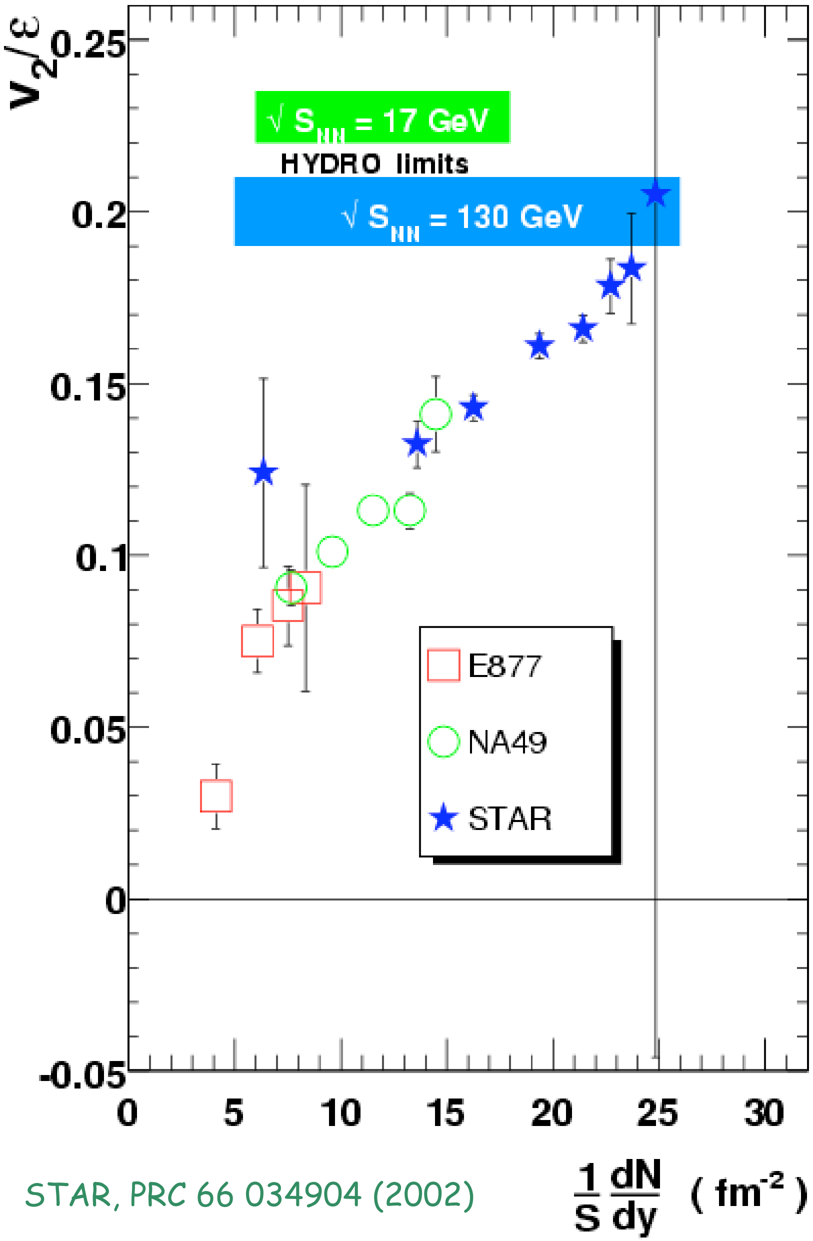
STAR, PRC 66 034904 (2002)



Y. Lu et al. Journal of Phys. G 32 1121 (2006)

v_2/ε approaches the limit of ideal hydrodynamics
Hadronic interaction alone does not produce enough v_2

Revisit the v_2/ε plot



STAR, PRC 66 034904 (2002)

Part I :

- There are many v_2 methods, what is the relation among them?

- There are many ways to calculate the eccentricity, which one to choose?

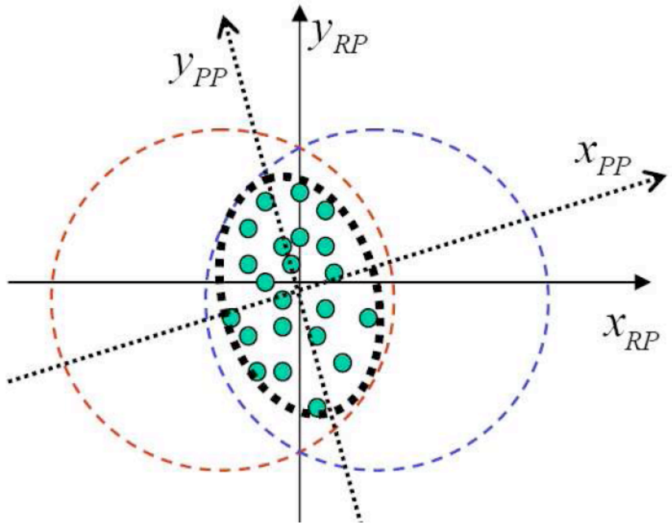
Part II :

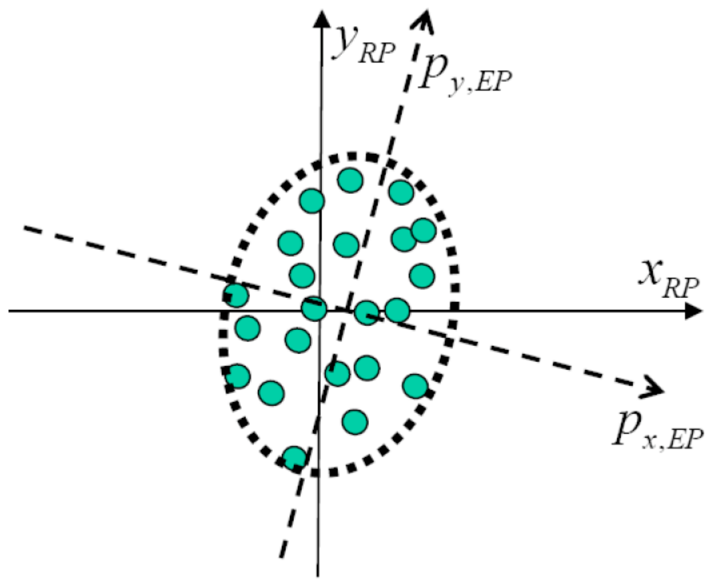
- Is the hydrodynamic limit really saturated?

- What is the trend we should expect if the requirement on local equilibrium is relaxed?

Part I:
Choose the Right $\{v_2,\varepsilon\}$ Pairs

Definition of Planes





ψ_{RP} - Reaction plane. Defined by the direction of the impact parameter.

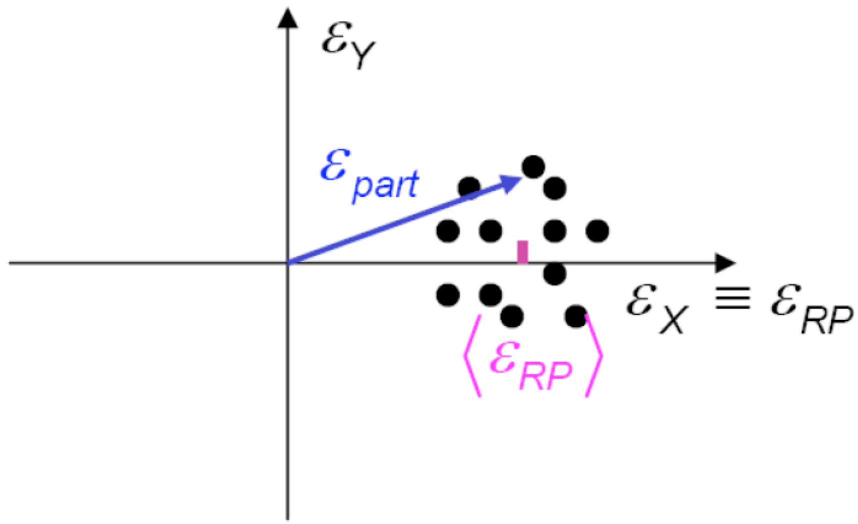
ψ_{PP} - Participant plane. Defined by the principle axis of the participant zone.

