Shape analysis of HBT correlations at STAR

XIV WORKSHOP ON PARTICLE CORRELATIONS AND FEMTOSCOPY

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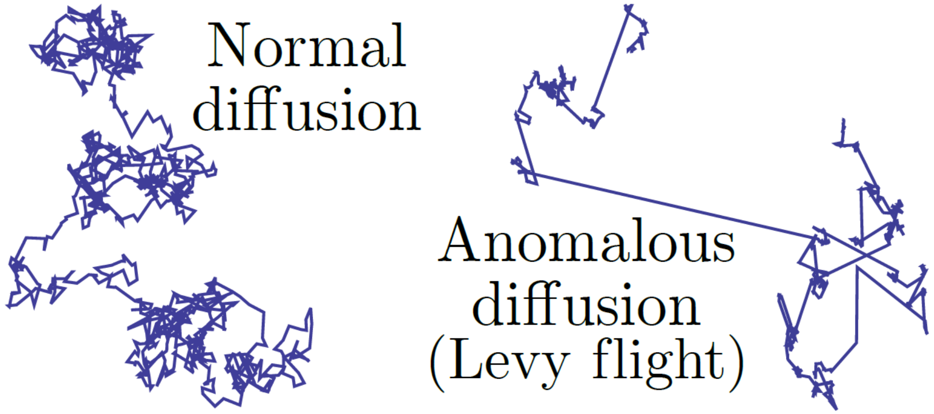
Introduction, motivation

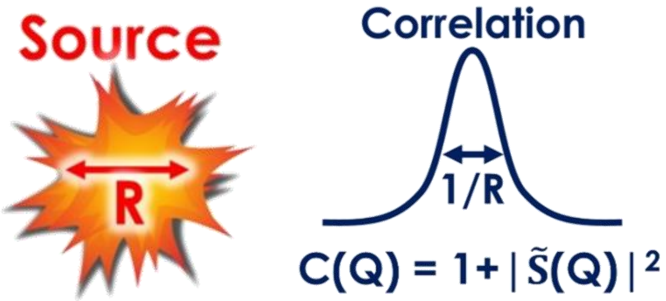
- THE HBT EFFECT AND LÉVY DISTRIBUTION
- LÉVY DISTRIBUTION AND THE CRITICAL POINT
- PARAMETERS OF A LÉVY-TYPE CORRELATION FUNCTION

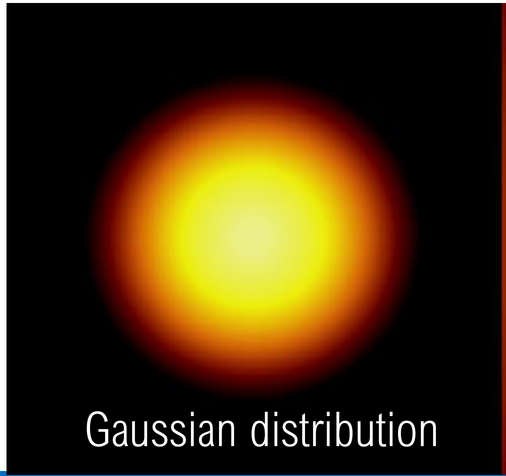
The HBT effect and the Lévy distribution

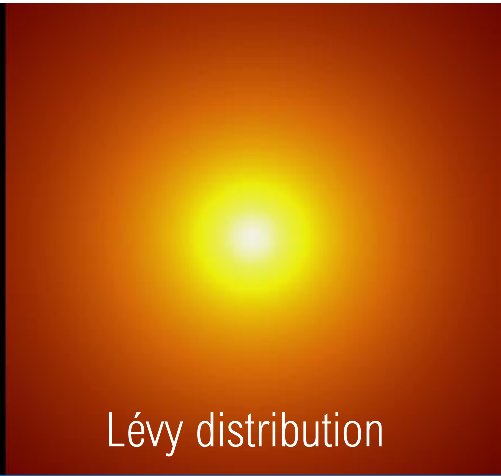
- Momentum correlations of identical pions
- Possible to map out the source on the fm scale
- Usually assumed source shape: Gaussian
- Generalization: **Lévy distribution**
- Lévy-type corr. func.:

