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Beam-energy Dependent Pion Interferometry with Levy-Stable Sources at STAR

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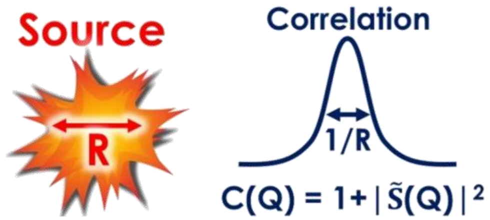
U.S. DEPARTMENT OF ENERGY

Part I.
Introduction, Motivation

Basic definitions of femtoscopic correlation funtions



- Single particle momentum distribution: $N_1(p) = \int d^4x\, s(x,p)$

s: Single particle phase-space density (emission func.)
x: particle coordinate
p: particle momentum

- Pair momentum distribution: $N_2(p_a,p_b) = \int d^4x_a d^4x_b s(x_a,p_a)s(x_b,p_b)\left|\psi_{p_a,p_b}(x_a,x_b)\right|^2$

- **Correlation function:** $C(p_a,p_b) = \dfrac{N_2(p_a,p_b)}{N_1(p_a)N_1(p_b)}$

- **Pair source/spatial correlation:** $D_{\boldsymbol{K}}(\boldsymbol{r}) = \int d^4\rho\, s\left(\rho+\frac{r}{2},K\right) s\left(\rho-\frac{r}{2},K\right)$

pair separation: $r = x_a - x_b$

pair avg. mom.: $K = (p_a+p_b)/2$

pair center-of-mass: $\rho = (x_a+x_b)/2$

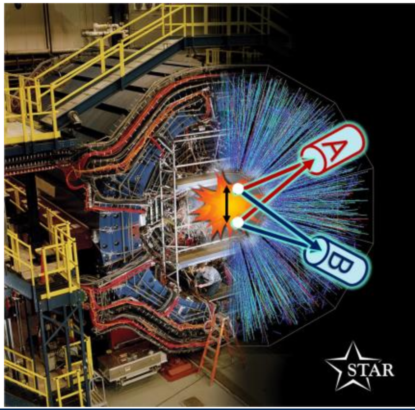
relative pair momentum | *average pair momentum** | *Pair wave function, **containing FSI!***

$$C(\boldsymbol{Q},\boldsymbol{K}) = \int d^3r D_{\boldsymbol{K}}(\mathbf{r})\left|\psi_{\boldsymbol{Q}}(\mathbf{r})\right|^2$$

**Instead of K, m_T is often used:*

$$m_T = \sqrt{k_T^2+m_\pi^2},\ k_T = \sqrt{K_x^2+K_y^2}$$

- **Experiments: measuring C(Q) → information about D(r) and FSI**
- Experimental (and phenomenological) indications: power-law tail for pions, **non-Gaussianity?**

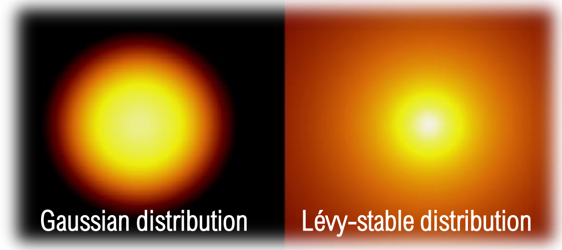


Ann.Rev.Nucl.Part.Sci.55(2005) 357-402; Phys.Lett.B398 (1997), pp. 252–258.; Phys.Rev.Lett.98 (2007), p. 132301.

s: single particle phase-space density (emission function)

$$s(\boldsymbol{x},\boldsymbol{p}) = \mathcal{L}(\alpha, R; \boldsymbol{x}) = \frac{1}{(2\pi)^3}\int d^3\boldsymbol{q}\, e^{i\boldsymbol{q}\boldsymbol{x}} e^{-\frac{1}{2}\left|\boldsymbol{q}^T\boldsymbol{R}^2\boldsymbol{q}\right|^{\alpha/2}}$$

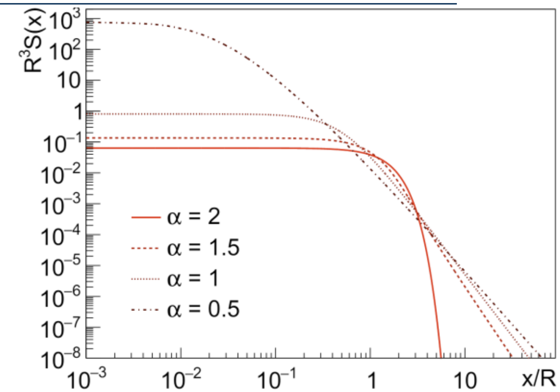
$\boldsymbol{p}$ dependence through α, R — *spherical symmety: R^2 diagonal*



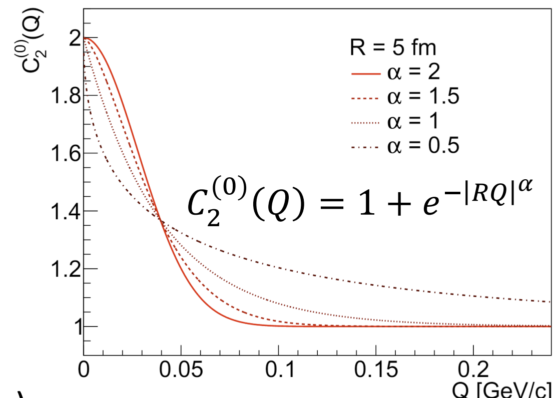
What is the shape of the source? Gaussian & Lévy distributions in heavy-ion physics

- **Symmetric Lévy-stable distribution** *Eur.Phys.J.C 36 (2004) 67*
  - From generalized central limit theorem (GCLT), **power-law tail (if $\alpha < 2$)** $\sim r^{-(1+\alpha)}$
  - $\alpha = 2$ Gaussian, $\alpha = 1$ Cauchy
  - Retains the same α under convolution

$$s(\boldsymbol{x},\boldsymbol{p}) = \mathcal{L}(\alpha, R; \boldsymbol{x})$$
$$\Downarrow$$
$$D_{\boldsymbol{K}}(\boldsymbol{r}) = \mathcal{L}\left(\alpha, 2^{1/\alpha}R; \boldsymbol{r}\right)$$



- **Experimental indications – Lévy source for pion pairs?**
  - **RHIC (PHENIX, STAR), LHC (CMS), SPS (NA61/SHINE)**
    Phys.Rev.C 97 (2018) no.6, 064911; Universe 10 (2024) 3, 102
    Phys.Rev.C 109 (2024) 2, 024914; Eur.Phys.J.C 83 (2024) 10, 919



- **Possible reasons for the $\alpha < 2$ Lévy exponent at RHIC:**
  - Critical behavior, resonance decays, anomalous diffusion (rescattering)

New data analyses and phenomenological investigations are needed to gain a better understanding!

$$s(\boldsymbol{x},\boldsymbol{p}) = \mathcal{L}(\alpha,R;\boldsymbol{x}) = \frac{1}{(2\pi)^3}\int d^3\boldsymbol{q}\, e^{i\boldsymbol{q}\boldsymbol{x}} e^{-\frac{1}{2}\left|\boldsymbol{q}^T\boldsymbol{R}^2\boldsymbol{q}\right|^{\alpha/2}}$$

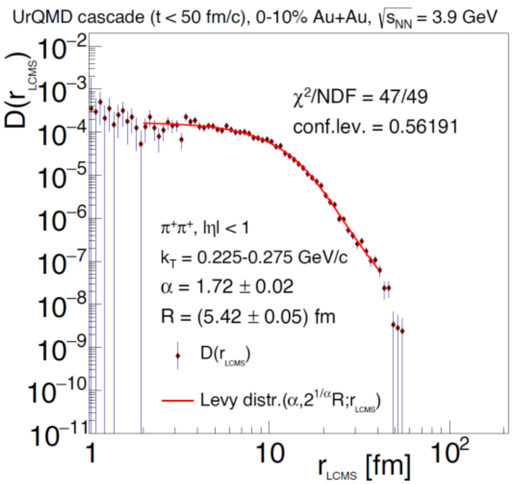
$\boldsymbol{p}$ dependence through α, R — *spherical symmety: R^2 diagonal*

Lévy source at RHIC – rescattering and decays

- **Hadronic rescattering** → Convolution of many elementary processes → **Stable distribution**
- **Test from the phenomenology side:**

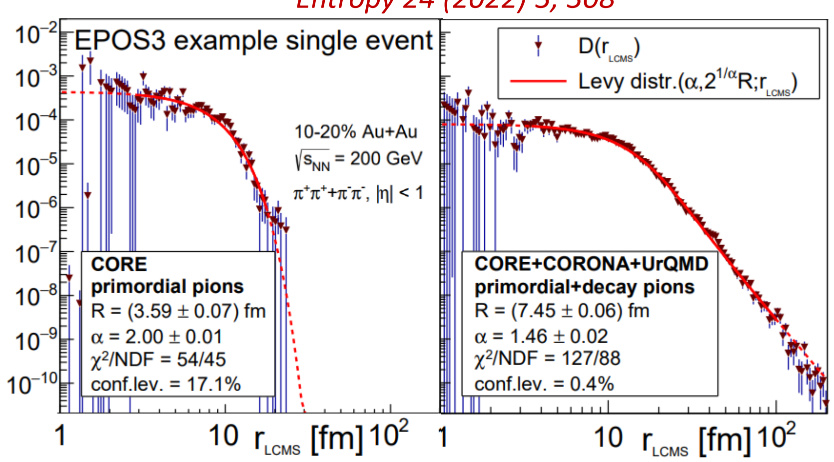
  Direct source-function reconstruction in event generator models including decays and rescattering
- **Power-law source seen in event generators** from RHIC FXT up to LHC energies
- **Global trend: power-law exponent decreases with increasing collision energy**

UrQMD, 3.9 GeV, α ~ 1.7



EPOS, 200 GeV, α ~ 1.5

Entropy 24 (2022) 3, 308



$$s(\boldsymbol{x},\boldsymbol{p}) = \mathcal{L}(\alpha, R; \boldsymbol{x}) = \frac{1}{(2\pi)^3}\int d^3\boldsymbol{q}\, e^{i\boldsymbol{q}\boldsymbol{x}} e^{-\frac{1}{2}\left|\boldsymbol{q}^T\boldsymbol{R}^2\boldsymbol{q}\right|^{\alpha/2}}$$

p *dependence through α, R* — *spherical symmety: R^2 diagonal*

Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828 (2006) 1, 525-532

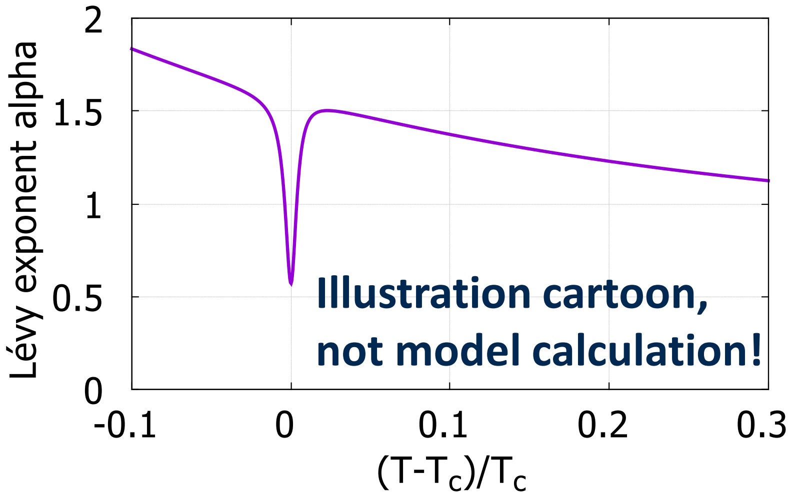
Lévy source at RHIC – critical behavior?