ψ_{EP} - Event plane. Defined by the flow vector Q = $\{Q_x, Q_y\}$.

$$Q_n \cos(n\Psi_n) = X_n = \sum_i w_i \cos(n\phi_i)$$

$$Q_n \sin(n\Psi_n) = Y_n = \sum_i w_i \sin(n\phi_i)$$

Definition of Eccentricities



Define

$$\varepsilon = \{\varepsilon_x, \varepsilon_y\} = \{\left\langle \frac{\sigma_y^2 - \sigma_x^2}{\sigma_x^2 + \sigma_y^2} \right\rangle_{part}, \left\langle \frac{2\sigma_{xy}}{\sigma_x^2 + \sigma_y^2} \right\rangle_{part} \}$$

We have

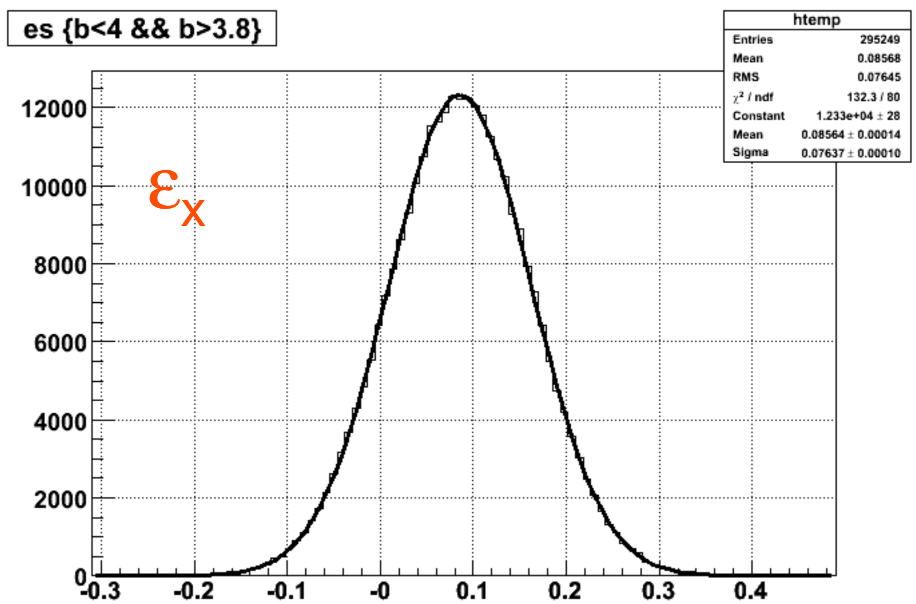
$$\varepsilon_x \equiv \varepsilon_{RP}$$

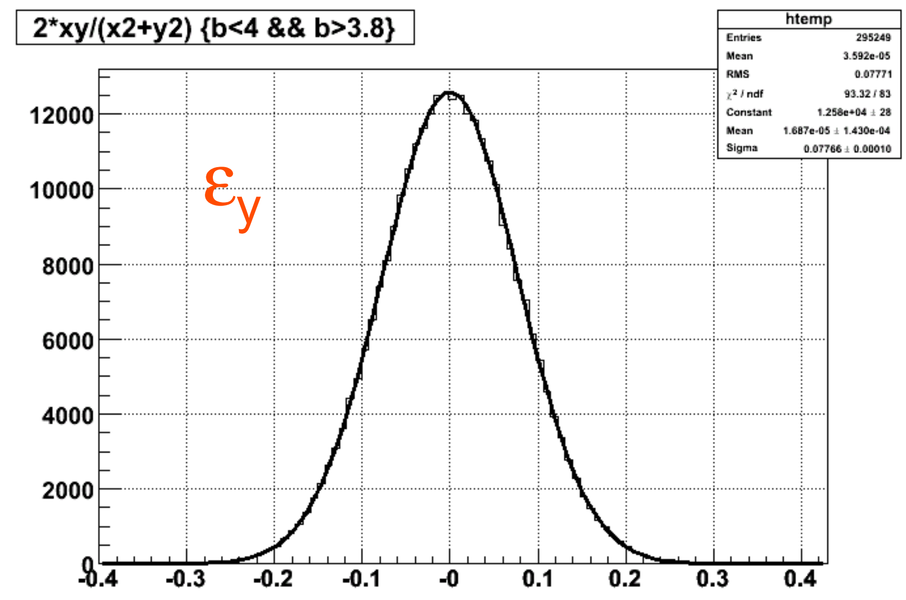
$$\langle \varepsilon_x \rangle \approx \varepsilon_{optical}$$

$$\varepsilon_{part} \equiv \varepsilon_{PP} = \sqrt{\varepsilon_x{}^2 + \varepsilon_y{}^2}$$

The angle between Ψ_{RP} and Ψ_{PP} is given by $\Delta\Psi = \frac{1}{2}\arctan(-\frac{\varepsilon_y}{\varepsilon_x})$

ε_x and ε_y





When small, the distribution of ε_x and ε_y can be well approximated by Gaussian, and it is found that the width of them are very close to each other.

The distribution of participant eccentricity

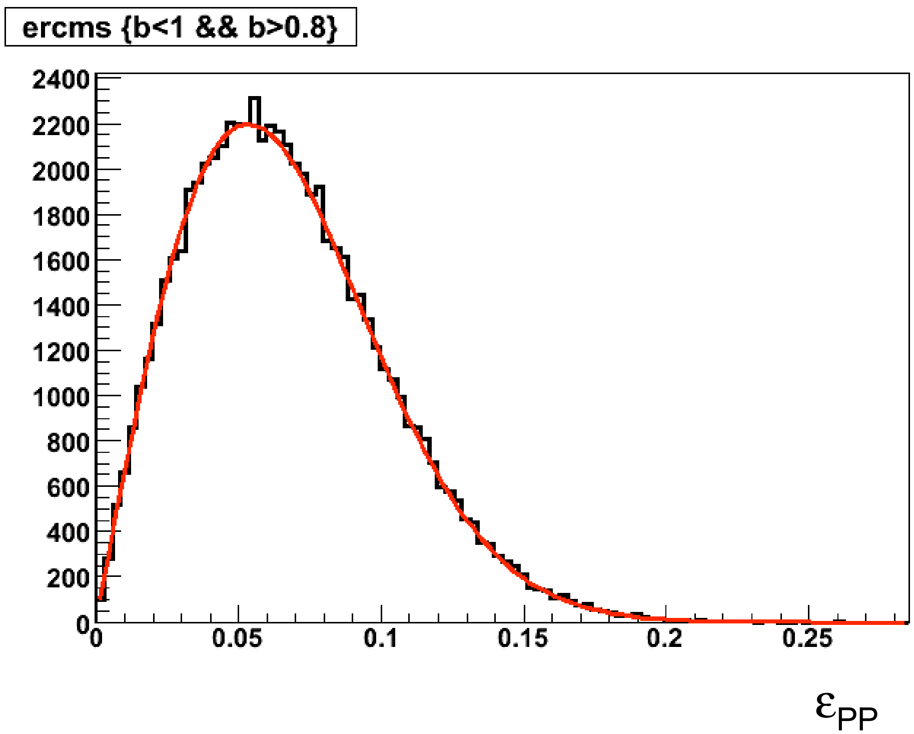
Given

$$\varepsilon_x \sim \frac{1}{\sigma_\varepsilon \sqrt{2\pi}} e^{-\frac{(\varepsilon_x - \langle \varepsilon_{RP} \rangle)^2}{\sigma_\varepsilon{}^2}}$$

$$\varepsilon_y \sim \frac{1}{\sigma_\varepsilon \sqrt{2\pi}} e^{-\frac{\varepsilon_y{}^2}{\sigma_\varepsilon{}^2}}$$