$$C(Q) = 1 + \lambda \cdot e^{-(RQ)^{\alpha}}$$









Lévy distribution and the critical point

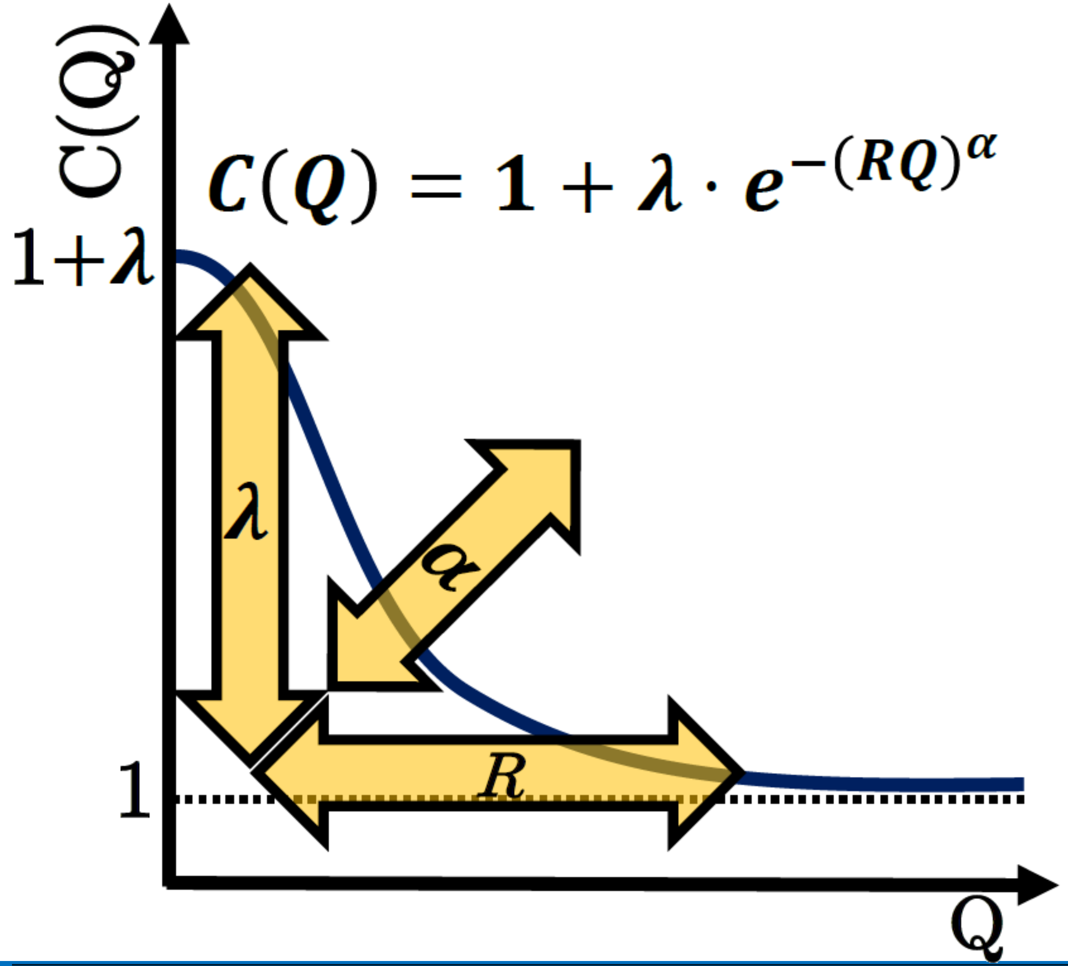
- **Lévy distribution**: $\mathcal{L}(\alpha,R,r) = \frac{1}{(2\pi)^3}\int d^3q\, e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$
- Critical behavior → critical exponents
- Spatial correlations at the CEP (in 3 dim.) $\propto r^{-1-\eta}$
- In case of Lévy source, spatial correlations $\propto r^{-1-\alpha}$
- QCD universality class → (rdf.) 3D Ising → $\boldsymbol{\eta \leq 0.5}$
- **Lévy-type parametrization**:

