Recent Results on Cumulants of Net-Particle Distributions in Au+Au Collisions at STAR

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STAR
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Outline

✓ Introduction

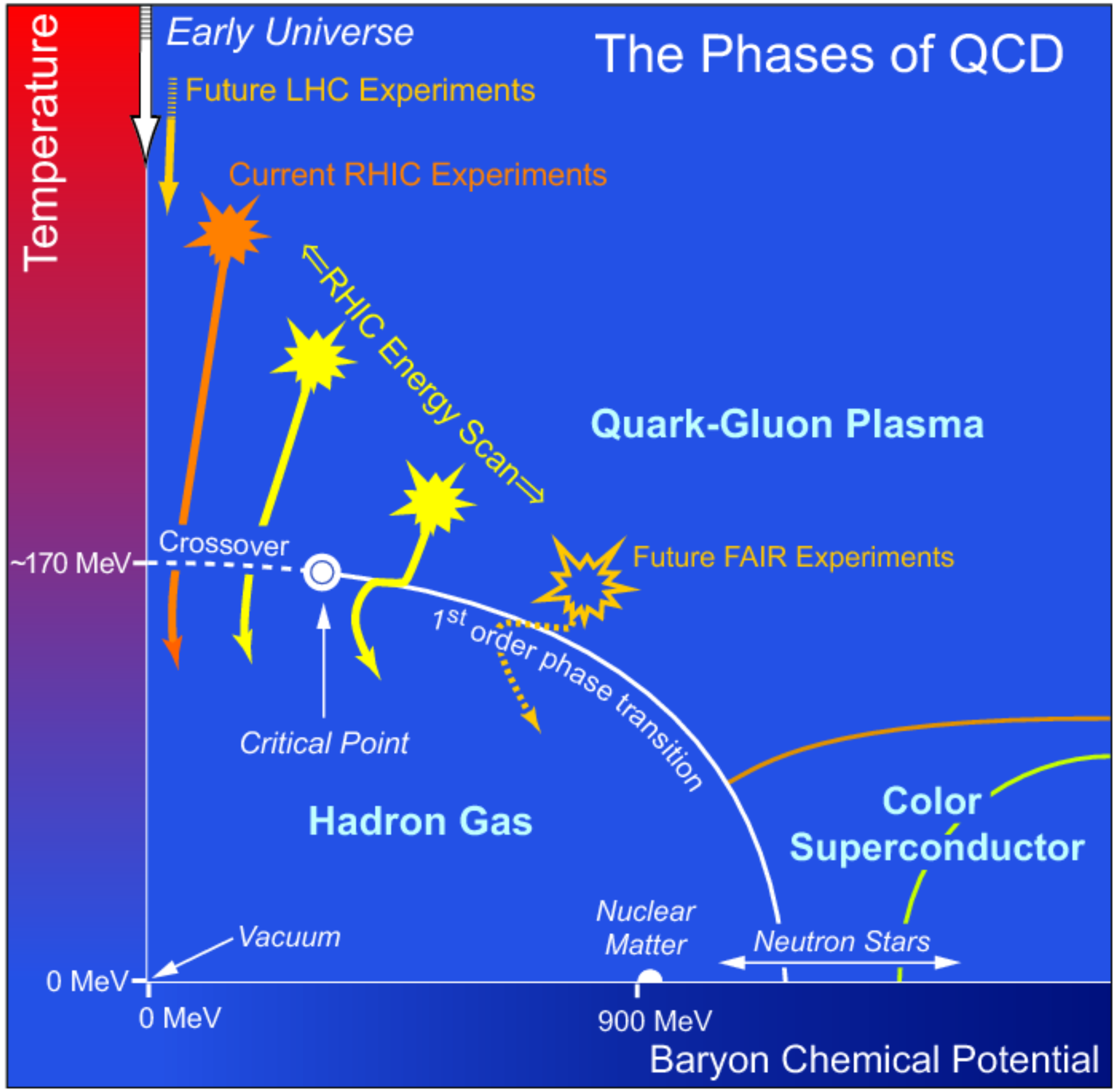
- Observables
- Analysis methods

✓ Experimental results

- Net-Λ cumulants
- Off-diagonal cumulants
- Sixth-order cumulants

✓ Non-binomial efficiencies

QCD phase diagram



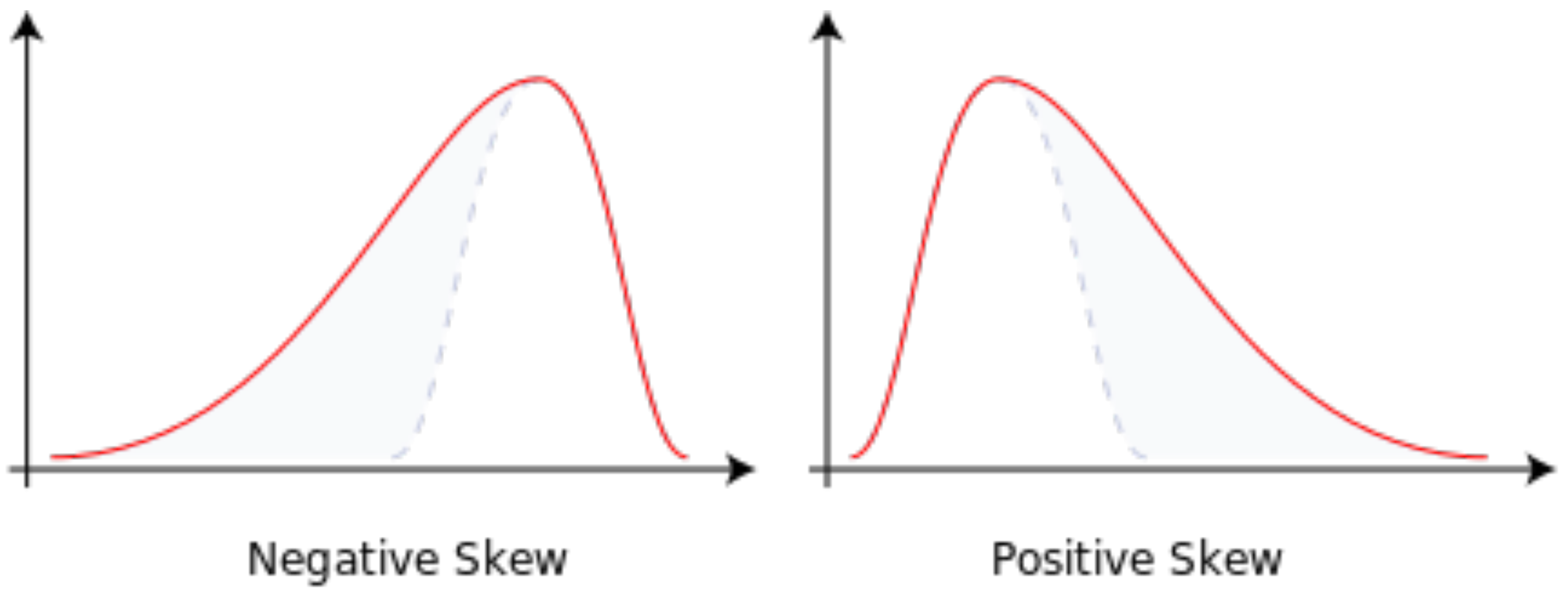
✓ Higher-order fluctuations of net-particle distributions.

- Crossover at μ_B=0
- 1st-order phase transition at large μ_B?
- Critical point?

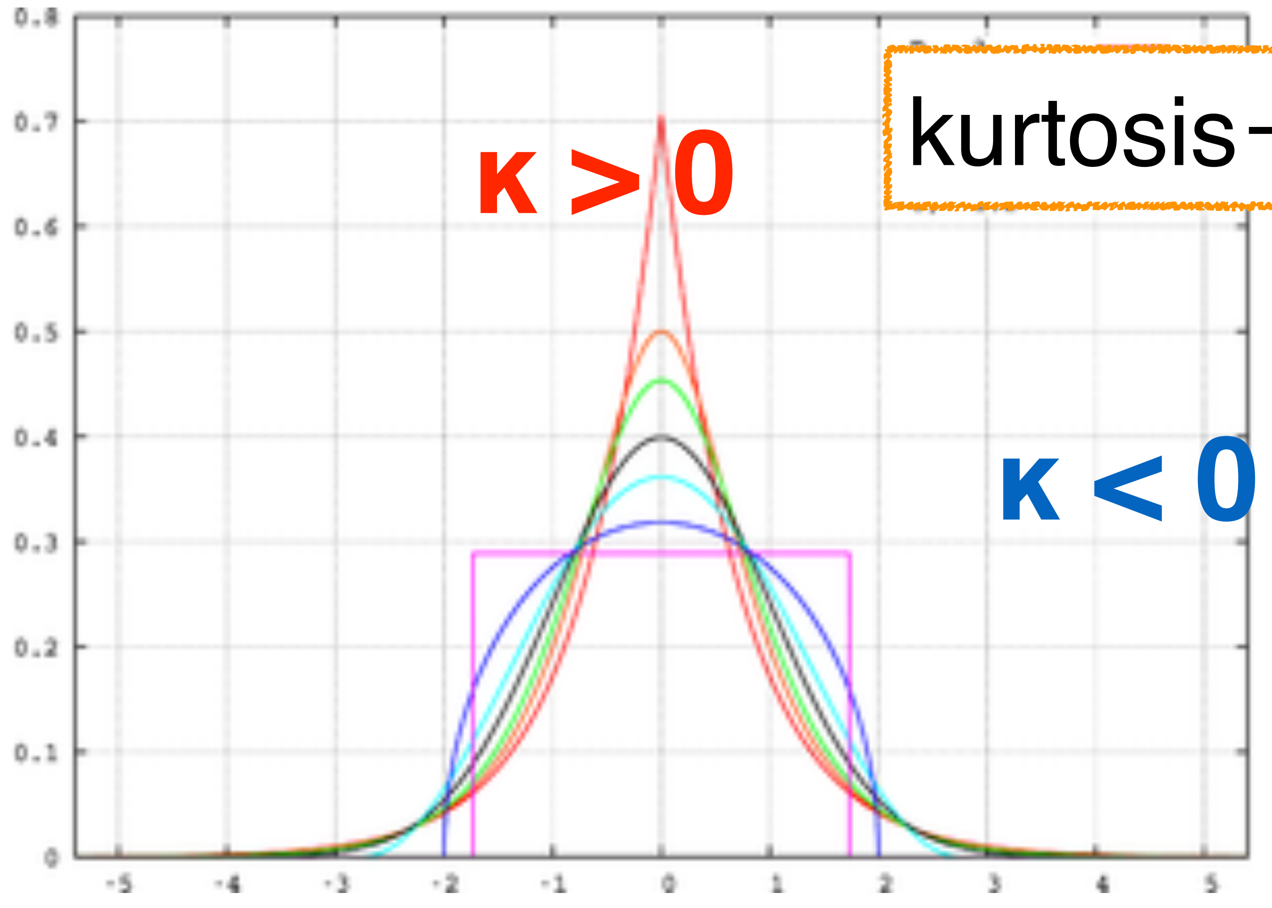
Higher-order fluctuation

- **Moments and cumulants are mathematical measures of "shape" of a distribution which probe the fluctuation of observables.**
  - ✓ **Moments: mean (*M*), standard deviation (*σ*), skewness (*S*) and kurtosis (κ).**
  - ✓ ***S and κ are non-gaussian* fluctuations.**

skewness→asymmetry



kurtosis→sharpness



from wikipedia

✓ **Cumulant ⇄ Moment**

$$< \delta N >= N - < N >$$
$$C_1 = M =< N >$$
$$C_2 = \sigma^2 =< (\delta N)^2 >$$
$$C_3 = S\sigma^3 =< (\delta N)^3 >$$
$$C_4 = \kappa\sigma^4 =< (\delta N)^4 > -3 < (\delta N)^2 >^2$$

✓ **Cumulant : additivity**

$$C_n(X+Y) = C_n(X) + C_n(Y)$$

→ **proportional to volume**

Existing analysis methods

✓ Centrality bin width averaging is done for the reduction of the initial volume fluctuation.

✓ Calculate the cumulants at each value of the multiplicity used for centrality, then weighted-average these in each centrality bin.

- *X.Luo, J. Xu, B. Mohanty and N. Xu. J. Phys. G40,105104(2013)*

$$C_n = \frac{\sum_{r=N_1}^{N_2} n_r C_n^r}{\sum_{r=N_1}^{N_2} n_r} = \sum_{r=N_1}^{N_2} \omega_r C_n^r \qquad \omega_r = n_r / \sum_{r=N_1}^{N_2} n_r$$

N_1, N_2 : lowest and highest multiplicity bin in the centrality
n_r : # of events in rth multiplicity bin

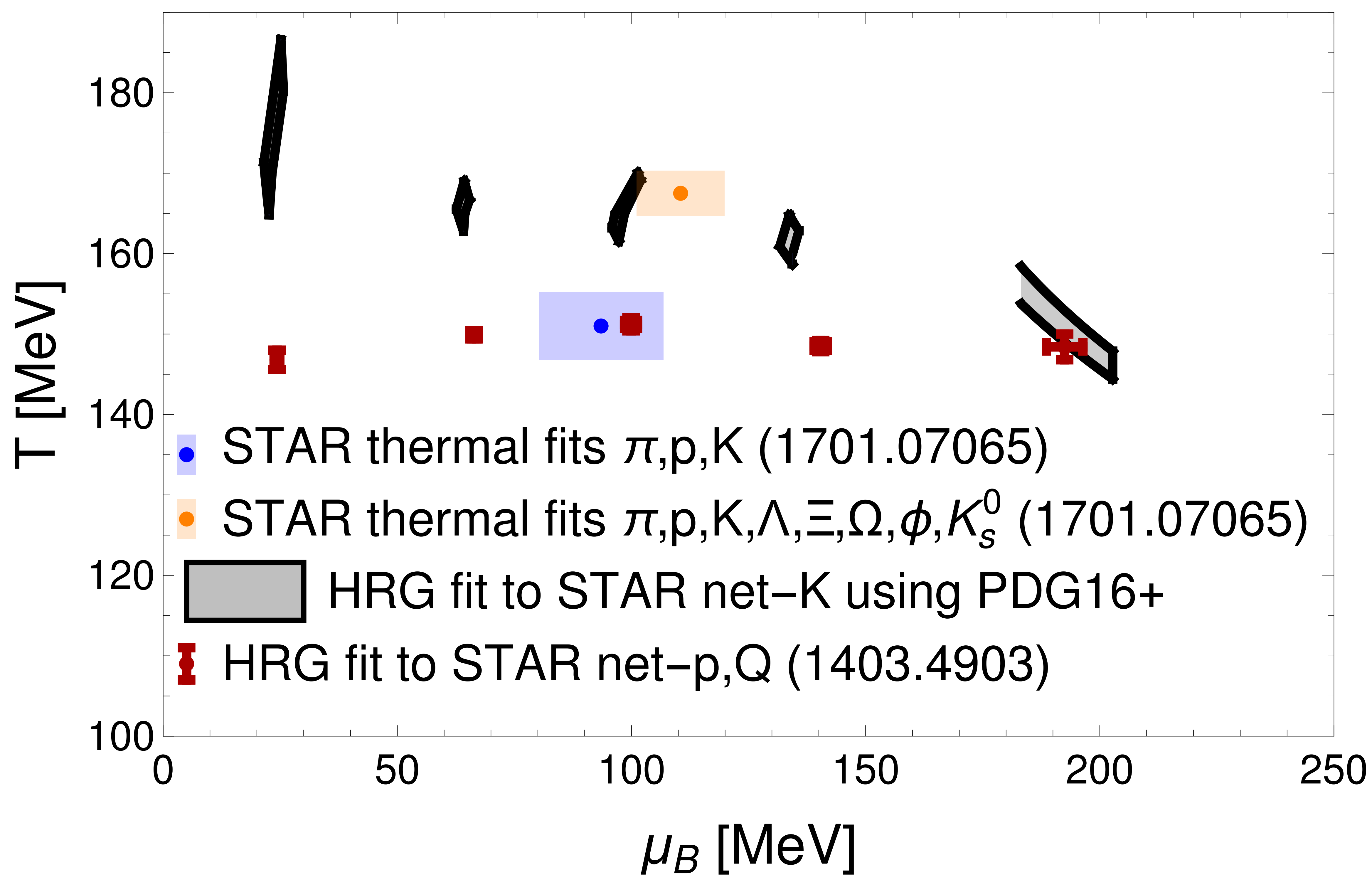
✓ Efficiency correction on cumulants have been done assuming the binomial efficiencies.

