The Critical Point and Onset of Deconfinement Conference
"CPOD 2018"

Off-diagonal cumulants of net-charge, net-proton and net-kaon multiplicity distributions in Au+Au collisions at $\sqrt{s_{NN}} = 7.7\text{-}200$ GeV from STAR

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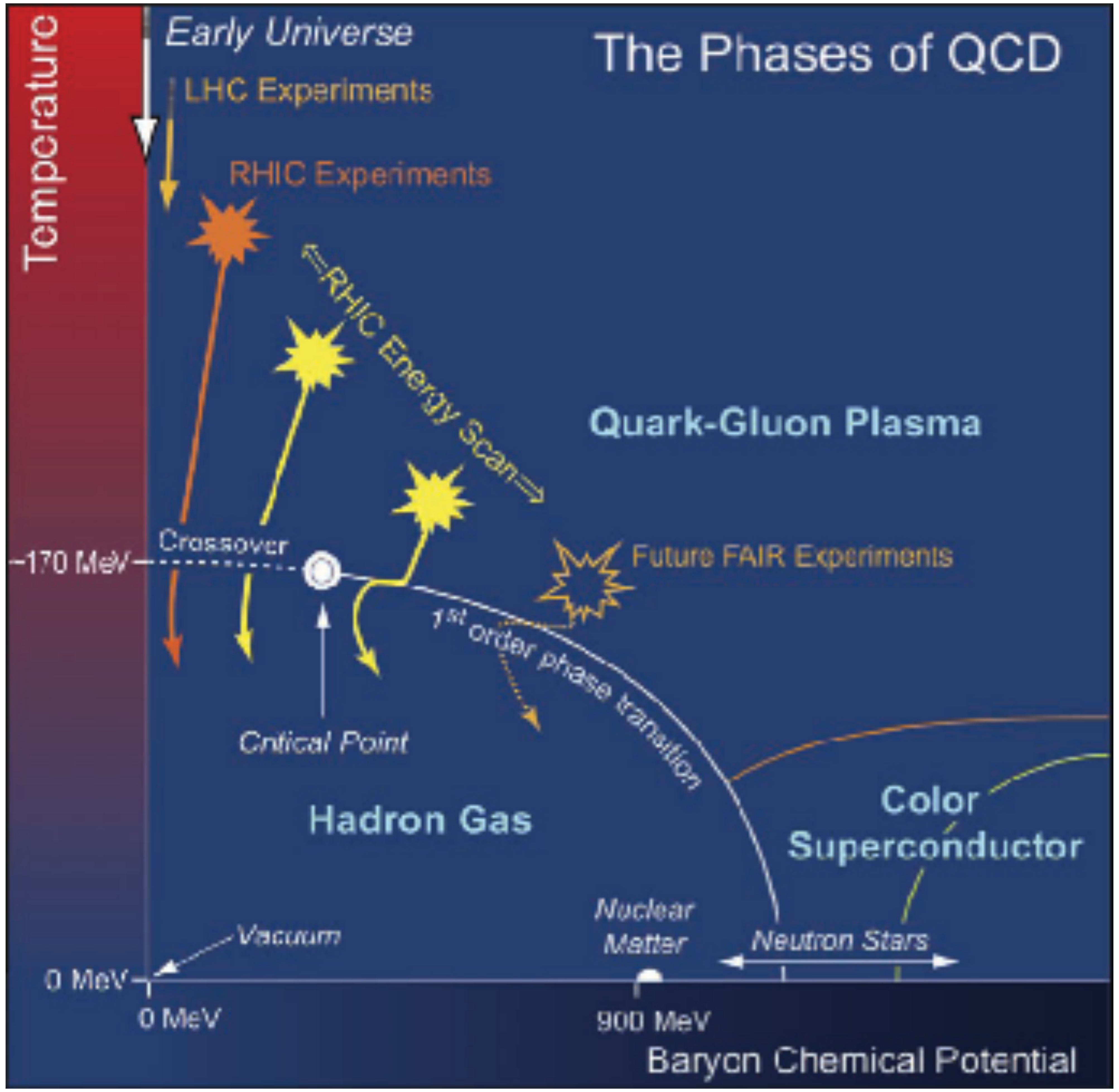
Outline

- Introduction
- Observables
- Analysis Technique
- Experiments results
  - Centrality dependence
  - η dependence
  - Energy dependence
- Summary

Introduction: mapping the QCD phase diagram



arXiv:0809.3137

- At large chemical potential and nearly T~0 MeV => 1st-order phase transition.
- At μ_B~0 MeV => Crossover.
- The end point of the 1st-order phase transition => QCD Critical Point.

S. Ejiri, Phys. Rev. D 78 074507 (2008)   Y. Aoki et al., Nature 443, 675--678(2006)

✓ Goal of Beam Energy Scan
- Explore the phase diagram.

✓ Susceptibilities are sensitive to phase boundary
- Sign change and diverge.

M. Stephanov, K. Rajagopal, E. Shuryak, Phys. Rev. Lett. 81, 4816 (1998). Y. Hatta, M. Stephanov, Phys. Rev. Lett. 91, 102003 (2003). V. Koch, A. Majumder, J. Randrup, Phys.Rev.Lett. 95, 182301 (2005). M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009). M. Asakawa, M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011).

Motivation: susceptibilities and number fluctuation

- Susceptibilities can be estimated by e-by-e fluctuations in HIC

In GCE: Partition function $Z = Z(T, \mu)$

$$< \Delta E^n >= A\frac{\partial^n Z}{\partial^n T} \longrightarrow < \Delta E^2 >= KT^2C_V$$

$$< \Delta N^n >= B\frac{\partial^n Z}{\partial \mu} \longrightarrow < \Delta N^n >= VT^3\chi_n$$

$$\mathbf{VT^3}\chi_1 = < \mathbf{N} >= \kappa_1$$

$$\mathbf{VT^3}\chi_2 = < (\mathbf{\Delta N})^2 >_c = \kappa_2$$

$$\mathbf{VT^3}\chi_3 = < (\mathbf{\Delta N})^3 >_c = \kappa_3$$

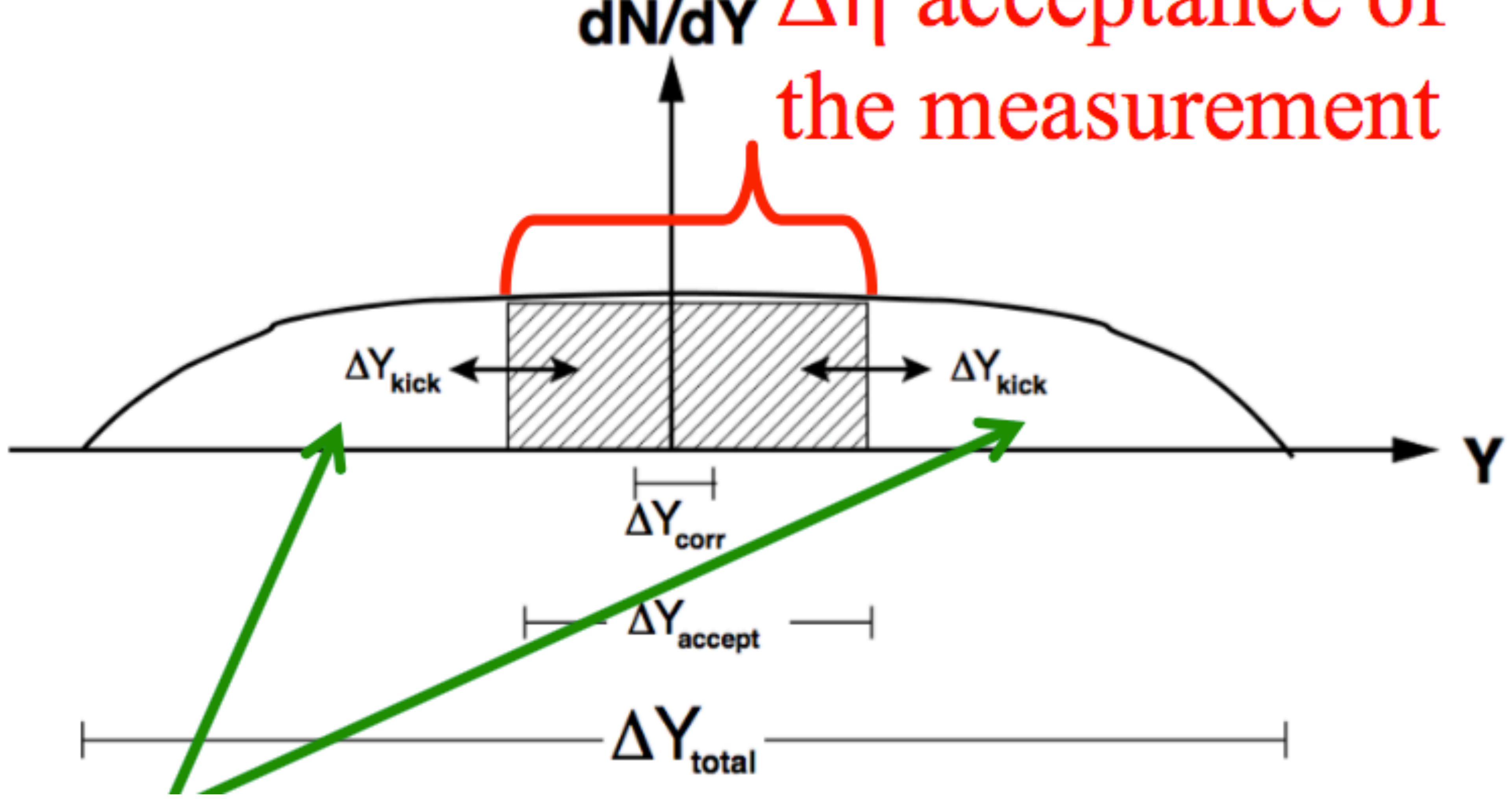
$$\mathbf{VT^3}\chi_{1,1} = < (\mathbf{\Delta M})(\mathbf{\Delta N}) >= \kappa_{1,1}$$

Theory: susceptibilities

Experiment: net-particle multiplicity distributions $N=N^+ - N^-$

Net-charge,
Net-proton (proxy),
Net-kaon (proxy)