$$C(Q) = 1 + \lambda \cdot e^{-(RQ)^\alpha}$$

- **Related references**:
  - Eur.Phys.J. C36 (2004) 67-78
  - Braz.J.Phys.37:1002-1013,2007
  - Acta Phys.Polon. B36 (2005) 329-337
  - AIP Conf.Proc. 828 (2006) no.1, 525-532

Parameters of a Lévy-type correlation function

- **Correlation strength λ**
  - Intercept of the corr. func.
  - Core-Halo model: $\sqrt{\lambda} = N_C/(N_C + N_H)$
- **Lévy-scale R**
  - Physical size of the source
  - Usually decreases with m_T
- **Lévy-exponent α**
  - Connected to critical exponent η
  - Could be a possible signal of CEP



Non-Gaussian HBT papers from other experiments

- **PHENIX:** Phys.Rev. C97 (2018) no.6, 064911 (Au+Au at 200 GeV, 0-30%, pions, Levy fits)
  + several different preliminary results
- **L3:** Eur.Phys.J. C71 (2011) 1648 (e^+e^-, Levy fits)
- **CMS:** Phys.Rev. C97 (2018) no.6, 064912 (pp, pPb, PbPb, $\alpha = 1$, exponential fits)

} Final

- **NA61:** arXiv:1811.05262 (Be+Be at 150A GeV/c, Levy fits)
- **LHCb:** Nucl.Phys. A982 (2019) 347-350 (pp at 7 TeV, 8 TeV, $\alpha = 1$, exponential fits)

} Preliminary

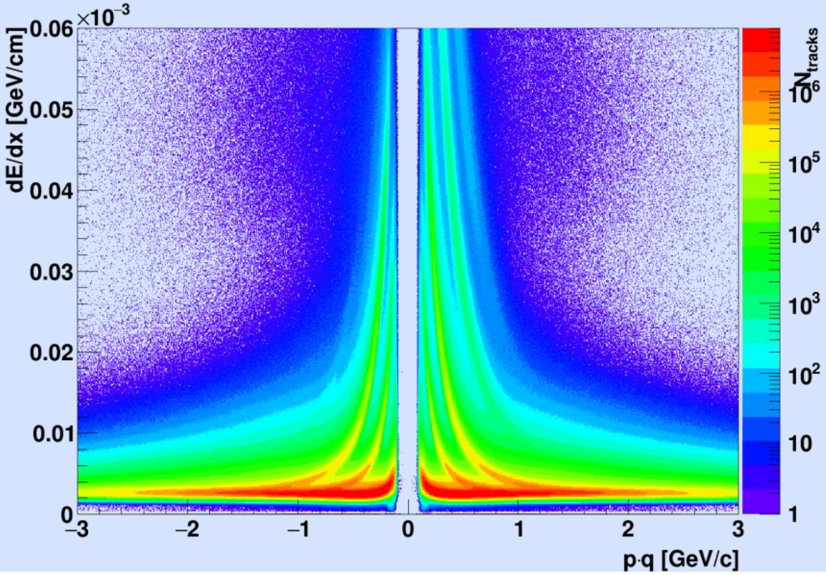
- Lévy fits **may help in CEP search** ($\alpha(\sqrt{s_{NN}})$) and can provide **better description of experimental data**