- *M. Kitazawa : PRC.86.024904, M. Kitazawa and M. Asakawa : PRC.86.024904*
- *A. Bzdak and V. Koch : PRC.86.044904, PRC.91.027901, X. Luo : PRC.91.034907*
- *T. Nonaka et al : PRC.94.034909, T. Nonaka, M. Kitazawa, S. Esumi : PRC.95.064912*

$$B_{p,N}(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^n$$

Net-Λ cumulants

- ✓ **Strange hadrons freeze-out earlier than light flavor hadrons?**
- ✓ **Net-Λ cumulants might provide additional constraints on freeze-out conditions.**

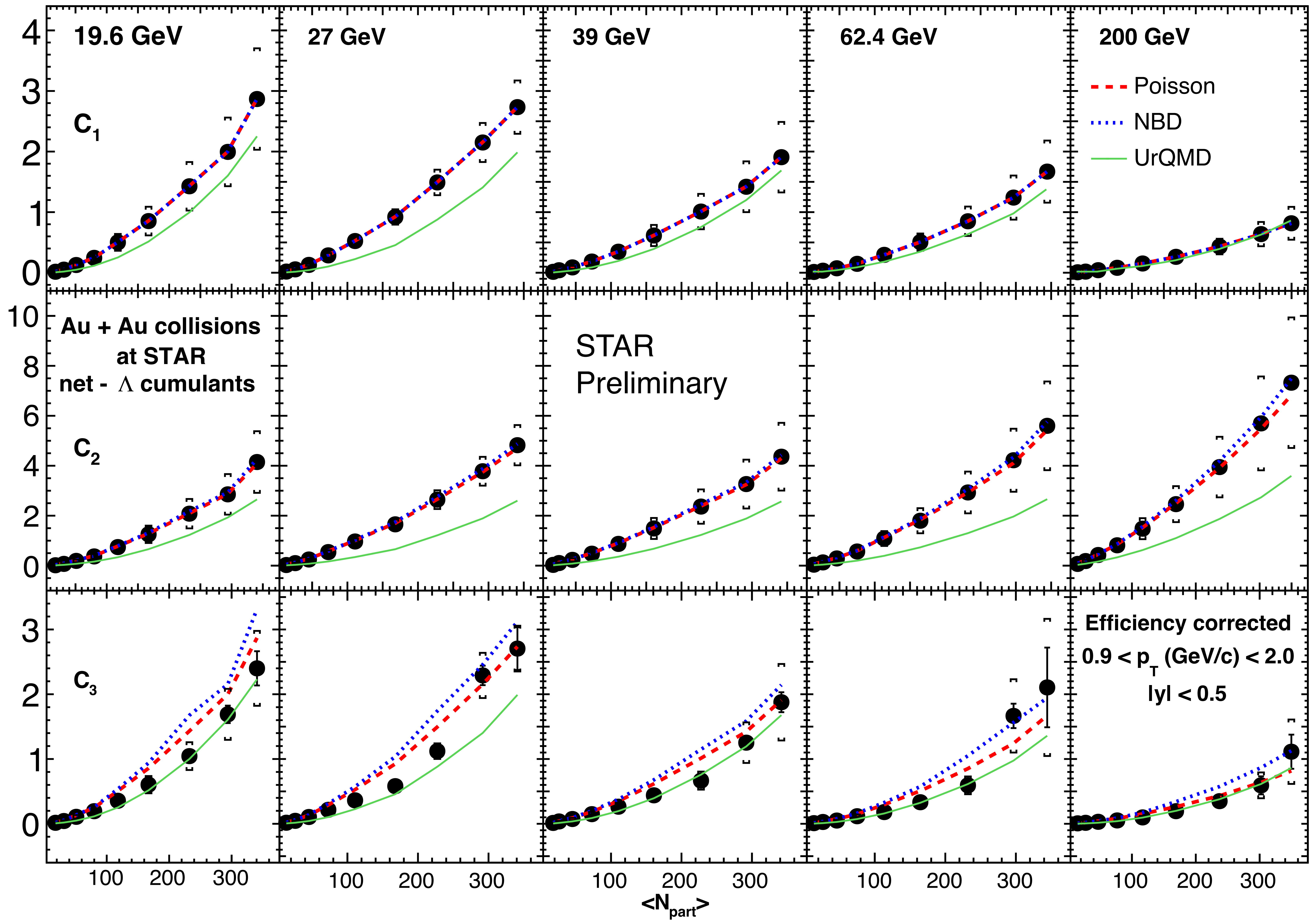


J. Noronha-Hostler, C. Ratti, P. Parotto, R. Bellwied, arXiv:1805.00088

Net-Λ cumulants

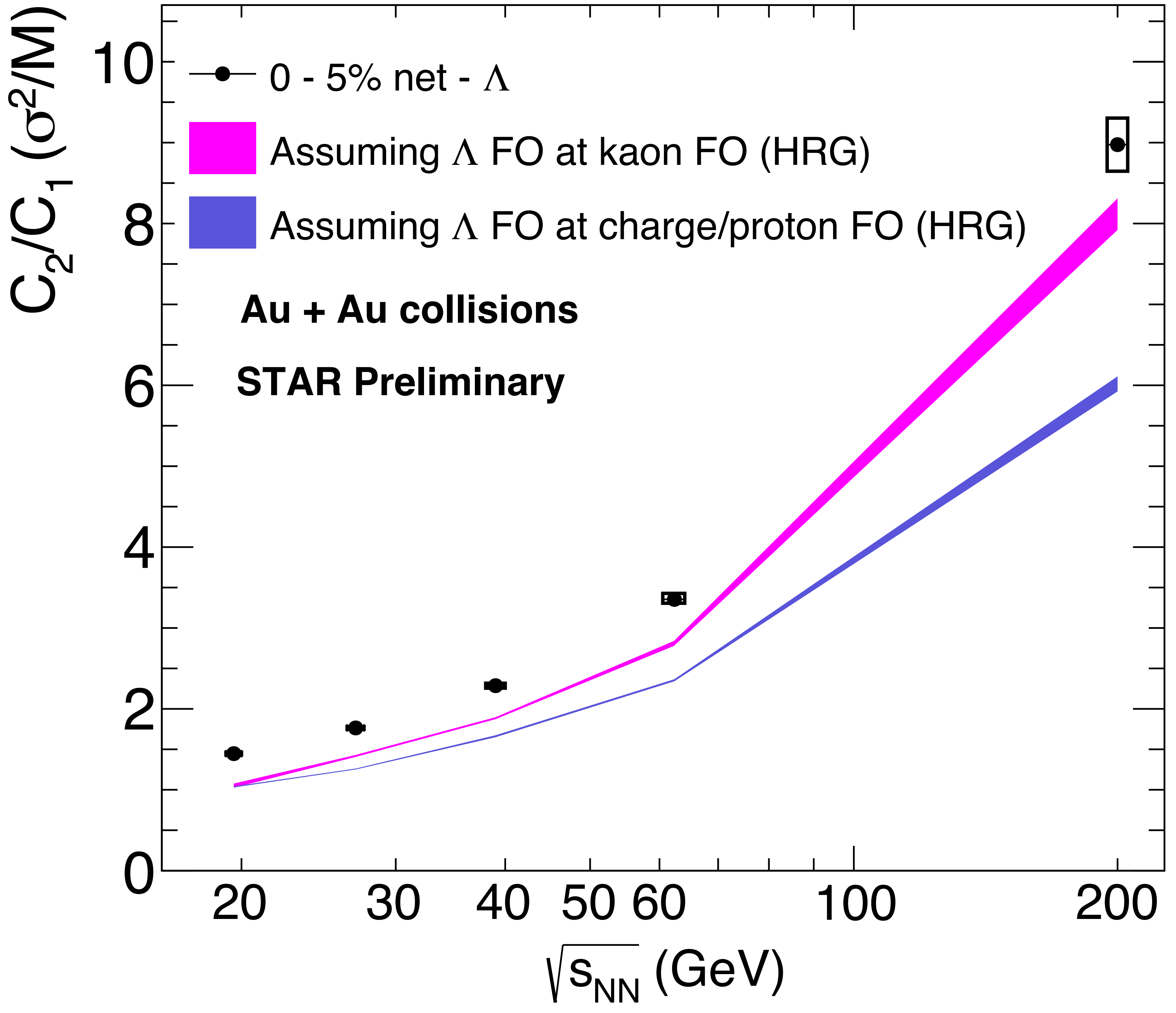
See N. Kulathunga, Poster #528

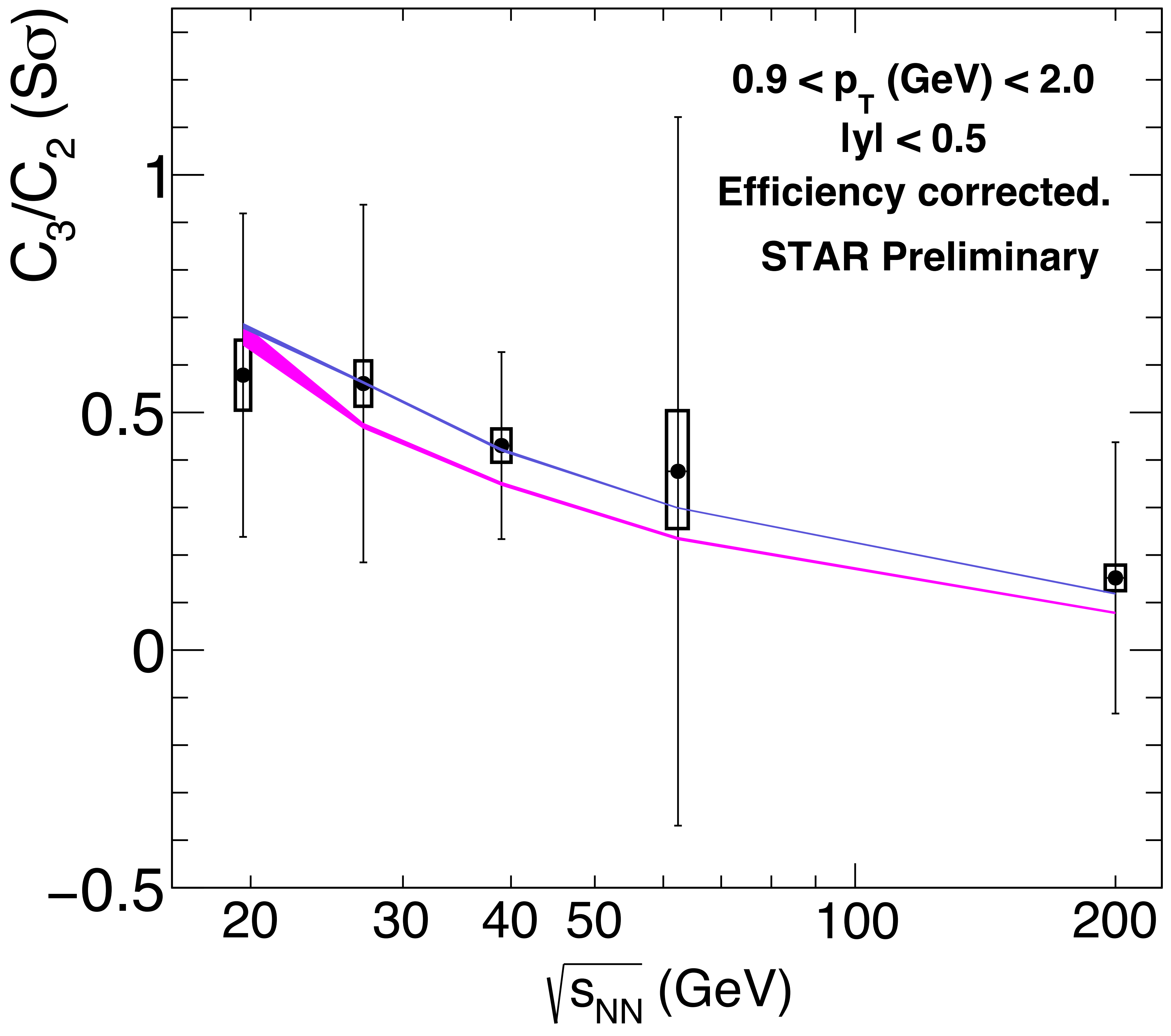
- ✓ Consistent with Poisson/NBD baselines.
- ✓ C_1 and C_2 are above UrQMD results.
- ✓ C_3 shows better agreement with UrQMD.



Net-Λ cumulants

- ✓ C_2/C_1 is close to HRG results (same kinematic range) with kaon freeze-out condition, and far away from those of charge and proton.
- ✓ The error on C_3/C_2 is presently too large to provide a meaningful constraint on the HRG model.





The 2nd-order off-diagonal cumulants

Off-diagonal cumulants of conserved charges will provide additional constraints on the freeze-out conditions.

$$\begin{pmatrix} \sigma_Q^2 & \sigma_{Q,p}^{1,1} & \sigma_{Q,k}^{1,1} \\ \sigma_{p,Q}^{1,1} & \sigma_p^2 & \sigma_{p,k}^{1,1} \\ \sigma_{k,Q}^{1,1} & \sigma_{k,p}^{1,1} & \sigma_k^2 \end{pmatrix}$$

$$\sigma_{x,y}^2 = \langle xy \rangle - \langle x \rangle \langle y \rangle$$

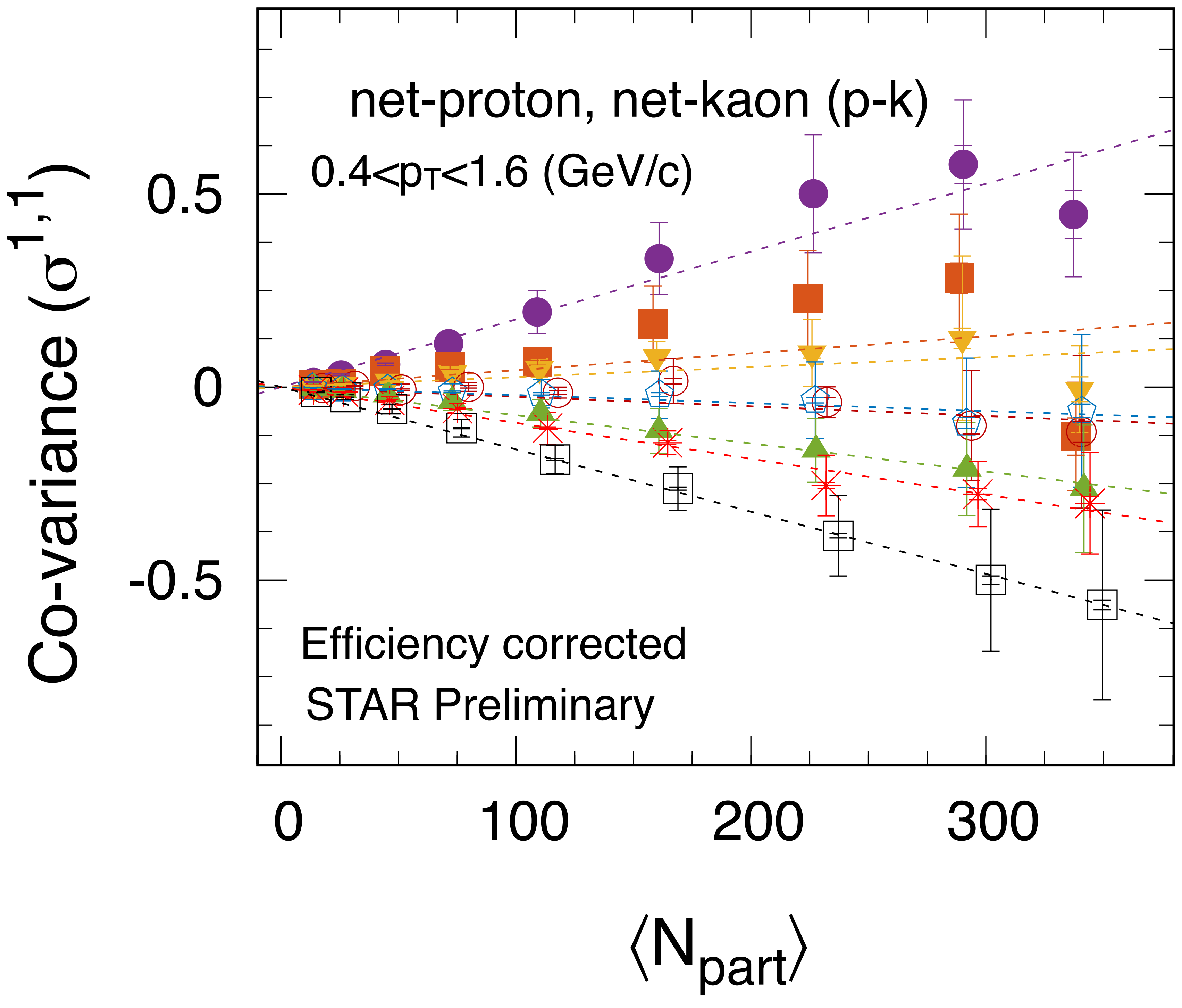
$$C_{x,y} = \frac{\sigma_{x,y}^{1,1}}{\sigma_y^2}$$

- *A. Majumder and B. Muller, Phy. Rev. C 74 (2006)*
- *A. Bazavov et al. Phys. Rev. D86 (2012) 034509*
- *A. Chatterjee et al. J. Phys. G: Nucl. Part. Phys. 43 (2016) 125103*
- *Z. Yang et al. Phys. Rev. C 95 014914 (2017)*