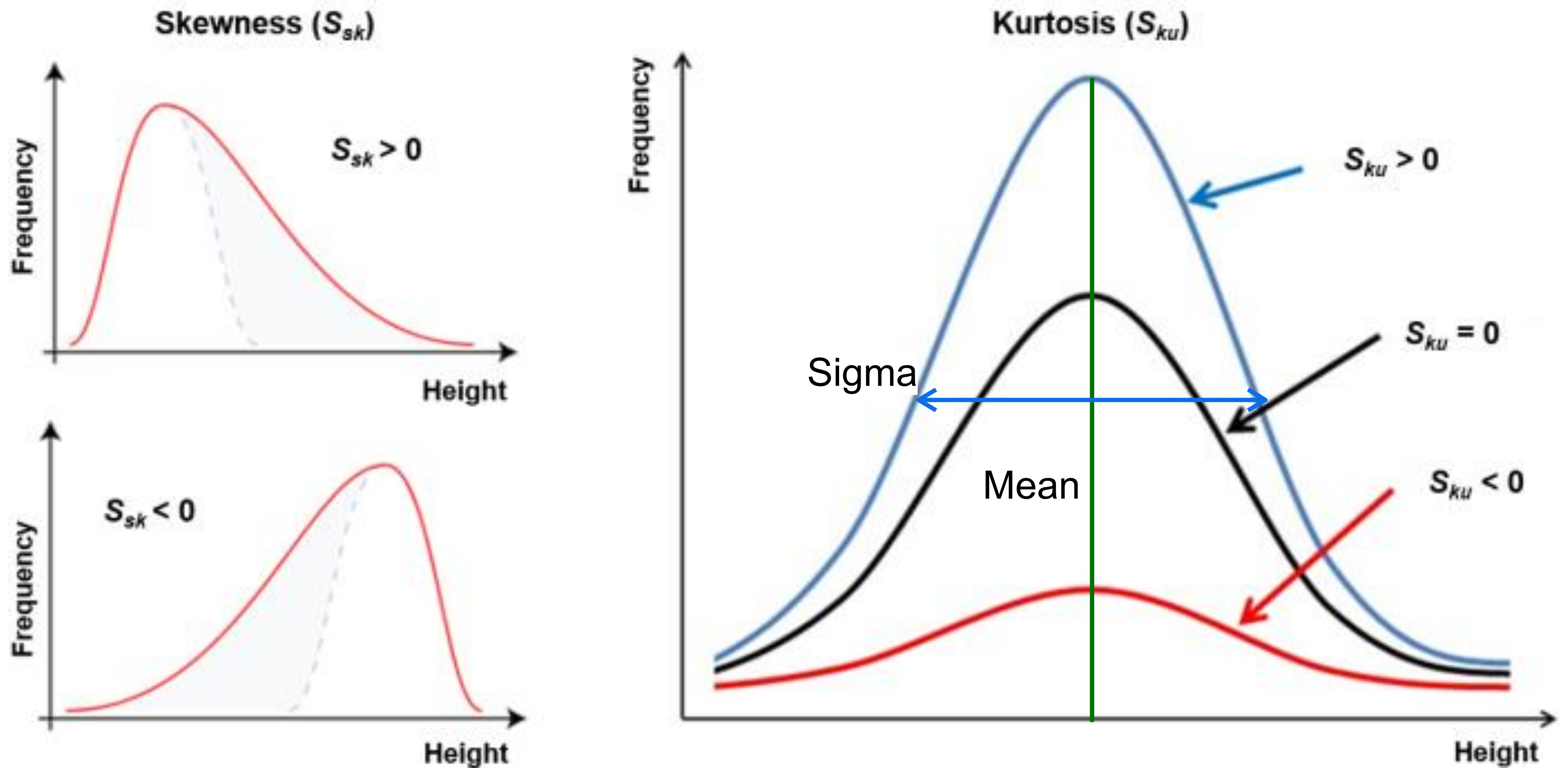
Δη acceptance of the measurement



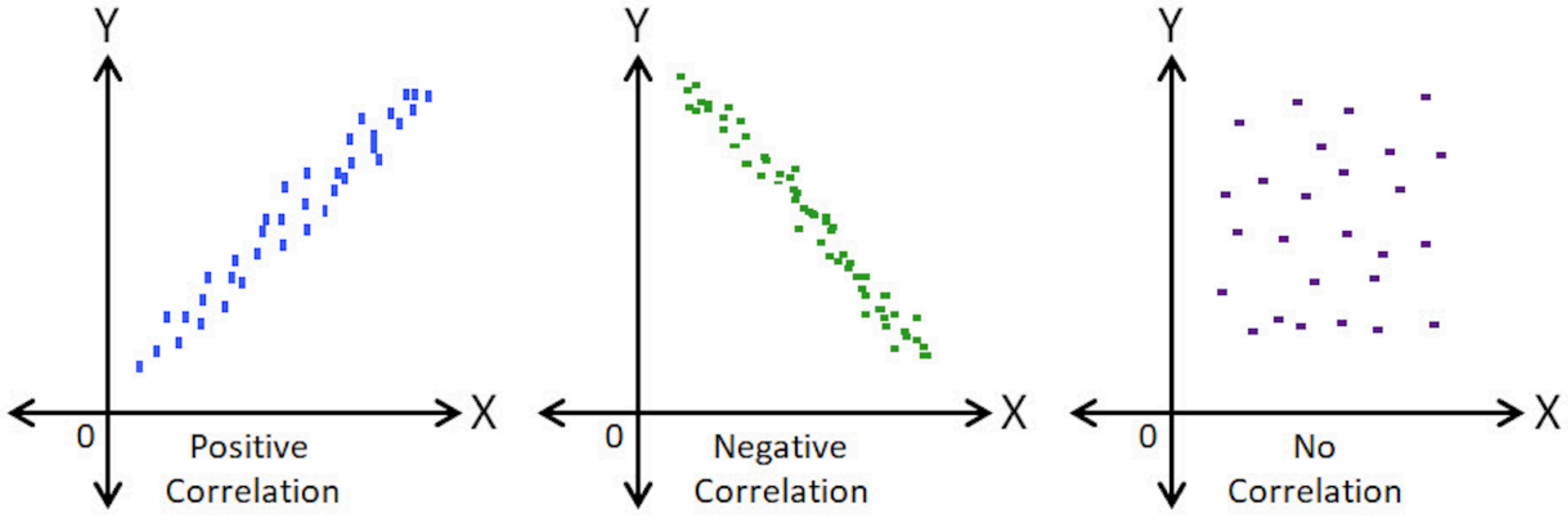
Particle bath

$$\Delta Y_{corr} \ll \Delta Y_{accept} \ll \Delta Y_{total}$$

Cumulants: measure of non-gaussian fluctuation and correlation



Figures are taken from DOI: 10.1109/SIITME.2015.7342289



- Diagonal cumulants are mathematical measures of a single-variate distribution "**shape**".
  - Mean (C_1), Width ($\sqrt{C_2}$).
  - skewness ($C_3/C_2^{3/2}$), kurtosis (C_4/C_2^2): non-gaussian fluctuation.
    - — Higher sensitivity to divergence correlation length (ξ).

$$C_3 \sim \xi^{4.5}$$

$$C_4 \sim \xi^{7}$$

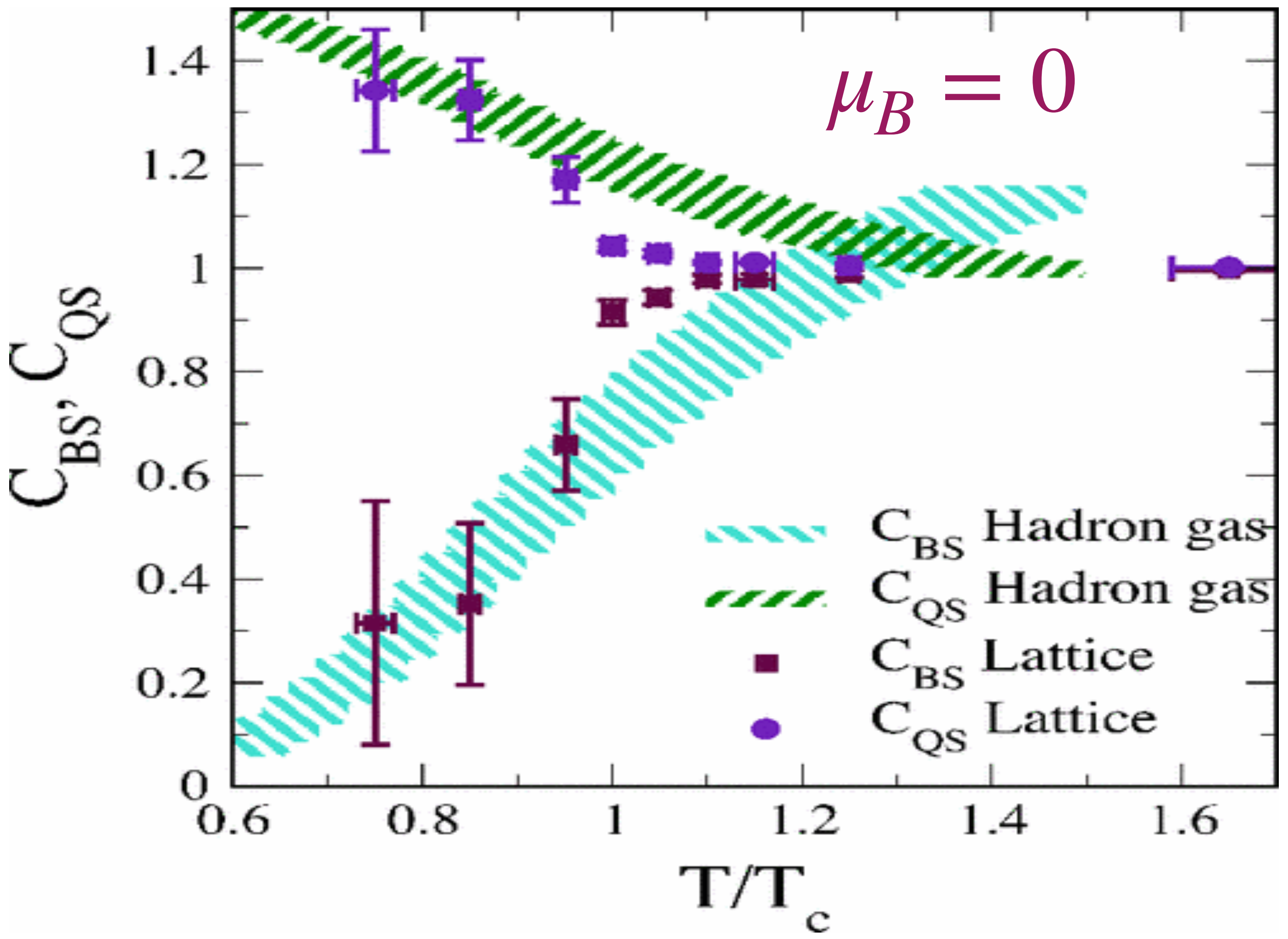
M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009).
M.Asakawa, S. Ejiri and M. Kitazawa, Phys. Rev. Lett. 103, 262301 (2009).
M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011).

✓ Net-proton: STAR Collaboration, PRL. 112, 032302 (2014).
X. Luo, PoS(CPOD14)019; QM (15)
✓ Net-charge: STAR Collaboration, PRL. 112, 032302 (2014).
✓ Net-kaon: STAR Collaboration, arXiv:1709.00773.

- Off-diagonal cumulants are mathematical measures of "**correlation**" between different net-particles.

- Off-diagonal cumulants are sensitive to the flavor carrying degrees of freedom.

Mixed cumulants:

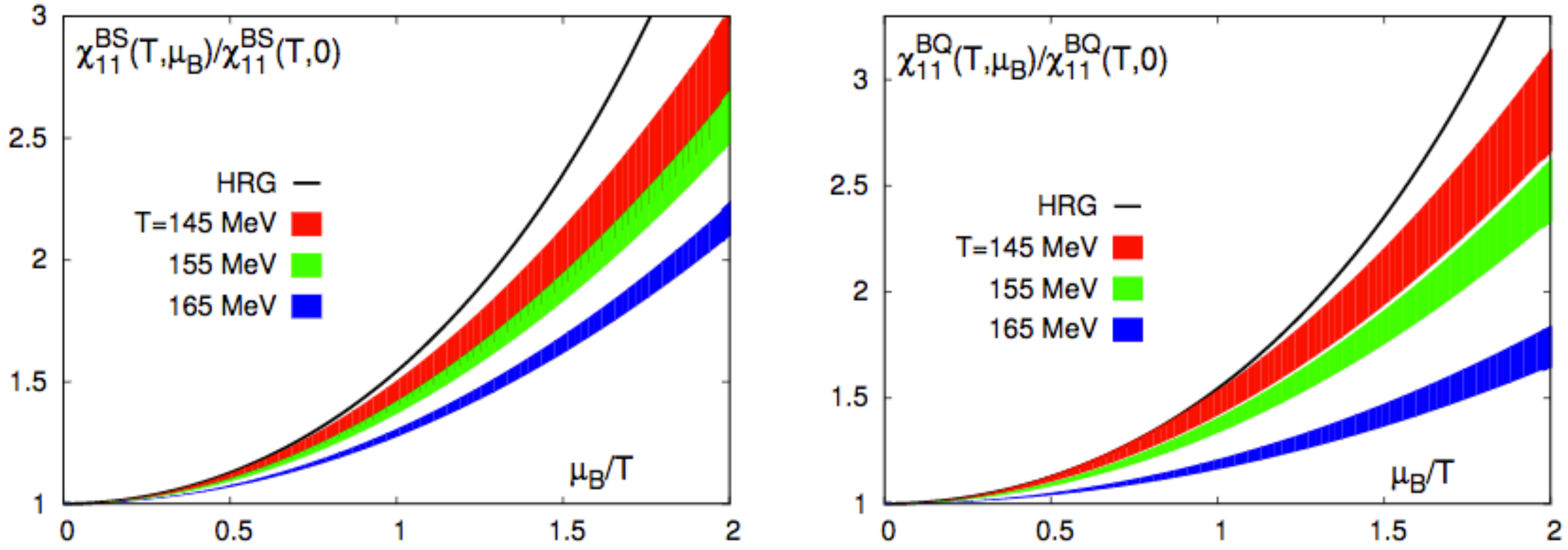


$$\chi^{ij}_{XY} = \frac{1}{VT^3} M_{XY} =< (X- < X >)^i (Y- < Y >)^j >$$

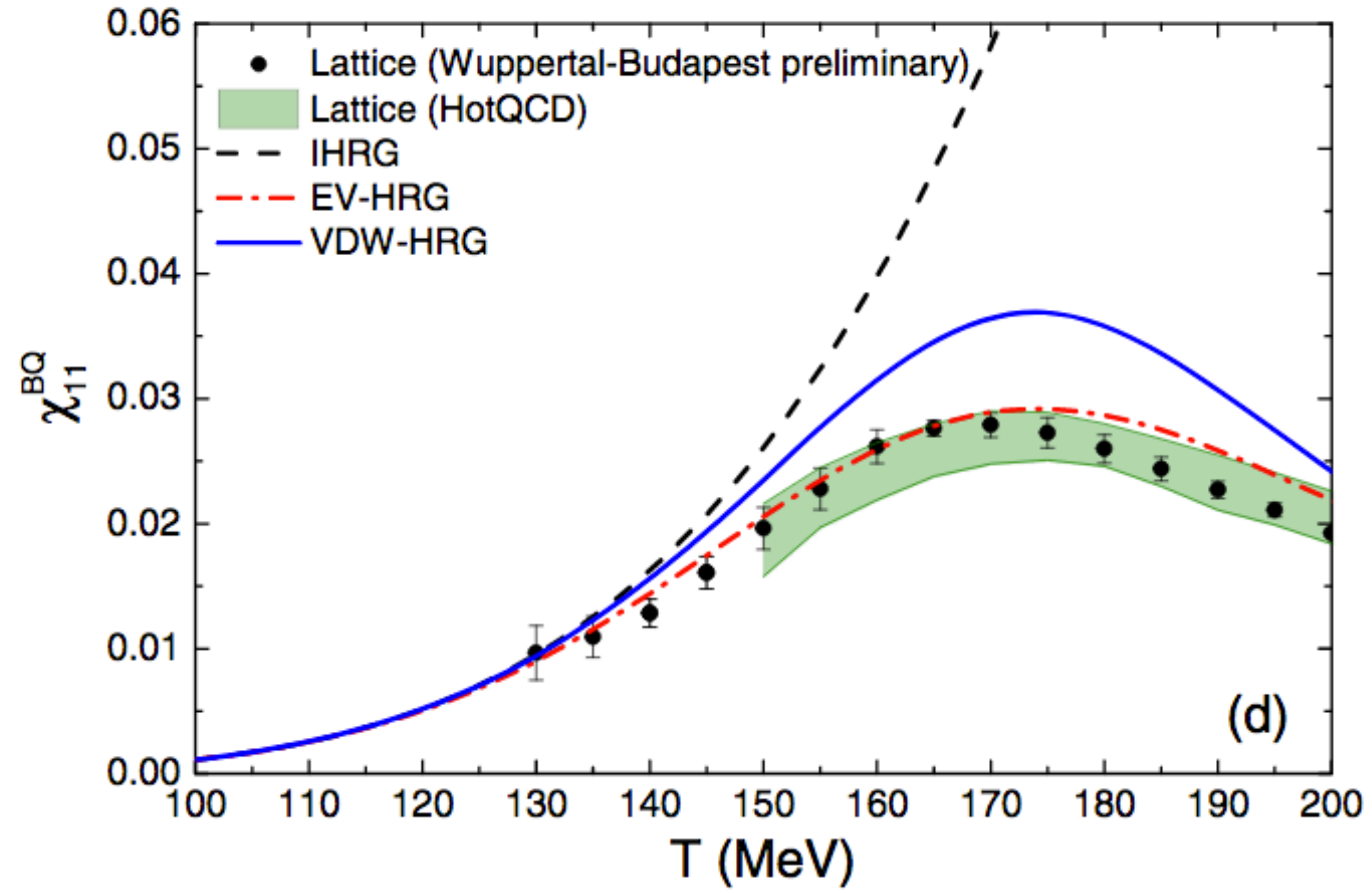
- Different T-dependent for partonic and hadronic phases.

V. Koch et al. PRL.95.182301 (2005), F. Karsch and K. Redlich, PLB 695 , 136142 (2011).

- Sensitivity to the difference between HRG and lattice calculations at the lowest order. New constrain on **freeze-out** condition.