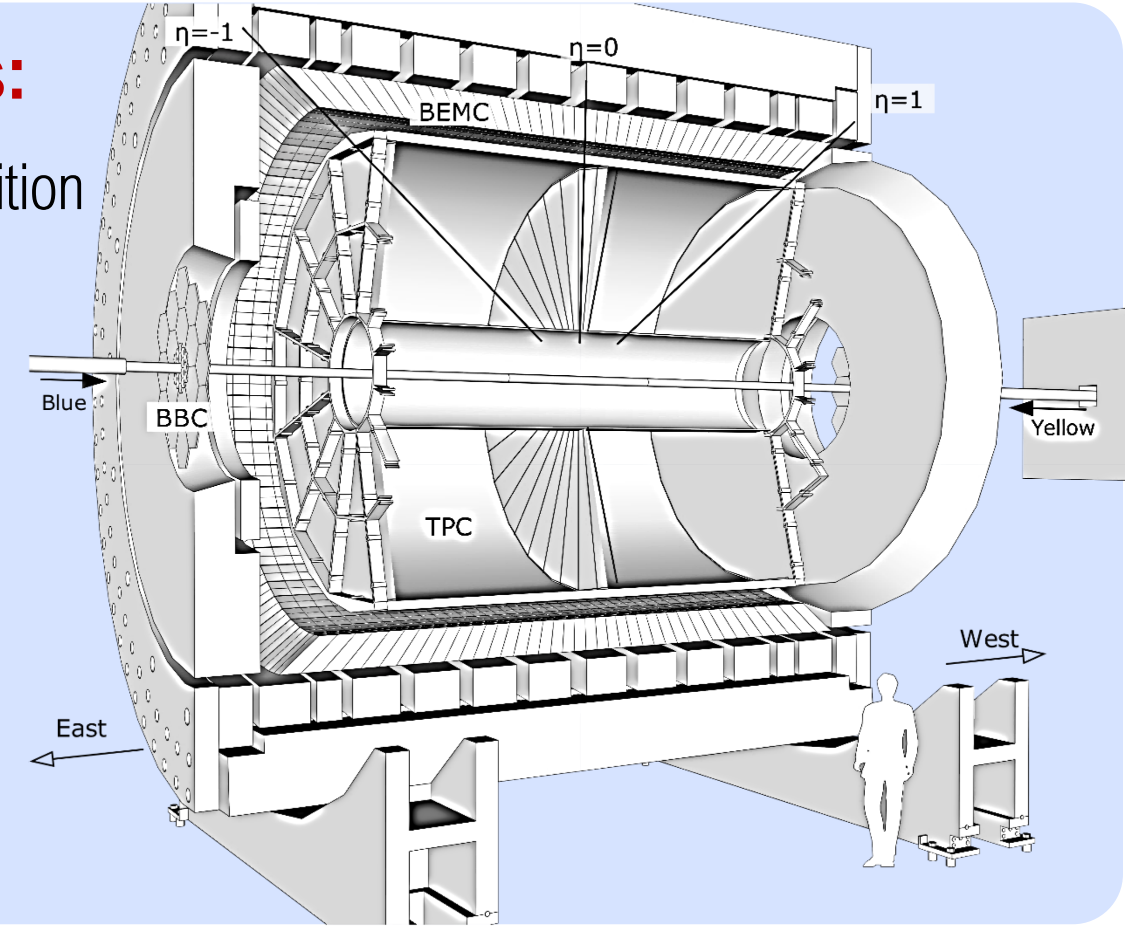
Experimental setup, measurement of the correlation functions

- THE STAR DETECTOR
- SUMMARY OF THE ANALYSIS CUTS
- KINEMATIC VARIABLES OF THE CORRELATION FUNCTIONS

The STAR detector

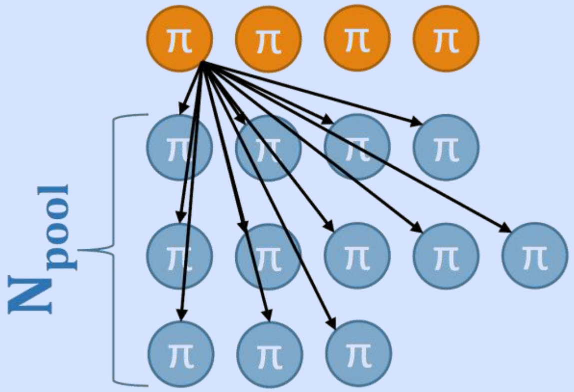
- **Detectors used for the analysis:**
  - **BBC, ZDC, VPD:** centrality, vertex position
  - **TPC:** tracking, dE/dx PID