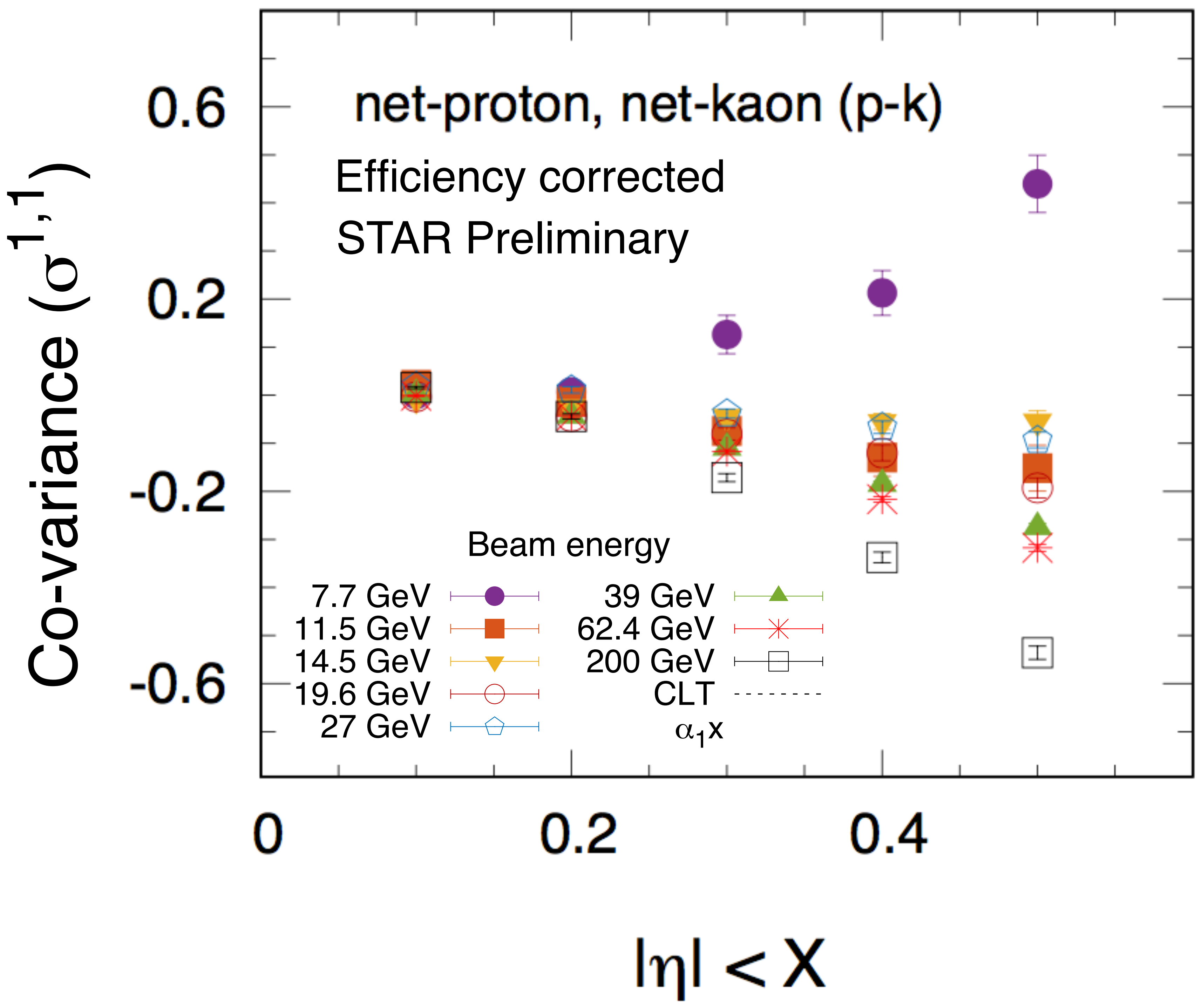
✓ **Covariance shows linear dependence with respect to the centrality and η acceptance.**

✓ **The correlation between net-protons and net-kaons changes from positive to negative with increasing beam energy.**

Centrality dependence, |η|<0.5



η acceptance dependence, 0-5%

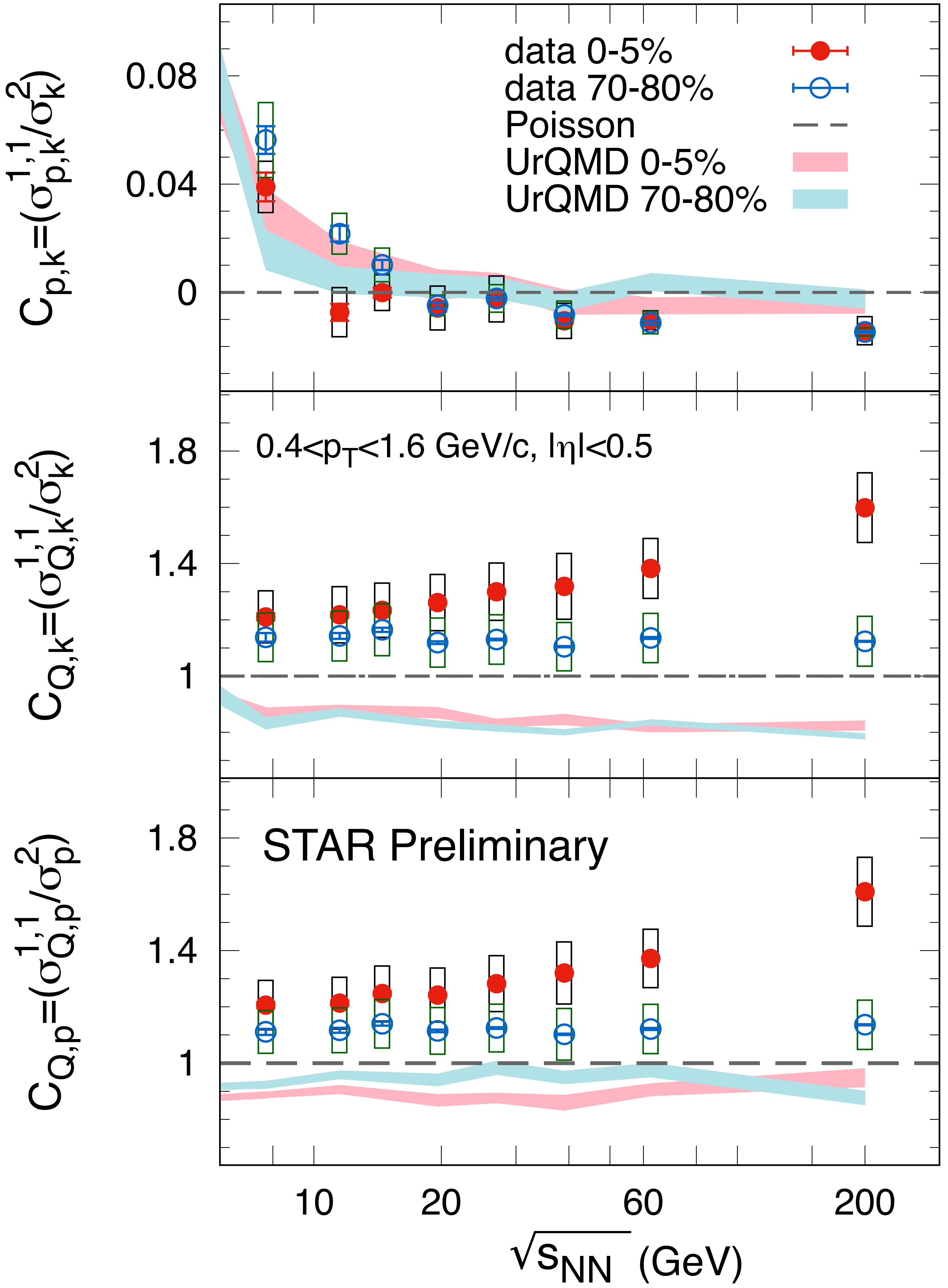


See A. Chatterjee, Poster #534

The 2nd-order off-diagonal cumulants

- ✓ **Normalized p-k correlation is positive at low energies and negative at high energies, which are also consistent with UrQMD.**
- ✓ **Significant excess is observed in Q-k and Q-p with respect to the Poisson baseline and UrQMD.**
- ✓ **This excess increases with the beam energy in central collisions compared to peripheral collisions.**

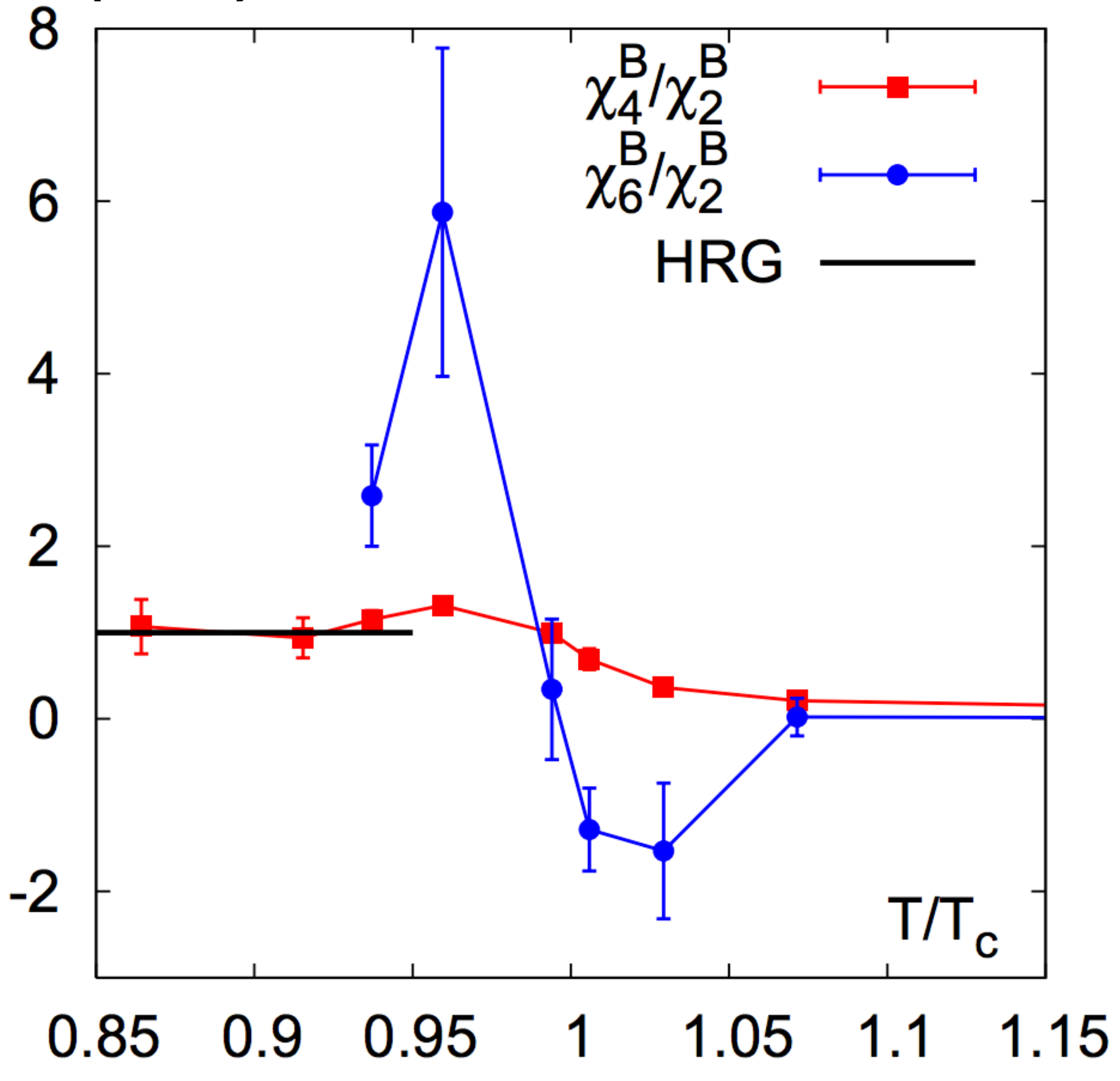
See A. Chatterjee, Poster #534



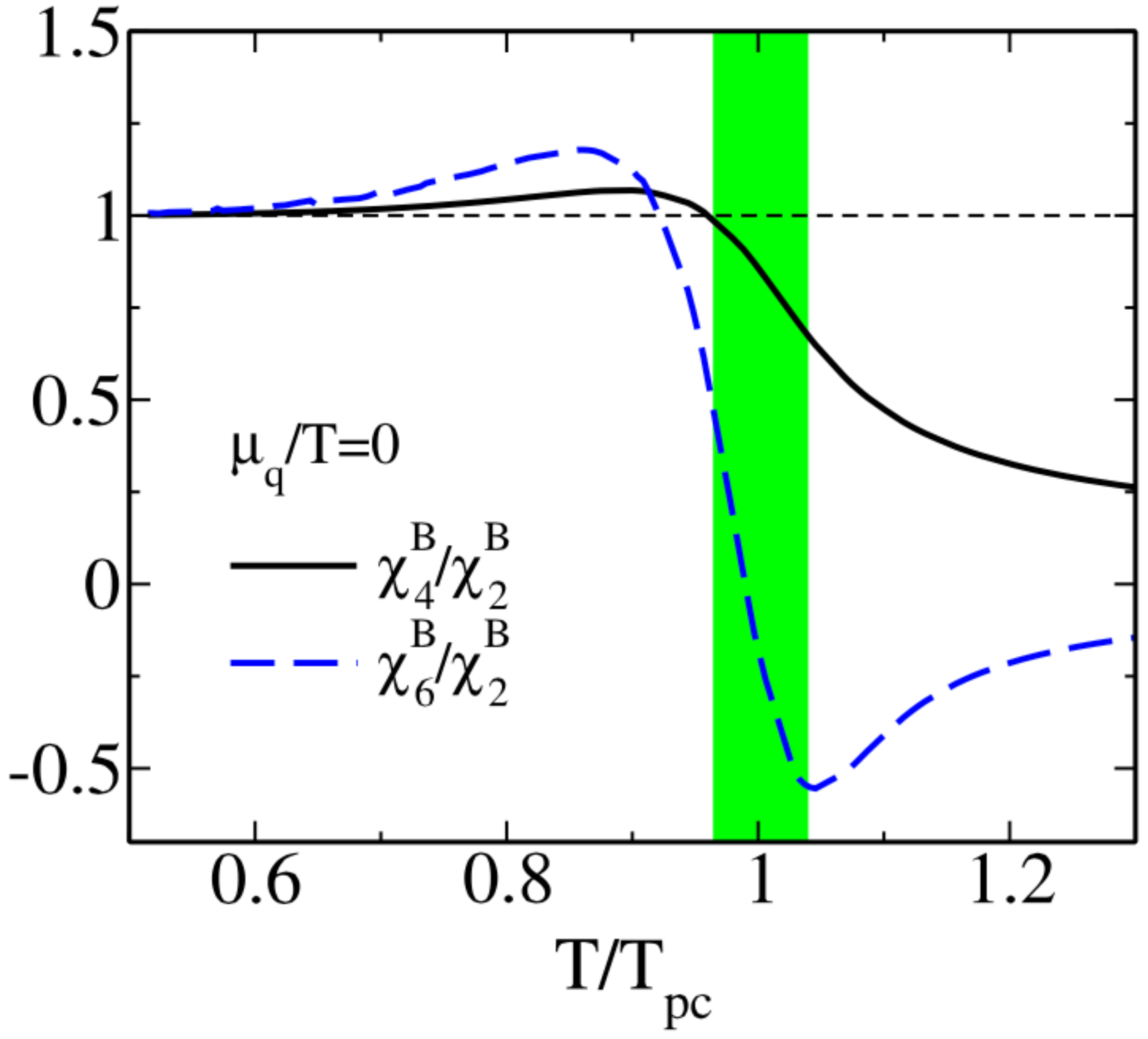
Net-charge sixth-order cumulant

- ✓ There isn't yet any direct experimental evidence for the smooth crossover at μ_B~0.
- ✓ Sixth-order cumulants of net-charge and net-baryon distributions are predicted to be negative if the chemical freeze-out is close enough to the phase transition.

Cheng et al, Phys. Rev. D 79, 074505 (2009) : Lattice QCD



Friman et al, Eur. Phys. J. C (2011) 71:1694 : O(4) scaling functions



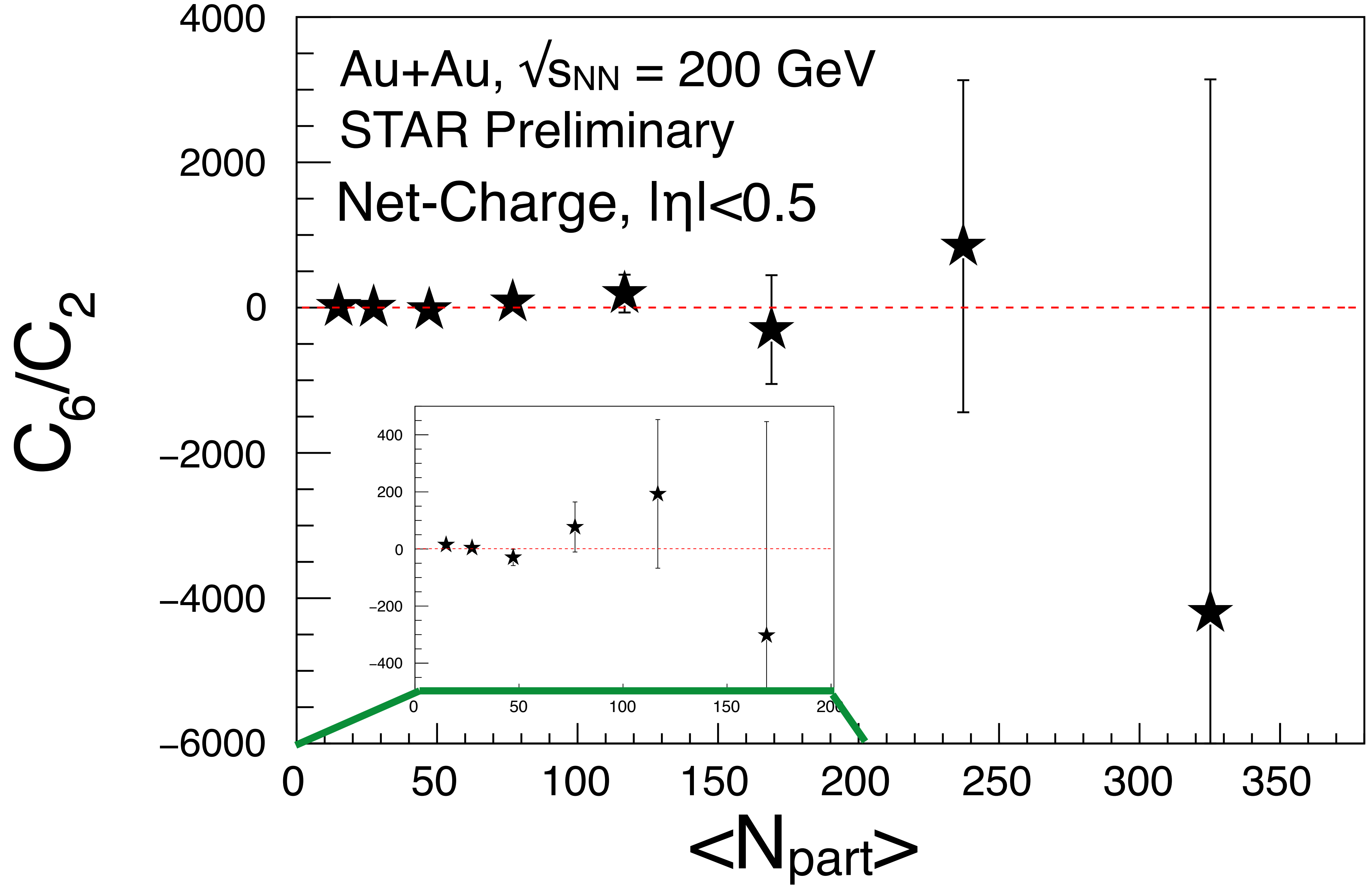
C.Schmidt,Prog.Theor.Phys.Suppl.186,563–566(2010)
Cheng et al, Phys. Rev. D 79, 074505 (2009)
Friman et al, Eur. Phys. J. C (2011) 71:1694