F. Karsch Nuclear Physics A 00 (2017) 1–4



A. Bazavov et al. Phys. Rev. D. 86. 034509, PRL.109.192302 (2012), V. Vovchenko phys. Rev. Lett. 118, 182301

Observables:

✓ "Variance" — Self correlation

$$c_2 = \sigma^2 = \langle(\delta X)^2\rangle$$

✓ "Co-variance" — Cross correlation

$$c_{1,1} = \sigma^{1,1} = \langle(\delta X)(\delta Y)\rangle$$

δX and δY can be:
Net-charge: ΔQ
Net-proton: Δp
Net-kaon: Δk

$\langle \ldots . \rangle \rightarrow$ average over the events

V. Koch et al. PRL.95.182301 (2005),

$$\begin{pmatrix} \sigma^2_Q & \sigma^{1,1}_{Q,p} & \sigma^{1,1}_{Q,k} \\ \sigma^{1,1}_{p,Q} & \sigma^2_p & \sigma^{1,1}_{p,k} \\ \sigma^{1,1}_{k,Q} & \sigma^{1,1}_{k,p} & \sigma^2_k \end{pmatrix}$$

- Bridge between theory and experiment

$$\sigma^{11}_{Q,k} \approx \sigma^{11}_{Q,S} \qquad \sigma^{11}_{Q,p} \approx \sigma^{11}_{Q,B}$$

$$\sigma^{11}_{p,k} \not\approx \sigma^{11}_{B,S}$$

A. Chatterjee, et. al, J. Phys. G43, 125103 (2016)

- What is now?

2^{nd}-order cumulant matrix within a common uniform acceptance.

Observables:

$$\chi_{i,j,...} = \frac{1}{VT^3} < (\Delta N)^i (\Delta N)^j ... >_c$$

V and T dependence

Ratio : $$C_{XY} = \frac{\sigma^{11}_{XY}}{\sigma^2_Y}$$

V. Koch et al. PRL.95.182301 (2005),

Volume independent correlation compared to self correlation

We study

- Beam energy and centrality dependence variance, covariance and ratio.
- acceptance (Iηl) dependence:

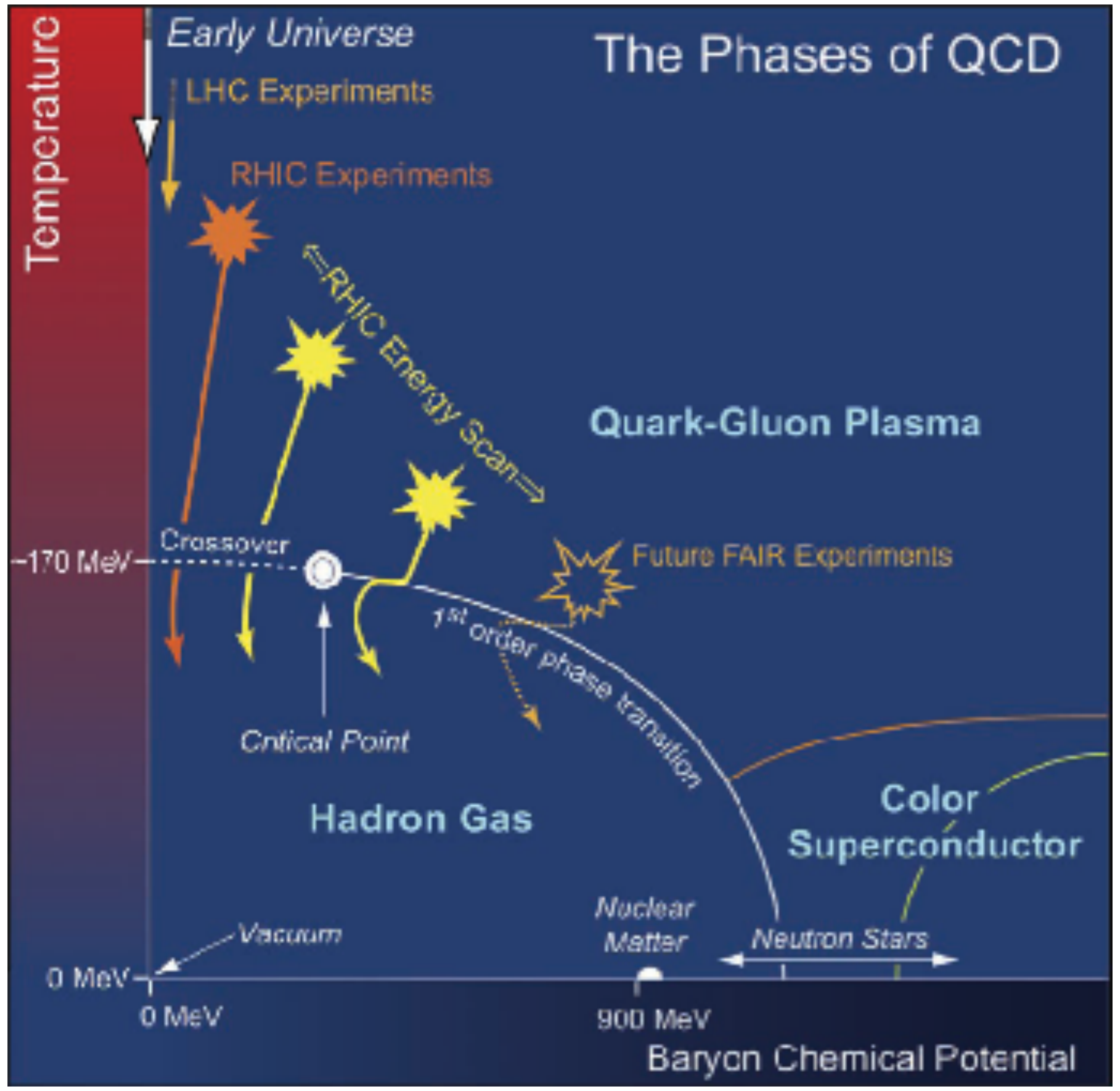
Provide useful information about baryon stopping, global charge conservation effect.

η acceptance

No signal 0 % $\Delta\eta=0$	No signal 100 % $\Delta\eta=\Delta Y_{beam}$
Poisson fluctuation ration ~ 1	fluctuation = 0 baryon/charge/ strangeness # conservation

J. Brewer et al, arXiv:1804.10215 [hep-ph]
A. Chatterjee, et. al, J. Phys. G43, 125103 (2016)

RHIC beam energy scan I:



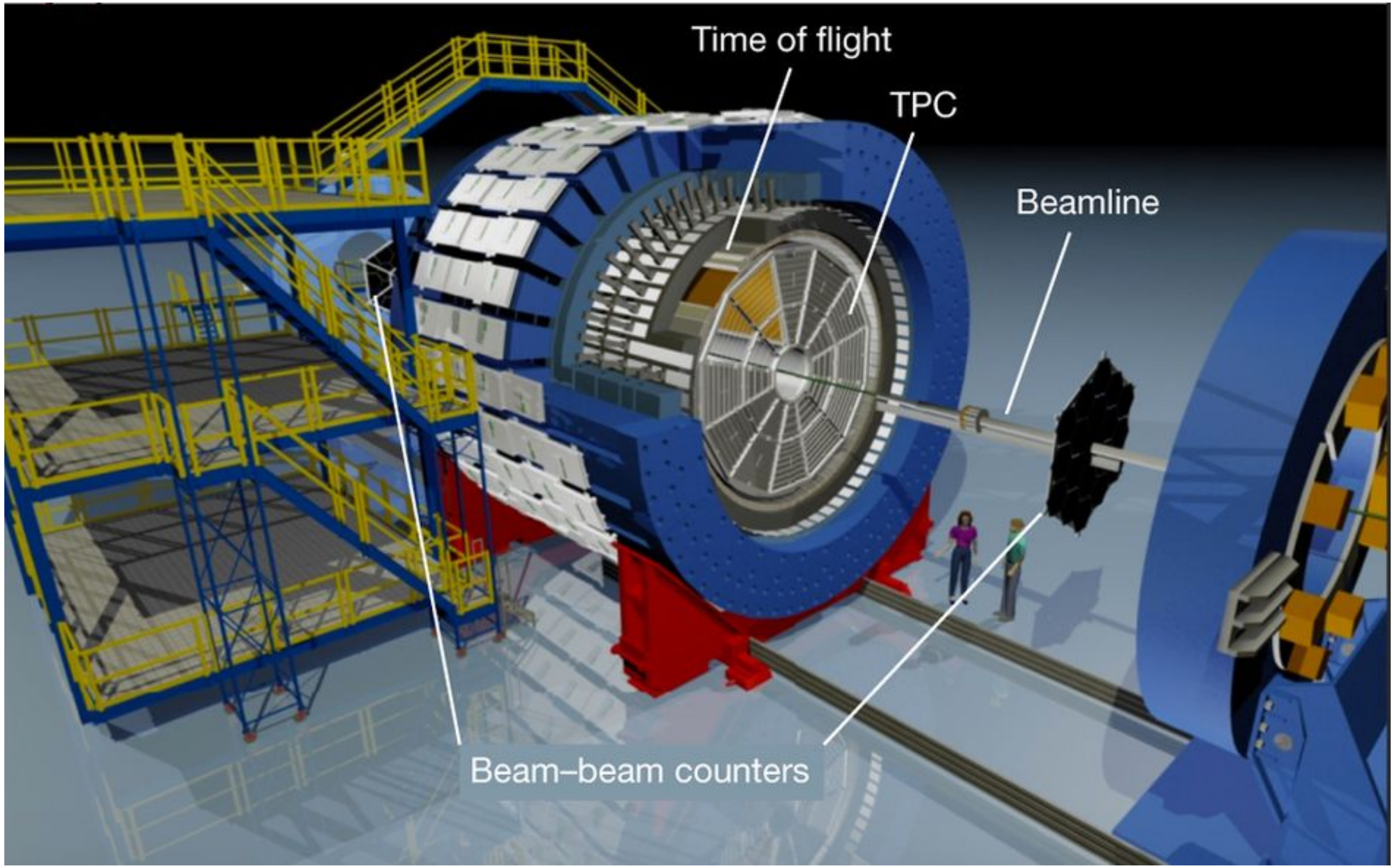
arXiv:0809.3137

$\sqrt{s_{NN}}$ (GeV)	μ_B (MeV)	T (MeV)	μ_B/T	Statistics (M) (0-80%)
7.7	422	140	3.020	~4
11.5	316	152	2.084	~12
14.5	316	152	1.639	~20
19.6	206	160	1.287	~36
27	156	163	0.961	~70
39	112	164	0.684	~130
62.4	73	165	0.439	~67
200	25	166	0.150	~350

T, μ_B values from J. Cleymans, et al. Phys. Rev. C 73, 034905

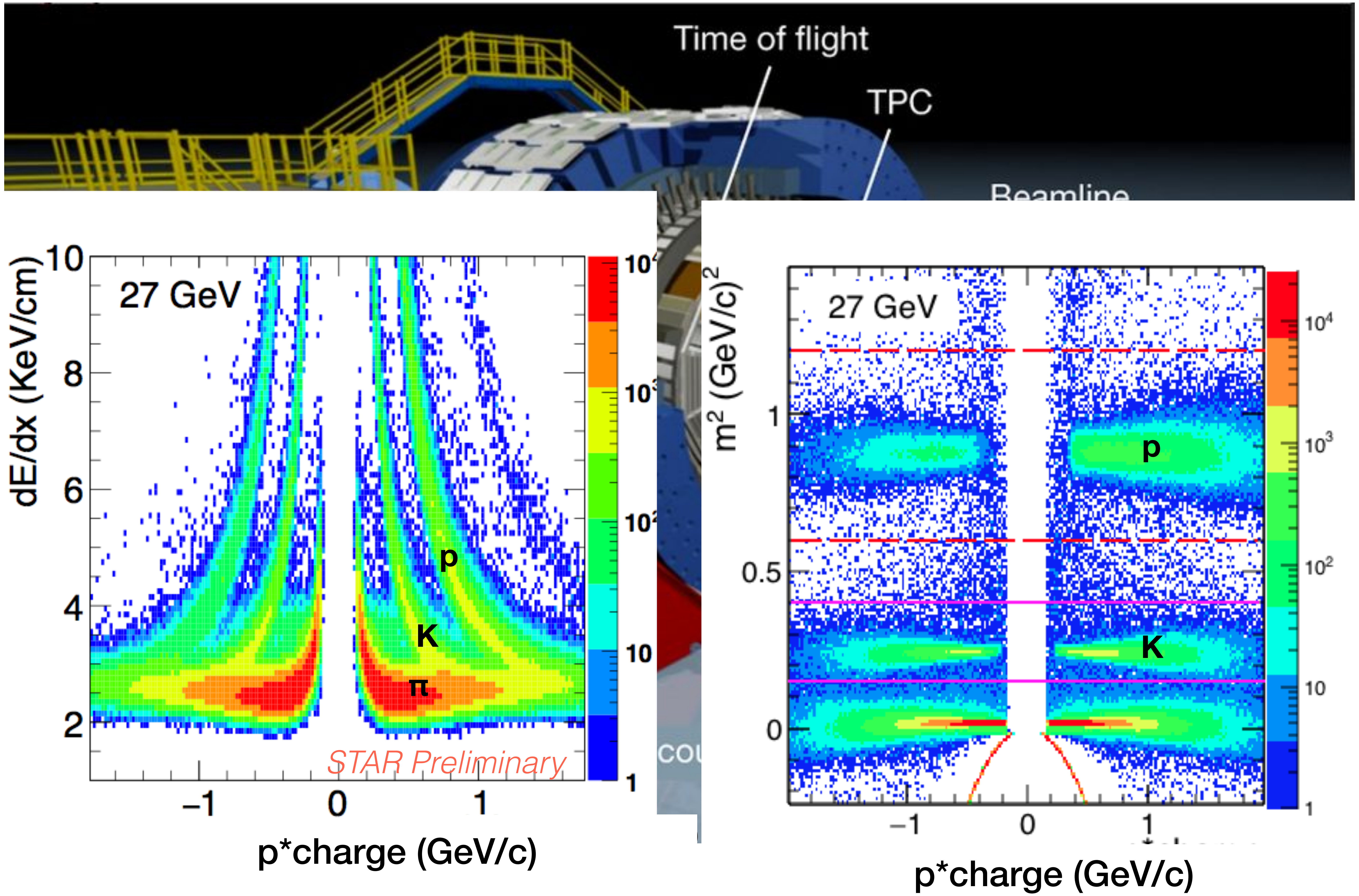
- Varying beam energy, we can access broad region of the QCD phase diagram.
  - QCD phase diagram can be mapped between μ_B values 20 to 425 MeV.

The STAR experiment:



- Full 2π coverage.
- Uniform acceptance.
  $-1<\eta<1$ & $0<\varphi<2\pi$
- Time Projection Chamber
  - Momentum determination.
  - PID through dE/dx.
- Time-of-Flight (TOF)
  - PID through mass^2 cut.