Measurement of the correlation functions

- Analyzing data from **200 GeV Au+Au, Run-10**
- Measurements of **1D two-pion HBT correlation functions**
- **Event mixing** is done with the conventional method
  - 2 cm wide z vertex bins, 5% wide centrality bins



<table>
<tr><th>Event cuts</th><th>Single track cuts</th><th>Pair cuts</th></tr>
<tr><td>Vertex position</td><td>PID</td><td rowspan="2">Splitting level</td></tr>
<tr><td rowspan="2">Charged particle multiplicity</td><td>Number of TPC hits</td></tr>
<tr><td>pT</td><td rowspan="2">Fraction of merged hits</td></tr>
<tr><td>Centrality</td><td>Distance of Closest Approach</td></tr>
</table>

Event and track cuts

- **Vertex position cuts:**
  - vpd $|v_z|$ < 30 cm
  - TPC $|v_z|$ < 30 cm
  - |vpd v_z - TPC v_z| < 3 cm
  - v_r ($\sqrt{v_x^2 + v_y^2}$) < 2 cm
- **Charged particle multiplicity cut:**
  - TOF mult. < 7.8 · Ref mult. + 100
  - TOF mult. > 3.57 · Ref mult. $- 71.43$
- **Centrality cut:**
  - 0-30%

- **PID cut:**
  - N_σ (π) < 2, N_σ (K, p, e) > 2
- **Number of TPC hits cut:**
  - N_{hits} > 15
- **pT cut:**
  - 0.15 GeV < p_T < 1.0 GeV
- **Distance of Closest Approach cut:**
  - DCA global < 3 cm

Pair cuts

J. Adams et al. (STAR Collaboration), Phys. Rev. C 71, 044906 (2005)

- **Splitting level < 0.6**

$$SL \equiv \frac{\sum_i S_i}{N_{hits,1} + N_{hits,2}},$$

$$S_i = \begin{cases} -1 & \textit{if one track leaves a hit on pad row} \\ +1 & \textit{if both tracks leave a hit on pad row} \\ 0 & \textit{if neither track leaves a hit on pad row} \end{cases}$$

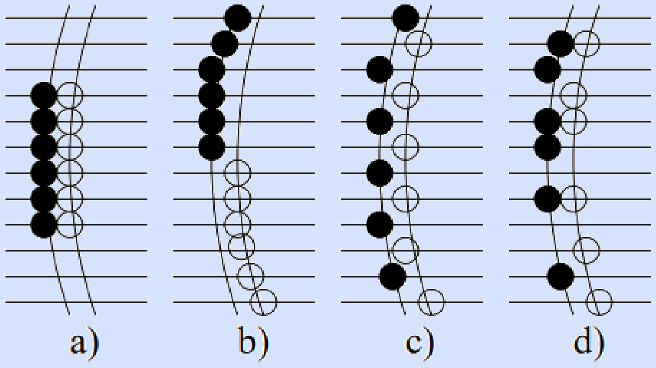


FIG. 1: Distribution of same number of hits in two tracks for four possible cases. Closed circles are hits assigned to one track, open circles are assigned to the other. a) SL = –0.5 (clearly two tracks) b) SL = 1 (possible split track) c) SL = 1 (possible split track) d) SL = 0.08 (likely two tracks).