The probability for

$$\varepsilon_{part} \equiv \varepsilon_{PP} = \sqrt{\varepsilon_x{}^2 + \varepsilon_y{}^2}$$



is

$$\frac{dn}{d\varepsilon_{part}} = \frac{\varepsilon_{part}}{\sigma_\varepsilon^2} I_0\left(\frac{\varepsilon_{part}\langle\varepsilon_{RP}\rangle}{\sigma_\varepsilon^2}\right)\exp\left(-\frac{\varepsilon_{part}^2 + \langle\varepsilon_{RP}\rangle^2}{2\sigma_\varepsilon^2}\right) \equiv BG\left(\varepsilon_{part};\langle\varepsilon_{RP}\rangle,\sigma_\varepsilon\right)$$

Some useful mathematical identities

$$\frac{dn}{d\varepsilon_{part}} = \frac{\varepsilon_{part}}{\sigma_\varepsilon^2} I_0\left(\frac{\varepsilon_{part}\langle\varepsilon_{RP}\rangle}{\sigma_\varepsilon^2}\right)\exp\left(-\frac{\varepsilon_{part}^2 + \langle\varepsilon_{RP}\rangle^2}{2\sigma_\varepsilon^2}\right) \equiv BG\left(\varepsilon_{part};\langle\varepsilon_{RP}\rangle,\sigma_\varepsilon\right)$$

$$\langle x^2\rangle = \bar{x}^2 + 2\sigma^2,$$

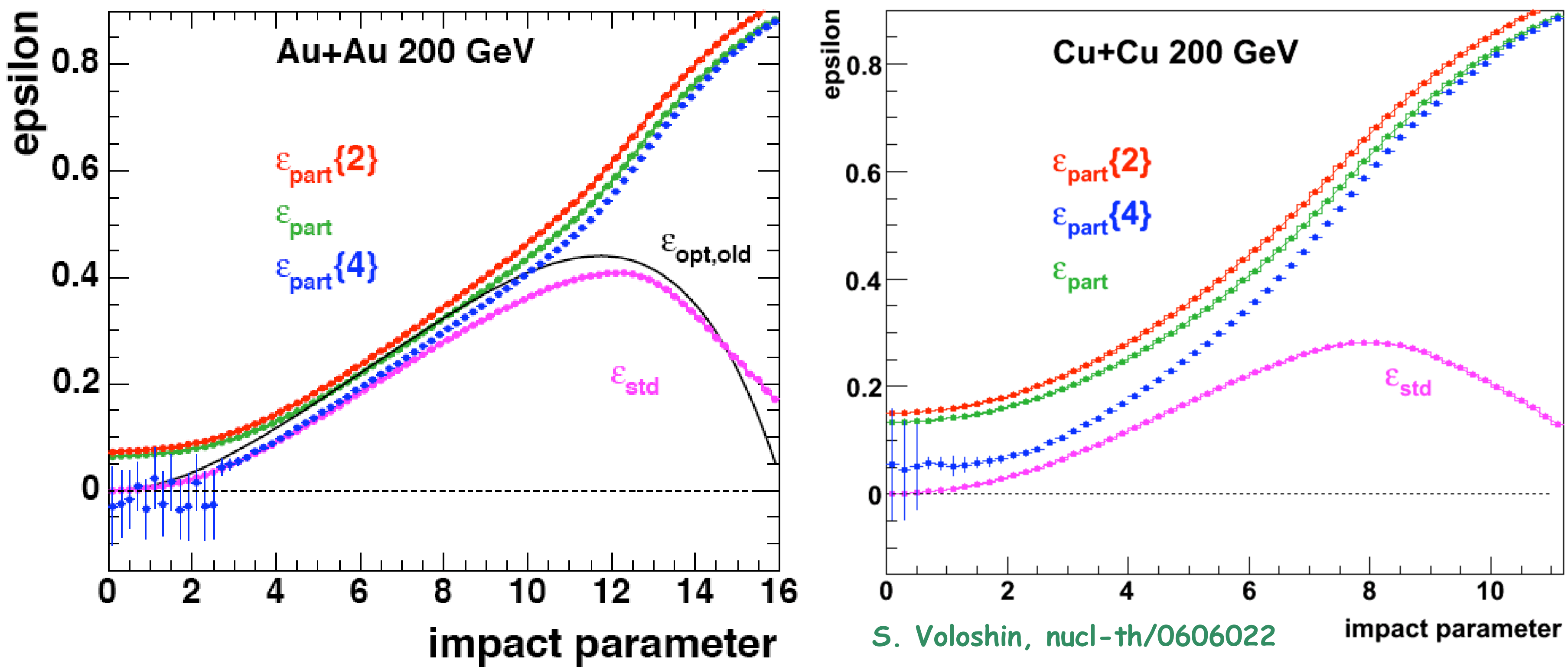
$$2\langle x^2\rangle^2 - \langle x^4\rangle = \bar{x}^4$$

$$\langle x^6\rangle - 9\langle x^4\rangle\langle x^2\rangle + 12\langle x^2\rangle^3 = 4\bar{x}^6$$

The distribution of participant eccentricity

$$\frac{dn}{d\varepsilon_{part}} = \frac{\varepsilon_{part}}{\sigma_\varepsilon^2} I_0\left(\frac{\varepsilon_{part}\langle\varepsilon_{RP}\rangle}{\sigma_\varepsilon^2}\right)\exp\left(-\frac{\varepsilon_{part}^2+\langle\varepsilon_{RP}\rangle^2}{2\sigma_\varepsilon^2}\right) \equiv BG\left(\varepsilon_{part};\langle\varepsilon_{RP}\rangle,\sigma_\varepsilon\right)$$

With this Pdf, both of the 4th order and 6th order cumulant ε_{part} equal to $\langle\varepsilon_{RP}\rangle$



S. Voloshin, nucl-th/0606022

The distribution of participant v_2

Define
$$v_2' = \langle \cos 2(\phi - \Psi_{PP}) \rangle$$
$$s_2' = \langle \sin 2(\phi - \Psi_{PP}) \rangle = 0$$

Assuming that on average, flow is proportional to ε, then the distribution of v_2'
is given by:

$$\frac{dn}{dv_2'} = \frac{v_2'}{\sigma^2_{v_2,dyn}} I_0\left(\frac{v_2' \langle v_2 \rangle}{\sigma^2_{v_2,dyn}}\right) \exp\left(-\frac{v_2'^2 + \langle v_2 \rangle^2}{2\sigma^2_{v_2,dyn}}\right)$$

With this Pdf, the 4th order and 6th order cumulant are $-\langle v_2 \rangle^4$ and $4\langle v_2 \rangle^6$,

respectively.