Particle identification:



- Uniform acceptance for proton, kaon and charge selection using both TPC and TOF.

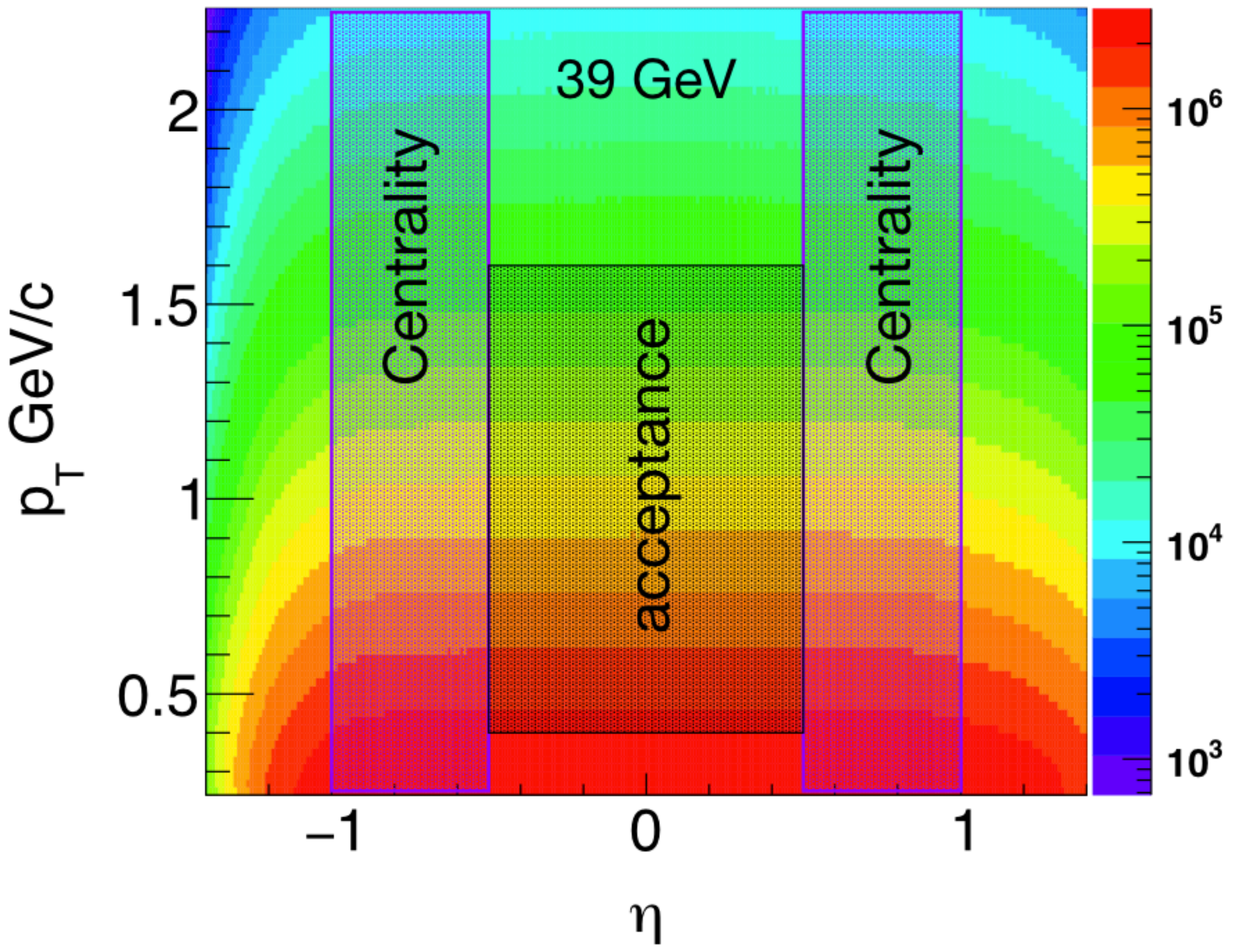
$$|\eta| < 0.5$$

$$0.4 < p_T < 1.6 \text{ GeV/c}$$

Analysis technique:

❖ Centrality: Reference Multiplicity 2

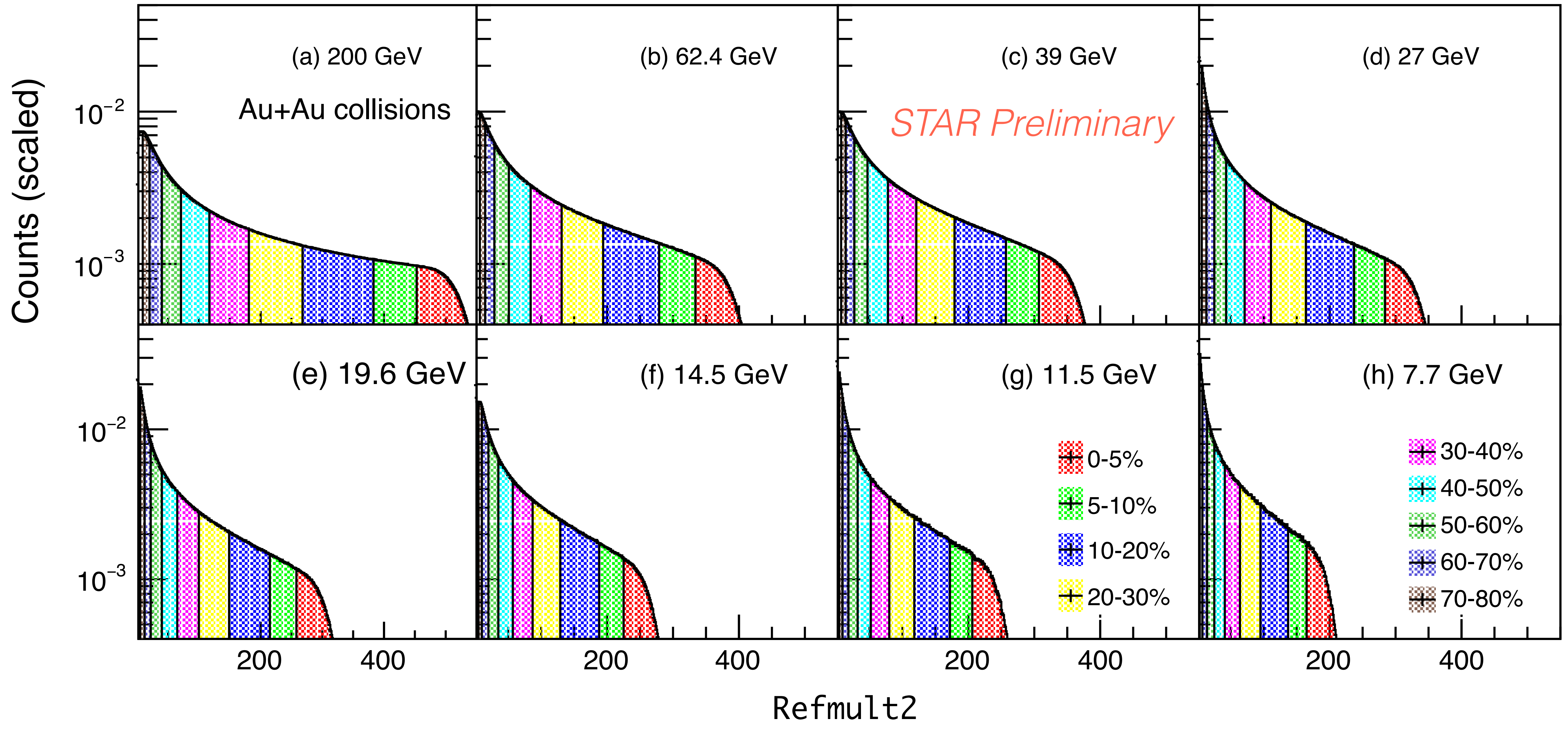
Use charged particles within 0.5<|η|<1.0 and -1.0<|η|<-0.5 (avoid track from analysis region), to avoid **auto-correlation.**



❖ Centrality bin width (**CBWC**) averaging : To suppress the artificial fluctuation due to initial variation in volume.

$$\kappa^{XY} = \sum_{i=N_1}^{N_2} n_i \kappa_i^{XY} \Big/ \sum_{i=N_i}^{N_2} n_i$$

N. R. Sahoo et. al, Phys. Rev. C 87.044906 (2013), X. Luo et. al., JPG40, 105104 (2013), Journal of Physics: Conf. Ser. 316, 012003



Efficiency correction:

- **Efficiency correction** on cumulants has been done using binomial detection response.

$$p(n) = \sum_{N=n}^{\infty} \frac{N!}{n!(N-n)!}(\varepsilon)^n(1-\varepsilon)^{N-n}P(N) = \sum_{N=n}^{\infty} B(n|N,\varepsilon)P(N)$$

Detected — Incident (true)

$$\begin{aligned}
\mu_1^{obs} &= \varepsilon\mu_1^{inc}\\
\mu_2^{obs} &= \varepsilon^2\mu_2^{inc} + \varepsilon(1-\varepsilon)\mu_1^{inc}\\
\mu_3^{obs} &= \varepsilon^3\mu_3^{inc} + 3\varepsilon^2(1-\varepsilon)\mu_2^{inc} + \varepsilon(1-2\varepsilon)\mu_1^{inc}
\end{aligned}$$

No clear pattern: **hard to handle**

Convert to **factorial moment**

$$\begin{aligned}
f_1^{obs} &= \varepsilon f_1^{inc}\\
f_2^{obs} &= \varepsilon^2 f_2^{inc}\\
f_3^{obs} &= \varepsilon^3 f_3^{inc}\\
f_n^{obs} &= \varepsilon^n f_n^{inc}
\end{aligned}$$

clear pattern: **easy** to correct

- Strategy is to convert cumulants to Irreducible factorial moments and correct for it.

$$F^{s,t,u,v,w,x,y,z}_{N_{(p,1)},N_{(p,2)},N_{(\bar{p},1)},N_{(\bar{p},2)},N_{(k_+,1)},N_{(k_+,2)},N_{(k_-,1)},N_{(k_-,2)}} = \frac{f^{s,t,u,v,w,x,y.z}_{n_{(p,1)},n_{(p,2)},n_{(\bar{p},1)},n_{(\bar{p},2)},n_{(k_+,1)},n_{(k_+,2)},n_{(k_-,1)},n_{(k_-,2)}}}{\varepsilon^s_{(p,1)},\varepsilon^t_{(p,2)},\varepsilon^u_{(\bar{p},1)},\varepsilon^v_{(\bar{p},2)},\varepsilon^w_{(k_+,1)},\varepsilon^x_{(k_+,2)},\varepsilon^y_{(k_-,1)},\varepsilon^z_{(k_-,2)}}$$

Corrected

Uncorrected

TPC (1) and TPC*TOF matching (2) efficiencies

M. Kendall, The Advanced Theory of Statistics No. v. 1 (1943)
X. Luo, Phys.Rev. C 91,034907 (2015)
A. Bzdakand V.Koch, PRC 86 044904, PRC 91 027901

Uncertainty estimation:

- **Statistical uncertainty:** **Precession limit** of the measurement. **Standard error (=√V(ϕ))**

Sampling variance

$$V(\mu_{r,s}) = \frac{1}{n}(\mu_{2r,2s} - \mu_{r,s}^2 + \ldots\ldots\ldots - 2s\mu_{r,s+1}\mu_{r,s-1})$$

Sampling covariance:

$$Cov(\mu_{r,s}, \mu_{u,v}) = \frac{1}{n}(\mu_{r+s,u+v} - \ldots\ldots\ldots - s\mu_{r,s+1}\mu_{u,v-1})$$

M. Kendall, The Advanced Theory of Statistics No. v. 1 (1943)

Ratio

$$V(\phi = C_{xy} = \mu_{1,1}/\mu_{0,2}) = (\frac{\partial\phi}{\partial\mu_{1,1}})^2 V(\mu_{1,1}) + (\frac{\partial\phi}{\partial\mu_{0,2}})^2 V(\mu_{0,2}) + 2\frac{\partial\phi}{\partial\mu_{1,1}}\frac{\partial\phi}{\partial\mu_{0,2}} Cov(\mu_{1,1}, \mu_{0,2})$$

n is the number of events,

Statistical uncertainty : Inversely proportional to #events

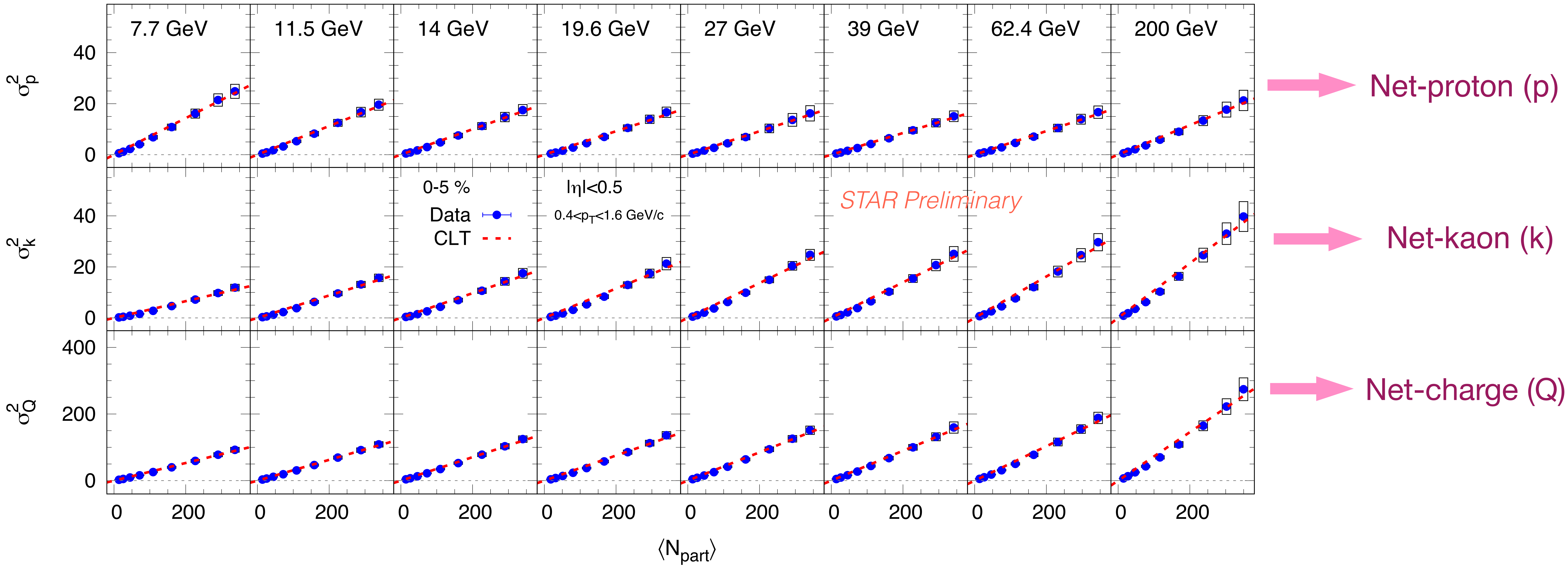
Proportional to its double power cumulant $error(\kappa_r) \propto \sqrt{\kappa_{2r}/n_{ev}}$

Less statistical uncertainty for the 2nd-order cumulants

X. Luo, J.Phys. G39,025008 (2012).
A. Chatterjee, et. al, J. Phys. G43, 125103 (2016)

- **Systematic uncertainty:** **Accuracy limit** of the measurement. How well we can control the experiment.
  vary different track selection criteria, like DCA, nSigma, nFitPoints
  Also vary efficiency by 5%.