- **Deviation from the trend predicted by rescattering → other effects in play!**
- Order parameter: $c = \langle \bar{q}q \rangle$, correlation function: $\rho(r) = \langle c(r+R)c(r)\rangle - \langle c \rangle^2$
- Critical spatial correlation: $\rho(r) \sim r^{-(d-2+\eta)}$;
- 1D Lévy source: $D(r) \sim r^{-(1+\alpha)}$; $\boldsymbol{\alpha \Leftrightarrow \eta}$?
- At the critical point:
  - Random field 3D Ising: $\eta = 0.50 \pm 0.05$

    Rieger, Phys.Rev.B52 (1995) 6659
  - 3D Ising: η = 0.03631(3)

    El-Showk et al., J.Stat.Phys.157 (4-5): 869
- **Change in α_{Levy} - proximity of CEP?**
- Note: finite-size effects are important, might not influence $\boldsymbol{\eta}$ that much

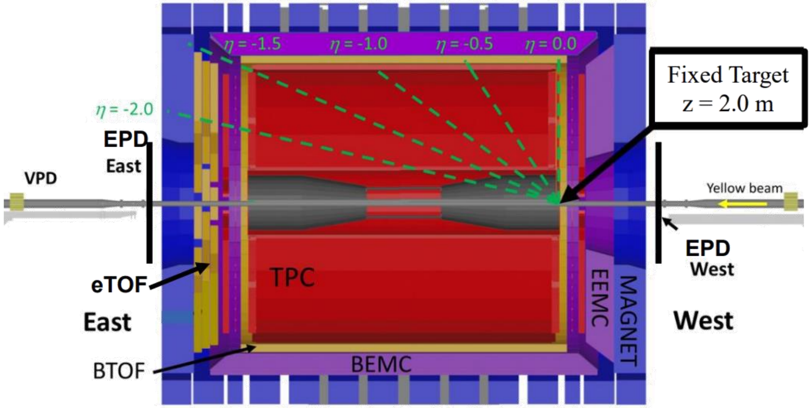
Phys. Lett. B 1996, 387, 125–131 *Phys. Rev. E 2024, 108, 044146*

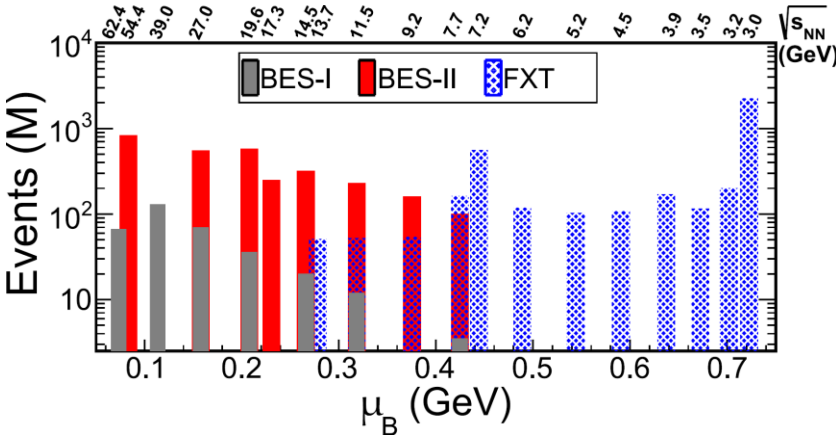


Part II.
Measurement and fitting of correlation functions at FXT energies

Lévy HBT analysis at STAR, Au+Au @ $\sqrt{s_{NN}}$ = 3.2 GeV, 3.9 GeV

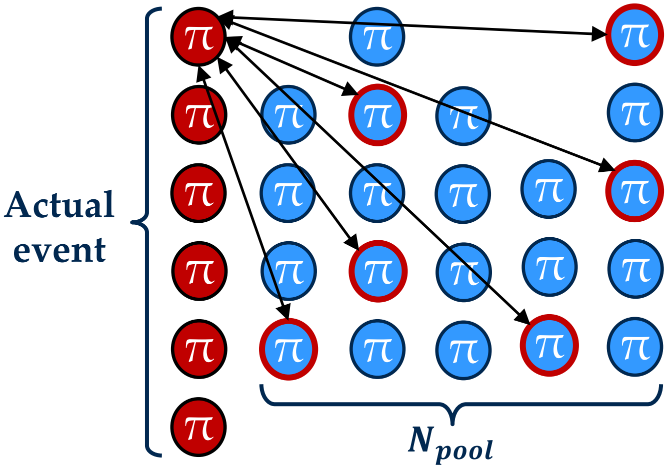
- **STAR FXT data analyzed**
  3.9 GeV (7.3 AGeV), 3.2 GeV (4.59 AGeV)
- **Detectors used for the analysis:**
  - **TPC**: centrality, vertex position, tracking, dE/dx Particle Identification (PID)
  - **TOF**: time-of-flight PID
- **Event selection:**
  - Pile-up cuts using TOF vs. TPC multiplicity
  - Vertex cuts:
    198 cm < v_z^{TPC} < 202 cm, $|v_r^{TPC}|$ < 2 cm,
    where $v_r^{TPC} = \sqrt{v_x^2 + (v_y + 2)^2}$





Measurement of two-pion correlation functions

- **Track-selection criteria**
  - Combined PID using TPC $N\sigma$ (based on dE/dx) and TOF $N\sigma$ (based on time-of-flight): $\sqrt{N\sigma^2_{TOF,\pi} + N\sigma^2_{TPC,\pi}} < 2.5$
  - Further single-track selection:
    TPC number of hits > 20, TPC number of hits/number of hits possible > 0.65,
    0.15 < p_T [GeV/c] < 1.0, -2.5 < η < -0.5, Distance of Closest Approach (DCA) < 2 cm
- **Pair-selection criteria** *J. Adams et al. (STAR Coll.), Phys. Rev. C 71, 044906 (2005)*
  - Splitting level (SL) < 0.6
  - Fraction of Merged Hits (FMH) < 5%
  - Average pair-separation (on TPC pad rows) Δr > 3 cm
- **Event mixing**
  - Similarly to *Phys.Rev. C97 (2018) no.6, 064911*
  - A(Q): pions from the same event
  - B(Q): pions from different events
  - C(Q)=A(Q)/B(Q), appropriately normalized
- **Average transverse momentum k_T selection:**
  - 8 k_T bins, (0.175 - 0.575) GeV/c



Fitting process with Lévy parametrization

- **Lévy parametrization without final state effects:**

$$C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^{\alpha}}$$

Lévy exponent

Lévy scale parameter

Intercept parameter (correlation strength)

LCMS three-momentum difference

$$Q = |\boldsymbol{q}_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + \left(p_{1y} - p_{2y}\right)^2 + q_{long,LCMS}^2}$$

- **Formula used for fitting procedure:**

Coulomb correction

Possible linear background (usually negligible)

$$C(Q) = \left(1 - \lambda + \lambda \cdot K(Q;\alpha,R) \cdot \left(1 + e^{-|RQ|^{\alpha}}\right)\right) \cdot N \cdot (1 + \varepsilon Q)$$

- **Coulomb-correction:**

Two-particle wave function (with Coulomb interaction)

$$K(Q;\alpha,R) = \frac{\int D_K(r) \left|\psi_Q^{Coul}(r)\right|^2 dr}{\int D_K(r) \left|\psi_Q^{(0)}(r)\right|^2 dr}$$

→ **calculated semi-analytically**

Nagy, Purzsa, Csanád, Kincses, Eur. Phys. J. C 83, 1015 (2024)

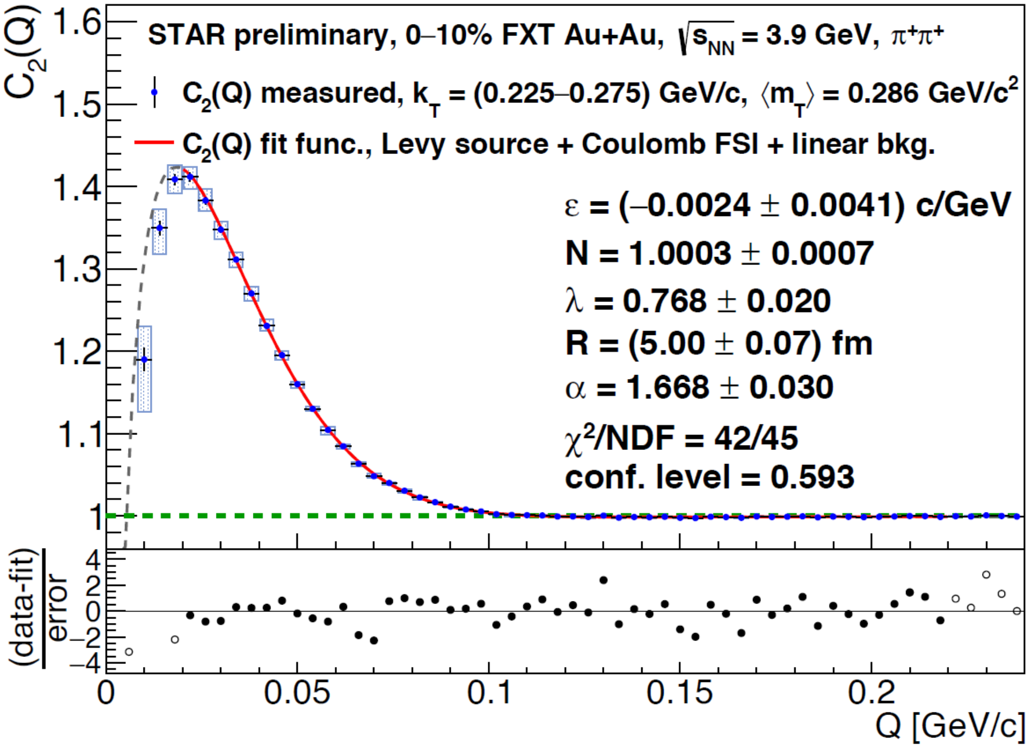
Spatial correlations

Two-particle wave function (plane wave)

$$C(Q) = \left(1 - \lambda + \lambda \cdot K(Q;\alpha,R) \cdot \left(1 + e^{-|RQ|^{\alpha}}\right)\right) \cdot N \cdot (1 + \varepsilon Q)$$

An example fit to a two-pion correlation function

- Example fit to a $\pi^+\pi^+$corr.func., k_T = (0.220-0.275) GeV/c
- **Iterative fitting method**, Coulomb FSI + Lévy-source
- Track and pair **syst. uncertainties illustrated with boxes**
- Total systematic uncertainties also include fit range study
- **Fits converged, conf.level > 0.1%**
- Confidence levels approx. uniformly distributed

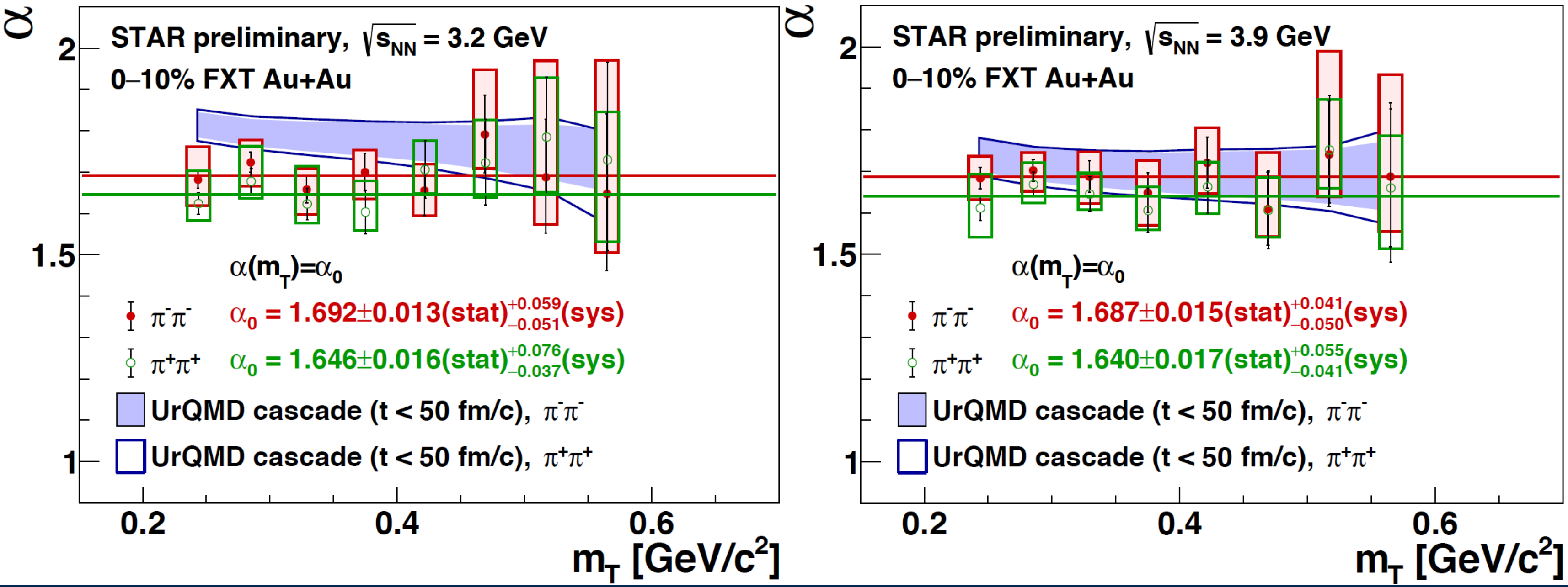


- Similar fits done in 8 k_T bins, separately for $\pi^-\pi^-$ and $\pi^+\pi^+$, $\boldsymbol{m_T}$ **dependence of the source parameters investigated** at both energies

Part III.
Preliminary results
at $\sqrt{s_{NN}}$ = 3.9 and 3.2 GeV
(m_T dependence)