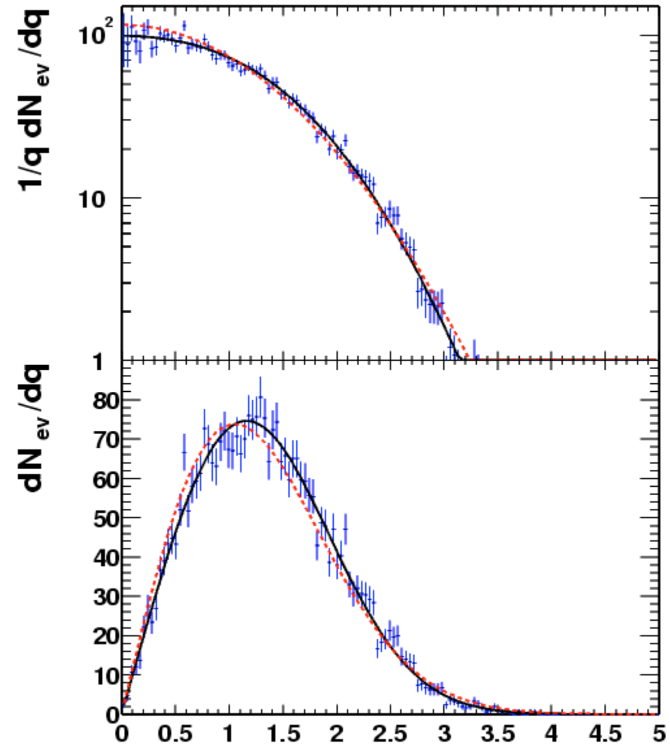
Although higher order cumulant v_2 can not give us the unbiased v_2' in the participant plane due to nonflow and fluctuations, they can give us v_2 in the reaction plane (not the participant plane) !

The q distribution method

$$q_{n,x} = \frac{1}{\sqrt{M}} \sum_{i=1}^{M} \cos(n\varphi_i)$$

$$q_{n,y} = \frac{1}{\sqrt{M}} \sum_{i=1}^{M} \sin(n\varphi_i)$$

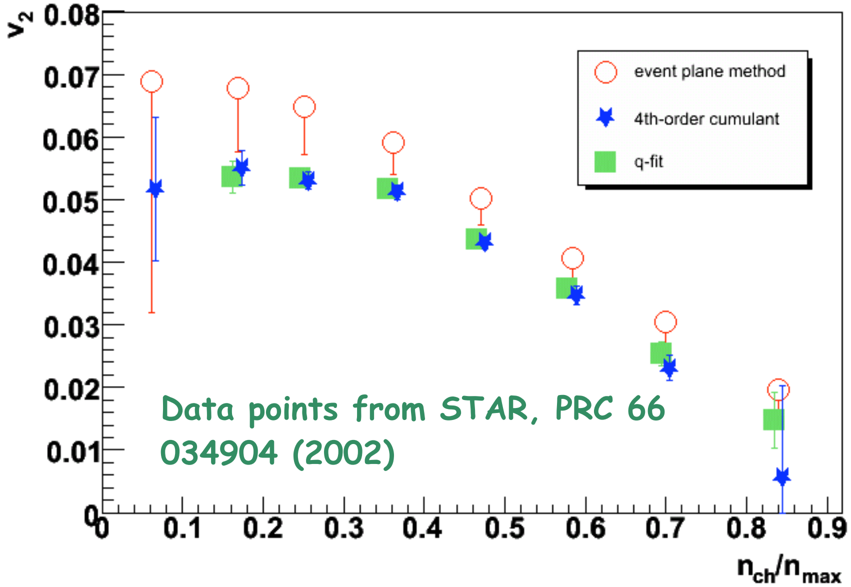
$$\frac{dn}{dq} = \frac{q}{\sigma^2} I_0\left(\frac{v_2 q\sqrt{M}}{\sigma^2}\right)\exp\left(-\frac{q^2 + Mv_2^2}{\sigma^2}\right)$$



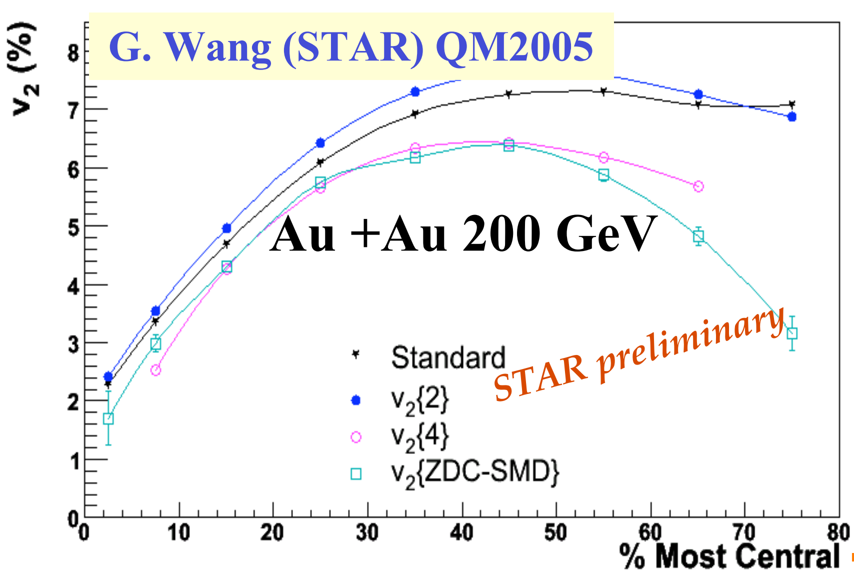
q distribution has the same formula as that for v_2' , that means that v_2 obtained from fitting the q-distribution will give us v_2 in the reaction plane (not the participant plane) too. $v_2\{qDist\}=v_2\{4\}$

Here we see that if we calculate $v_2\{q\ Cumulat\}$ $(=1/\sqrt{M}\ (2\langle q^2\rangle^2 - \langle q\rangle^4)^{1/4})$, it will give us the same answer as $v_2\{4\}$

Check with data



$v_2\{4\}=v_2\{qDist\}$



$v_2\{4\}=v_2\{ZDC\text{-}SMD\}$

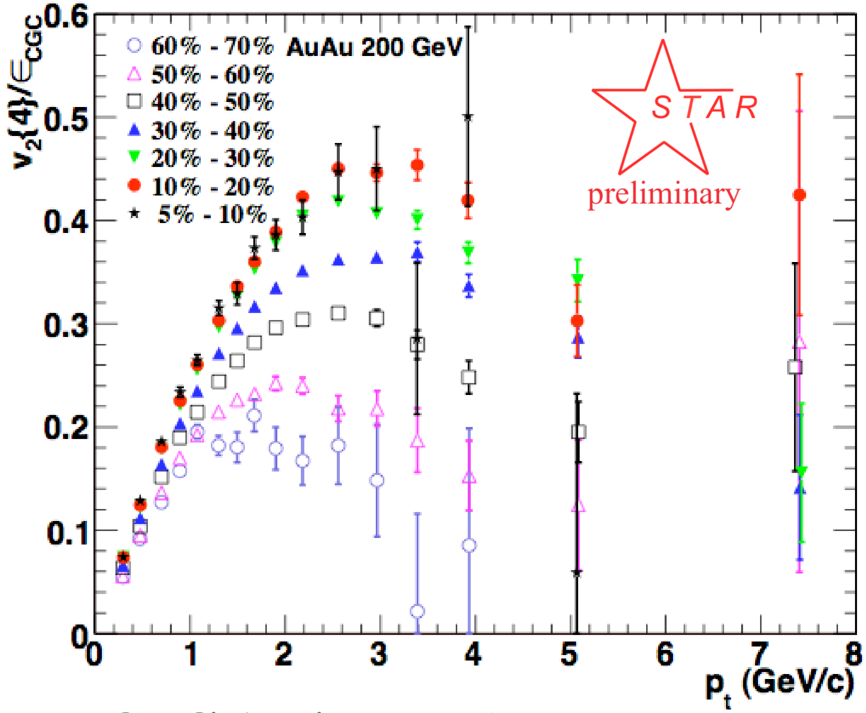
Choose the right $\{v_2, \varepsilon\}$ pairs