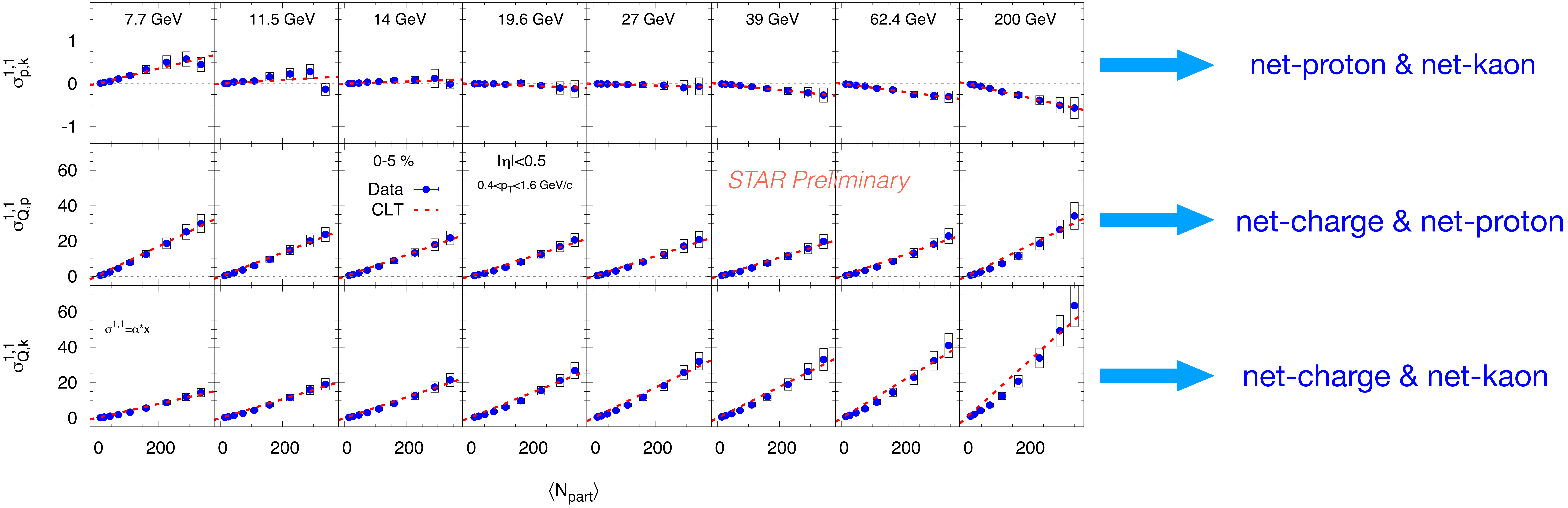
Diagonal cumulant:



- Variance increases with the number of colliding nucleons and consistent with CLT expectation.

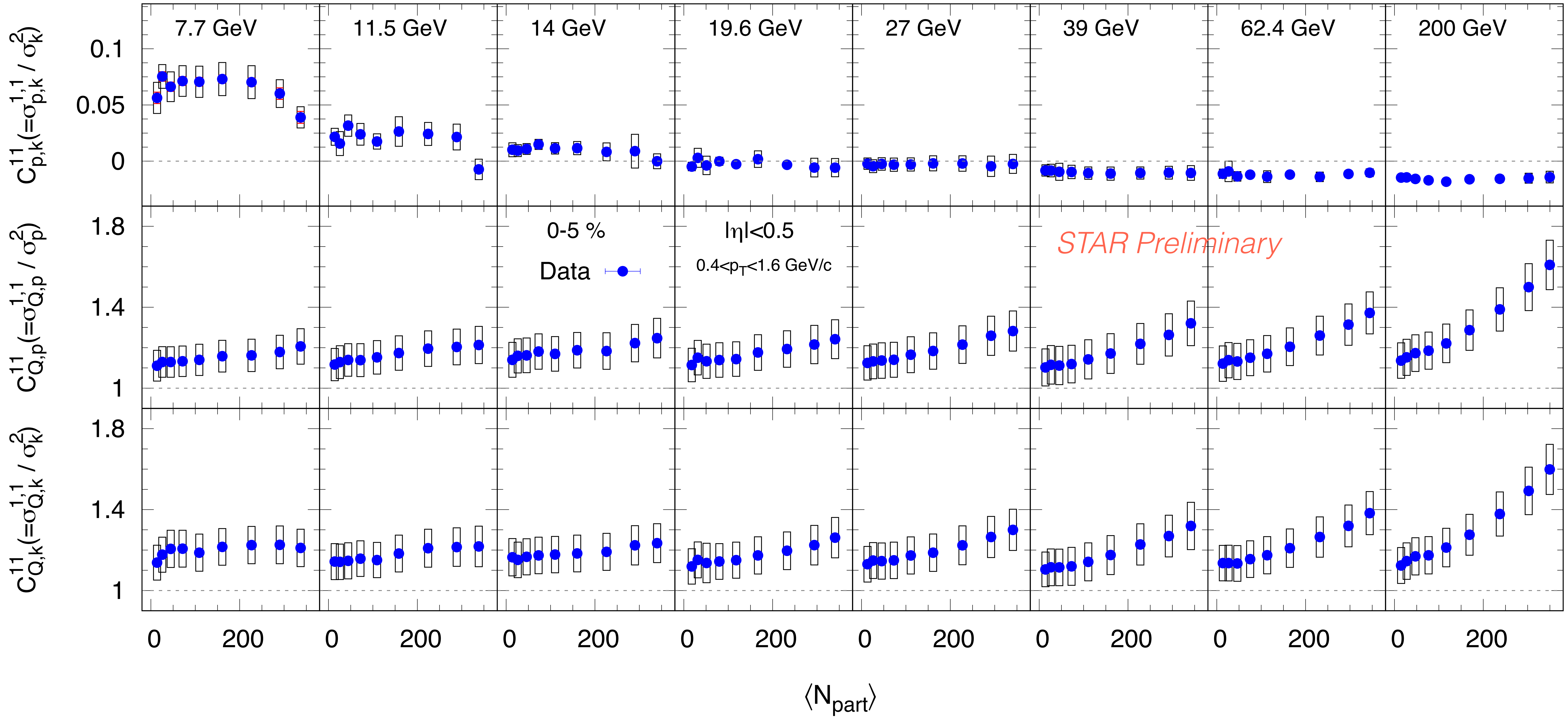
$$\sigma^2 \propto \langle N_{part} \rangle$$

Off-diagonal cumulant:



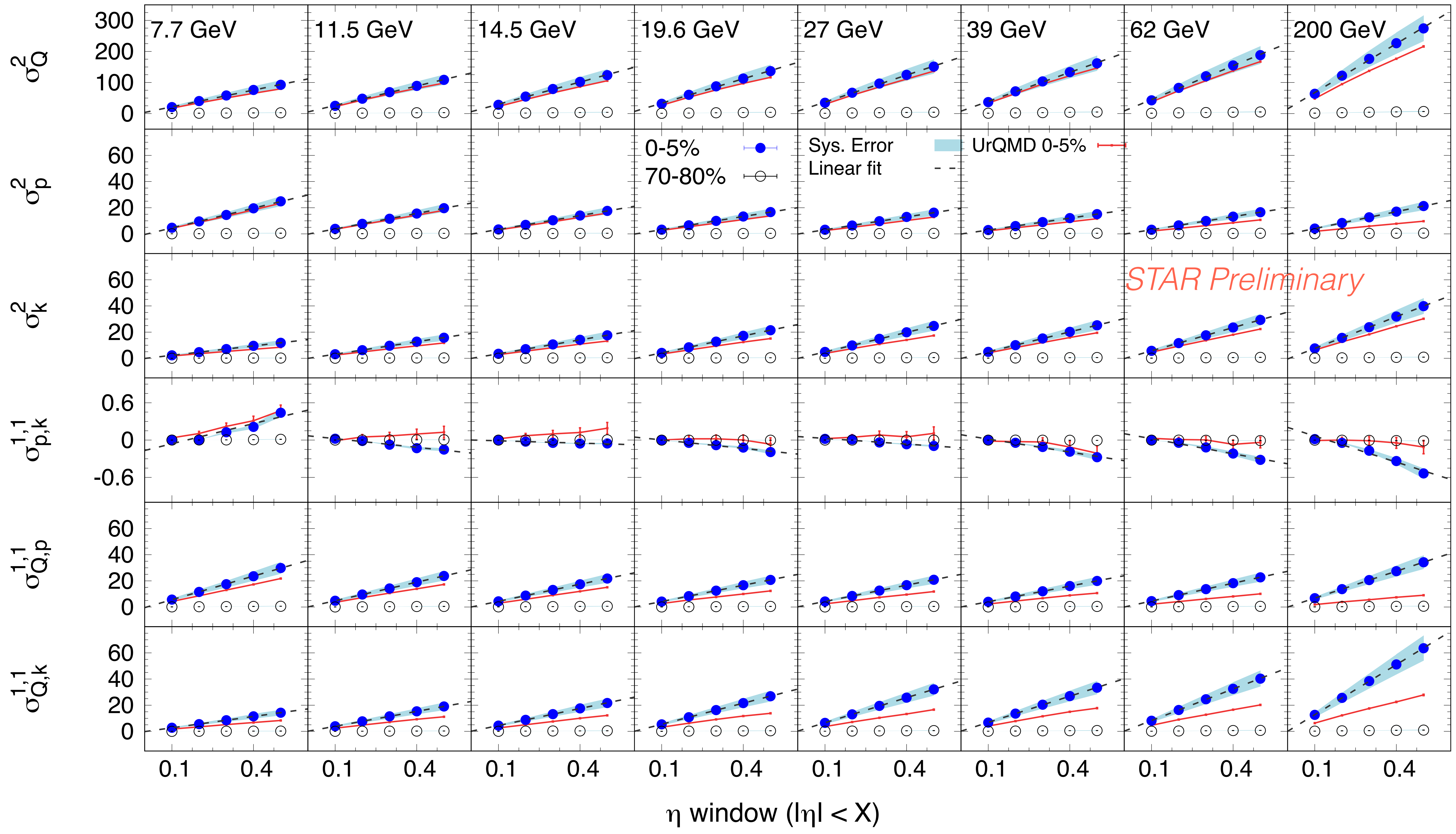
- σ_{Qp} and $\sigma_{Q,k}$ show linear dependence with respect to centrality.
- The covariance between net-proton and net-kaon is positive at low energy and negative at higher energy — Indicating net-p and net-k are anti-correlated at high energy.
- Covariance follows the CLT like variance : $\sigma^{11} \propto \langle N_{part} \rangle$

Centrality dependence of cumulant ratio:



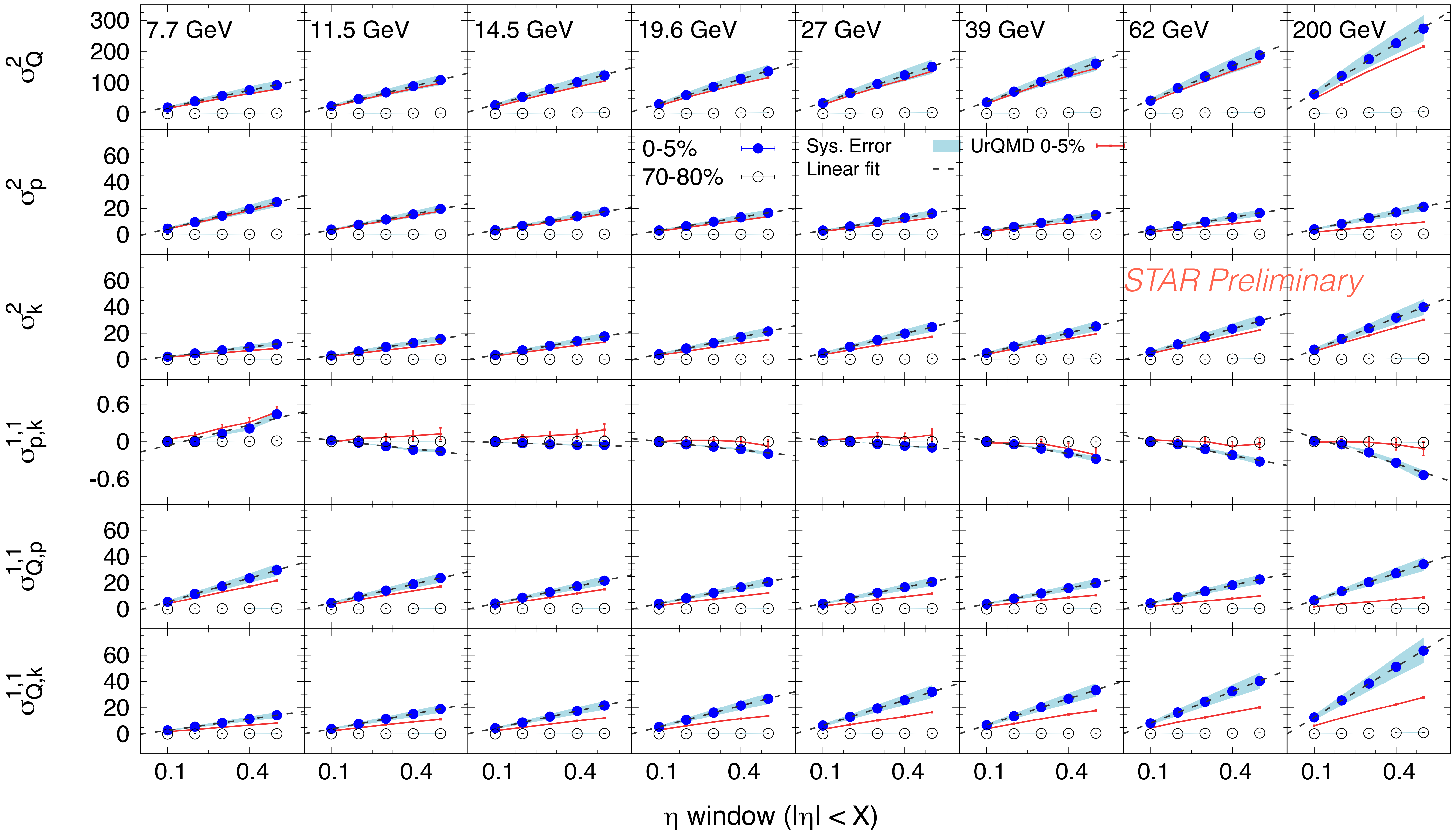
- C_{pk} sign changes around 14.5-19.6 GeV.
- C_{Qk} and C_{Qp} > 1. Increase with centrality for BES energies.