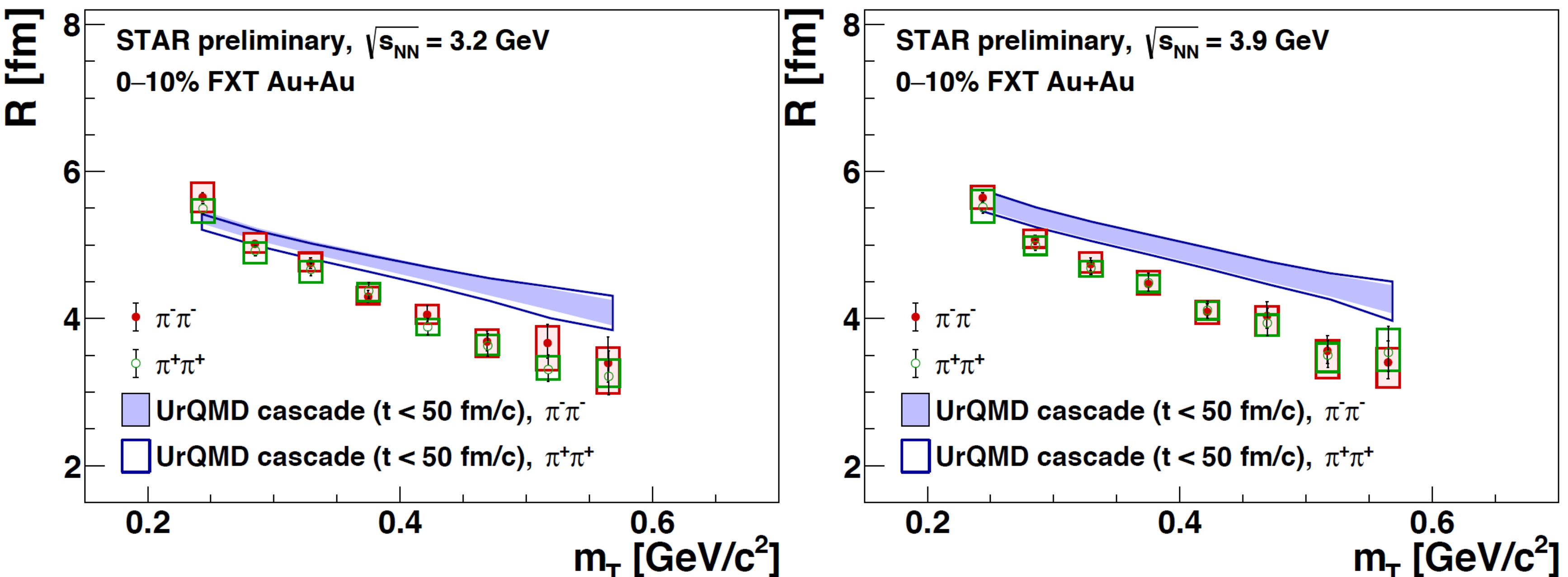
Lévy exponent α

- **Non-gaussian values ($\alpha < 2$)**
- Small systematic difference between $\pi^-\pi^-$ and $\pi^+\pi^+$ pairs
- 3.9 and 3.2 GeV compatible, no m_T dependence observed
- **UrQMD within uncert.** – no other effect but rescattering and decays, good agreement



Lévy scale R

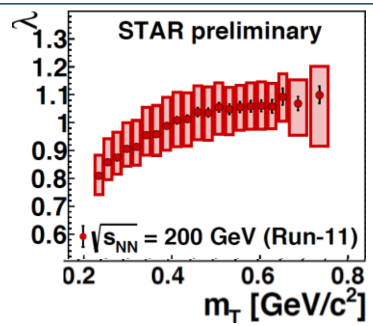
- Decreases towards higher m_T and lower energies
- Small systematic difference between $\pi^-\pi^-$ and $\pi^+\pi^+$ pairs
- Both energies compatible
- **UrQMD describes the trends qualitatively well, moderate quantitative mismatch**

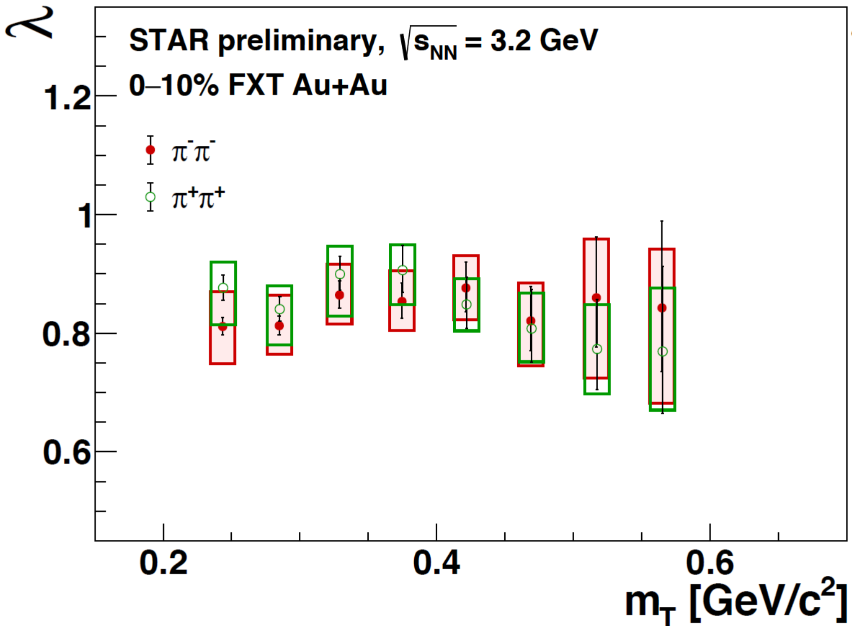


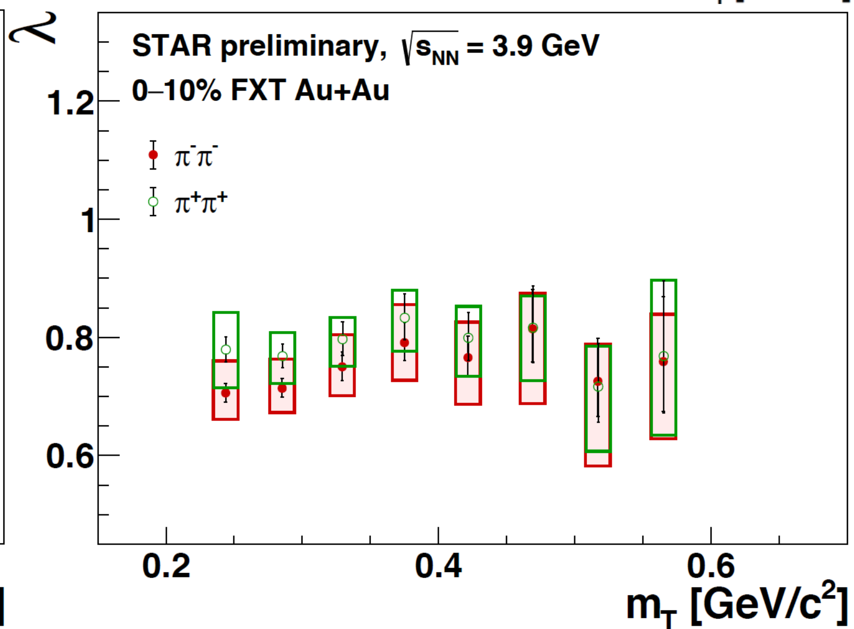
Correlation strength λ

$$\lambda \equiv \lim_{Q\to 0} C_2^{(0)}(Q) - 1$$

- Small systematic difference between $\pi^-\pi^-$ and $\pi^+\pi^+$ pairs
- Low-m_T decrease seen at higher energies not as pronounced (see next slide!)



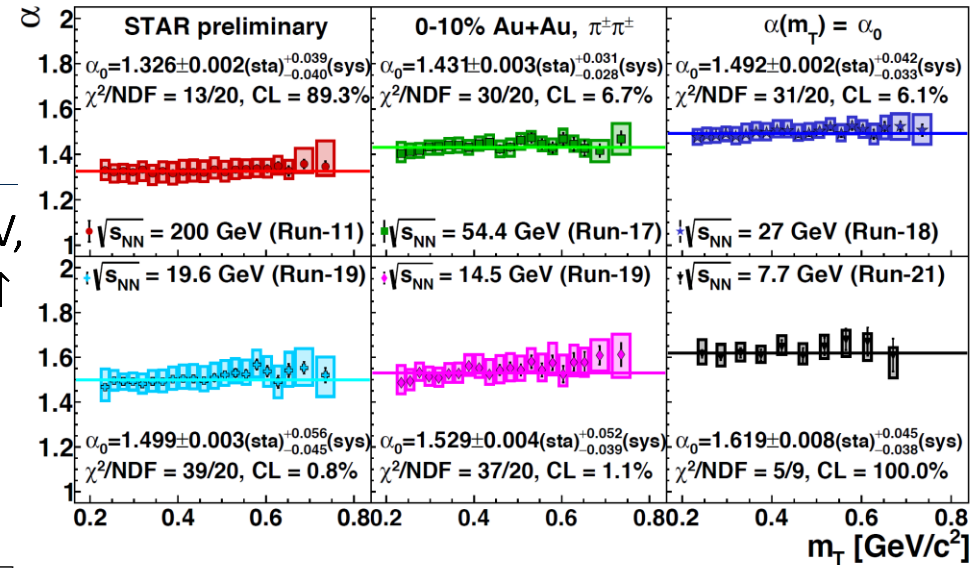


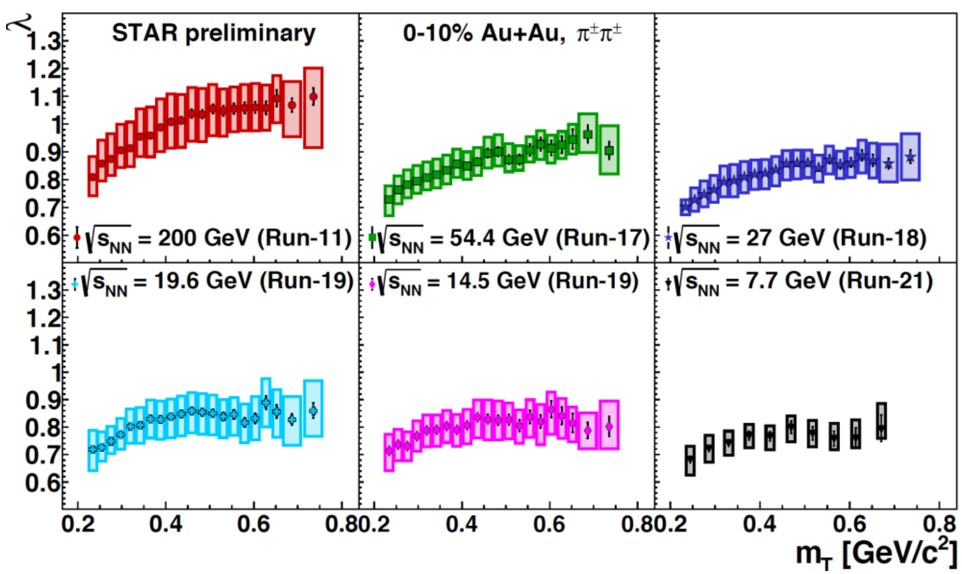


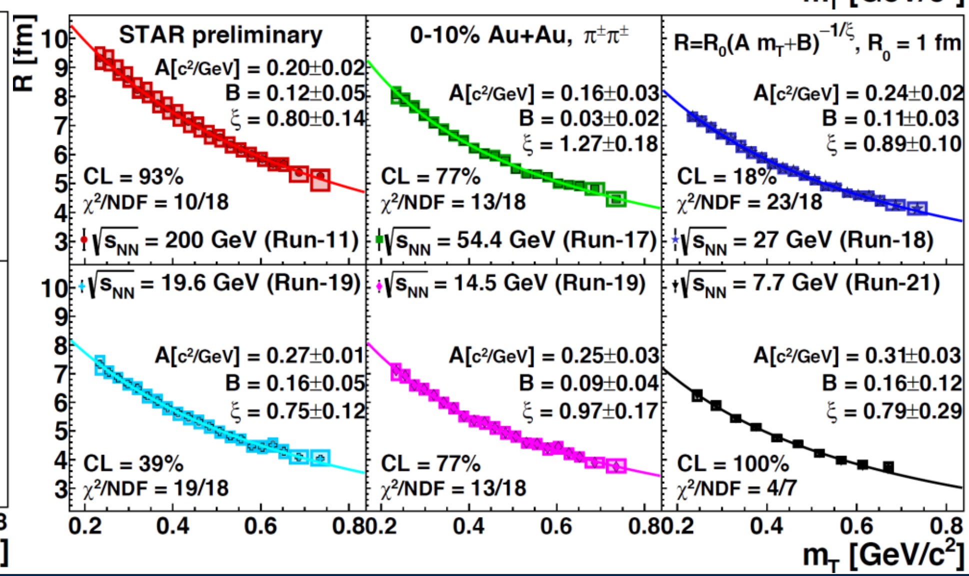
Part IV.
Preliminary results at BES-II
($\sqrt{s_{NN}}$, m_T dependence)

$\sqrt{s_{NN}}$ and m_T dependence of the source parameters

- $\sqrt{s_{NN}}$ dependence from 7.7 to 200 GeV, due to increasing density? $\boldsymbol{\alpha}\downarrow, \boldsymbol{R}\uparrow, \boldsymbol{\lambda}\uparrow$
- m_T dependent trends at all energies: $\boldsymbol{\alpha}$ const., $\boldsymbol{R}\downarrow$ (connection to flow), $\boldsymbol{\lambda}\uparrow$ (change in decay contribution?)