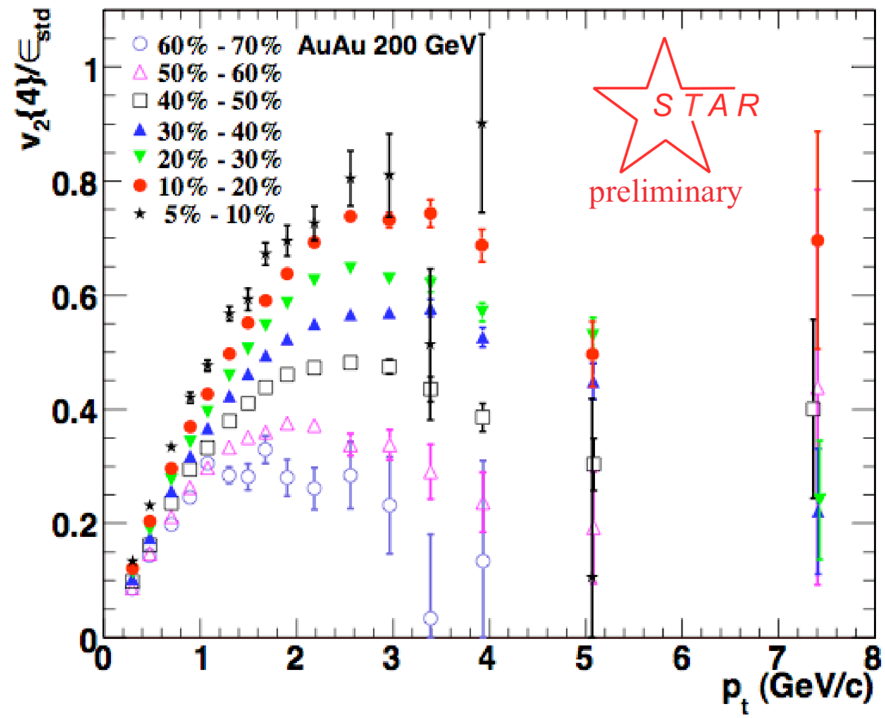
v_2 that are sensitive to anisotropy w.r.t. the **Reaction Plane v_2:** $v_2\{4\}$, $v_2\{qDist\}$, $v_2\{qCumulant4\}$, $v_2\{ZDCSMD\}$		v_2 that are sensitive to anisotropy w.r.t. the **Participant Plane :** $v_2\{2\}$,$v_2\{EP\}$,$v_2\{uQ\}$ etc.
ε that are sensitive to anisotropy w.r.t. the **Reaction Plane**: $\varepsilon\{std\}$, $\varepsilon\{4\}$		ε That are sensitive to anisotropy w.r.t. the **Participant Plane:** $\varepsilon\{part\}$ $\varepsilon\{2\}$

In this slide (and throughout this talk as well), I assume that nonflow has been suppressed by external techniques (such as pseudorapidity gap etc.) in v_2 measurements that are based on two particle correlations ($v_2\{2\}$,$v_2\{EP\}$,$v_2\{uQ\}$).

S.Voloshin, A.Poskanzer,A.Tang and G.Wang, Phys. Lett. B 659 (2008) 537
R.Bhalerao and J-Y. Ollitrault, Phys. Lett. B 614 (2006) 260

Flow Increases



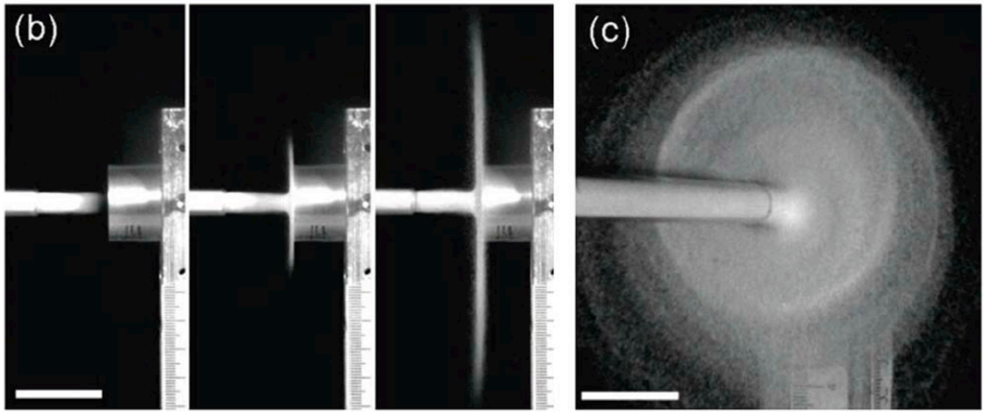


Y. Bai, Ph.D. Thesis, STAR.

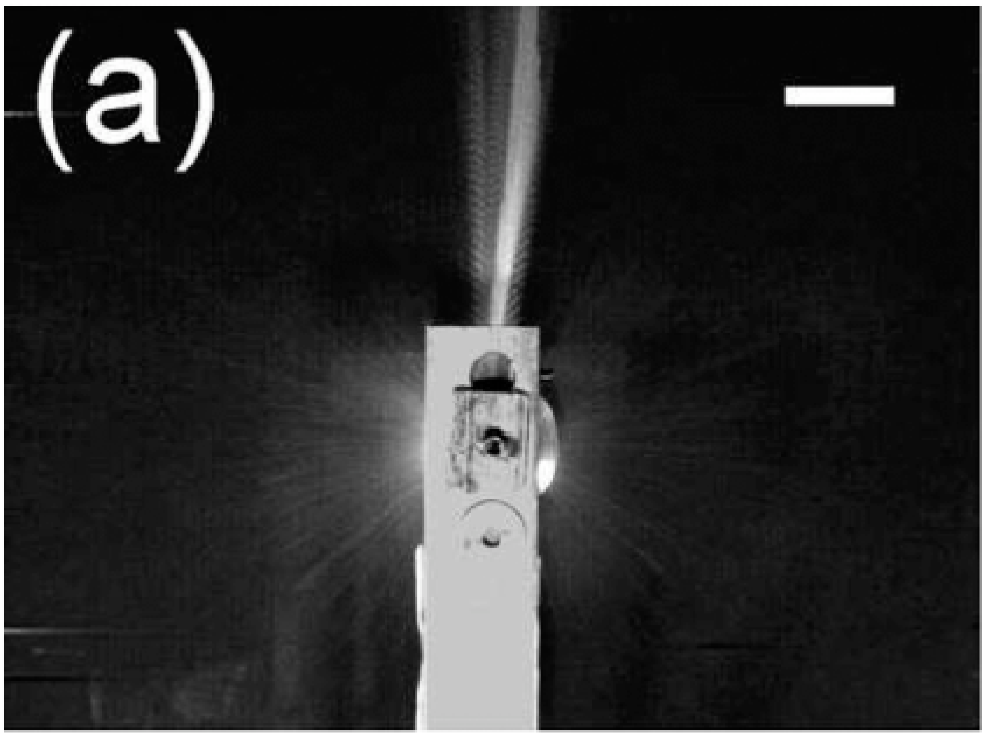
$v_2\{4\}/\varepsilon_{std}$ increases with centrality over large p_t range ($v_2\{2\}$ did not allow for this study due to strong nonflow at high p_t).
Peak position of $v_2\{4\}$ moves to higher transverse momentum with increasing centrality

Part II:
What if we relax the requirement for local equilibrium ?

Is hydro limit saturated ? Let's check a classical example



A jet of sand deforms into an extraordinarily thin symmetric granular sheet clearly resembling a spreading liquid.