- **Fraction of merged hits < 0.1**
  - For each pair, the fraction of hits that are close enough so they would appear merged is computed

Variable choice and Coulomb effect

- KINEMATIC VARIABLES OF THE CORRELATION FUNCTION
- LÉVY FITS AND THE COULOMB EFFECT
- SHAPE OF THE CORRELATION FUNCTIONS WITH COULOMB EFFECT INCLUDED

Kinematic variables of the correlation function

- Usually used one-dimensional variable (with the Bertsch-Pratt variables in the LCMS frame):

$$\boldsymbol{q_{inv}} = \sqrt{(\boldsymbol{1}-\boldsymbol{\beta_T^2})\boldsymbol{q_{out}^2}+\boldsymbol{q_{side}^2}+\boldsymbol{q_{long}^2}}, \qquad \beta_T = 2k_T/(E_1+E_2)$$

- If β_T is close to 1 (intermediate-high k_T) q_{inv} can be small even if q_{out} is not
- Radius extracted from q_{inv} dependent two pion HBT correlations (in Au+Au) overestimates the 3D LCMS ($R_{out}, R_{side}, R_{long}$) results (see e.g. the thesis of A. Enokizono) *http://inspirehep.net/record/673843/*
- Another approach: **LCMS three-momentum difference:**

$$Q = |\boldsymbol{q}_{LCMS}| = \sqrt{(p_{1x}-p_{2x})^2+\left(p_{1y}-p_{2y}\right)^2+q_{long,LCMS}^2}$$

Lévy fits and the Coulomb effect

- **Single particle distribution:** $N_1(p) = \int dx\, S(x,p)$
- **Pair momentum distribution:** $N_2(p_1,p_2) = \int dx_1 dx_2 S(x_1,p_1)S(x_2,p_2)|\psi(x_1,x_2)|^2$
- **Correlation function:** $C(p_1,p_2) = \frac{N_2(p_1,p_2)}{N_1(p_1)N_2(p_2)}$
- **Pair source/spatial correlation:**

$$D(r,K) = \int d^4\rho S\left(\rho + \frac{r}{2}, K\right) S\left(\rho - \frac{r}{2}, K\right)$$

$$C(Q,K) = \frac{\int D(r,K)\left|\psi_Q(\mathrm{r})\right|^2 dr}{\int D(r,K)dr}$$

relative coordinate (r); relative pair momentum (Q); average pair momentum (K); Pair wave function (ψ)

- **Core-Halo model:** $S = \sqrt{\lambda}\, S_C + (1-\sqrt{\lambda})S_H \xrightarrow{R_H\ large} C(Q) = 1 - \lambda + \lambda \cdot \frac{\int D_C(r)\left|\psi_Q(\mathrm{r})\right|^2 dr}{\int D_C(r)dr}$

Lévy fits and the Coulomb effect

- **Lévy parametrization without final state effects**: $C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^{\alpha}}$
- **Bowler-Sinyukov procedure:**

$$C(Q) = \left(1 - \lambda + \lambda \cdot K(Q;\alpha,R) \cdot \left(1 + e^{-|RQ|^{\alpha}}\right)\right) \cdot N \cdot (1 + \varepsilon Q)$$

Intercept parameter (correlation strength) → λ

Lévy exponent → α

Possible linear background (usually negligible) → $N \cdot (1 + \varepsilon Q)$

Coulomb correction → $K(Q;\alpha,R)$

Lévy scale parameter → R

- **Coulomb-correction:**

$$K(Q;\alpha,R) = \frac{\int D(r) \left|\psi^{Coul}(r)\right|^2 dr}{\int D(r) \left|\psi^{(0)}(r)\right|^2 dr} \rightarrow \text{calculated numerically}$$