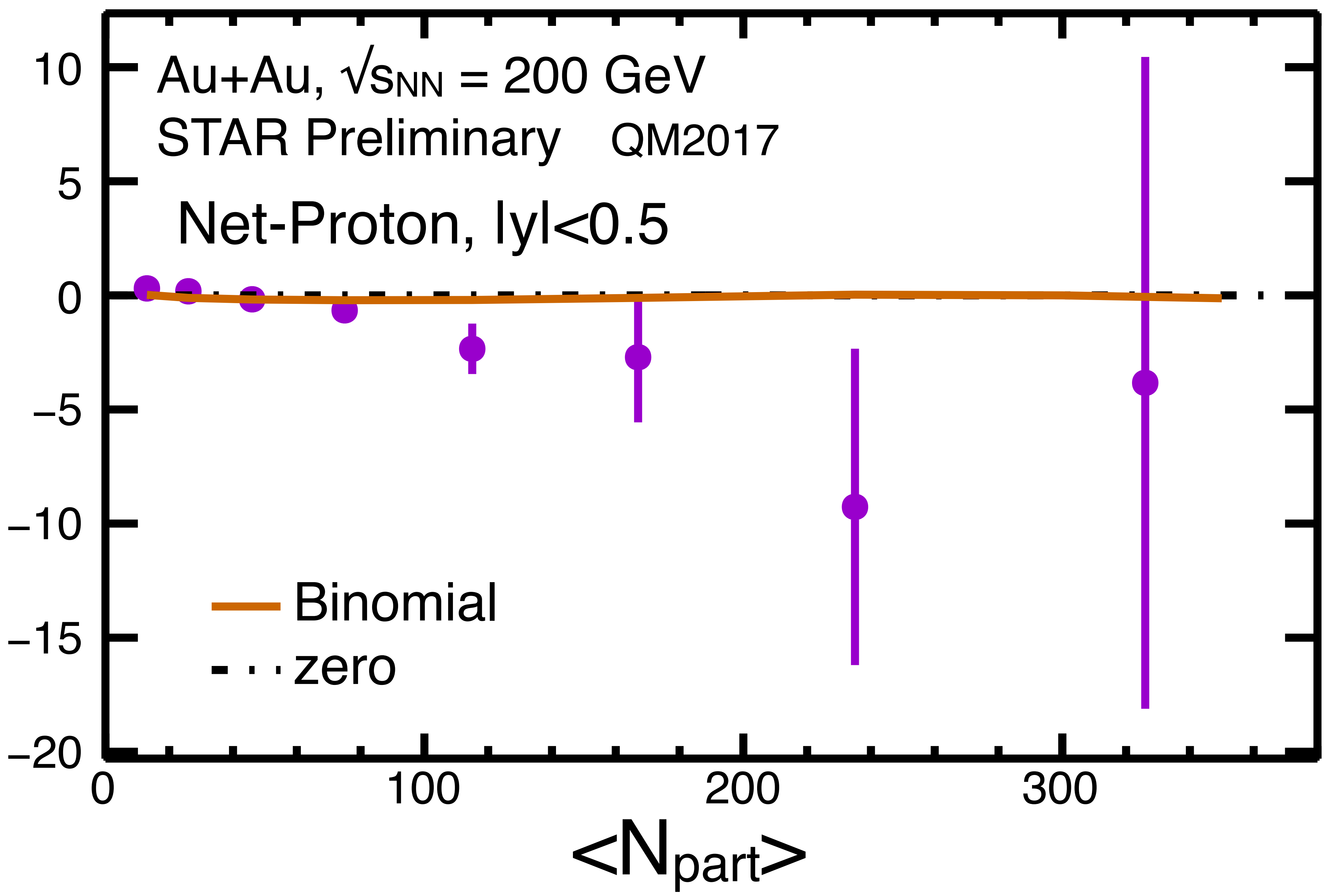
Freeze-out conditions	χ_4^B/χ_2^B	χ_6^B/χ_2^B	χ_4^Q/χ_2^Q	χ_6^Q/χ_2^Q
HRG	1	1	~2	~10
QCD: $T^{freeze}/T_{pc} \lesssim 0.9$	$\gtrsim 1$	$\gtrsim 1$	~2	~10
QCD: $T^{freeze}/T_{pc} \simeq 1$	~0.5	<0	~1	<0

Predicted scenario for this measurement

Net-charge sixth order cumulant

- ✓ **The first result of C_6/C_2 of net-charge.**
- ✓ **Net-charge has much larger errors than net-proton because the error strongly depends on the standard deviation.** $error(C_r) \propto \frac{\sigma^r}{\sqrt{N_{eve}}}$
- ✓ **Results of net-charge C_6/C_2 are consistent with zero within large statistical errors.**
- ✓ **Negative values are observed in net-proton C_6/C_2 systematically from peripheral to central collisions.**

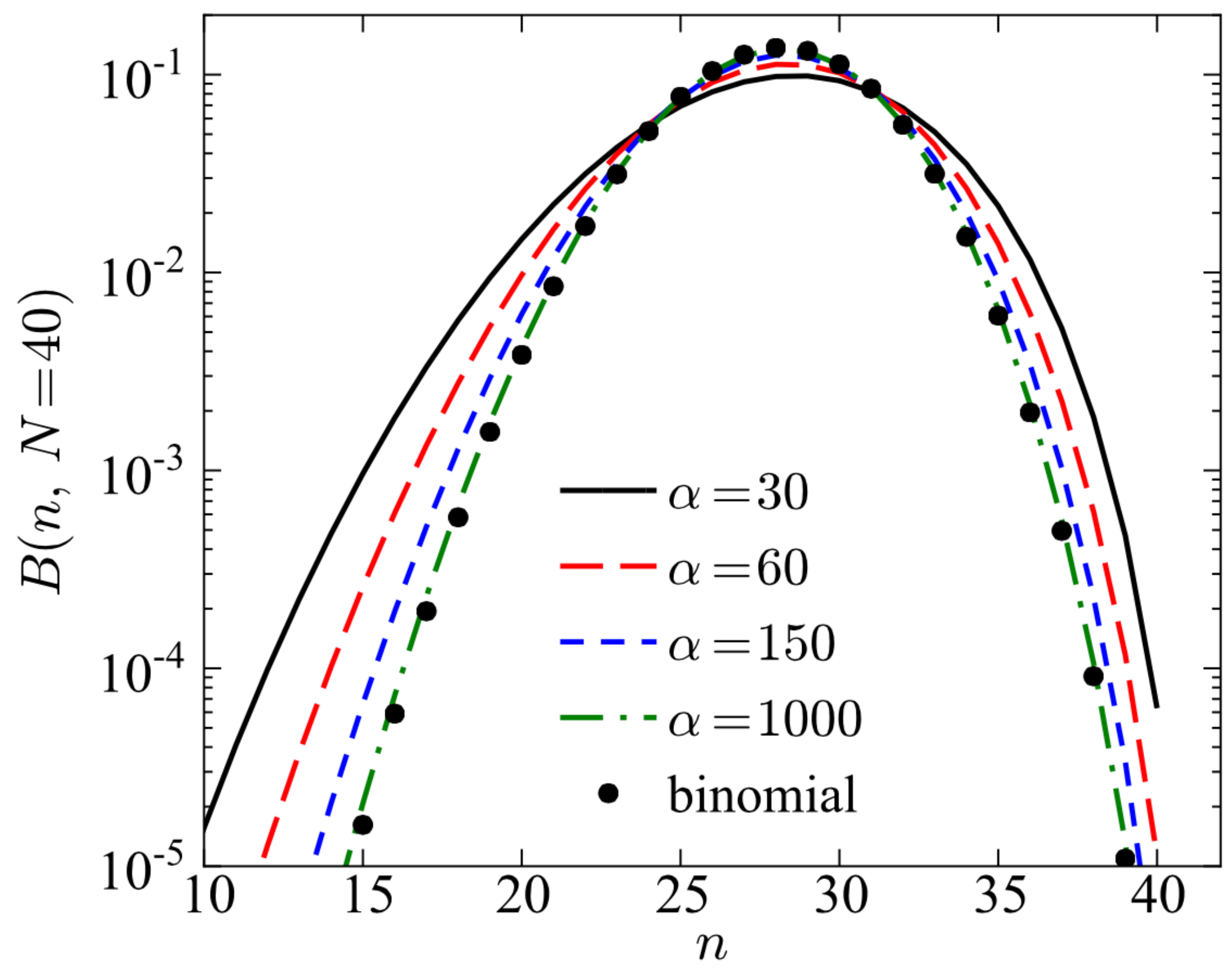




See T. Sugiura, Poster #532

Non-binomial efficiencies

A. Bzdak, R. Holzmann, V. Koch : PRC.94.064907



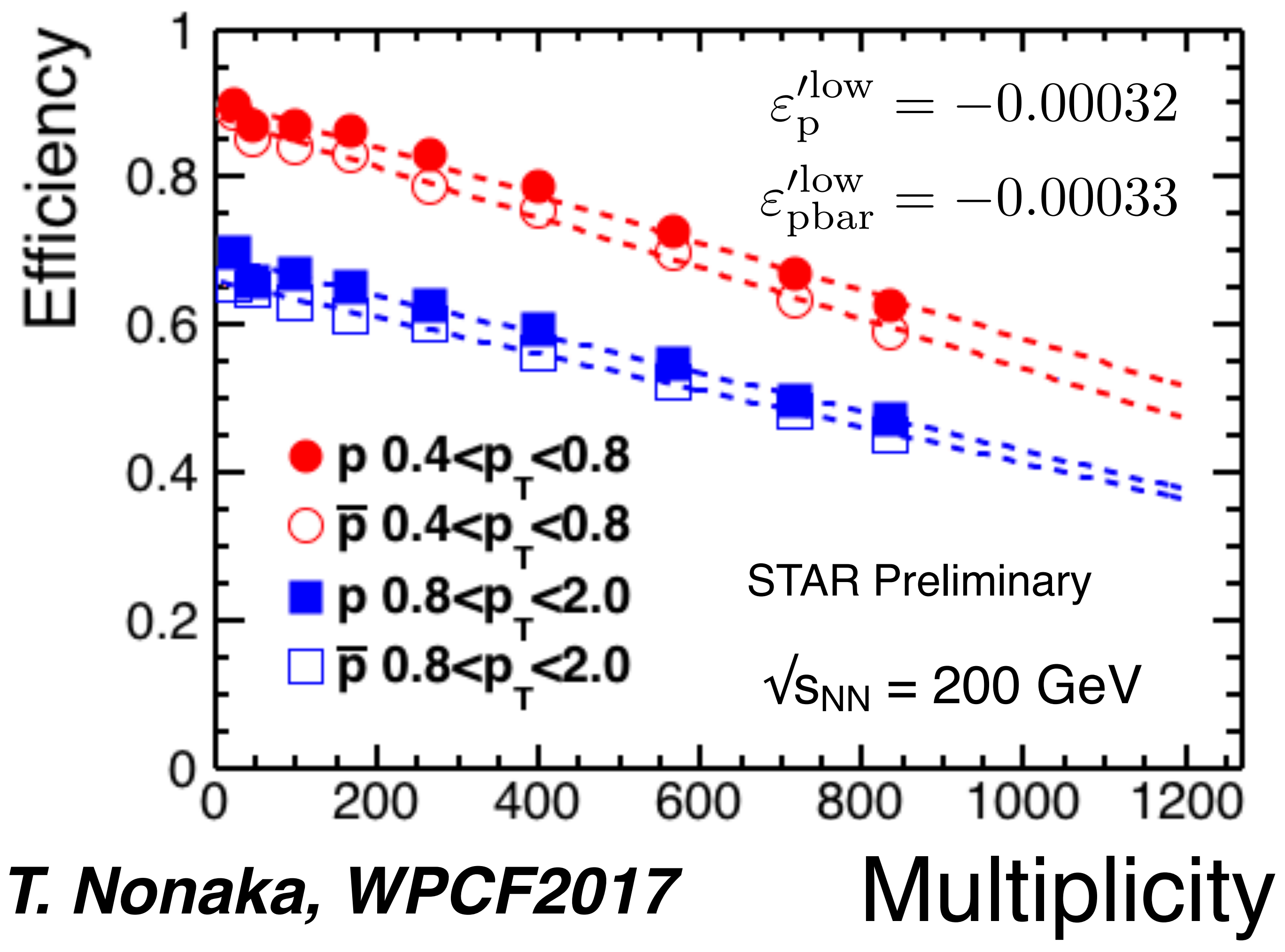
➡ *One example of non-binomial distribution, Beta-binomial, is wider distribution than binomial*

1. Experimental effects

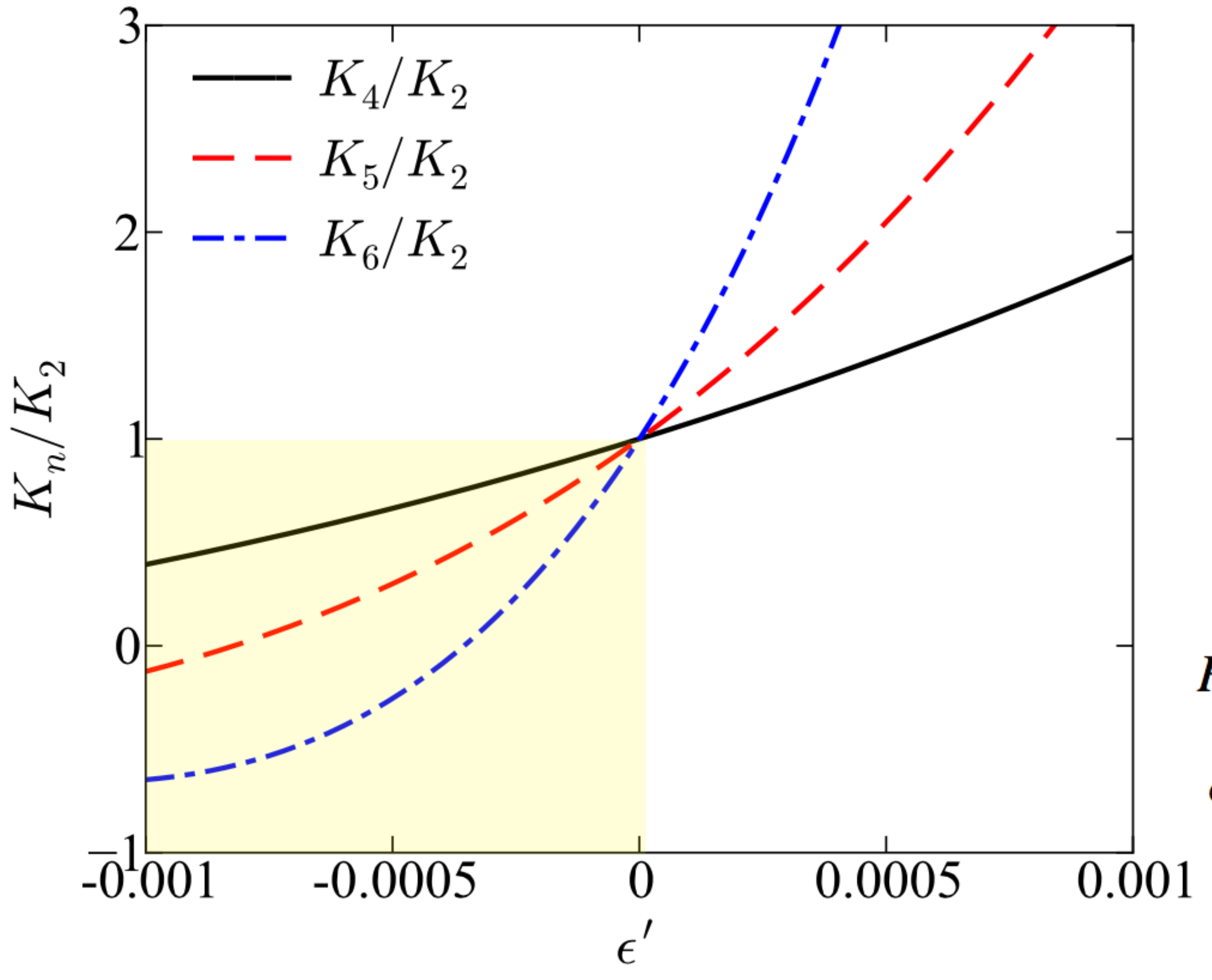
✓ **The detector efficiency may not be binomial, which would be due to the particle mis-identification, track splitting/merging effects, and many other reasons.**