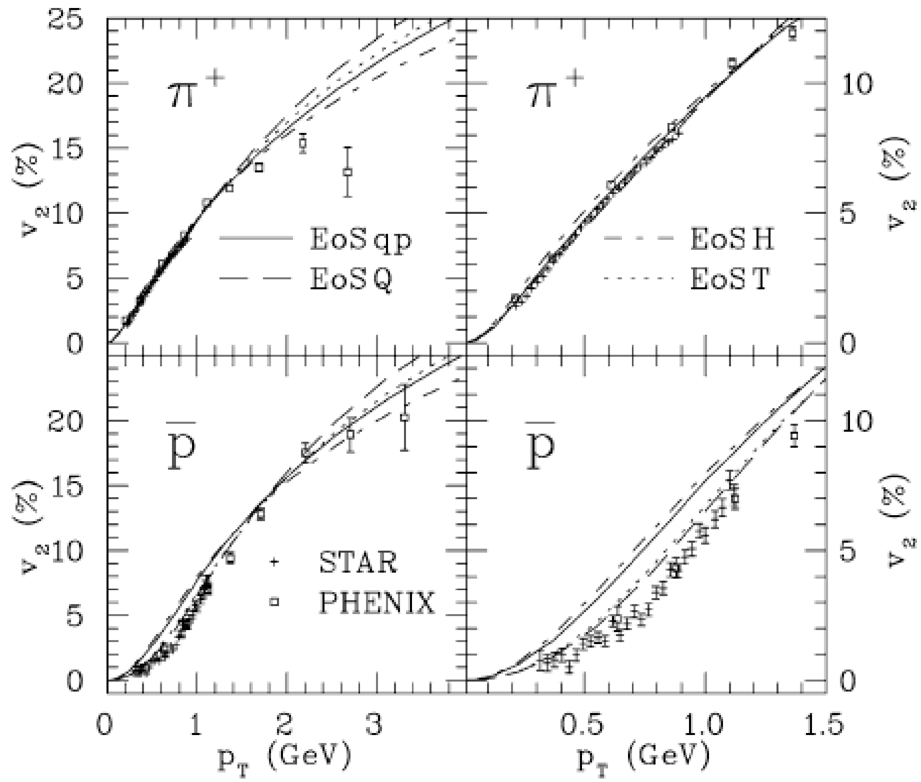


A sharply focused azimuthal pattern is seen if the target has a rectangular shape.

v_2/ε = 0.26 ~ comparable to central AuAu collisions at RHIC ! (shall we believe that it behaves like ideal hydro as well ? ☺)

X. Cheng, G. Varas, D. Citron, H. Jaeger and S. Nagel, Phys. Rev. Lett. 99 188001 (2007)

Is hydro limit saturated ? Let's check different EoSs



P. Houvine Nucl. Phys. A 761 296 (2005)

An EoS with a rapid crossover over predicted the flow

How to view the hydro behavior better ? - Move away from it

- Ideal fluid and low viscosity ⇔ local equilibrium (small λ or large σ)

- **To study the local equilibrium, we have to move away from it**, say, check what if we relax the constraint of local equilibrium

- How to get a complete view? Study Boltzman equation for diluted system. It recovers Hydro when λ becomes small.

"To have a complete view of Lu Mountain, one has to move away from it."
- Shi Su (1037~1101)

Transport Theory and Hydrodynamics

Transport Theory	Hydrodynamics
Microscopic	Macroscopic
Applicable out of equilibrium	Local equilibrium
Cannot describe phase transition	Can treat phase transition
D<<1	K<<1

D (Dilution parameter) = $\frac{\text{Typical distance between two particles}}{\text{Mean free path}}$

K (Knudsen number) = $\frac{\text{Mean free path}}{\text{System size}}$

Boltzmann Equation will be reduced to Hydrodynamics when both D<<1 and K<<1

Connecting Pieces

$$D \equiv \frac{n^{-1/3}}{\lambda} = \sigma n^{2/3}$$

$$\frac{1}{K} \equiv \frac{R}{\lambda}$$

← Number of collisions. Local thermal equilibrium is achieved if k^{-1} >>1

$$\lambda = \frac{1}{\sigma n}$$

$$n = \frac{1}{ct}\frac{1}{S}\frac{dN}{dy}$$

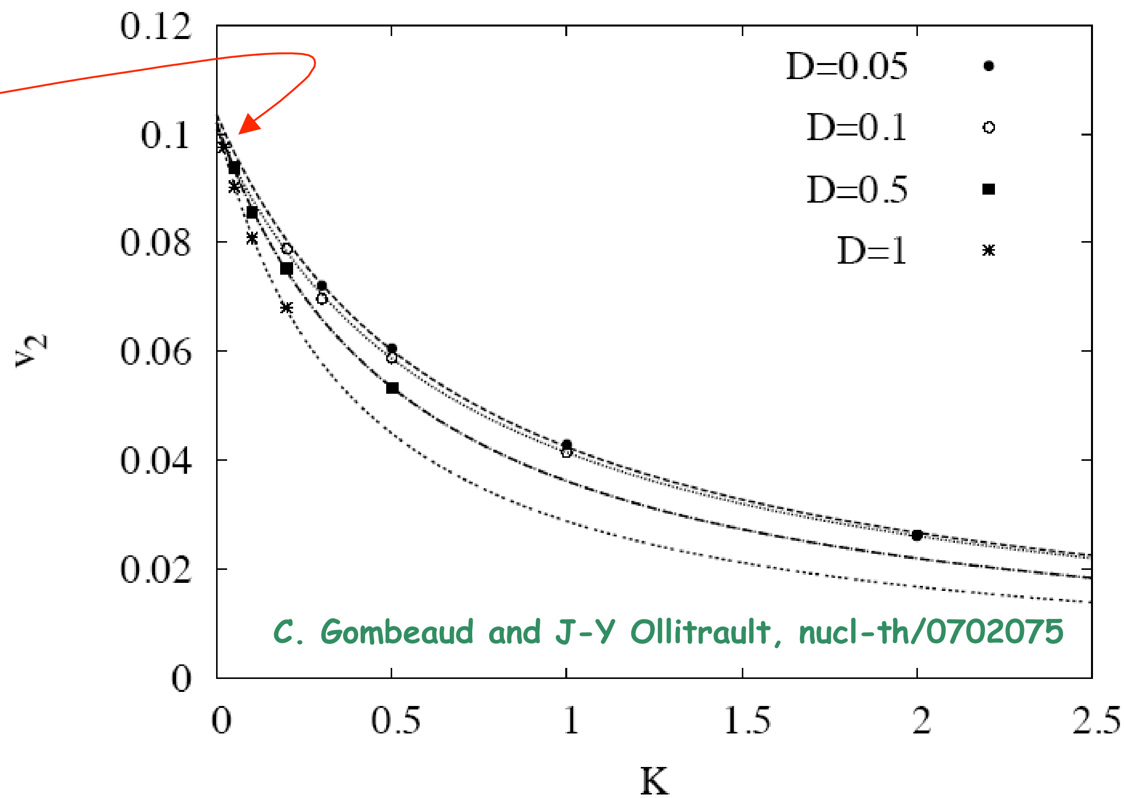
$$t \sim R / c_s$$

$$\frac{1}{K} = \sigma \boxed{\frac{1}{S}\frac{dN}{dy}} \frac{c_s}{c}$$

n: particle density
σ: parton cross section
R: system size
λ: mean free path

v_2 from Solving the Boltzmann Equation

Hydro limit is recovered when D<<1 and K <<1



$v_2 \propto 1/K$, and v_2 saturates eventually when the system reaches local equilibrium

$$\Rightarrow \quad \frac{v_2}{\varepsilon} = \frac{v_2^{hydro}}{\varepsilon} \frac{1}{1 + K / K_0}$$

See next slide

How much deviation from ideal hydro ?