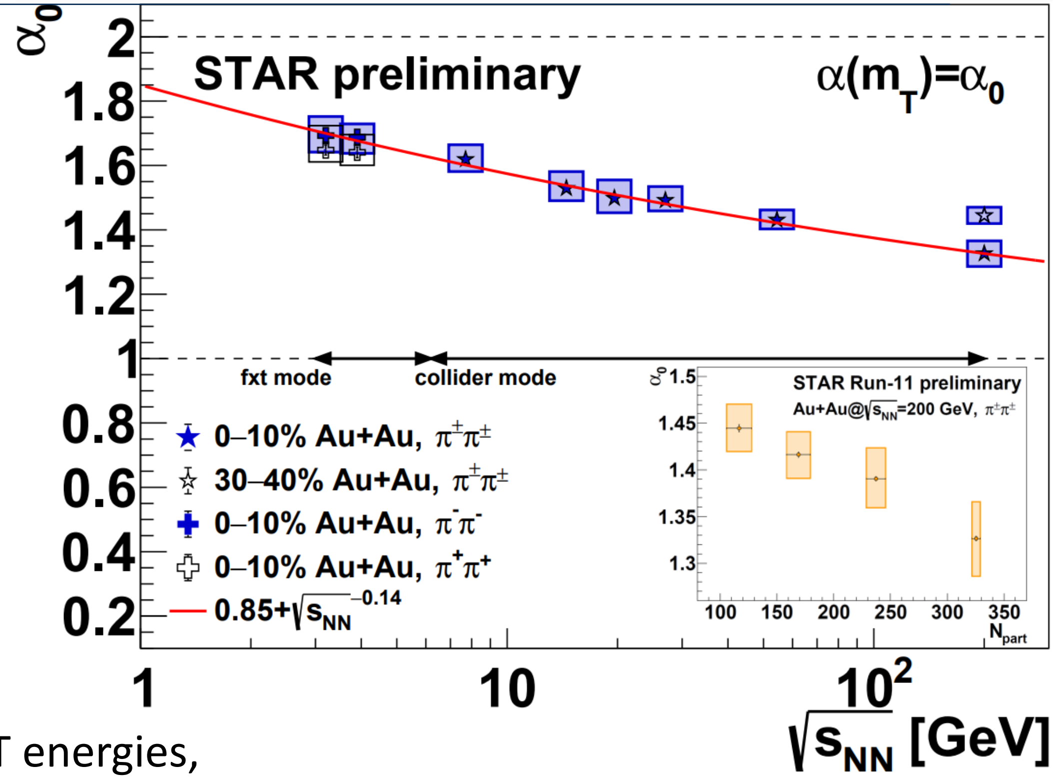






Excitation function of the m_T average Lévy exponent α_0

- Non-gaussian values ($\alpha \ll 2$)
- Increasing density → rescattering decreases $\boldsymbol{\alpha}$?
- 200 GeV centrality dependence: same trend!
- Trend illustrated by power-law: $\alpha_0 \sim 0.85 + \sqrt{s_{NN}}^{-0.14}$

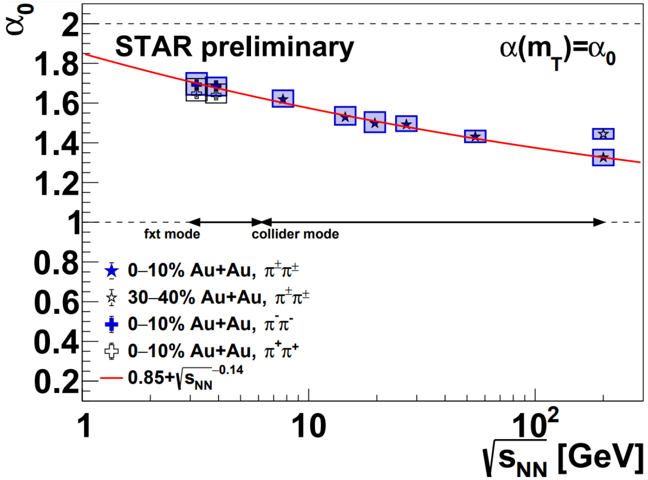


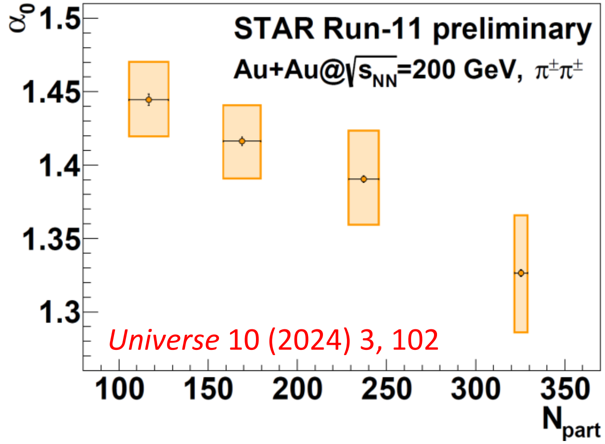
- Good description by UrQMD at FXT energies, a comprehensive energy scan is ongoing
- **No non-monotonic trend in α observed yet, no signs of criticality in Levy exponent**

If interested in a follow-up, come to Zimányi School in December!
http://zimanyischool.kfki.hu/24/

Summary

- 1-dim. two-pion correlation functions investigated
- Lévy-source + Coulomb FSI → good description
- **Observation**:
  - 0-10% Au+Au: **200 GeV → 3.2 GeV**
    200 GeV Au+Au: **central → peripheral**
    } **trends:** $\boldsymbol{\alpha} \uparrow, \boldsymbol{R} \downarrow, \boldsymbol{\lambda} \downarrow$
- **Possible interpretation:**
  - $\alpha < 2$ connected to decays and rescattering
  - Increasing particle density → rescattering decreases $\boldsymbol{\alpha}$?
- **Next steps: even more energies (fxt&col), 3D analysis!**





Thank you!

Further details, backup slides

$$\lambda \equiv \lim_{Q\to 0} C_2^{(0)}(Q) - 1$$

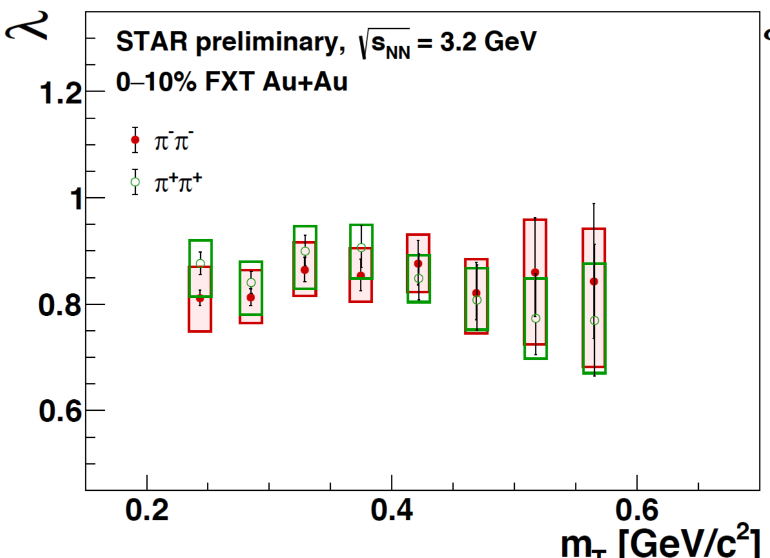
Correlation strength λ

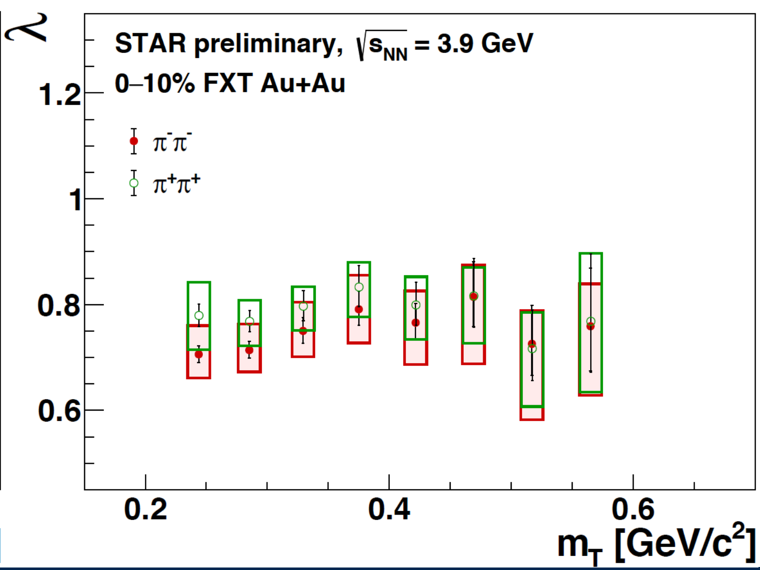
- **Core-halo* picture** - two-component source: $S = S_{core} + S_{halo} \Rightarrow D = D_{(c,c)} + D_{(c,h)} + D_{(h,h)}$
  - Halo: decays of long-lived resonances → unresolvable
  - For power-law sources, more complicated picture!

Single particle source:
$S_{core}(x) = \mathcal{L}(\alpha, R; x)$,
$S_{halo}(x) = \mathcal{L}(\alpha_h, R_h; x)$

Pair-source:
$D_{(c,c)}(r) = \mathcal{L}\left(\alpha, 2^{1/\alpha}R; r\right)$
$R_h \gg R \Longrightarrow D_{(c,h),\,(h,h)}$ unresolvable

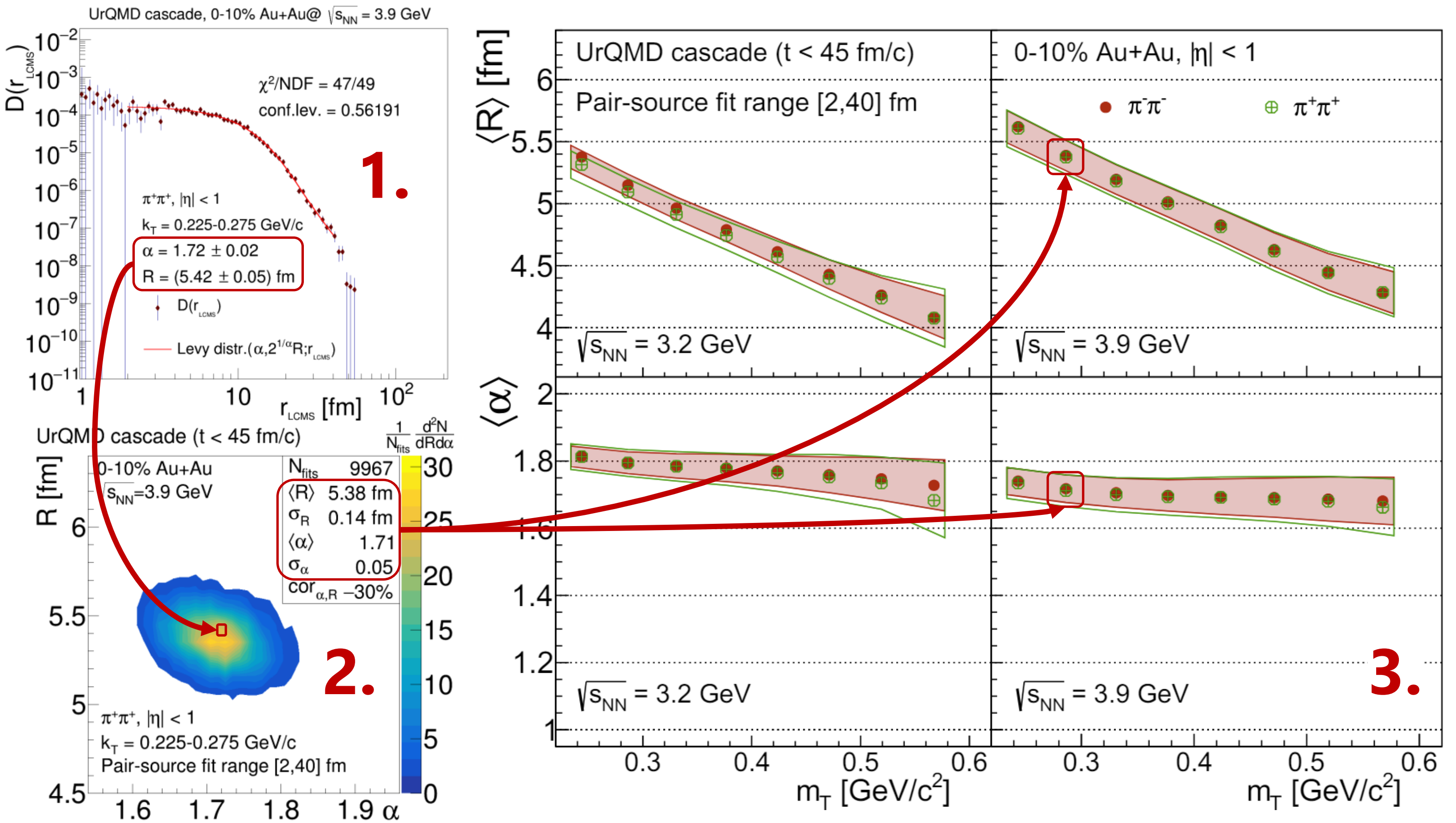
*note that core-halo here is not the same as EPOS core-corona, it is just a coincidence in nomenclature