2. Multiplicity dependent efficiency

✓ **Residual dependence of efficiency inside one multiplicity bin (for centrality) needs to be taken into account.**



T. Nonaka, WPCF2017



A. Bzdak, R. Holzmann, V. Koch : PRC.94.064907

$$P(N) = \frac{\langle N \rangle^N}{N!} e^{-\langle N \rangle},$$

$$\epsilon(N) = \epsilon_0 + \epsilon'(N - \langle N \rangle),$$

Non-binomial efficiencies

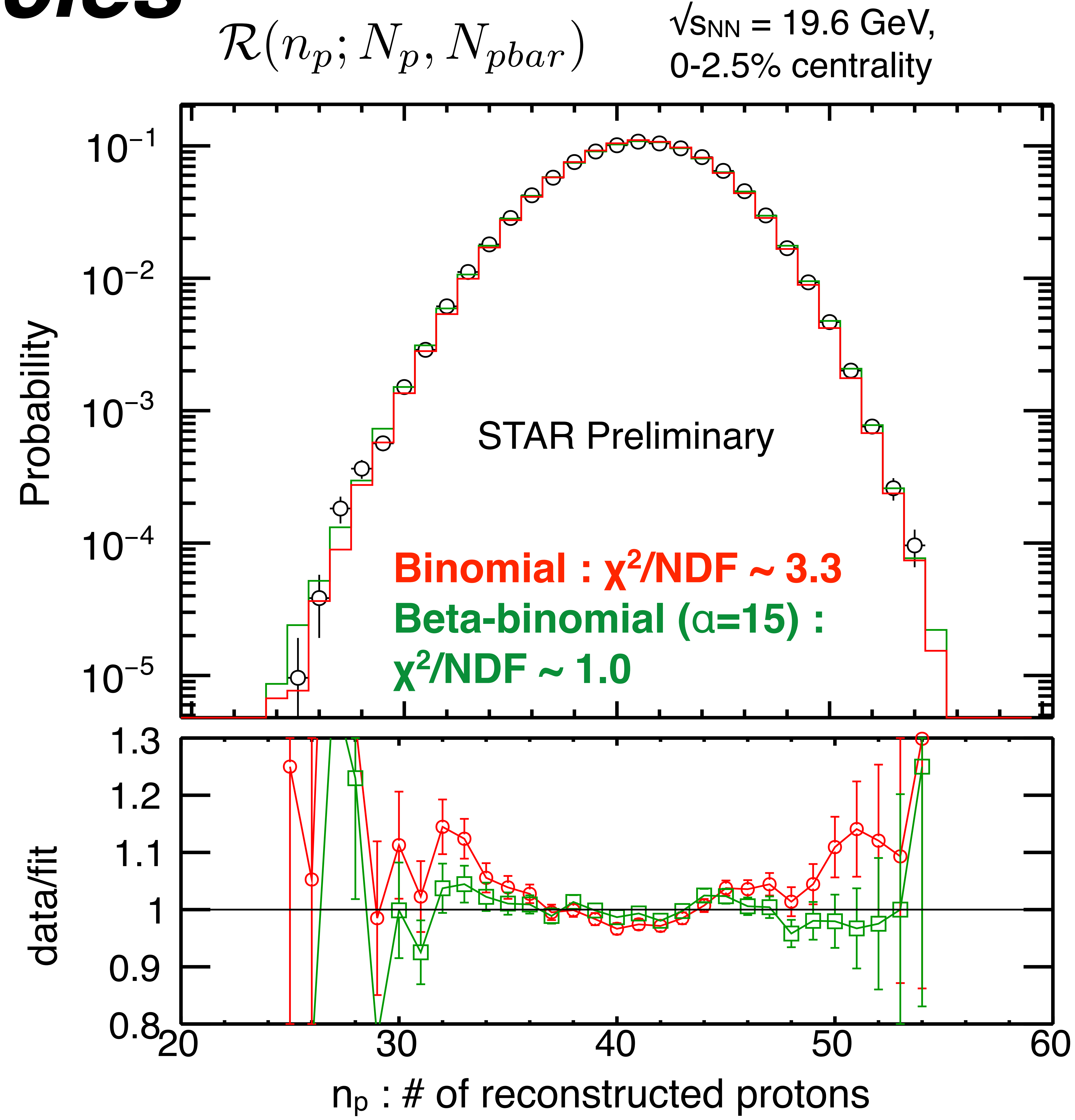
- **We performed MC simulations by embedding protons and antiprotons, e.g., N_p=60 and N_{pbar}=15 (which would be an extreme number), and see whether those particles can be reconstructed or not.**
- **The response matrix is close to the beta-binomial distribution, which is wider than binomial.**
  - ➡ "Urn model" for beta-binomial distribution, where the parameter α controls the deviation from binomial.

N_w : white balls, N_b: black balls, ε: efficiency

$$N_w = \alpha N_p \quad \varepsilon = N_w/(N_w + N_b)$$

See T. Nonaka, Poster #453

$\mathcal{R}(n_p; N_p, N_{pbar})$

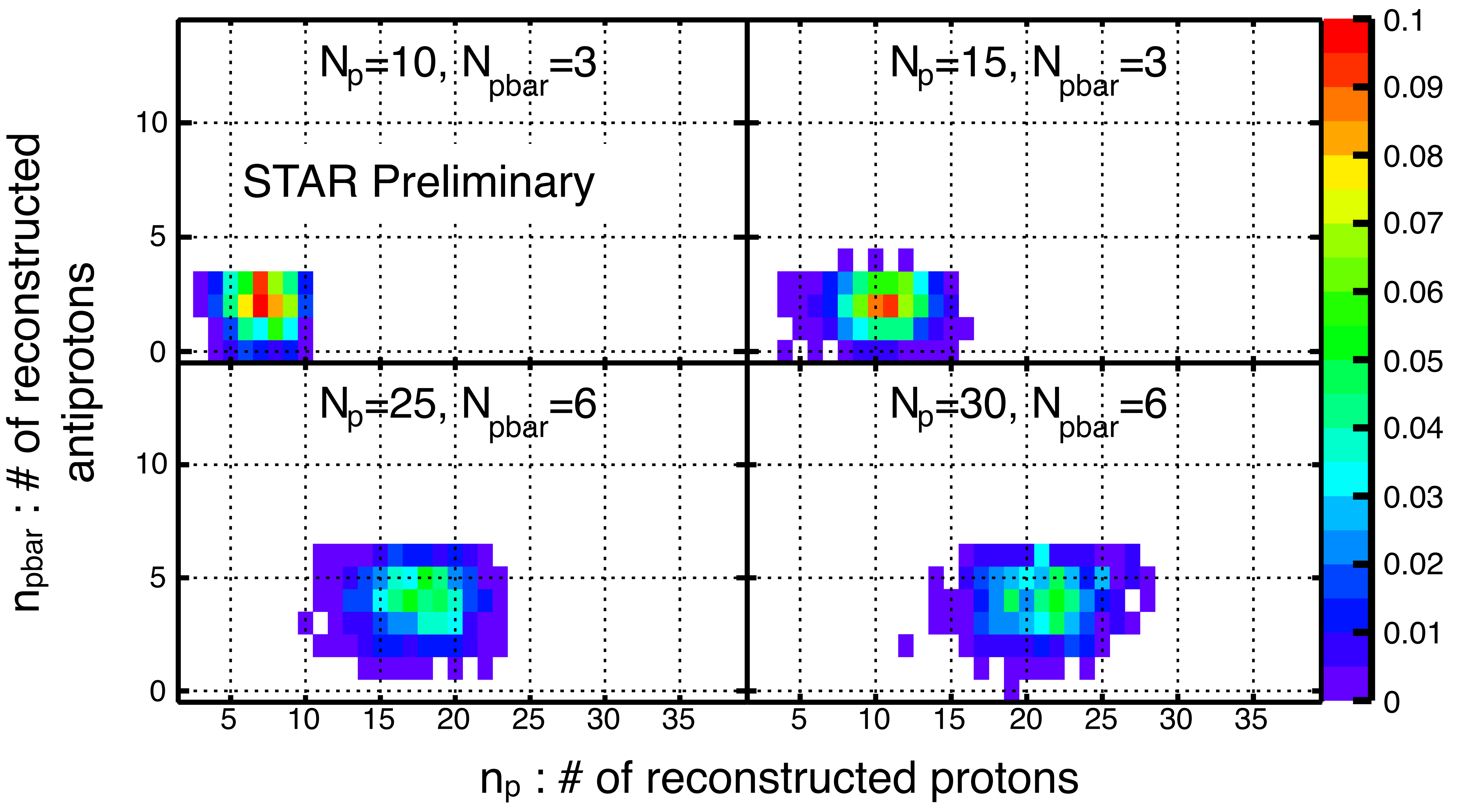


Response matrices

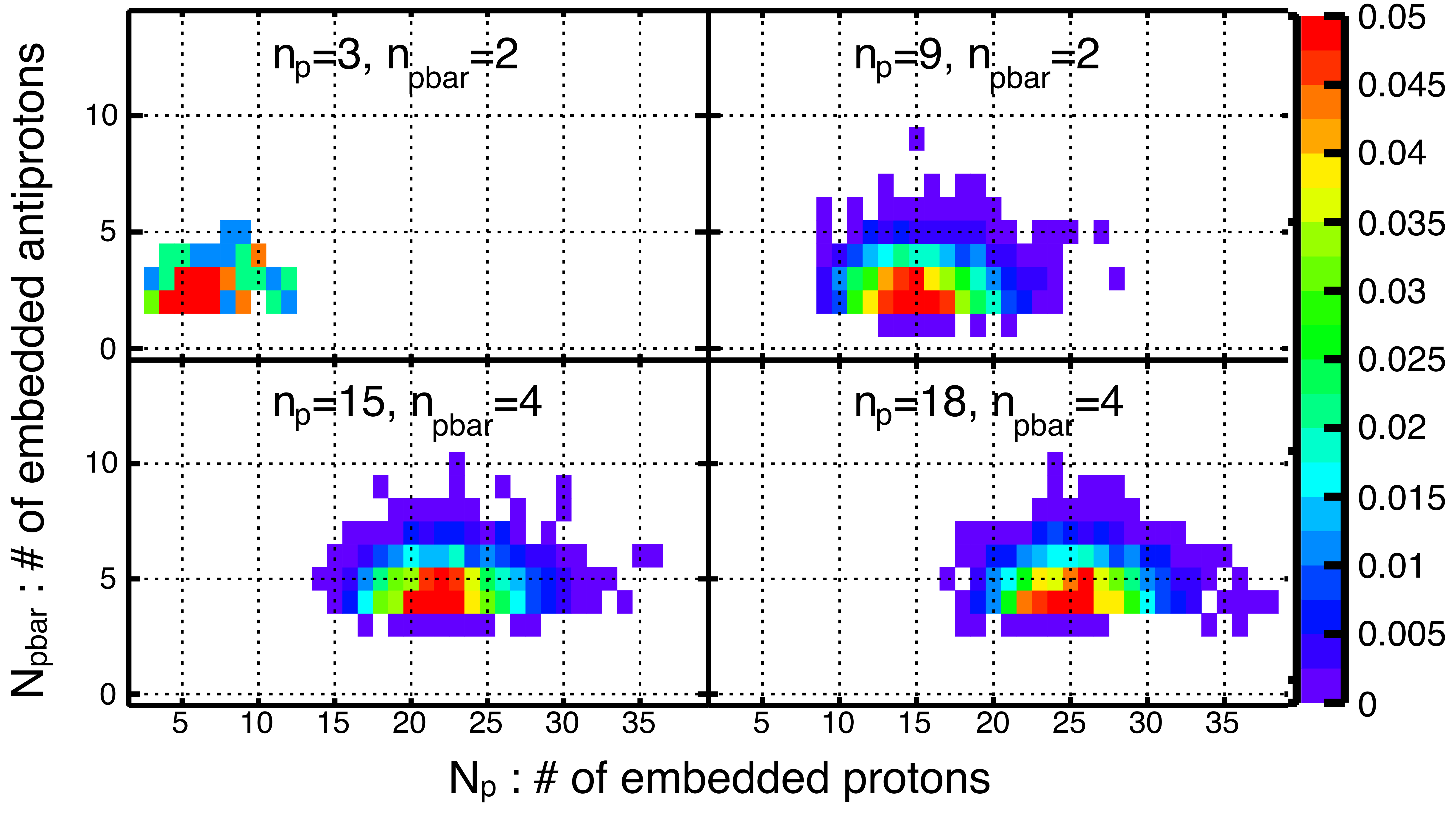
✓ The deviation from binomial would depend on the # of embedded protons and antiprotons.

✓ 4-D response matrices are determined by embedding simulation, which can be directly used for unfolding in order to reconstruct the distribution itself.

$\mathcal{R}(n_p, n_{pbar}; N_p, N_{pbar})$ **Forward response matrix**



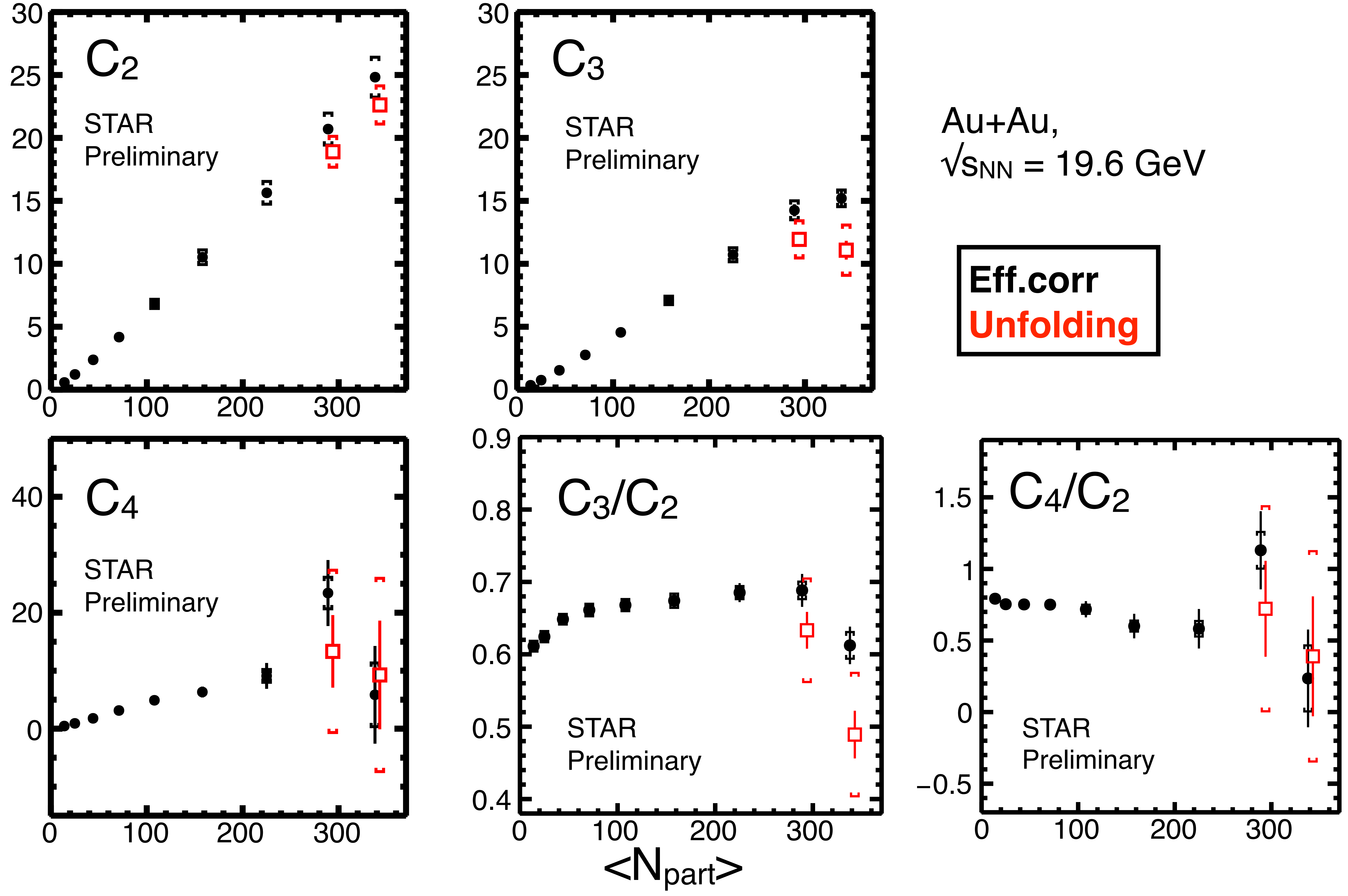
$\mathcal{R}(N_p, N_{pbar}; n_p, n_{pbar})$ **Reversed response matrix**



$\sqrt{s_{NN}}$ = 19.6 GeV, 0-5% centrality, |y|<0.5,
0.4 < p_T < 2.0, embedding simulation

See T. Nonaka, Poster #453

Results of unfolding



- ✓ For unfolding, 2.5% centrality width averaging has been done.
- ✓ Systematic suppression is observed for C_2 and C_3 with respect to the results of efficiency correction assuming binomial efficiencies.
- ✓ C_4, C_3/C_2 and C_4/C_2 are consistent within large systematic uncertainties limited by embedding samples.