Acceptance dependence:



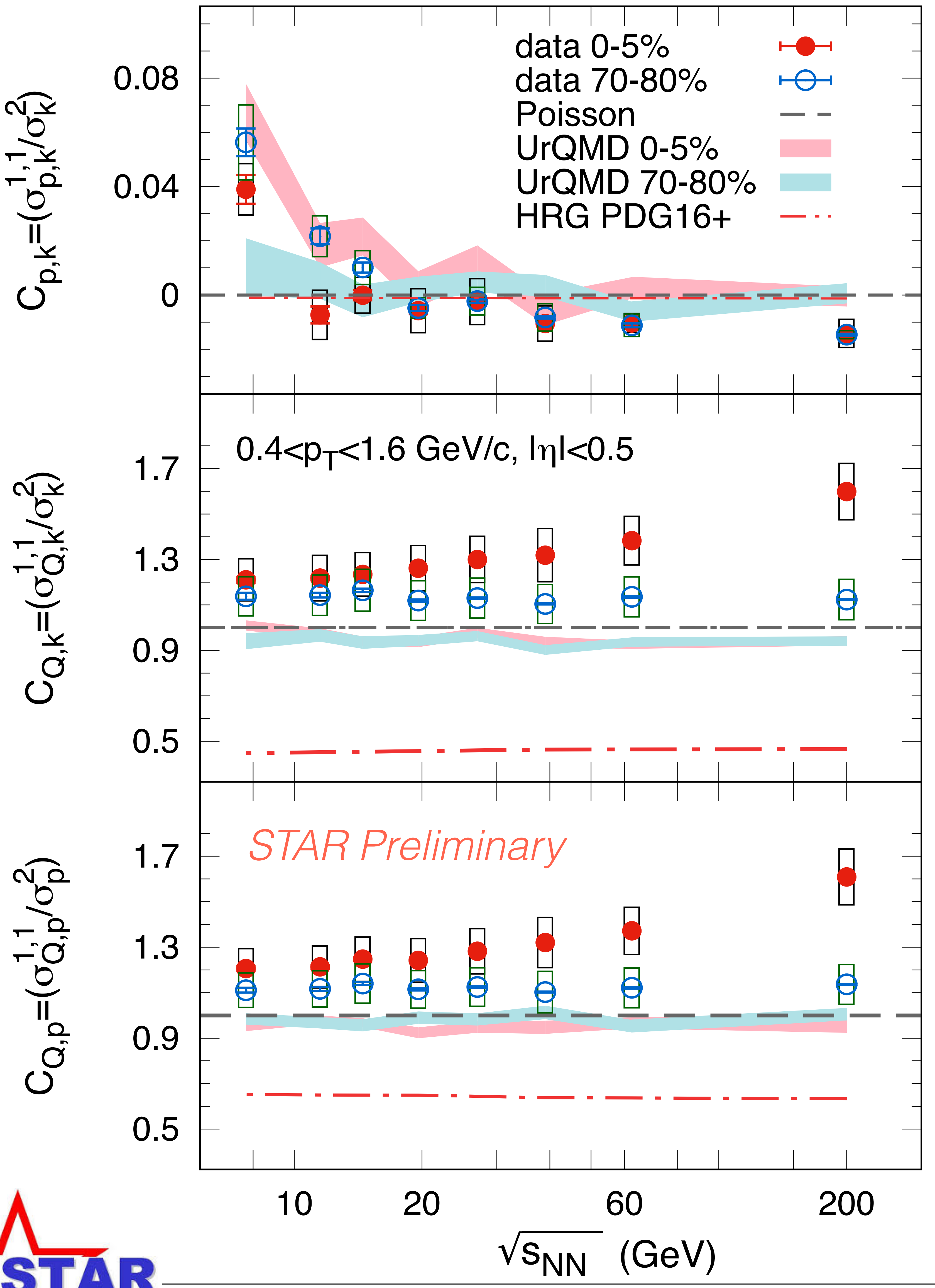
- Both diagonal and off-diagonal cumulants of Q, p and k linearly increase with the |η|-window for central (0-5%) collisions and show almost a constant dependence for peripheral (70-80%) collisions.
- No non-monotonic variation as a function of acceptance is observed.
- UrQMD does a relatively better job for diagonal cumulants. Fails to explain off-diagonal cumulants.

Acceptance dependence:



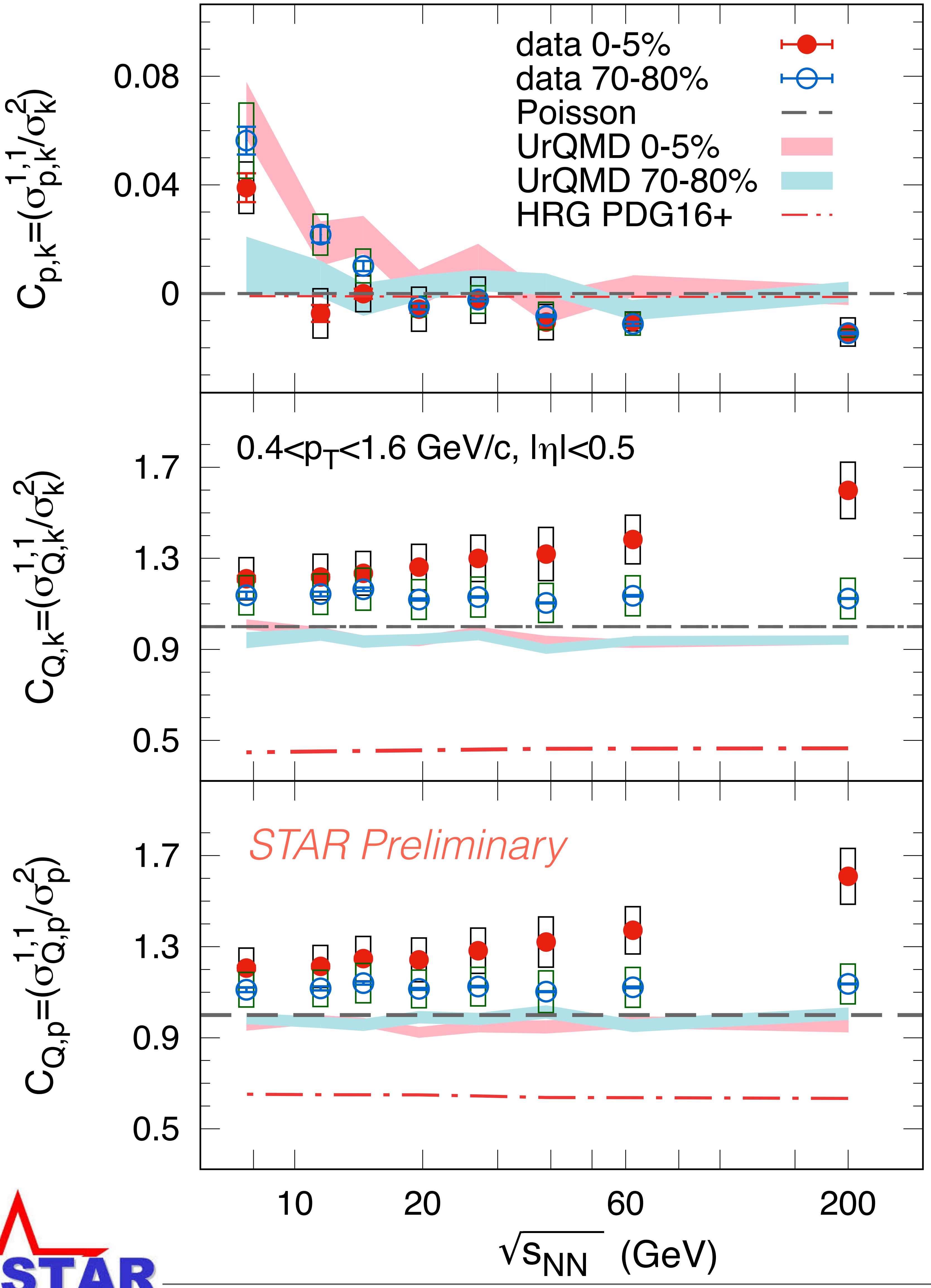
- Both diagonal and off-diagonal cumulants of Q, p and k linearly increase with the |η|-window for central (0-5%) collisions and show almost a constant dependence for peripheral (70-80%) collisions.
- No non-monotonic variation as a function of acceptance is observed.
  - With the current experimental setup we can only explore a small portion of the acceptance dependence.
  - BES-II may provide better insight on η-dependence.

Beam energy dependence ratio:



- Normalized cumulants between net-p and net-k are positive at low energies and negative at high energies.
  - negative contribution in hadronic medium can be affected by resonance production, like Λ(1520)—>pK⁻ (22%).
  - @ lower energies, pp—>pΛK⁺ process may lead to positive correlation. J. T. Balewski et. al, PLB 420, 211 (1998)
  - negative correlation @ 200 GeV cannot be explained by UrQMD.
  - In QGP phase, B-S correlation is negative. We also observed negative p-K correlation. Although direct quantitate comparison is not possible. V. Koch et al. PRL.95.182301 (2005),
  - C_{pk} may provide some important information on baryon-strange correlation.

Beam energy dependence ratio:



- Normalized cumulants between net-p and net-k are positive at low energies and negative at high energies.
- C_{Qp} and C_{Qk} both show significantly higher correlation in central collisions as compared to the peripheral ones and Poisson baseline.
- The increasing trend in C_{Qp} and C_{Qk} not observed in both UrQMD and HRG.
- More theoretical studies are needed to understand this excess correlation.

Summary

- First measurement of all 2nd-order cumulant matrix elements as a function of collision centrality for Au+Au collisions $\sqrt{s_{NN}}$= 7.7, 11.5, 14.5, 19.6, 27, 39, 62.4 and 200 GeV are presented. Results are shown for the kinematic range of $|\eta| < 0.5$ and $0.4 < pT < 1.6$ GeV/c as well as different $|\eta|$-windows.
- An excess correlations between net-charge and net-kaon, and net-charge and net-proton are observed for central collisions with respect to the peripheral ones. The correlations increase with increasing the beam energy. This increase is larger compared to the Poisson baseline and has not been observed for the UrQMD event generator.
- In QGP phase, B-S correlation is negative. We also observed negative p-K correlation. Although direct quantitate comparison is not possible.
- Both diagonal and off-diagonal cumulants of net-charge, net-kaon and net-proton show linear dependence with the $\Delta\eta$ acceptance window.

Thank You

Backup

Back up slides

p_T-integrated efficiency

p_T-integrated efficiency

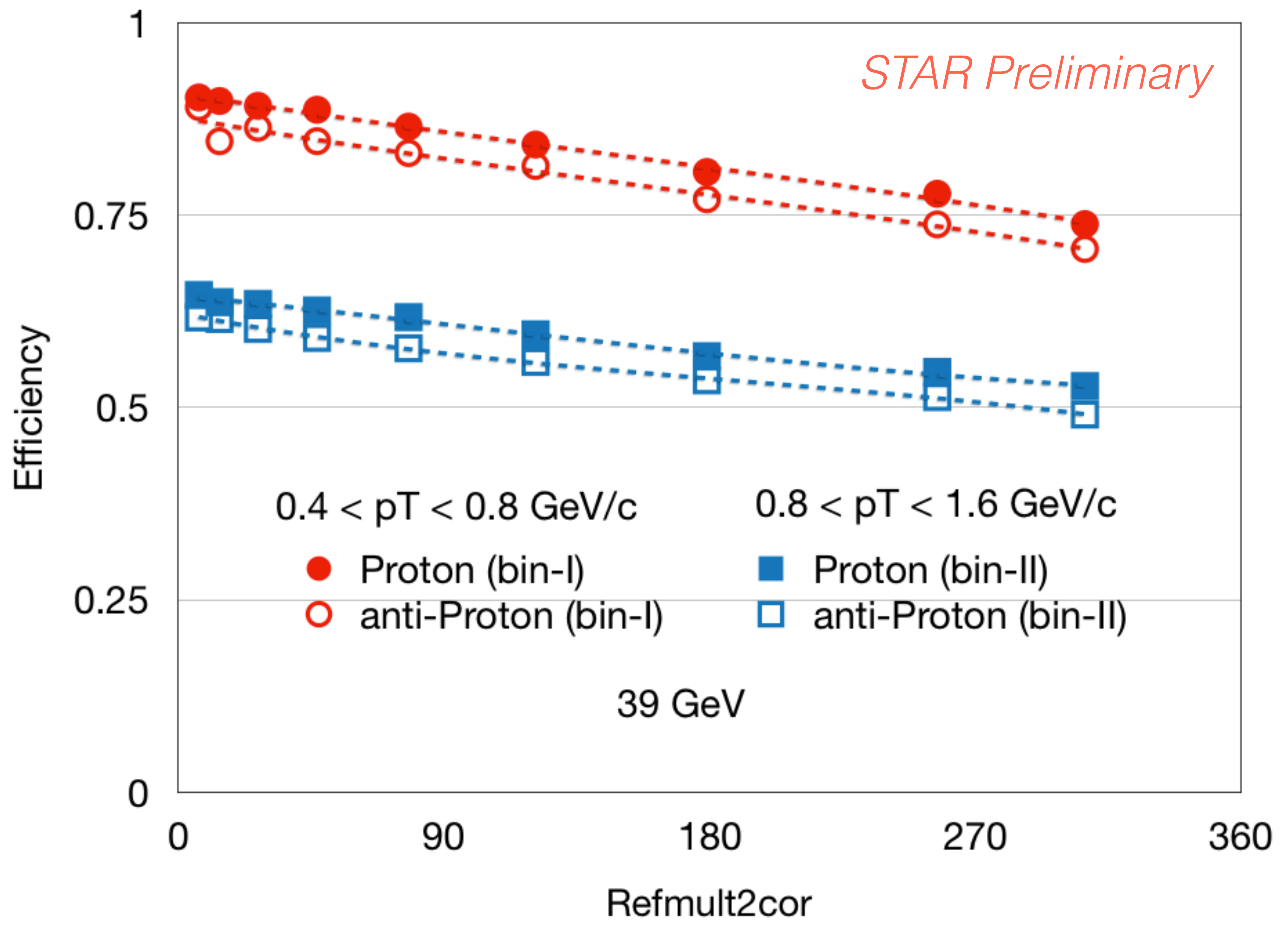
$\varepsilon' = p_T$ dependence efficiency