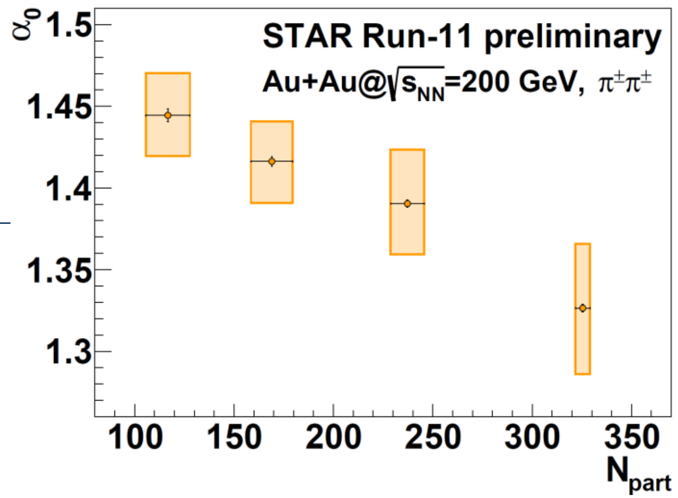


UrQMD cascade analysis (t < 50 fm/c)

D(r) event-by-event → (α, R) distribution → mean and std.dev vs m_T



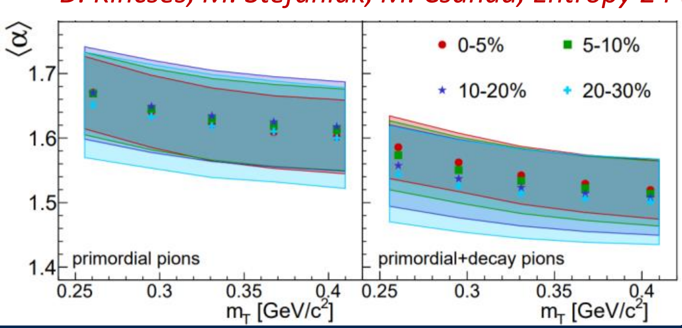
$\sqrt{s_{NN}}$ = 200 GeV

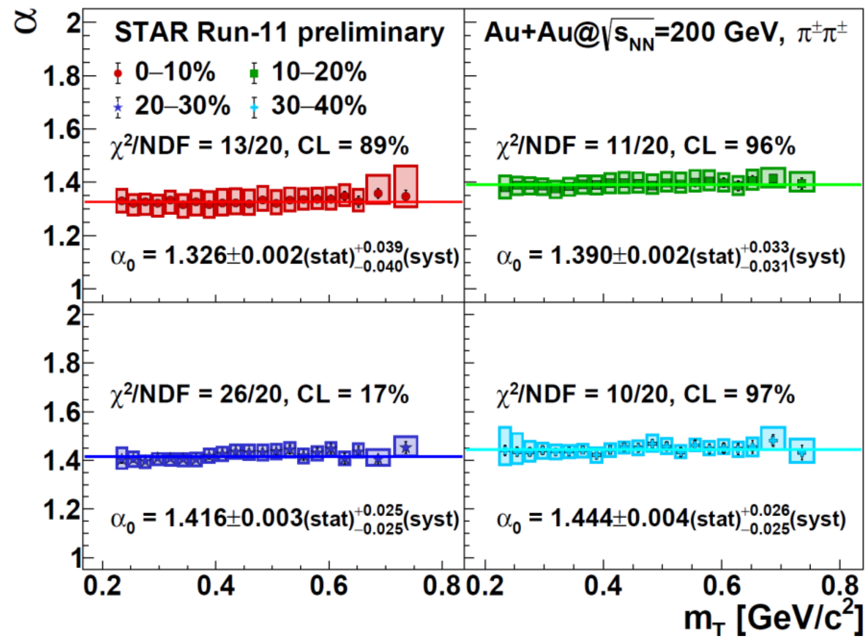


Lévy exponent $\alpha(m_T, \text{centrality})$

- **Non-gaussian values ($\alpha \ll 2$)**
- No dependence on m_T, slight centrality dep.
- $\boldsymbol{\alpha_0}$ vs N_{part} (m_T average values from constant fit)
  - Decreasing trend due to anti-correlation with λ and R?
  - CMS observed opposite trend, see *Phys.Rev.C* 109 (2024) 2, 024914
- EPOS model: slightly higher α values

D. Kincses, M. Stefaniak, M. Csanád, Entropy 24 (2022) 3, 308





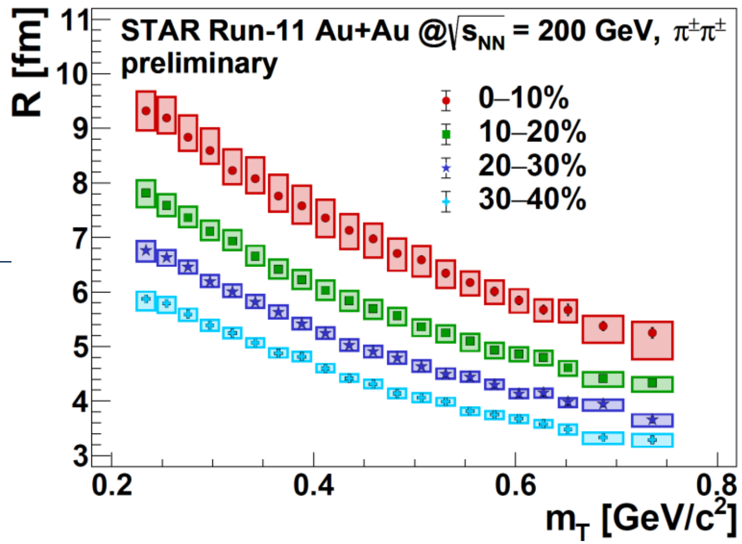
$\sqrt{s_{NN}}$ = 200 GeV

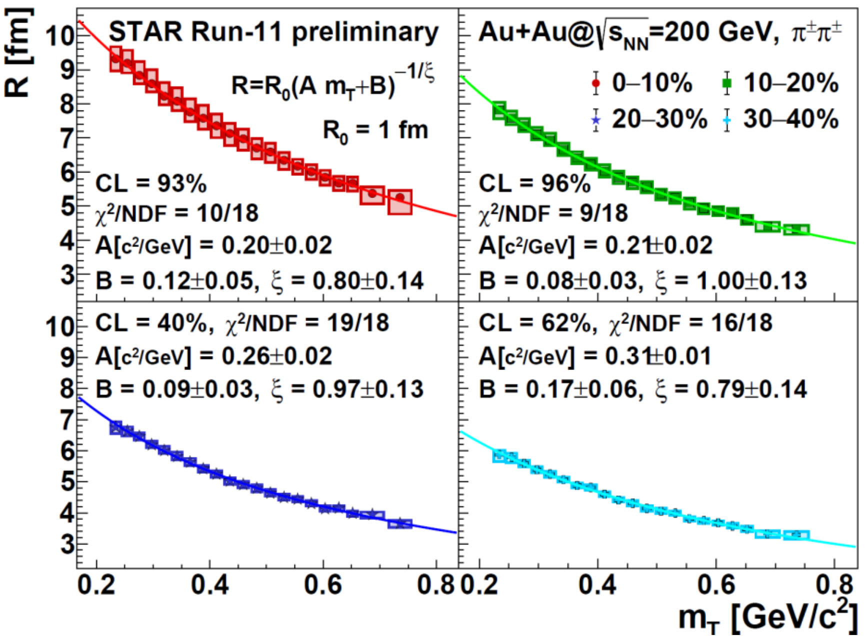
Lévy scale $R(m_T, \text{centrality})$

- **Decreasing trend with m_T, also with centrality**
  - Connection to flow and initial geometry?
- Fits with $\boldsymbol{R = R_0(Am_T + B)^{-1/\xi}}, R_0 = 1$ fm
  - Hydro calculations (R_{Gauss}): $\xi = 2$

Makhlin, Sinyukov, Z. Phys. C 39, 69 (1988)
Csörgő, Lörstad, Phys. Rev. C 54, 1390 (1996)
Chapman, Scotto, Heinz, Acta Phys.Hung.A 1 (1995) 1-31
Csanád, Csörgő, Lörstad, Ster, J. Phys. G 30, S1079 (2004)

  - **Our case (R_{Levy}): ξ close to 1**





$\sqrt{s_{NN}}$ = 200 GeV

Correlation strength $\lambda(m_T, \text{centrality})$

- Without FSI: $C_2^{(0)}(Q=0) = 2 \longrightarrow \lambda \equiv \lim_{Q\to 0} C_2^{(0)}(Q) - 1$, experimentally often $\lambda < 1$
- **Core-halo picture** - two-component source: $S = S_{core} + S_{halo} \Rightarrow D = D_{(c,c)} + D_{(c,h)} + D_{(h,h)}$
  - Core: primordial + decays of short-lived resonances
  - Halo: decays of long-lived resonances → large R → small Q → measurement limited

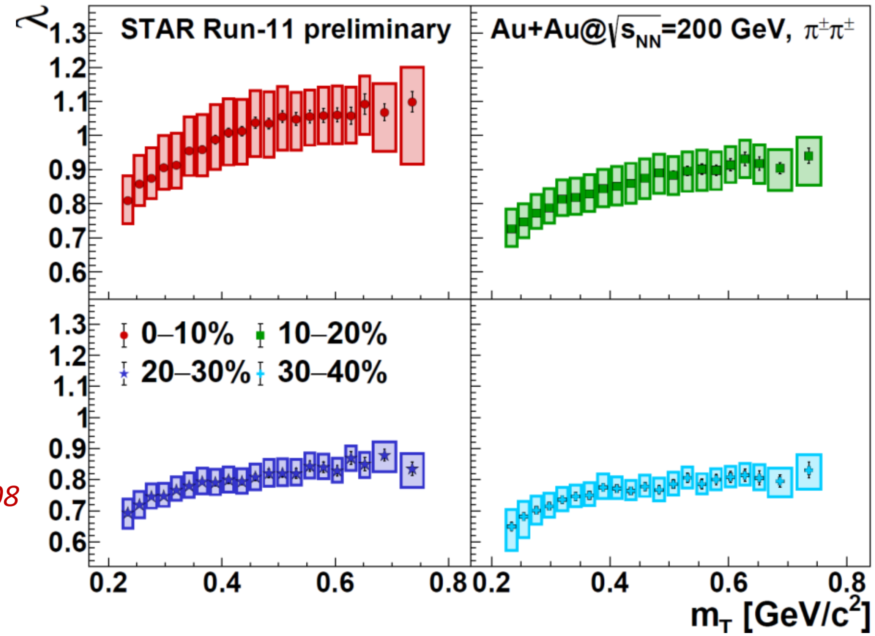
$$\lambda = N_{core}^2/(N_{core} + N_{halo})^2$$

Csörgő, Lörstad, Zimányi, Z.Phys. C71 (1996) 491-497
Bolz et al, Phys.Rev. D47 (1993) 3860-3870;

- **For power-law sources, more complicated picture!**
- **Increase from low to high m_T**
  - More decay products at low m_T?
  - In-medium mass modification of η'?
  - Partially coherent particle emission?