See T. Nonaka, Poster #453

Summary

1. Net-Λ cumulants up to 3rd-order
   - Consistent with Poisson/NBD baselines.
   - The result of C_2/C_1 is closer to those of HRG with kaon freeze-out condition rather than light flavor hadrons.
2. Second-order off-diagonal cumulants
   - Q-k and Q-p correlations are in excess of the UrQMD results.
3. Sixth-order cumulant of net-charge and net-proton
   - Negative value is observed (although with extremely large uncertainties) in consistency with expectations from O(4) scaling functions.
4. Influence of non-binomial efficiencies
   - One example of the response matrix is tested by embedding simulation, which is closer to beta-binomial than binomial.
   - Unfolding has been applied at $\sqrt{s_{NN}}$ = 19.6 GeV in central collisions, where results show systematic suppression for C_2 and C_3 compared to the efficiency correction assuming binomial efficiencies, while C_4, C_3/C_2 and C_4/C_2 are consistent within large systematic uncertainties limited by embedding samples.

Thank you for your attention

Back up

Centrality definition

✓ **In order to avoid the auto-correlation, the centrality is defined by using different paricle species or acceptance.**

- Net-proton
- Net-Λ

Charged paricles excluding (anti)protons in $|\eta|<1.0$

See : Phys. Rev. Lett. 112, 032302(2014)

- Net-charge
- Off-diagonal (π, K, p)

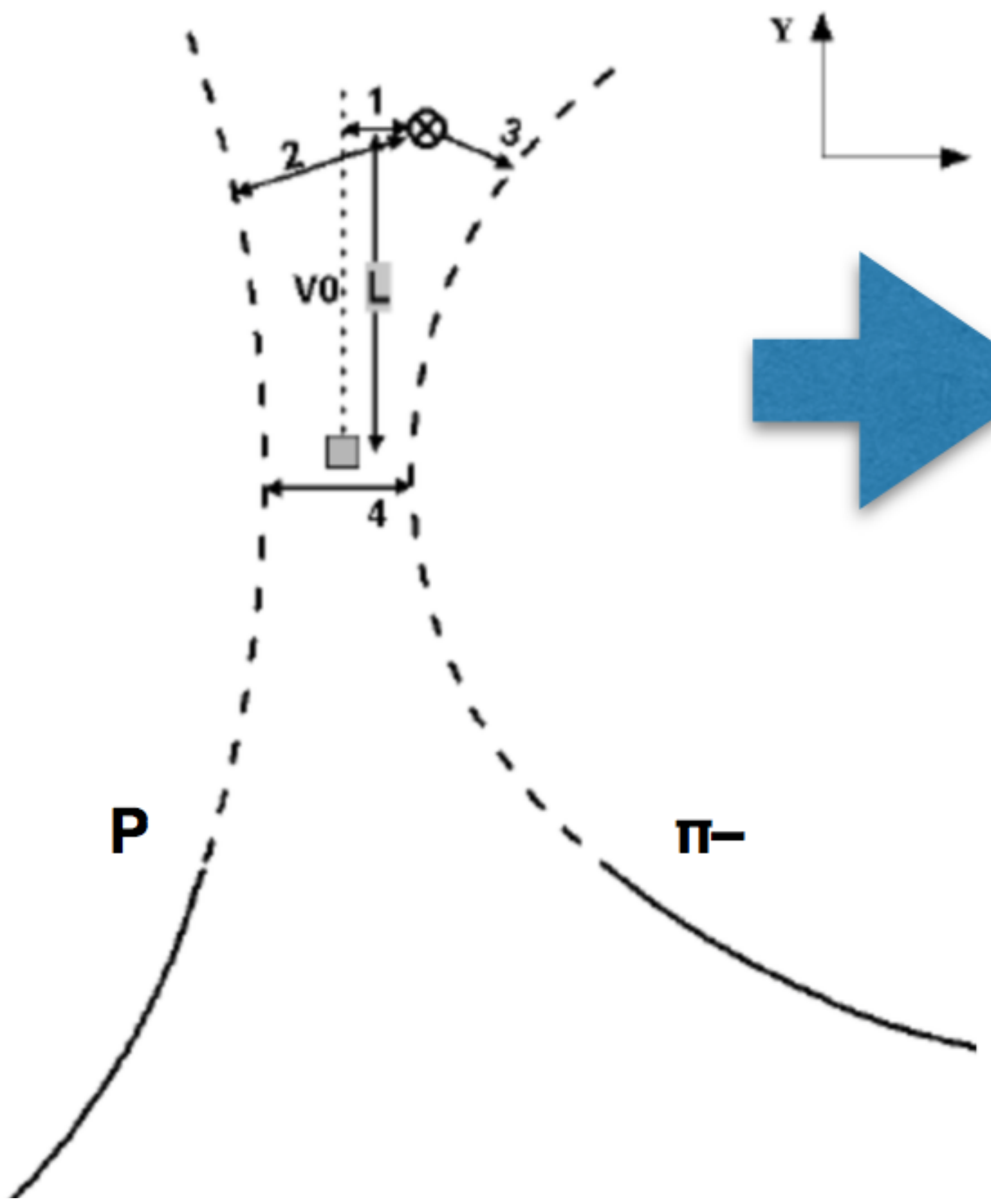
Charged paricles excluding (anti)protons in $|\eta|>0.5$

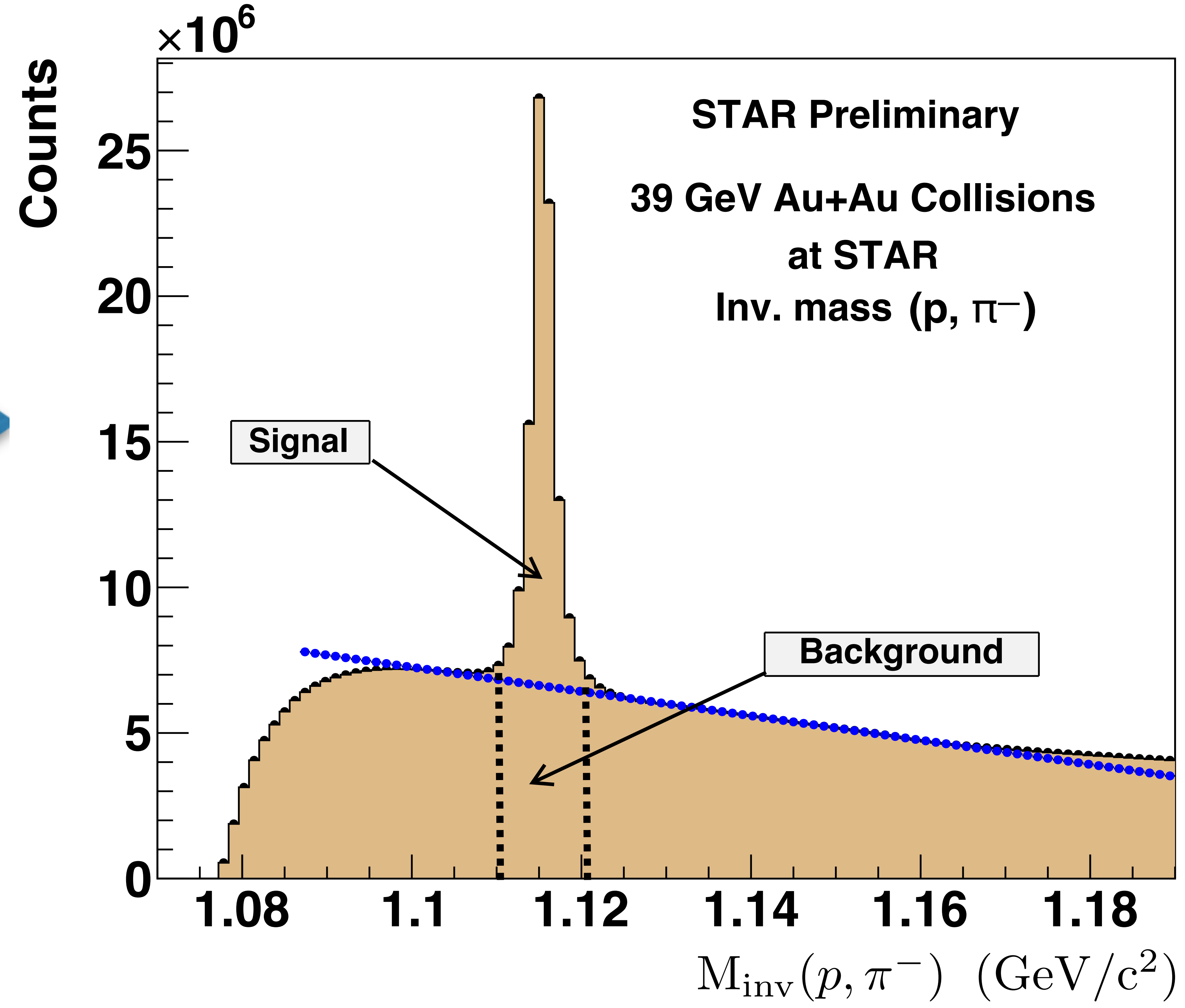
See : Phys. Rev. Lett. 113, 092301(2014)

Lambda reconstruction

✓ **Lambda is reconstructed event-by-event based on the decay topology.**

Lambda decay topology

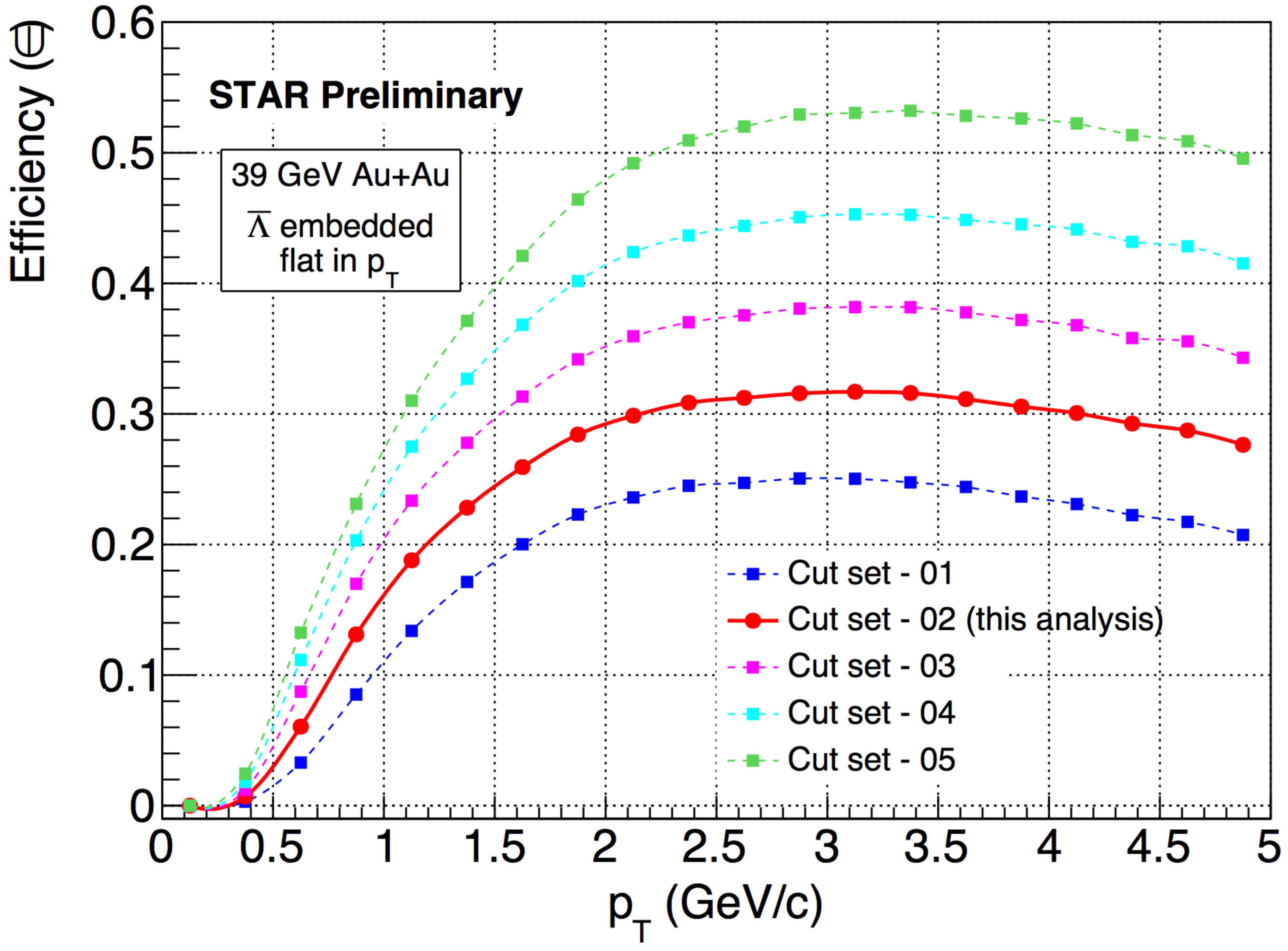


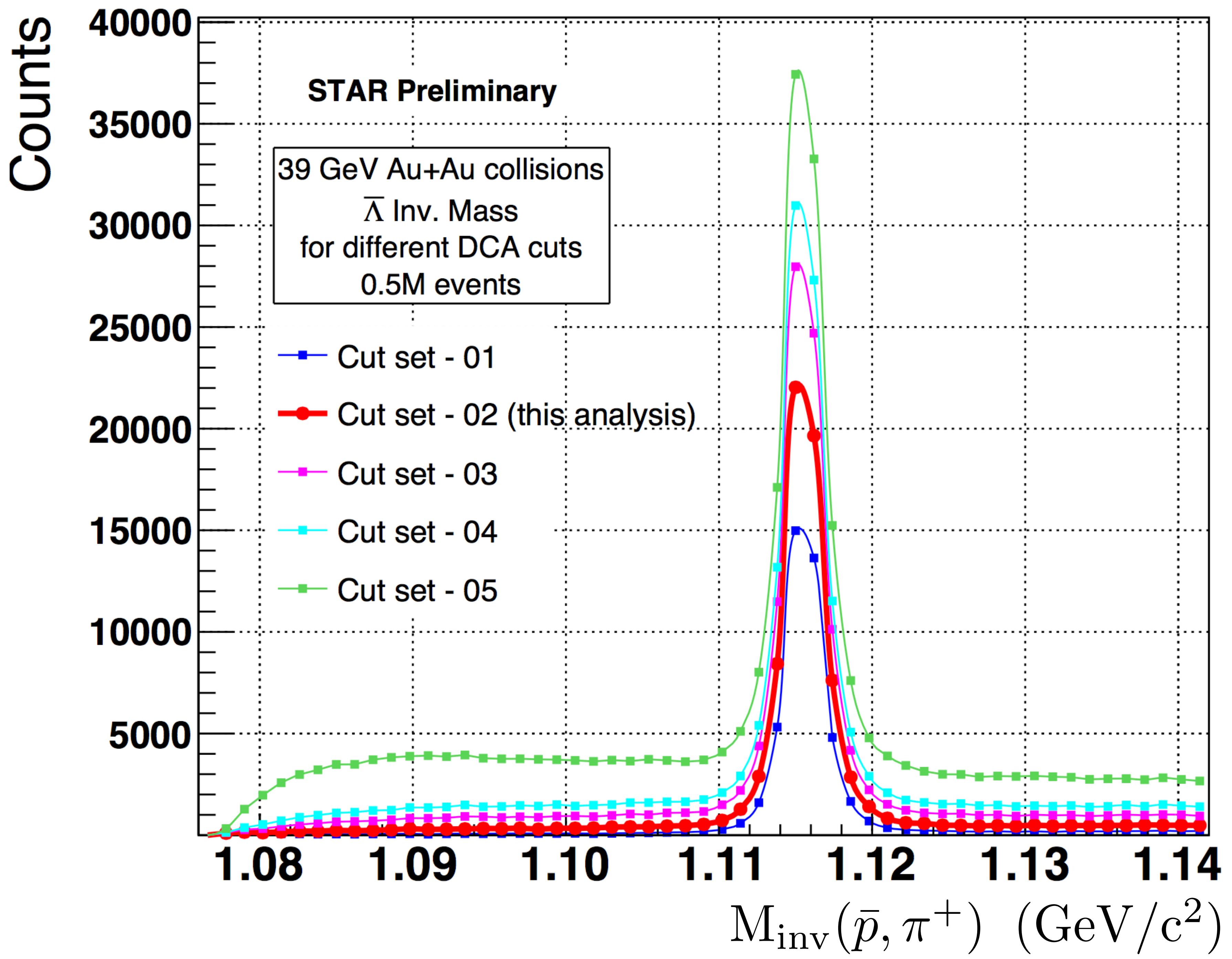


Lambda reconstruction

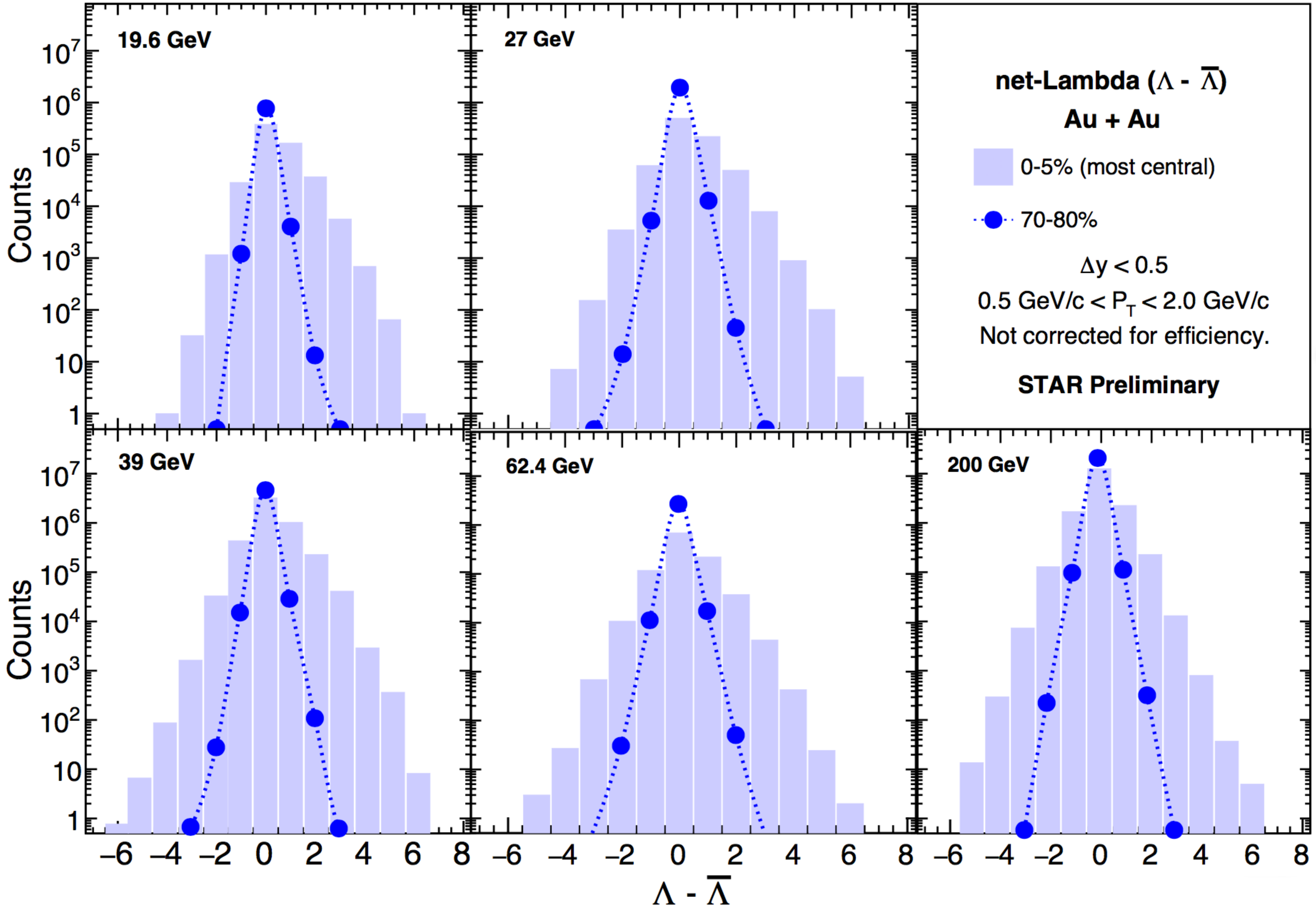
✓ **The geometrical cuts were varied in order to obtain high pure (>90%) sample.**