Two-particle wave function (with the Coulomb interaction) → $\psi^{Coul}(r)$

Spatial correlations → $D(r)$

Two-particle wave function (plane wave) → $\psi^{(0)}(r)$

Lévy fits and the Coulomb effect

- **Coulomb-correction (calculated numerically):**

Confluent hypergeometric function

$$D(r) = \mathcal{L}\left(\alpha, 2^{\frac{1}{\alpha}}R, r\right)$$

$$\psi^{Coul}(\boldsymbol{r}) = \frac{1}{\sqrt{2}} \frac{\Gamma(1+i\eta)}{e^{\pi\eta/2}} \{e^{i\boldsymbol{qr}} F(-i\eta, 1, i(kr - \boldsymbol{qr})) + [\boldsymbol{r} \leftrightarrow -\boldsymbol{r}]\}$$

$$K(Q;\alpha,R) = \frac{\int D(r) \left|\psi^{Coul}(r)\right|^2 dr}{\int D(r) |\psi^{(0)}(r)|^2 dr}$$

$$\eta = \frac{\alpha_{EM} m_\pi c^2}{2\hbar q c}$$

3-dim. mom. diff. in pair rest frame ($\boldsymbol{q_{PCMS}}$)

$$q = Q/2$$

Plane wave

- **Two options for fitting:**
  - Numerically pre-calculated table for different Q, α, R values
  - Parametrizing the Coulomb-correction, using an empirical formula:

$$K(q;\alpha,R)^{-1} = F(q) \cdot K_{Gamow}^{-1}(q) \cdot K_{mod}^{-1}(q;\alpha,R) + \left(1 - F(q)\right) \cdot E(q)$$

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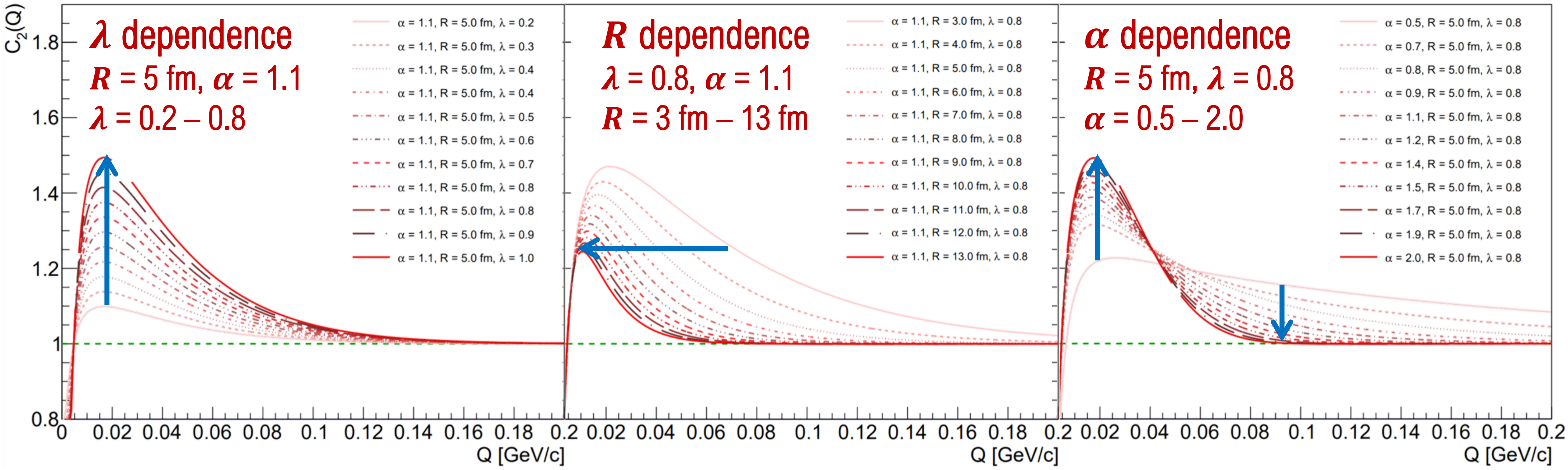
Shape of the correlation functions with Coulomb effect included