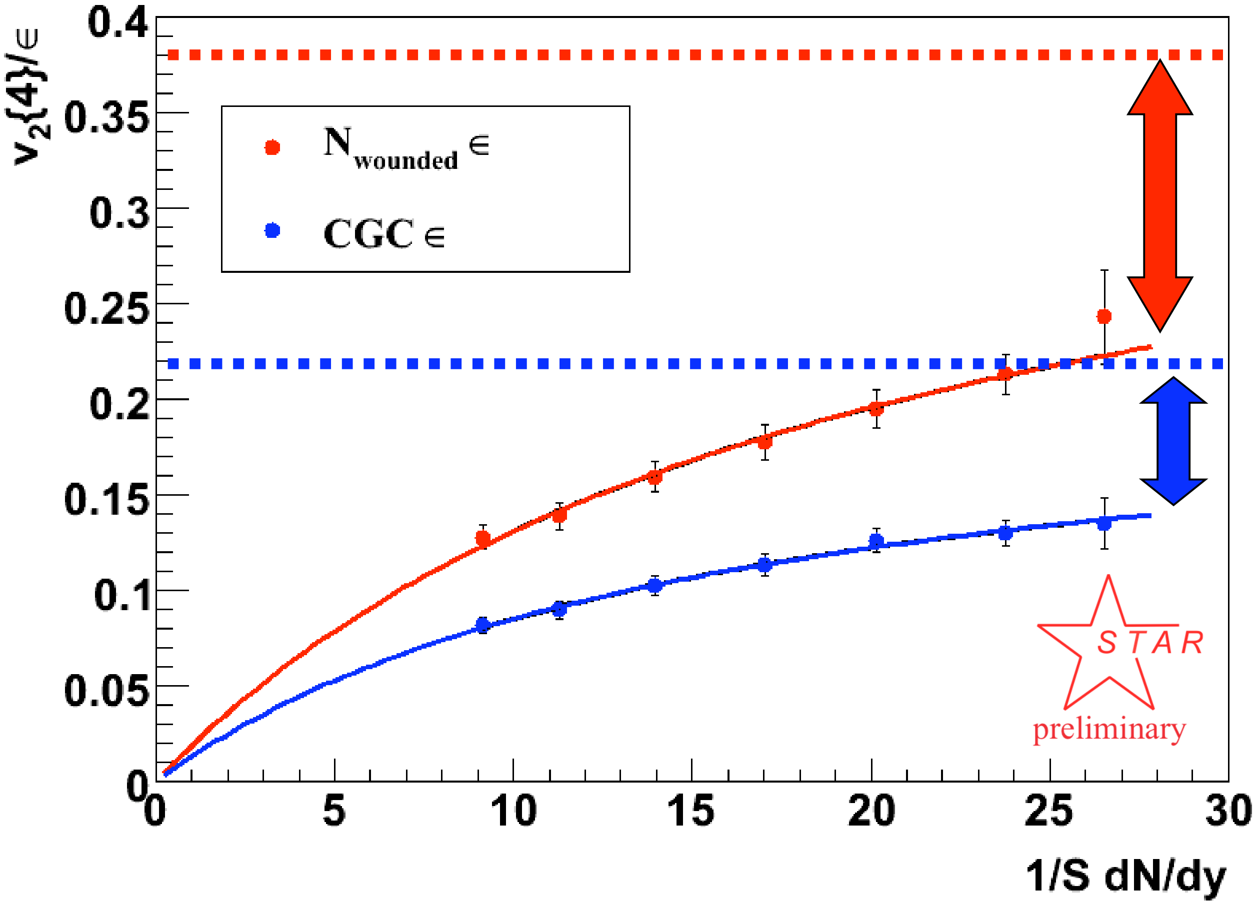
$$\frac{v_2}{\varepsilon} = \frac{v_2^{hydro}}{\varepsilon} \frac{1}{1 + K / K_0}$$

$$K = \lambda / R$$

$$\frac{1}{K} = \frac{\sigma}{S} \frac{dN}{dy} C_s$$

For the case with Standard ε : σ=4.9mb, v_2/ε=0.38.

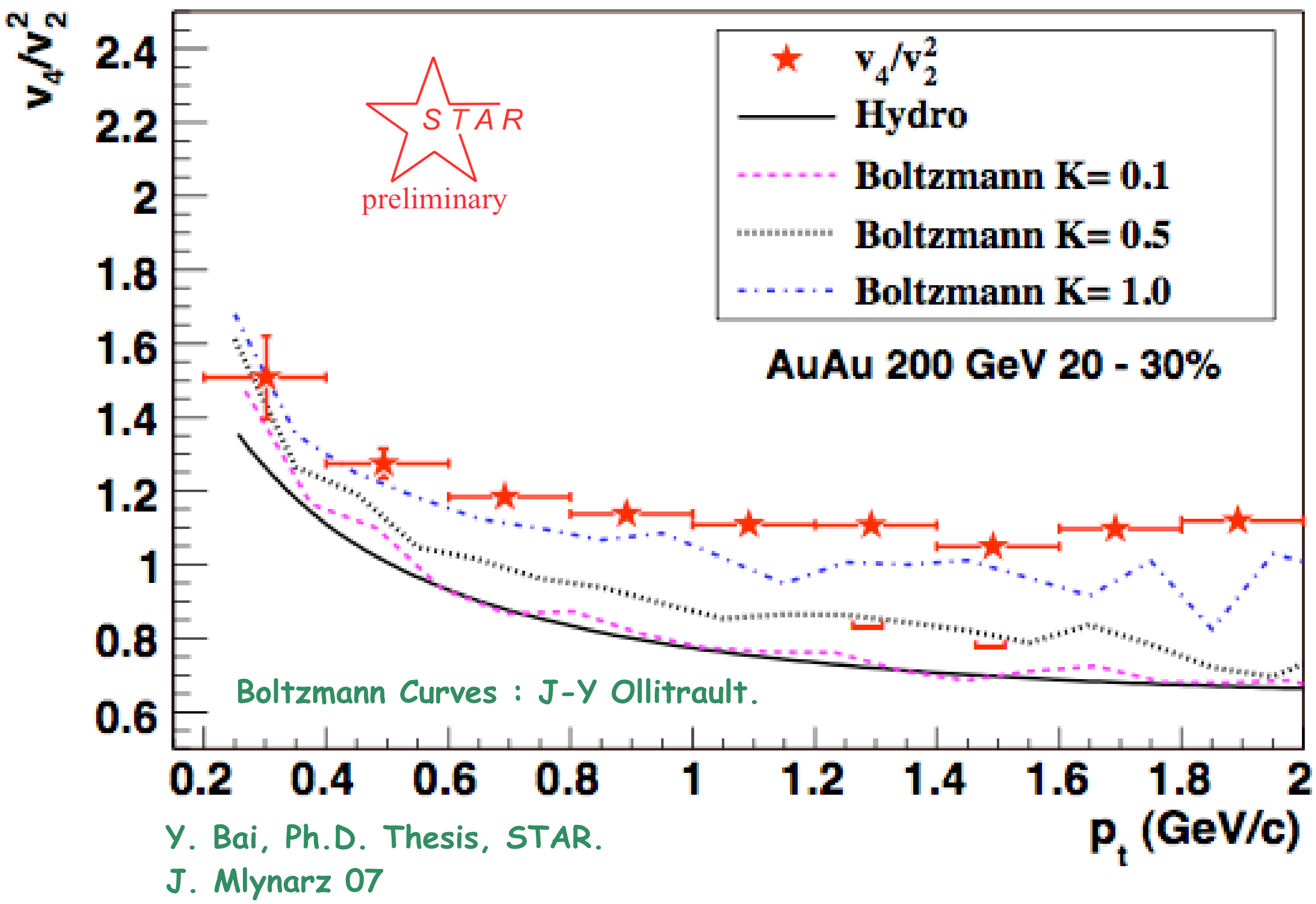
For the case with CGC ε : σ=6.2mb, v_2/ε=0.22.



Fitting function from Drescher, Dumitru, Gombeaud, J.Ollitrault, Phys. Rev. C76, 024905(2007)
CGC ε obtained from A.Adil, H-J Drescher, A.Dumitru, A.Hayashigaki and Y.Nara, Phys. Rev. C 74 044905 (2006)

Dashed lines are hydro limit from fitting the data (as opposed to a pure theoretical calculation as as adopted before)
~40% away from ideal hydro even in central collisions

How much deviation from ideal hydro ?