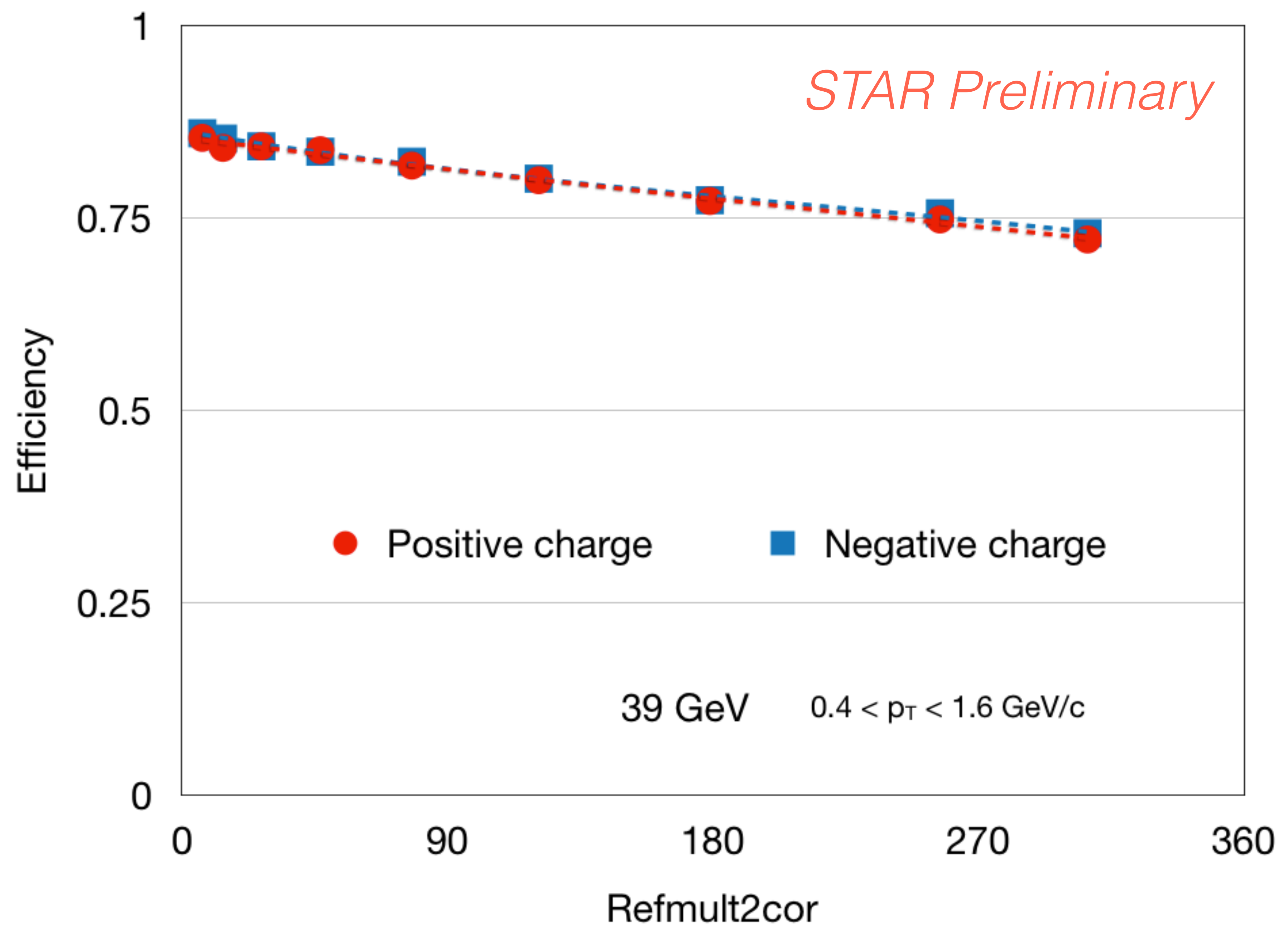
$f(p_T) =$ Transverse momentum spectra for $p^{\pm}, K^{\pm}$ or $\pi^{\pm}$

$$\langle \varepsilon \rangle = \frac{\int_a^b \varepsilon'(p_T) f(p_T) p_T dp_T}{\int_a^b f(p_T) p_T dp_T}$$

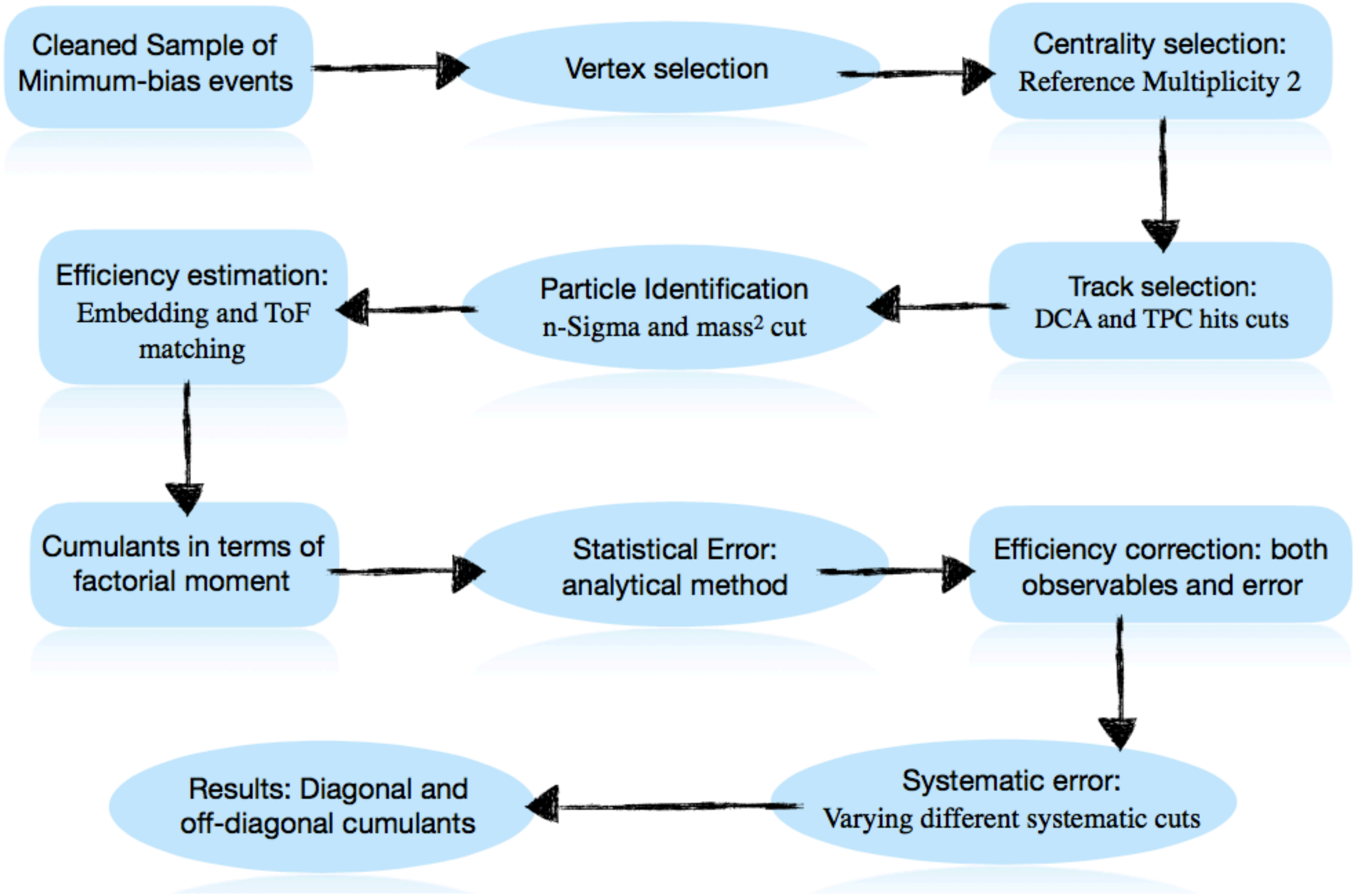
Average charge efficiency are based on for an integrated p_T range a—> b

$$\langle \varepsilon_{ch} \rangle = \frac{\int_a^b \left[\varepsilon_p(p_T) f_p(p_T) + \varepsilon_K(p_T) f_K(p_T) + \varepsilon_\pi(p_T) f_\pi(p_T)\right] p_T dp_T}{\int_a^b \left[f_p(p_T) + f_K(p_T) + f_\pi(p_T)\right] p_T dp_T}$$