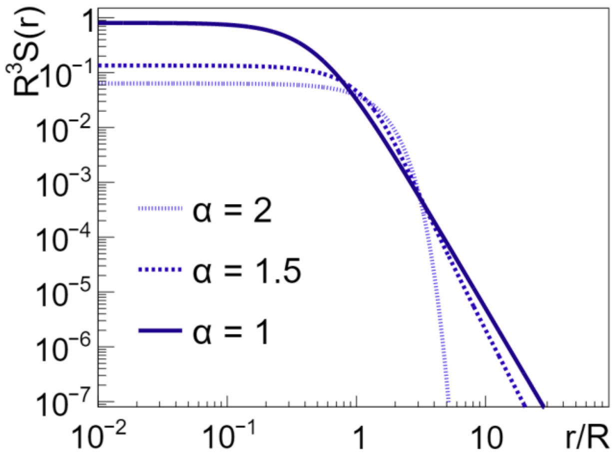
Vance, Csörgő, Kharzeev, Phys. Rev. Lett.81 (1998), pp. 2205–2208
Bolz et al., Phys. Rev. D47 (1993), pp. 3860–3870

- **Decrease from central to peripheral**



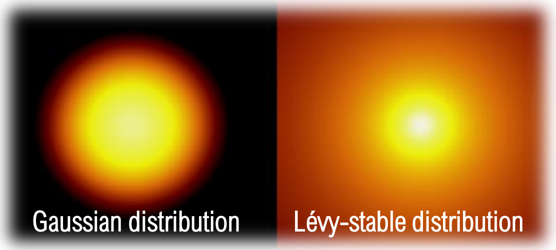
What is the shape of the source?
Gaussian & Lévy distributions in heavy-ion physics

- **Possible interpretations of the α Lévy exponent based on:**
  - Jet fragmentation *Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36*
  - Critical behavior *Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828*
  - Event averaging *Cimerman, Tomasik, Plumberg, Phys.Part.Nucl. 51 (2020) 3, 282*
  - Resonance decays *Kincses, Stefaniak, Csanád, Entropy 24 (2022) 3, 308*
  - Anomalous diffusion *Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002;*



$$S(\boldsymbol{r}) = \mathcal{L}(\alpha, R; \boldsymbol{r}) = \frac{1}{(2\pi)^3}\int d^3\boldsymbol{q}\, e^{i\boldsymbol{q}\boldsymbol{r}} e^{-\frac{1}{2}\left|\boldsymbol{q}^T\boldsymbol{R}^2\boldsymbol{q}\right|^{\alpha/2}}$$

spherical sym.: $R_{ij}^2 = R^2\delta_{ij}$



$$S(\boldsymbol{r}) = \mathcal{L}(\alpha, R; \boldsymbol{r}) = \frac{1}{(2\pi)^3}\int d^3\boldsymbol{q}\, e^{i\boldsymbol{q}\boldsymbol{r}} e^{-\frac{1}{2}\left|\boldsymbol{q}^T\boldsymbol{R}^2\boldsymbol{q}\right|^{\alpha/2}}$$

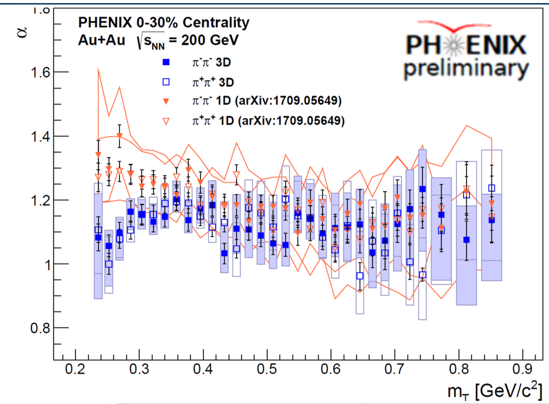
spherical sym.: $R_{ij}^2 = R^2\delta_{ij}$

Lévy source at RHIC energies not because of event averaging!

- **Not spherically sym. source: 3D vs. 1D α compatible! α < 2 in 1D analyses not because of angle averaging!**

  Kurgyis, Acta Phys. Pol. B Proc. Suppl. vol. 12 (2), 477 (2019)

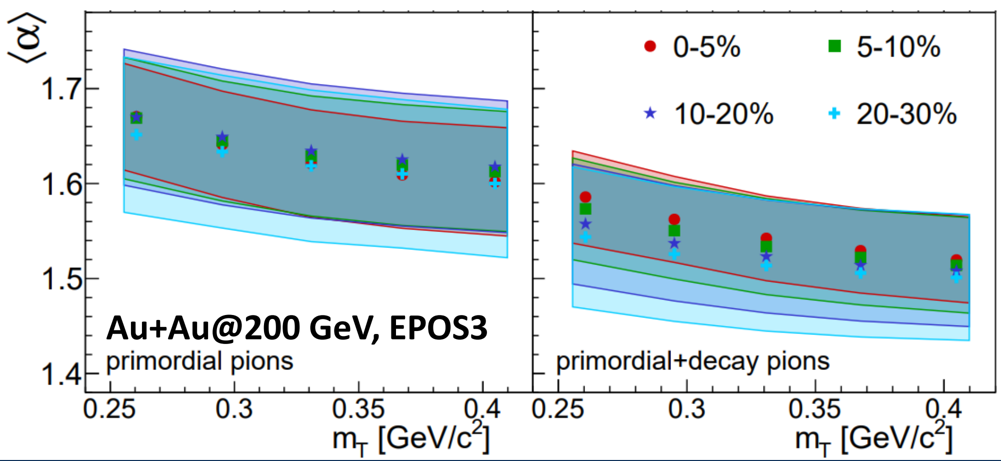
$$\boldsymbol{R}^2 = \begin{pmatrix} R_{out}^2 & 0 & 0 \\ 0 & R_{side}^2 & 0 \\ 0 & 0 & R_{long}^2 \end{pmatrix} \quad \boldsymbol{q} = \begin{pmatrix} q_{out} \\ q_{side} \\ q_{long} \end{pmatrix}$$

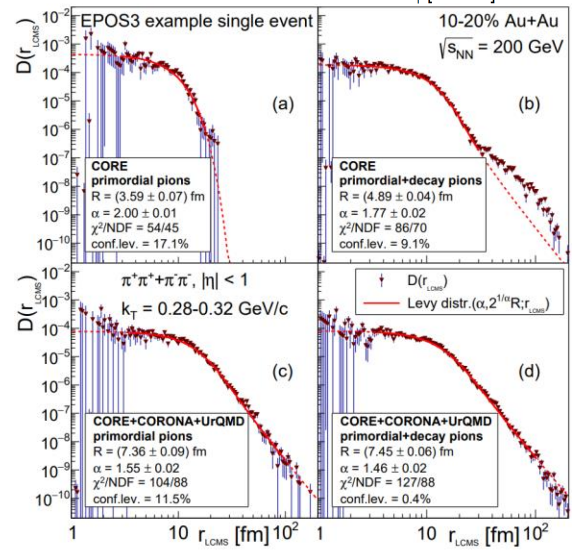


- **Event-by-event pion pair-source analysis at 200 GeV:**

  Decays and hadronic rescattering (UrQMD) play an important role!

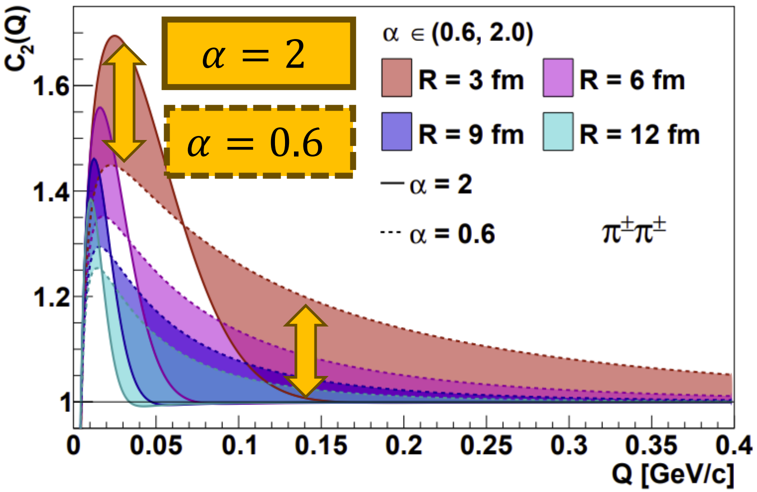
  Kincses, Stefaniak, Csanád, Entropy 24 (2022) 3, 308





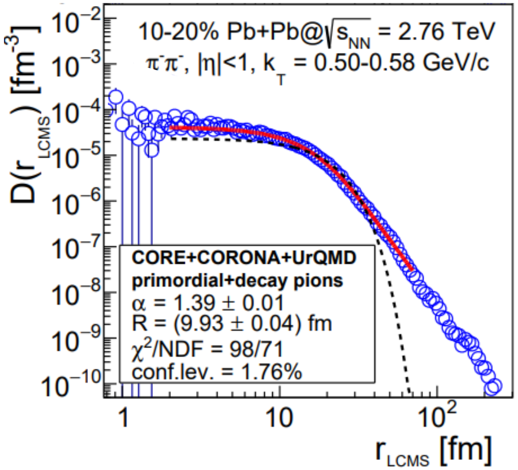
Recent phenomenological developments

- **Coulomb Corrections for Bose–Einstein Correlations from One- and Three-Dimensional Lévy-Type Source Functions** *Kurgyis, Kincses, Nagy, Csanád, Universe 9 (2024) 7, 328*
- **Event-by-Event Investigation of the Two-Particle Source Function in Heavy-Ion Collisions with EPOS** *Kincses, Stefaniak, Csanád, Entropy 24 (2022) 3, 308*
  Kórodi, Kincses, Csanád, Phys. Lett. B 847 (2024) 138295
- **A novel method for calculating Bose-Einstein correlation functions with Coulomb final-state interaction** *Nagy, Purzsa, Csanád, Kincses, Eur. Phys. J. C 83, 1015 (2024)*



EPOS, 2.76 TeV, $\alpha \sim 1.4$

Phys. Lett. B 847 (2024) 138295



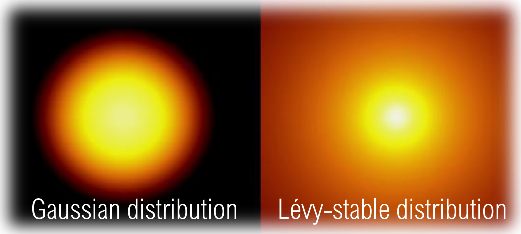
Properties of univariate stable distributions

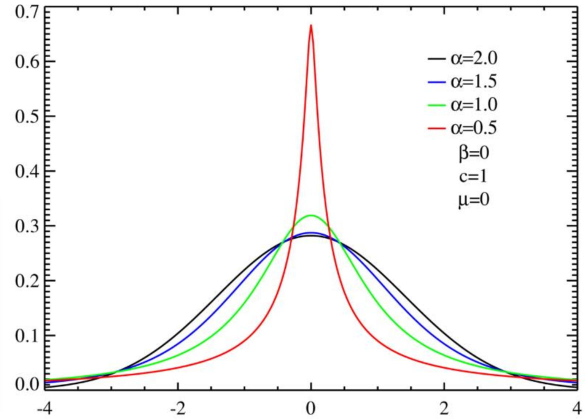
- **Univariate stable distribution:** $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\varphi(q)e^{-ixq}dq$, where the characteristic function:
- $\varphi(q;\alpha,\beta,R,\mu) = \exp(iq\mu - |qR|^{\alpha}(1 - i\beta\,\mathrm{sgn}(q)\Phi))$

$$\Phi = \begin{cases} \tan\left(\frac{\pi\alpha}{2}\right), \alpha \neq 1 \\ -\frac{2}{\pi}\log|q|, \alpha = 1 \end{cases}$$

- α: index of stability
- β: skewness, symmetric if $\beta = 0$
- R: scale parameter
- μ: location, equals the median,

  if $\alpha > 1$: $\mu =$ mean





In 3D: $$\mathcal{L}(\boldsymbol{r};\alpha,R) = \frac{1}{(2\pi)^3}\int d^3\boldsymbol{q}\, e^{i\boldsymbol{q}\boldsymbol{r}} e^{-\frac{1}{2}|\boldsymbol{q}\mathbf{R}\boldsymbol{q}|^{\alpha/2}}$$

$$R^2_{\sigma\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & R^2_{\mathrm{out}} & 0 & 0 \\ 0 & 0 & R^2_{\mathrm{side}} & 0 \\ 0 & 0 & 0 & R^2_{\mathrm{long}} \end{pmatrix}$$

- **Important characteristics of stable distributions:**
  - Retains same α and β under convolution of random variables
  - Any moment greater than α isn't defined

Kinematic variables of the correlation function I.