	Cut Set #1	Cut Set #2	Cut Set #3	Cut Set #4	Cut Set #5
DCA of V0 to PV	< 0.35	< 0.5	< 0.65	< 0.8	< 0.95
DCA of P to PV	> 0.6	> 0.5	> 0.4	> 0.3	> 0.2
DCA of pi- to PV	> 1.75	> 1.5	> 1.25	> 1.0	> 0.75
DCA of P to pi-	< 0.5	< 0.6	< 0.7	< 0.8	< 0.9
Background	3196	9819	22908	34184	82161
Signal	108654	157856	196537	213468	253431
Sig/Background	33.9969	16.0766	8.5794	6.2448	3.0845
Purity%	97.1426%	94.144%	89.5609%	86.1968%	75.5176%





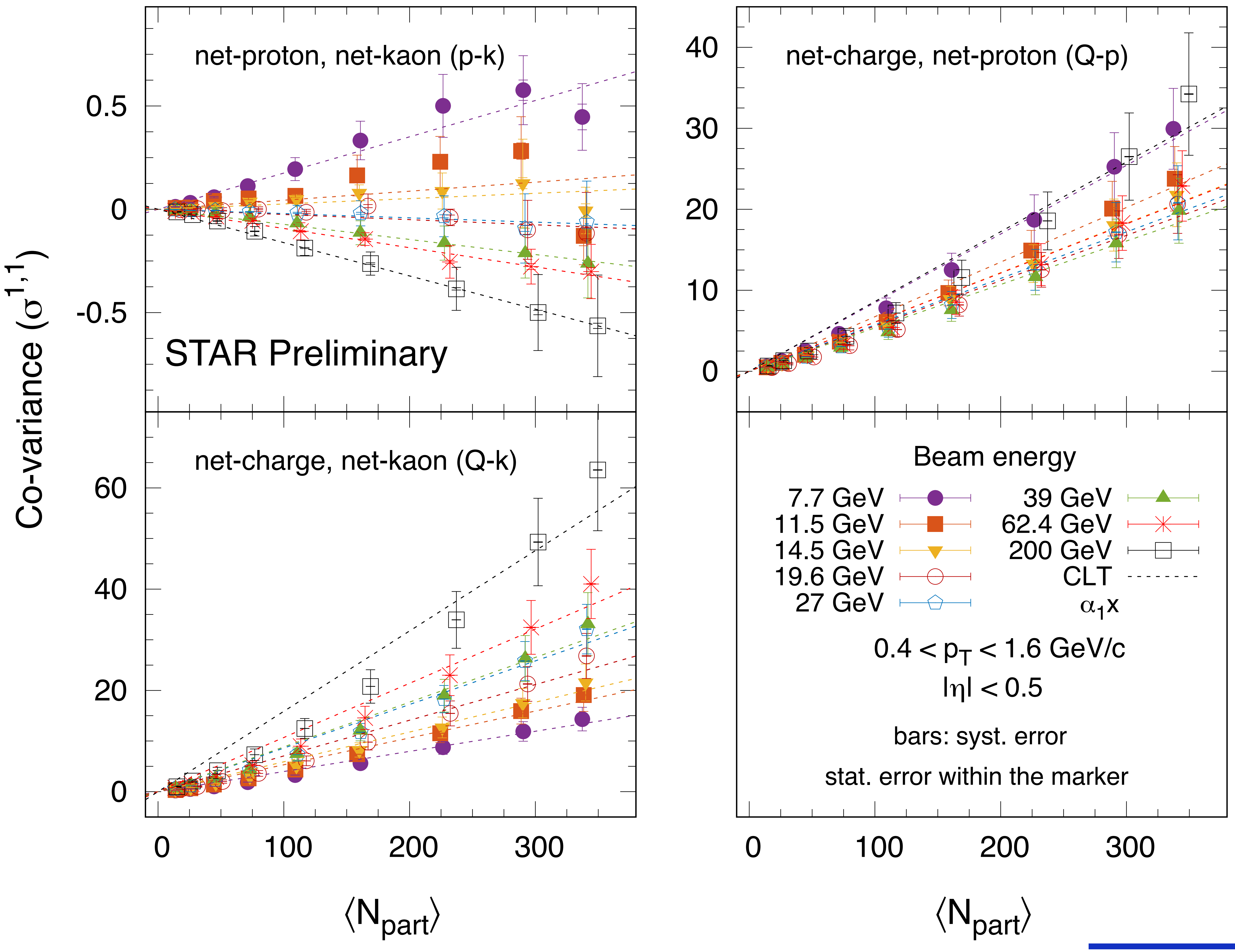
Net-lambda distribution



2nd-order off-diagonal cumulants

Centrality dependence

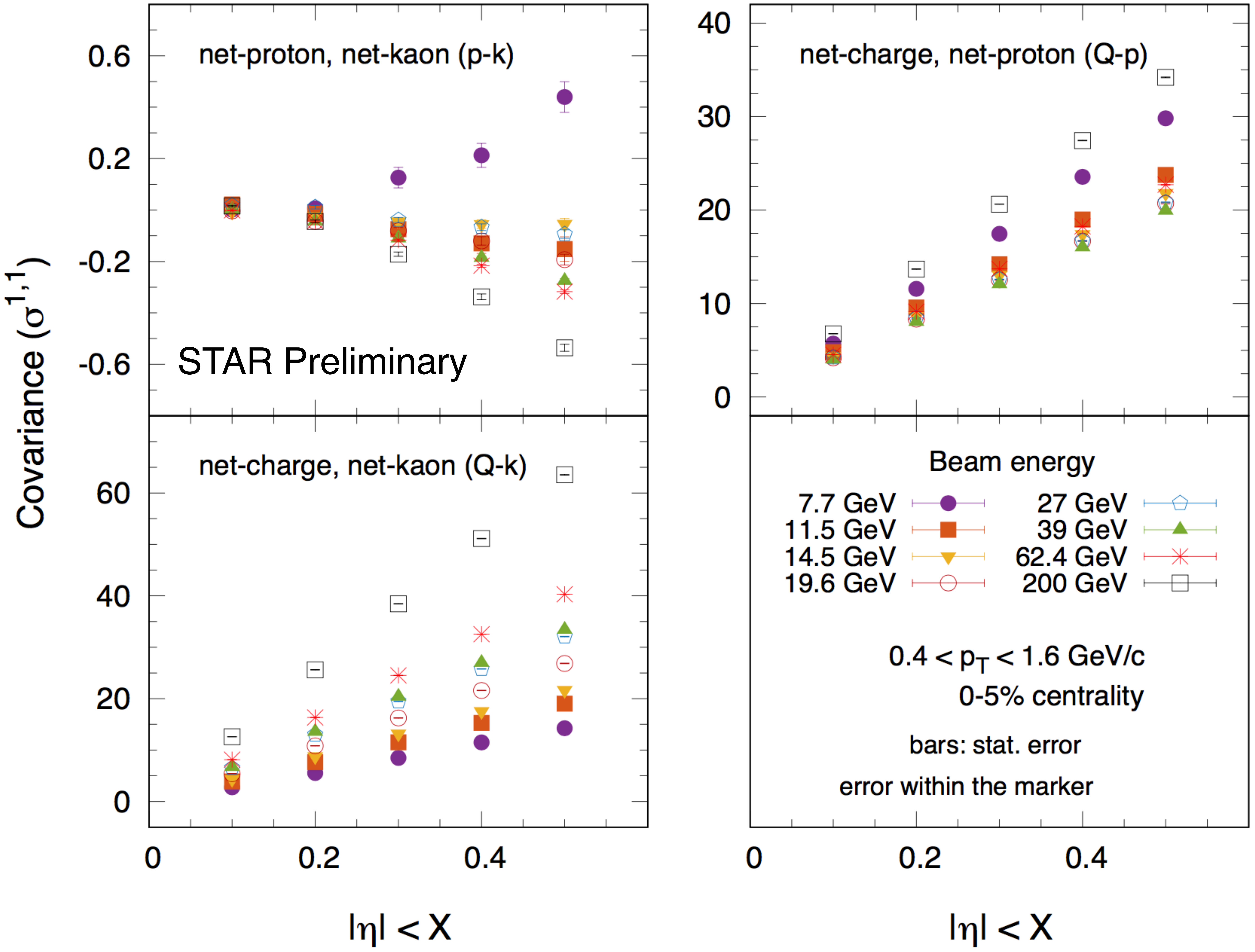
See A. Chatterjee, Poster #534



2nd-order off-diagonal cumulants

η acceptance dependence

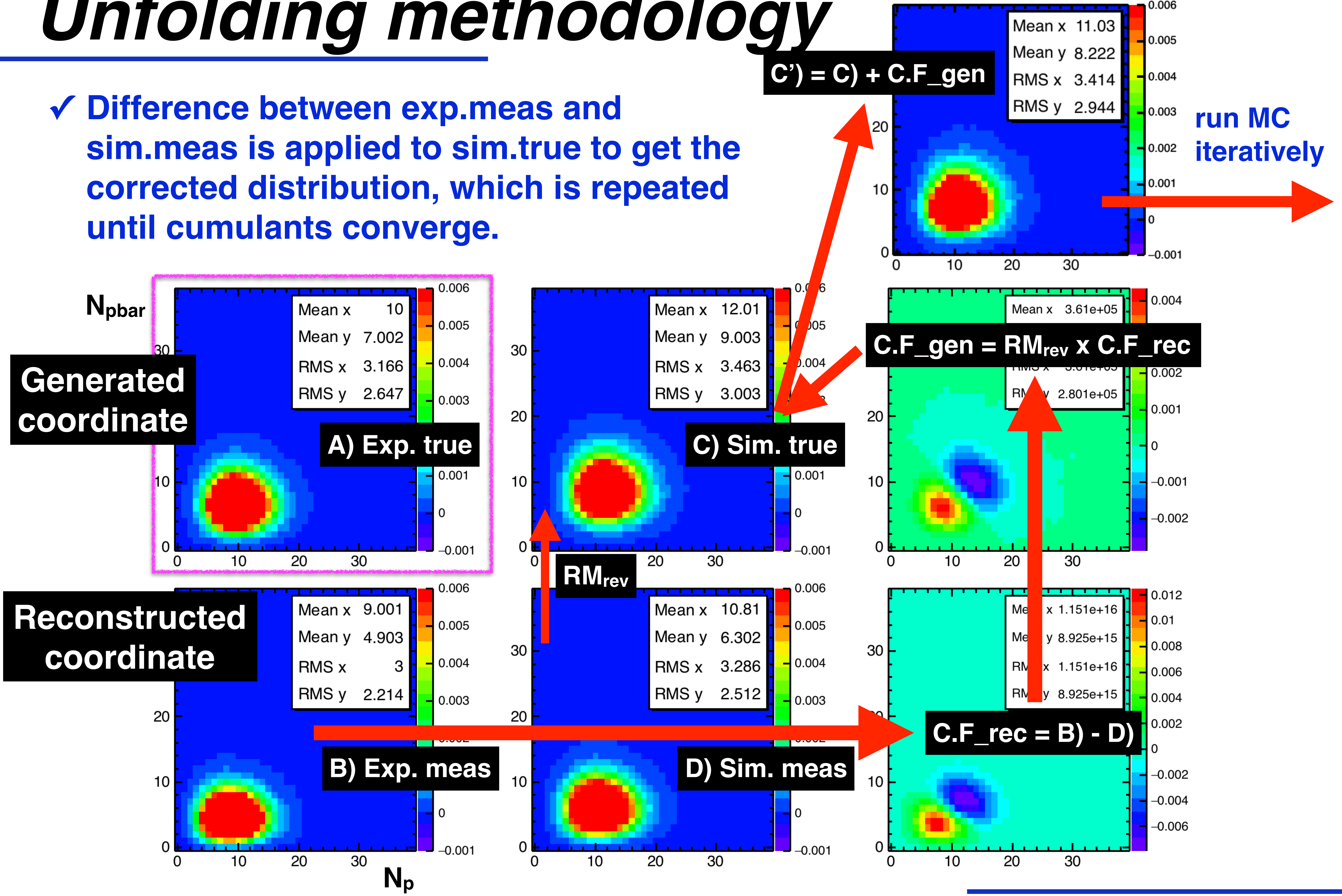
See A. Chatterjee, Poster #534



Unfolding methodology

S. Esumi *et al* : in preparation

✓ **Difference between exp.meas and sim.meas is applied to sim.true to get the corrected distribution, which is repeated until cumulants converge.**



Test with non-binomial distributions

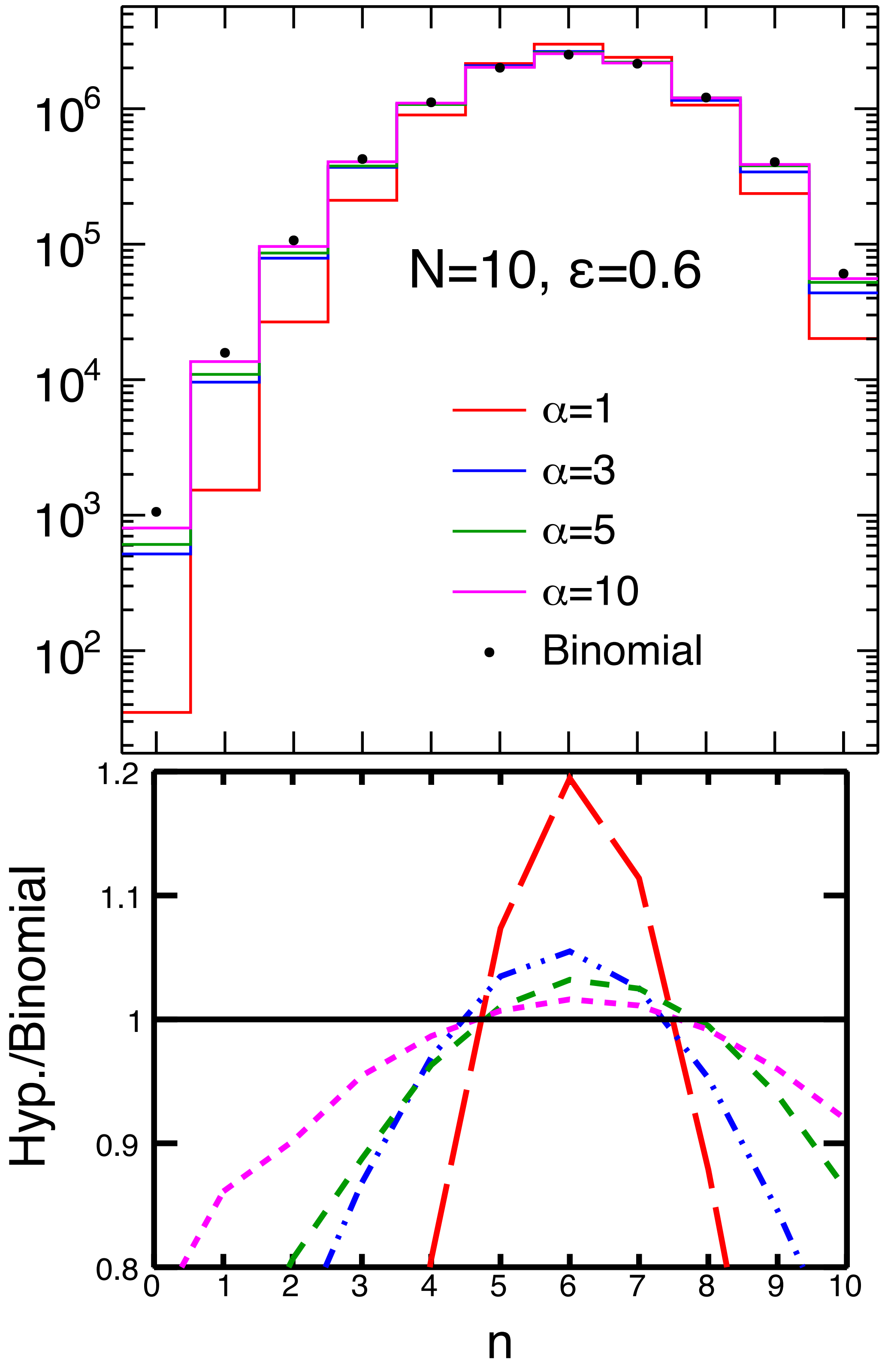
- ✓ **The beta-binomial and hypergeometric distributions can be defined by the urn model, in which the parameter α controls the devitation from the binomial distribution.**