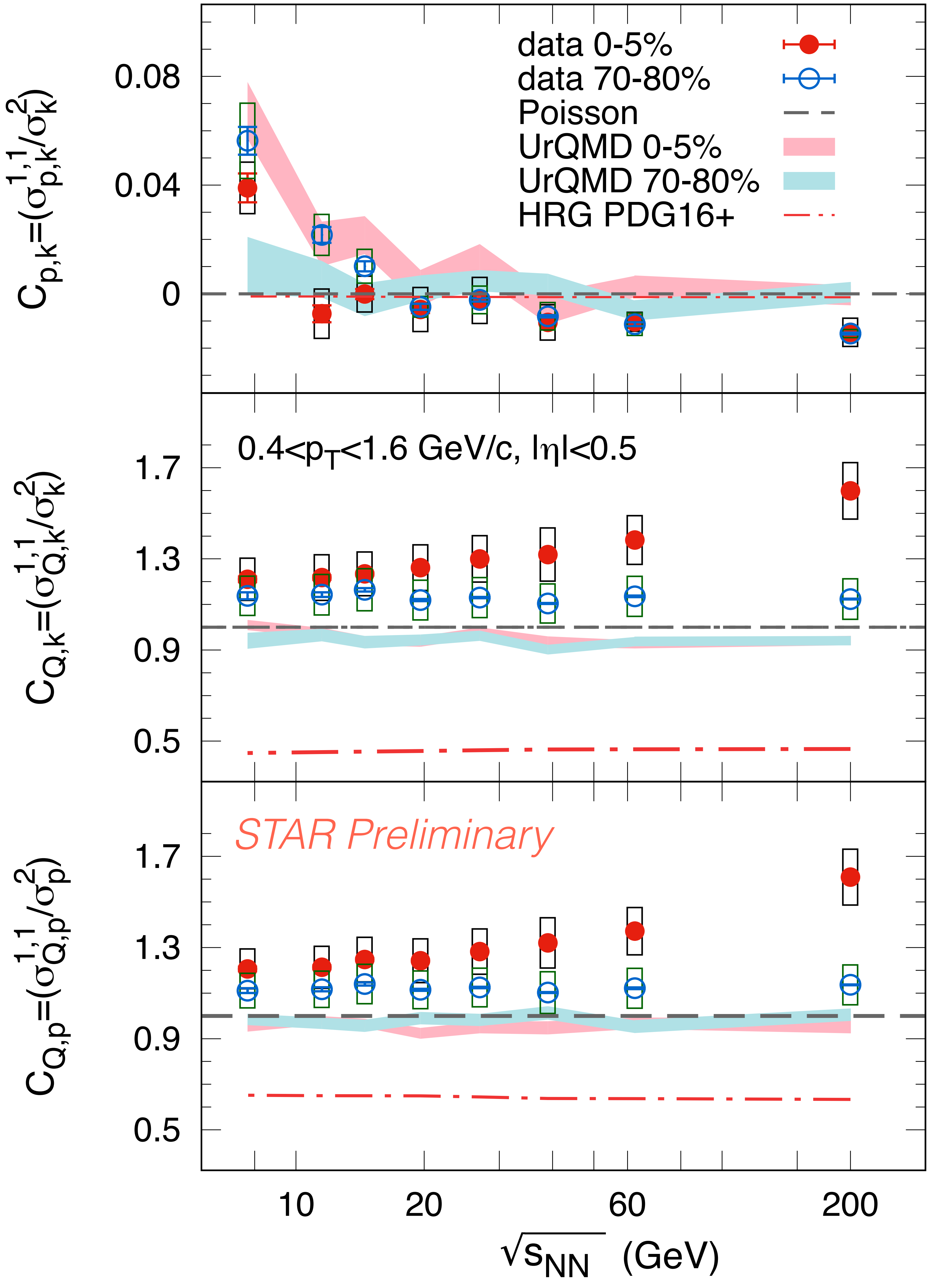


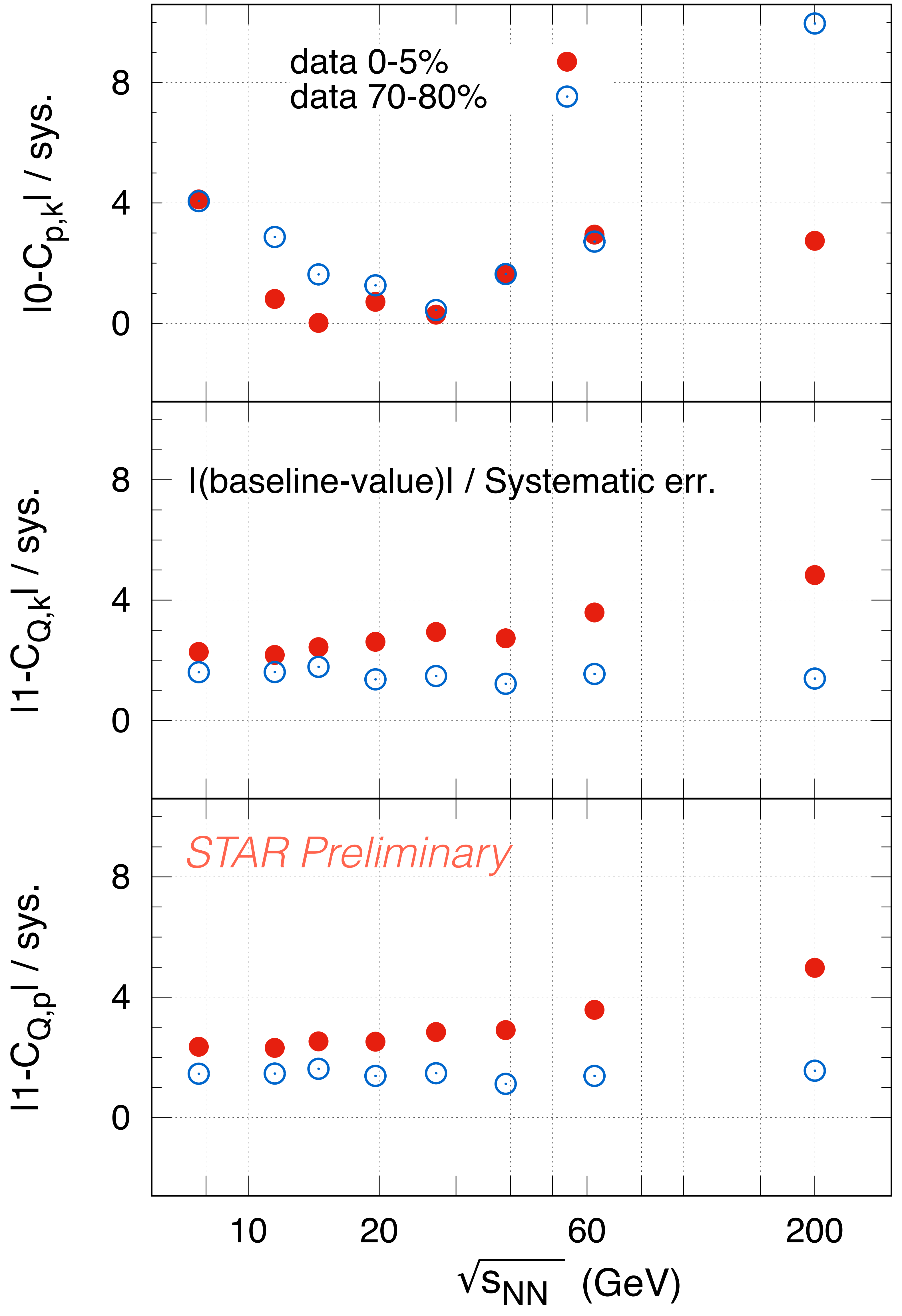


Analysis flowchart

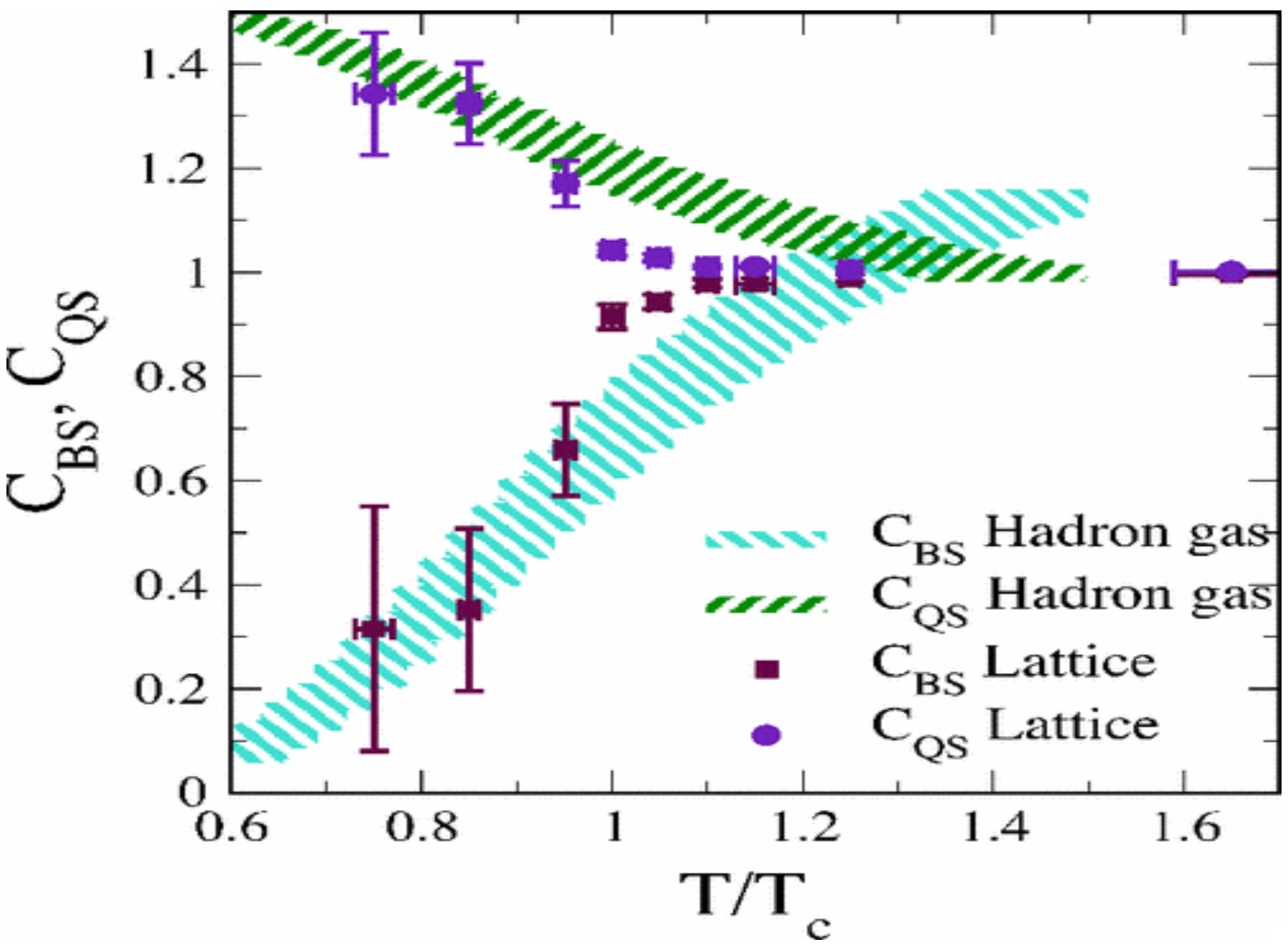


Cumulant ratio:





B-S correlation in Partonic medium



V. Koch et al. PRL.95.182301 (2005),

	Q	B	S
u	+2/3	1/3	0
d	-1/3	1/3	0
s	-1/3	1/3	-1

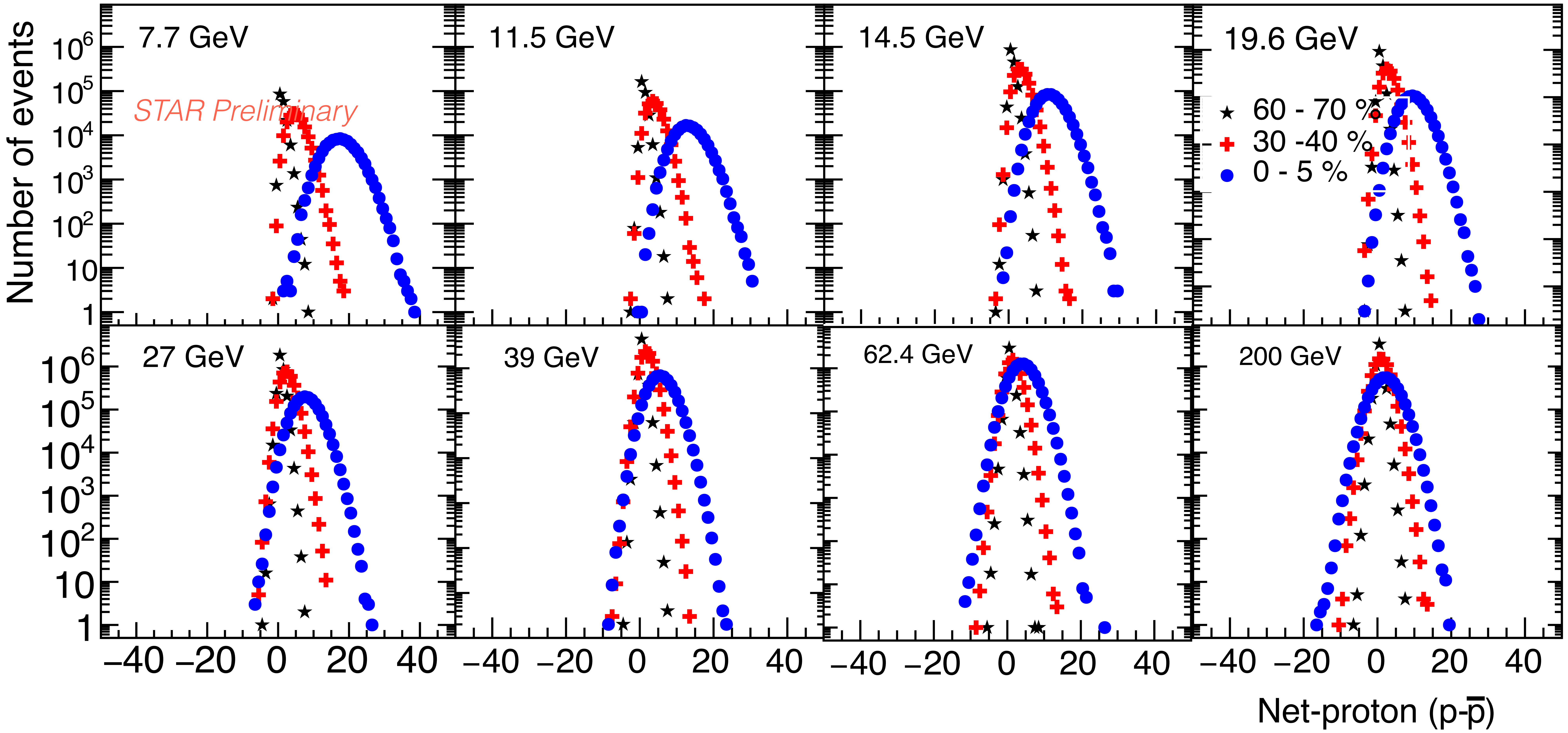
$$B = \frac{1}{3}(\Delta u + \Delta d + \Delta s),$$
$$Q = \frac{2}{3}\Delta u - \frac{1}{3}\Delta d - \frac{1}{3}\Delta s,$$
$$S = -\Delta s.$$

Partonic:

$$C_{BS} = -3\frac{\langle \delta B \delta S\rangle}{\langle \delta S^2\rangle} = -3 \times \frac{\frac{1}{3}(\Delta u + \Delta d + \Delta s)(-\Delta s)}{(\Delta s)^2} = 1$$

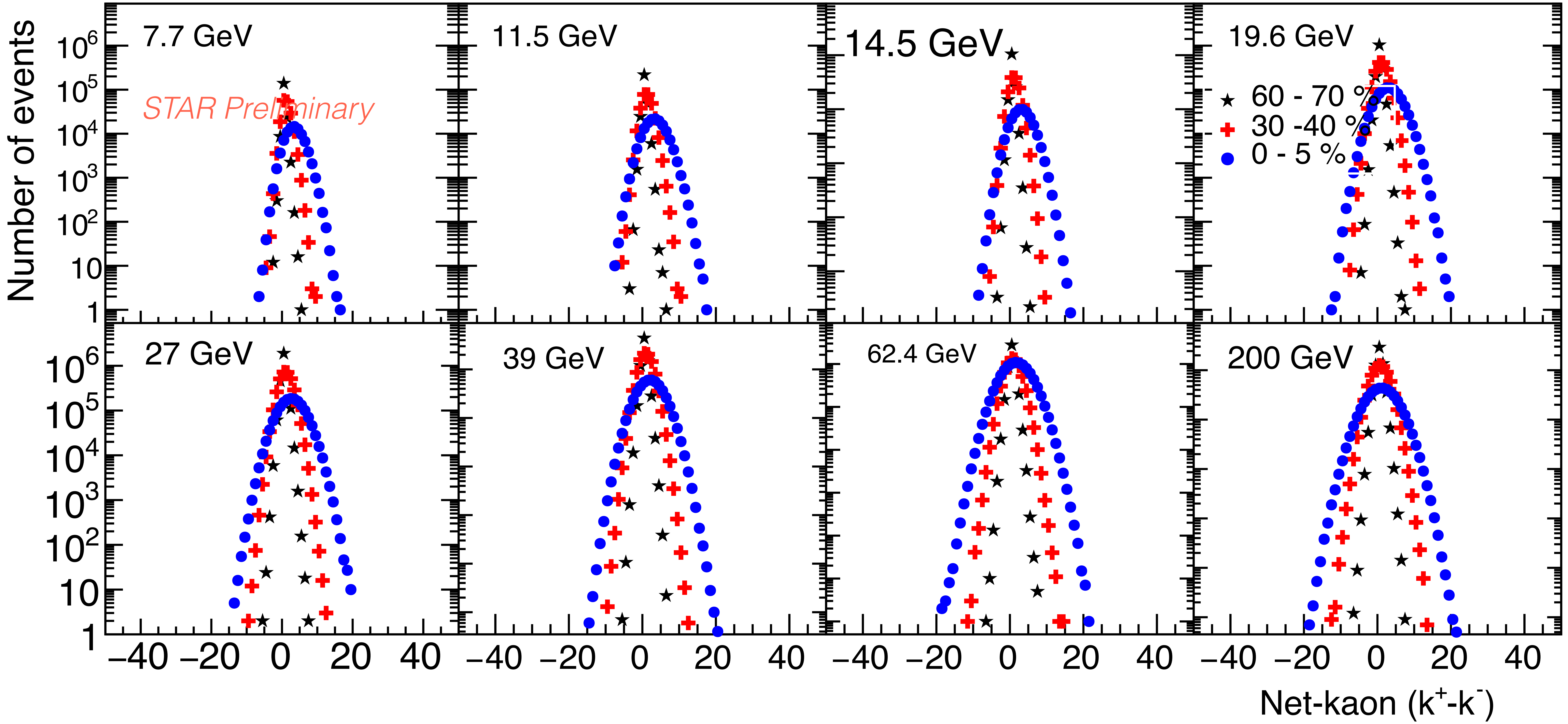
- In QGP phase, B-S correlation is negative, $\langle \delta B \delta S\rangle < 0$

Net-proton multiplicity distributions:



- Efficiency uncorrected
- $|\eta|<0.5$ and $0.4<p_T<1.6$ GeV/c

Net-kaon multiplicity distributions:



- Efficiency uncorrected
- |η|<0.5 and 0.4<pT<1.6 GeV/c