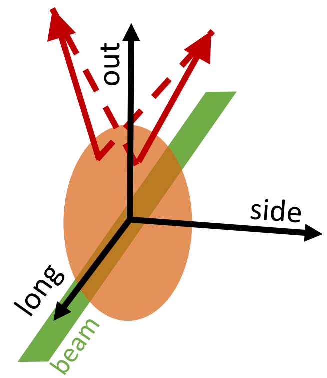
- Smoothness approximation ($p_1 \approx p_2 \approx K$): $S(x_1, K - q/2)\, S(x_2, K + q/2) \;\approx\; S(x_1, K)\, S(x_2, K)$
- $C_2(q, K) = \int d^4 r D(r, K) \left|\psi_q^{(2)}(\mathrm{r})\right|^2$
- Without any FSI $\left|\psi_q^{(2)}(\mathrm{r})\right|^2 = 1 + \cos(\mathrm{qr})$

$$\Bigg\} \; C_2^{(0)}(q, K) \simeq 1 + \frac{\widetilde{D}(q,K)}{\widetilde{D}(0,K)}, \text{ where } \widetilde{D}(q, K) = \int D(x, K) e^{iqx} d^4 x$$

- **HBT correlation function in direct connection with Fourier transform of the pair-source function**
- Important to determine the nature and dimensionality of the correlation function
- Lorentz-product of $q = (q_0, \boldsymbol{q})$ and $K = (K_0, \boldsymbol{K})$ is zero, i.e.: $qK = q_0 K_0 - \boldsymbol{qK} = 0$
- Energy component of q can be expressed as $q_0 = \boldsymbol{q} \frac{\boldsymbol{K}}{K_0}$
- If the energy of the particles are similar, K is approximately on shell
- **Correlation function can be measured as a function of three-momentum variables**

Kinematic variables of the correlation function II.

- $C_2(\boldsymbol{q}, \boldsymbol{K})$ as a function of three-momentum variables
- $\boldsymbol{K}$ dependence is smoother, $\boldsymbol{q}$ is the main kinematic variable
- Close to mid-rapidity one can use $k_T = \sqrt{K_x^2 + K_y^2}$, or $m_T = \sqrt{k_T^2 + m^2}$
- For any fixed value of m_T, the correlation function can be measured as a function of $\boldsymbol{q}$ only
- Usual decomposition: **out-side-long or Bertsch-Pratt (BP) coordinate-system**
  - $\boldsymbol{q} \equiv (q_{out}, q_{side}, q_{long})$
  - long: beam direction
  - out: k_T direction
  - side: orthogonal to the others
  - Essentially a rotation in the transverse plane
- Customary to use a Lorentz-boost in the long directon and change to the **Longitudnal Co-Moving System (LCMS)** where the average longitudinal momentum of the pair is zero



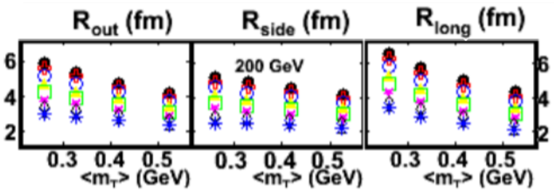
Kinematic variables of the correlation function III.

- Drawback of a 3D measurement: lack of statistics, difficulties of a precise shape analysis
- **What is the appropriate one-dimensional variable?**
- Lorentz-invariant relative momentum: $q_{inv} \equiv \sqrt{-q^\mu q_\mu} = \sqrt{q_x^2 + q_y^2 + q_z^2 - (E_1 - E_2)^2}$
- Equivalent to three-mom. diff. in Pair Co-Moving System (PCMS), where $E_1 = E_2$: $q_{inv} = |\boldsymbol{q}_{PCMS}|$
- In LCMS using BP variables: $q_{inv} = \sqrt{(1-\beta_T)^2 q_{out}^2 + q_{side}^2 + q_{long}^2}$ $\beta_T = 2k_T/(E_1+E_2)$
- **Value of q_{inv} can be relatively small even when q_{out} is large!**
- Experimental indications: **in LCMS source is ≈ spherically symmetric**
- Correlation function boosted to PCMS will not be spherically symmetric
- Let us introduce the following variable invariant to Lorentz boosts in the beam direction:

$$Q \equiv |\boldsymbol{q}_{LCMS}| = \sqrt{(p_{1x}-p_{2x})^2 + (p_{1y}-p_{2y})^2 + q_{z,LCMS}^2},$$

$$\text{where } q_{z,LCMS}^2 = \frac{4(p_{1z}E_2 - p_{2z}E_1)^2}{(E_1+E_2)^2 - (p_{1z}+p_{2z})^2}.$$

STAR, Phys.Rev.C 92 (2015) 1, 014904



Kinematic variables of the correlation function IV.

- Nature of the 1D variable in experiment: check correlation function in two dimensions!

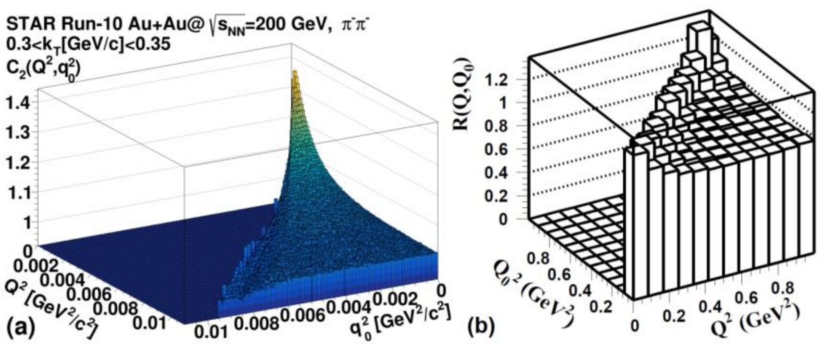


Figure 3.4: Example two-dimensional pion correlation functions for $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions (a) and $\sqrt{s} = 91$ GeV e^+e^- collisions (b). The latter figure is taken from the thesis of Tamás Novák [161].

Q dep. corr.func. $\boldsymbol{q_{inv}}$ dep. corr.func.

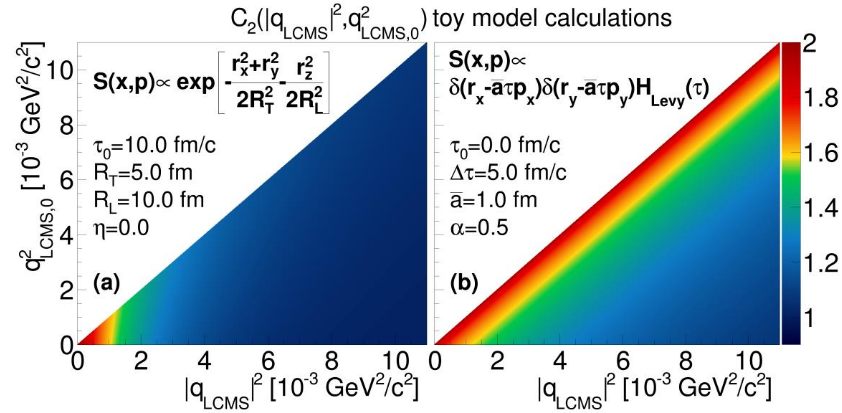
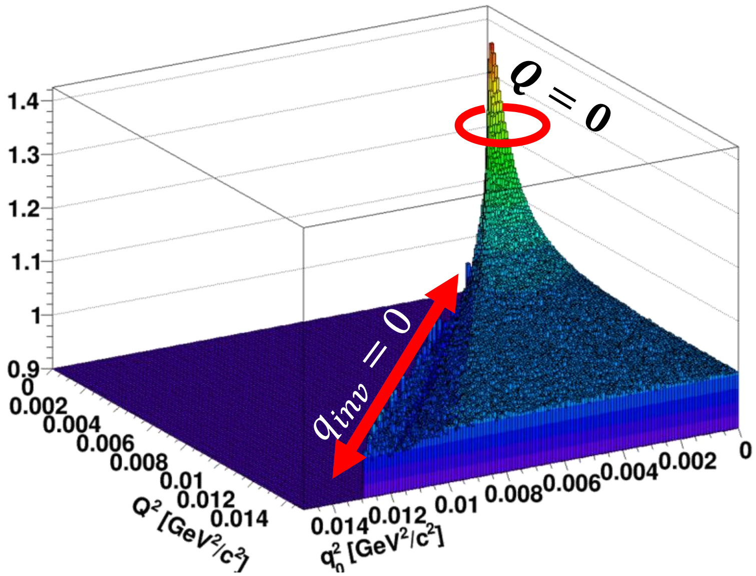


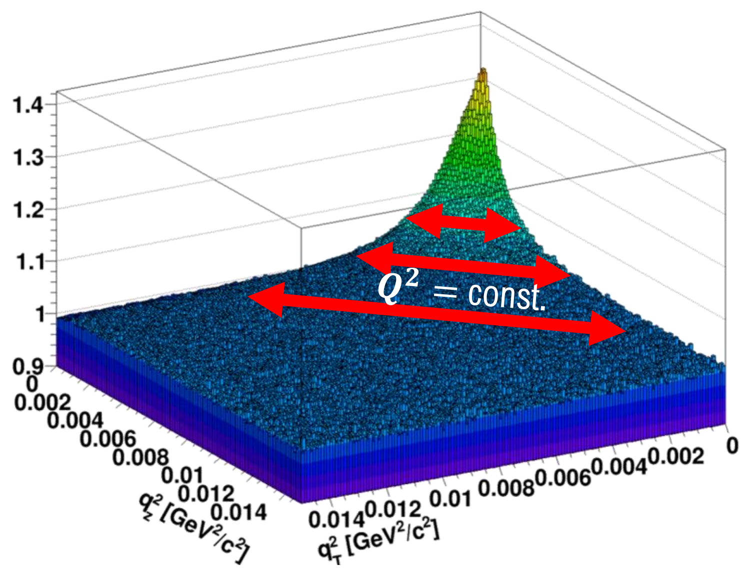
Figure 3.5: Toy model calculation for two different types of source functions. Taking a Gaussian source in both space and time leads to a correlation function that depends mostly on $|q_{LCMS}|$ (a), while a source that shows strong space-time and momentum space correlation leads to a q_{inv} dependent correlation function (b).

Kinematic variables of the correlation function V.

- Nature of the 1D variable in experiment: check correlation function in two dimensions!

$$Q = |\boldsymbol{q}_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + \left(p_{1y} - p_{2y}\right)^2 + q_{long,LCMS}^2}$$





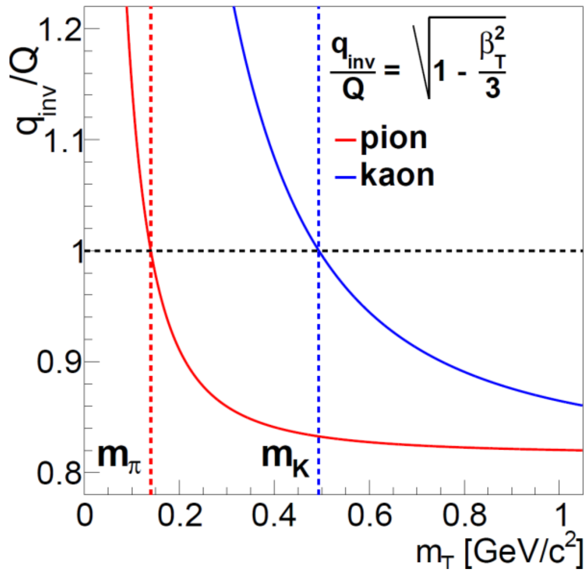
D. Kincses, Ph.D. thesis, 10.15476/ELTE.2022.164

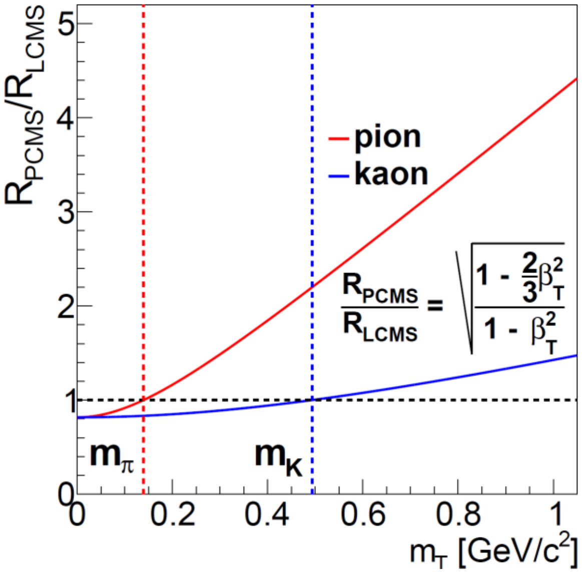
Kinematic variables of the correlation function VI.