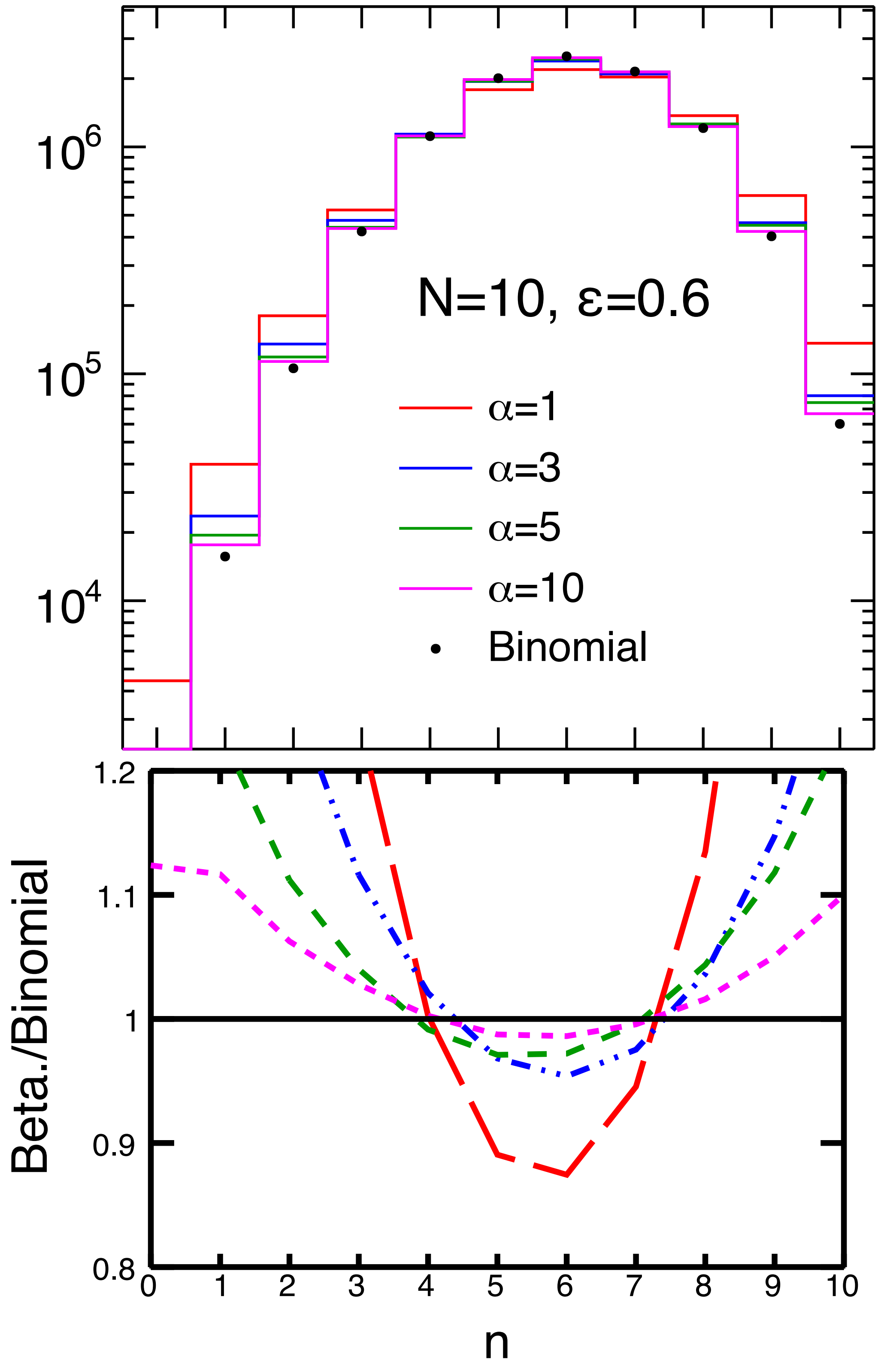
N_w : white balls, N_b: black balls, ε: efficiency

$$N_w = \alpha N \qquad \varepsilon = N_w/(N_w + N_b)$$

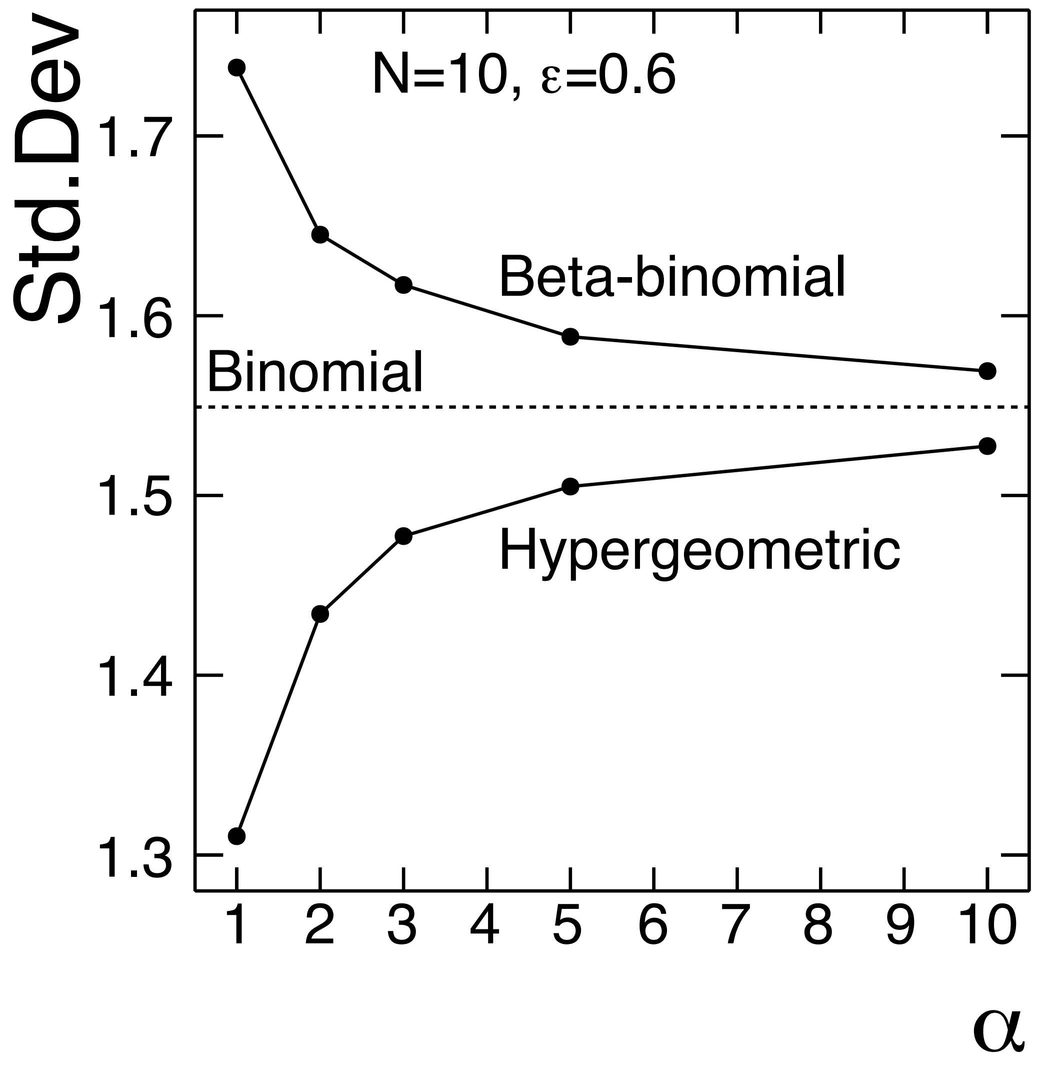
Hypergeometric distribution
Draw a ball from urn, if it is white, count particle. This is repeated without replacement.

Beta-binomial distribution
Draw a ball from urn, if it is white, count particle. And return two white balls to urn (similar for black balls).



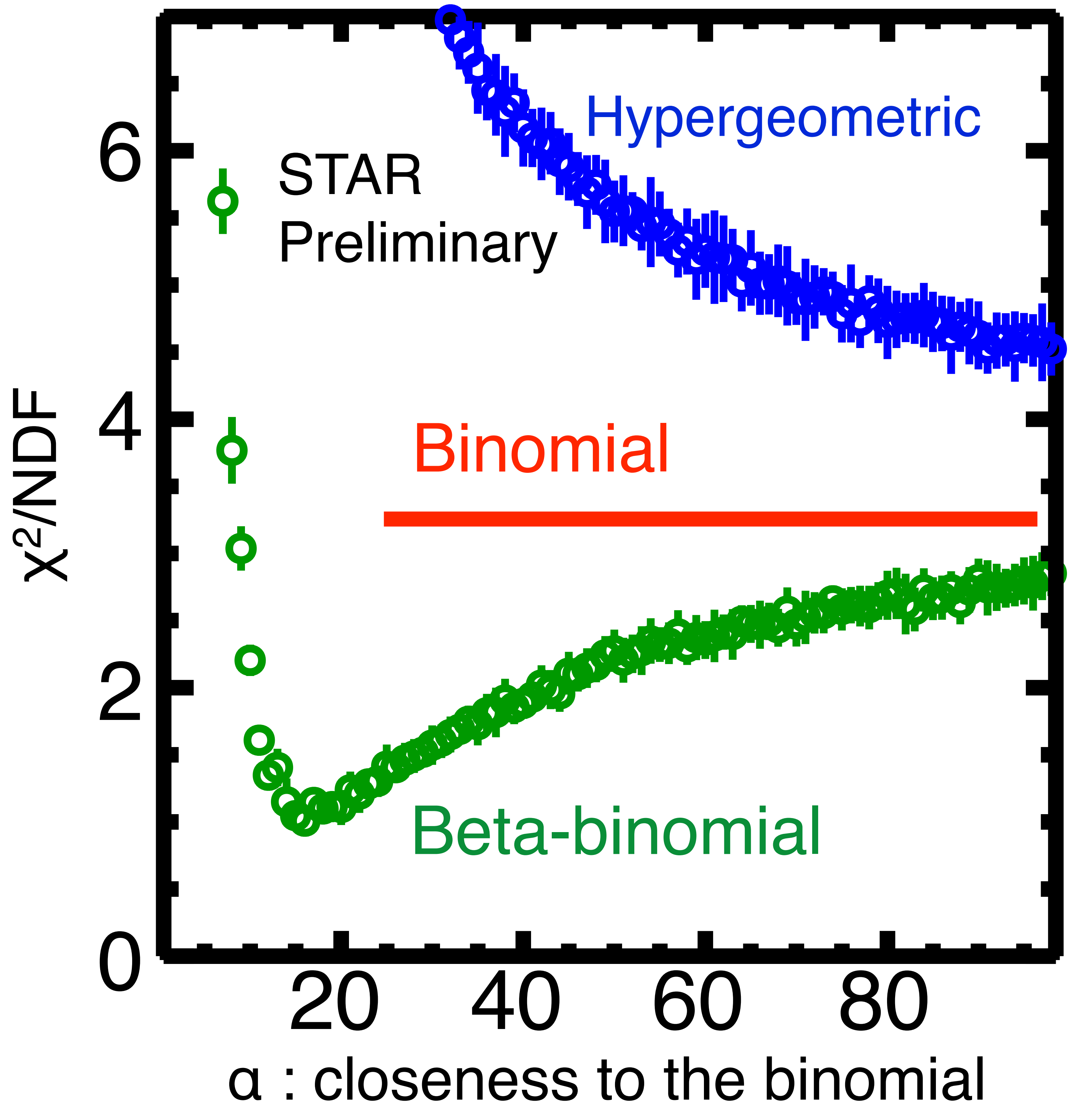


- ✓ **Smaller α for *Hypergeometric* distribution, becomes narrower than binomial distribution.**
- ✓ **Smaller α for *Beta-binomial* distribution, becomes wider than binomial distribution.**
- ✓ **Both non-binomial distributions become close to the binomial with large α.**

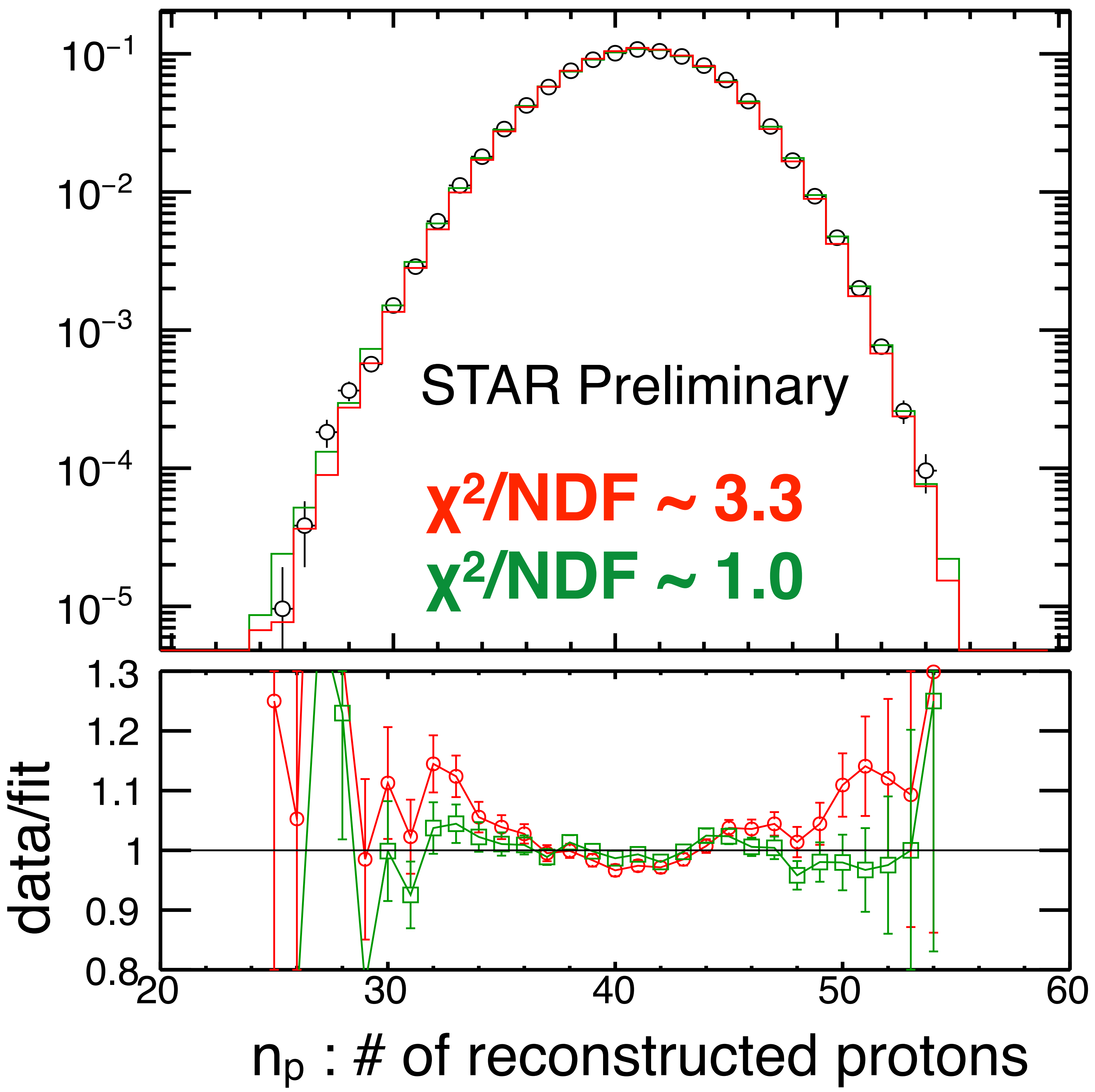


Response matrix from embedding simulation

- ✓ **Embed 60 protons and 15 antiprotons into the real data (which would be the extreme number), and see whether those particles can be reconstructed or not.**
- ✓ **The response matrix is wider than the binomial, and it is close to the beta-binomial distribution.**
- ✓ **More detailes can be found in the poster #453.**



$\sqrt{s_{NN}}$ = 19.6 GeV, 0-2.5% centrality,
60 protons and 15 antiprotons are embedded



Test with non-binomial distributions

- ✓ **The beta-binomial distribution is wider than binomial, which can be defined by the urn model.**
- ✓ **The parameter α controls the devitation from the binomial distribution.**

N_w : white balls, N_b: black balls, ε: efficiency

$$N_w = \alpha N \qquad \varepsilon = N_w/(N_w + N_b)$$