An improved analysis since QM06
Considerable deviation from ideal Hydro

v_4 Systematics From v_2

v_4 of particle i at a certain p_t can be obtained by three-particle (i, j, k) correlations:

$$\langle \cos(4\phi_i - 2\phi_j - 2\phi_k)\rangle = v_4(p_t)v_2^2, \qquad (4.2)$$

where the average is taken over all the particles and events. The dominant non-flow contribution to the three particle correlation can be estimated as follows: if particle i is correlated with particle j by non-flow and correlated with particle k by flow, the three-particle non-flow correlations can be written like:

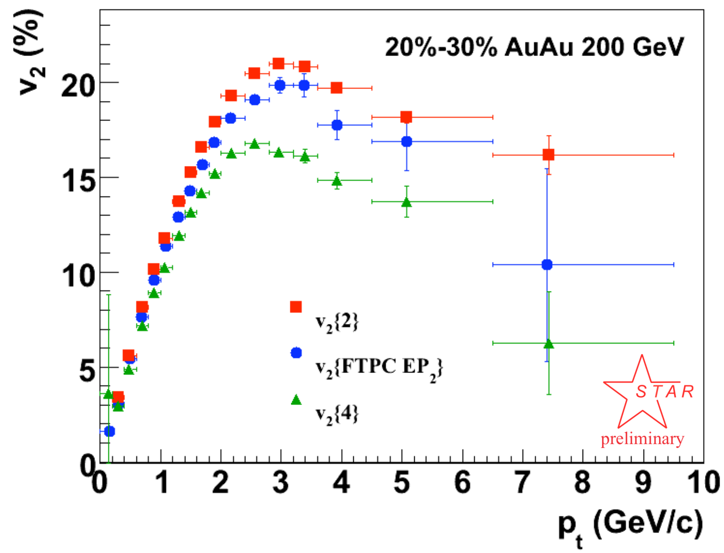
$$g_2 \times \langle \cos(2\phi_i - 2\phi_k)\rangle = g_2 \times v_2\{4\}(p_t)v_2 \qquad (4.3)$$

where g_2 is the non-flow contribution from two-particle correlations. It is shown that $g_2 \propto v_2^2\{2\}(p_t) - v_2^2\{4\}(p_t)$ [49, 95]. Therefore, the non-flow contributions to $v_4(p_t)$ is obtained by:

$$\frac{g_2 \times v_2\{4\}(p_t)v_2}{v_2^2} = \frac{(v_2^2\{2\}(p_t) - v_2^2\{4\}(p_t)) \times v_2\{4\}(p_t)}{v_2}. \qquad (4.4)$$

The non-flow contributions to v_4/v_2^2 is then estimated by

$$\frac{v_2^2\{2\}(p_t) - v_2^2\{4\}(p_t)}{v_2 v_2\{4\}(p_t)}. \qquad (4.5)$$



Y. Bai, Ph.D. Thesis, STAR.

The first order systematics in v_4 is from flow*nonflow term

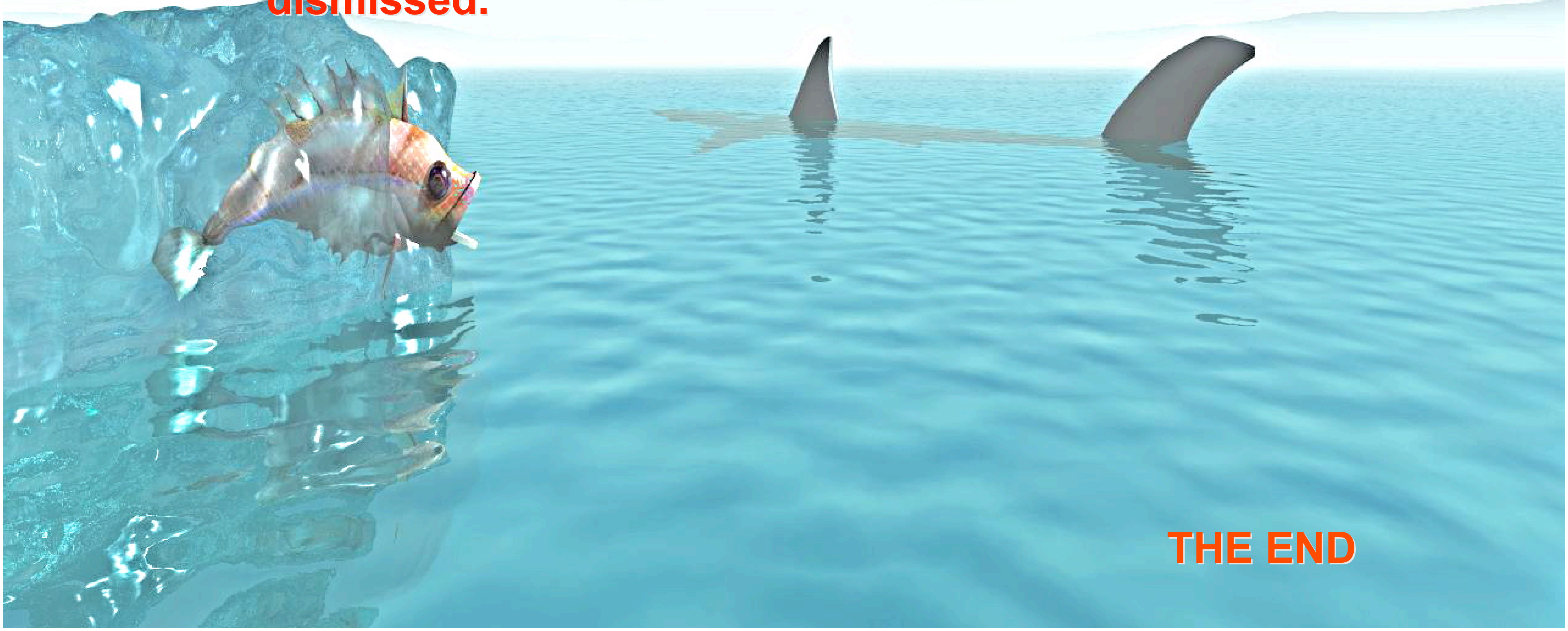
The nonflow term is from v_2 nonflow (not v_4)

The difference between v_2\{FTPC\} and v_2\{4\} is used in the estimation of nonflow of v_4.

An Inconvenient Truth

(not really related to global warming)

- While it is generally accepted that Hydrodynamics did a good job, for the first time, in describing RHIC's data, there are features that are not consistent with a complete thermalization, and they cannot be easily dismissed.



Backup Slides

Eccentricity Definitions

Standard eccentricity	$\varepsilon_{std} = \frac{\{y^2\} - \{x^2\}}{\{y^2\} + \{x^2\}}$
Participant eccentricity (rotation only)	$\varepsilon_{rot} = \frac{\sqrt{(\{y^2\} - \{x^2\})^2 + 4\{xy\}^2}}{\{y^2\} + \{x^2\}}$
Participant eccentricity (rotation + CM shift)	$\varepsilon_{part} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2},$ $\sigma_x^2 = \{x^2\} - \{x\}^2, \quad \sigma_y^2 = \{y^2\} - \{y\}^2, \quad \sigma_{xy} = \{xy\} - \{x\}\{y\}$
Transverse Area (fm^2)	$S = \pi\sqrt{\{x^2\}\{y^2\}}$
$\varepsilon\{2\} = \sqrt{\langle\varepsilon^2\rangle}, \quad \varepsilon\{4\} = \left(2\langle\varepsilon^2\rangle^2 - \langle\varepsilon^4\rangle\right)^{1/4}$	