- Correlation function measured in LCMS, Coulomb effect calculated in PCMS
- Approximation:

$$q_{\rm inv} \equiv q_{\rm PCMS} \approx q_{\rm LCMS} \cdot \sqrt{1 - \beta_T^2/3} \qquad R_{\rm PCMS} \approx R_{\rm LCMS} \cdot \sqrt{\frac{1 - 2\beta_T^2/3}{1 - \beta_T^2}}$$

- (Note $m_T < m$ not physical of course)





Coulomb correction and fitting of the corr. function

- Core-Halo model, Bowler-Sinyukov method: $C_2(Q,k_T) = 1 - \lambda + \lambda \int d^3\boldsymbol{r} D_{(c,c)}(\boldsymbol{r},k_T) \left|\Psi_Q^{(2)}(\boldsymbol{r})\right|^2$
- Neglecting FSI and using a Lévy-stable source function: $C_2^{(0)}(Q,k_T) = 1 + \lambda e^{-|RQ|^\alpha}$
- **Using numerical integral calculation as fit function results in numerically fluctuating χ^2 landscape**
- Treat FSI as correction factor: $K(Q,k_T) = \frac{C_2(Q,k_T)}{C_2^{(0)}(Q,k_T)}$
- An iterative method can be used: $C_2^{(fit)}(Q;\lambda,R,\alpha) = C_2^{(0)}(Q;\lambda,R,\alpha) \cdot K(Q;\lambda_0,R_0,\alpha_0)$
- Procedure continued until $\Delta_{\text{iteration}} = \sqrt{\frac{(\lambda_{\text{n}+1}-\lambda_{\text{n}})^2}{\lambda_{\text{n}}^2} + \frac{(R_{\text{n}+1}-R_{\text{n}})^2}{R_{\text{n}}^2} + \frac{(\alpha_{\text{n}+1}-\alpha_{\text{n}})^2}{\alpha_{\text{n}}^2}} < 0.01$
- **Iterations usually converge within 2-3 rounds, fit parameters can be reliably extracted**

Coulomb correction and fitting of the corr. function

- Lévy-type correlation function without final state effects: $C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^{\alpha}}$
- Bowler-Sinyukov method:

Intercept parameter (correlation strength)

Lévy scale parameter

Possible linear background (usually negligible)

$$C(Q_{\mathrm{LCMS}}; \lambda, R_{LCMS}, \alpha) = \left(1 - \lambda + \lambda \cdot K(q_{inv}; \alpha, R_{\mathrm{PCMS}}) \cdot (1 + e^{-|R_{\mathrm{LCMS}} Q_{\mathrm{LCMS}}|^{\alpha}})\right) \cdot N \cdot (1 + \varepsilon Q_{\mathrm{LCMS}})$$

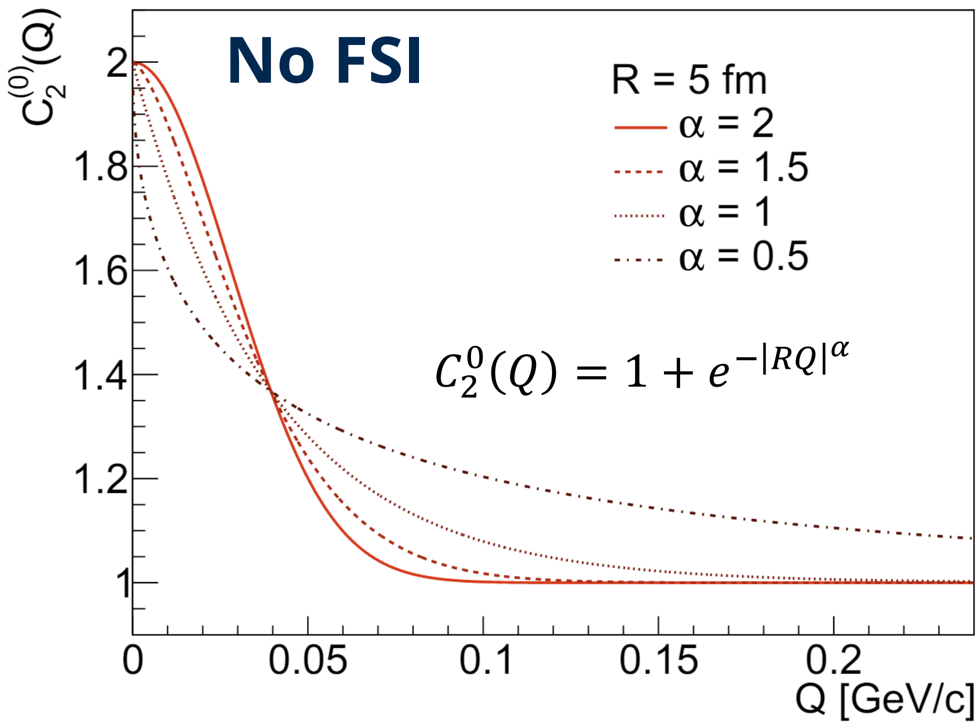
Coulomb correction

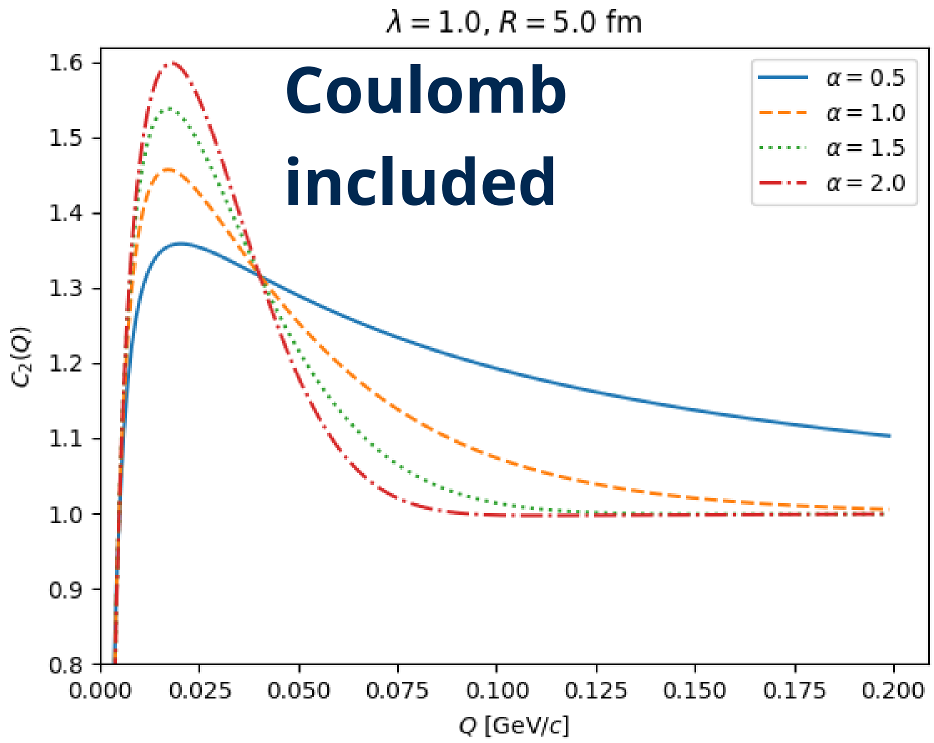
Lévy exponent

- Coulomb-correction calculated numerically (in PCMS)

$$q_{\mathrm{inv}} \equiv q_{\mathrm{PCMS}} \approx q_{\mathrm{LCMS}} \cdot \sqrt{1 - \beta_T^2/3} \qquad R_{\mathrm{PCMS}} \approx R_{\mathrm{LCMS}} \cdot \sqrt{\frac{1 - 2\beta_T^2/3}{1 - \beta_T^2}}$$

Shape of the correlation function

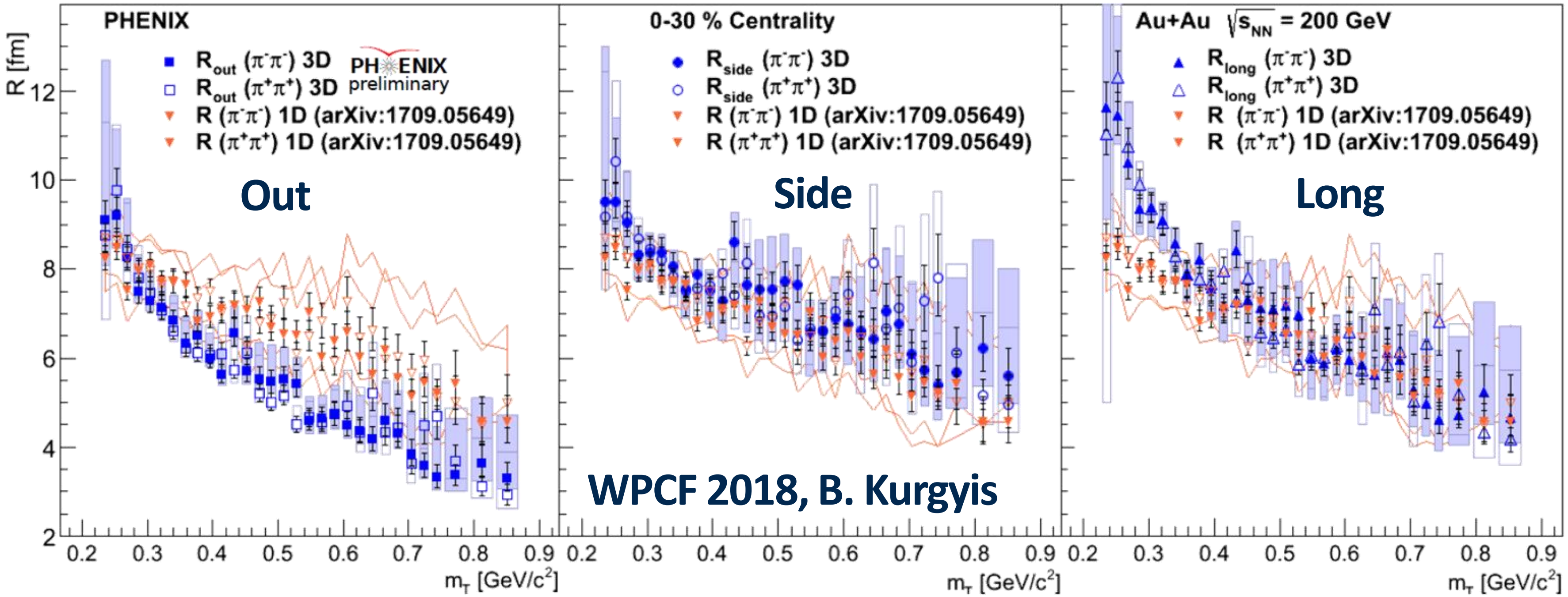




B. Kurgyis for the PHENIX Coll.,
Acta Phys. Pol. B Proc. Suppl. vol. 12 (2), pp. 477 - 482 (2019)

Cross-check with 3D analysis – PHENIX preliminary

$$C(Q) = \left(1 - \lambda + \lambda \cdot K(q_{inv}; \alpha, R_{inv}) \cdot \left(1 + e^{-\left|R_o^2 q_o^2 + R_s^2 q_s^2 + R_l^2 q_l^2\right|^{\alpha/2}}\right)\right) \cdot N \cdot (1 + \varepsilon Q)$$



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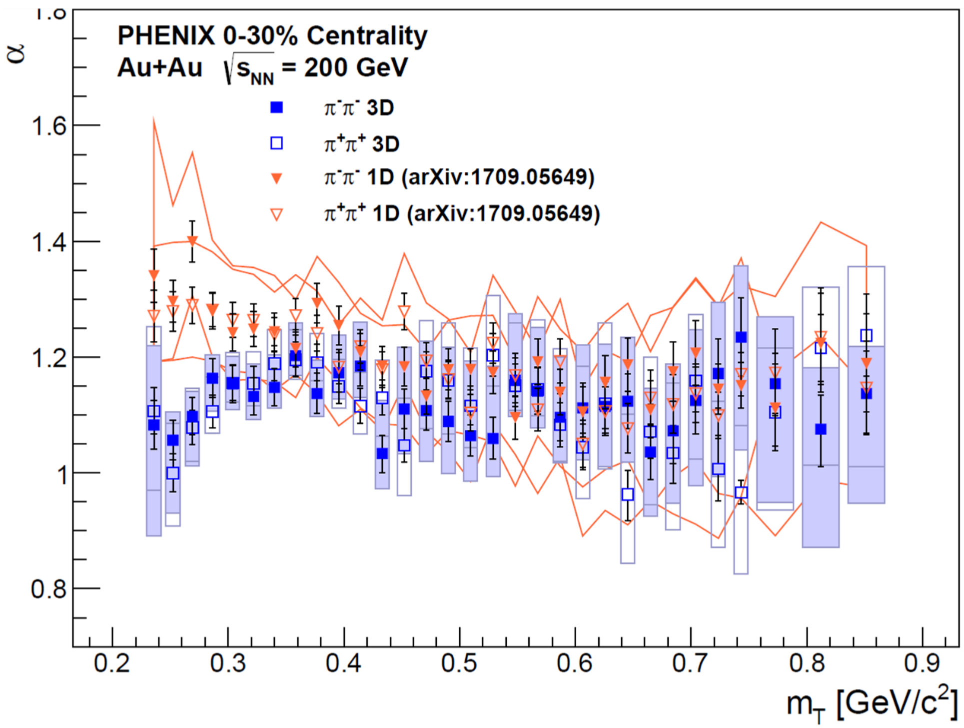

- **Compatible with 1D (Q_{LCMS}) measurement** of Phys. Rev. C 97, 064911 (2018)
- Small discrepancy at small m_T: due to large R_{long} at small m_T?

3D Gaussian vs 1D Levy

- **Angle averaged 3D Gaussian ≠ 1D Levy!**
  - Difference: several percent
  - Available experimental precision: much better than this difference