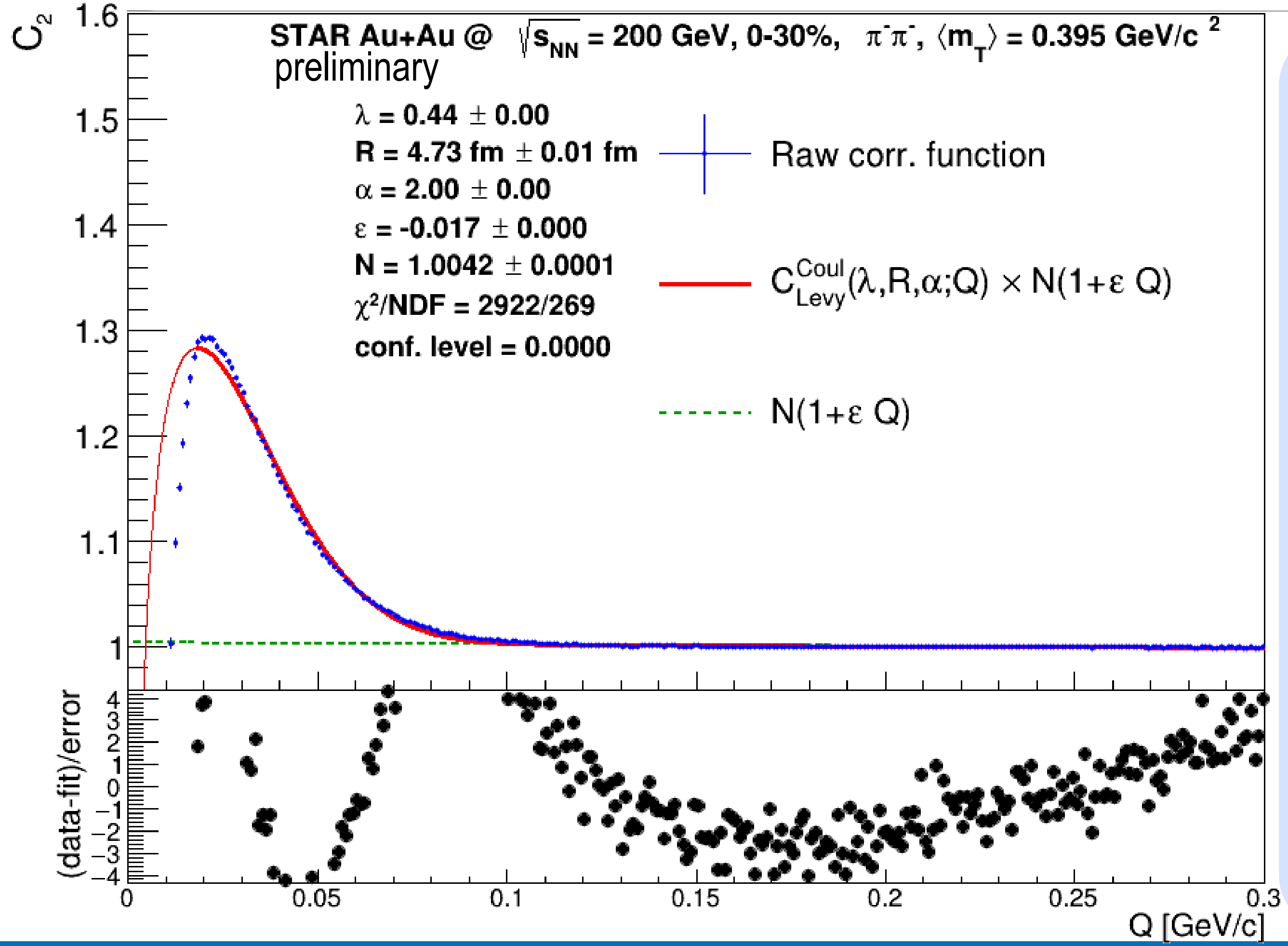
$$C(Q) = 1 - \lambda + \lambda \cdot K(Q;\alpha,R) \cdot \left(1 + e^{-|RQ|^{\alpha}}\right)$$



Fitting of the correlation functions

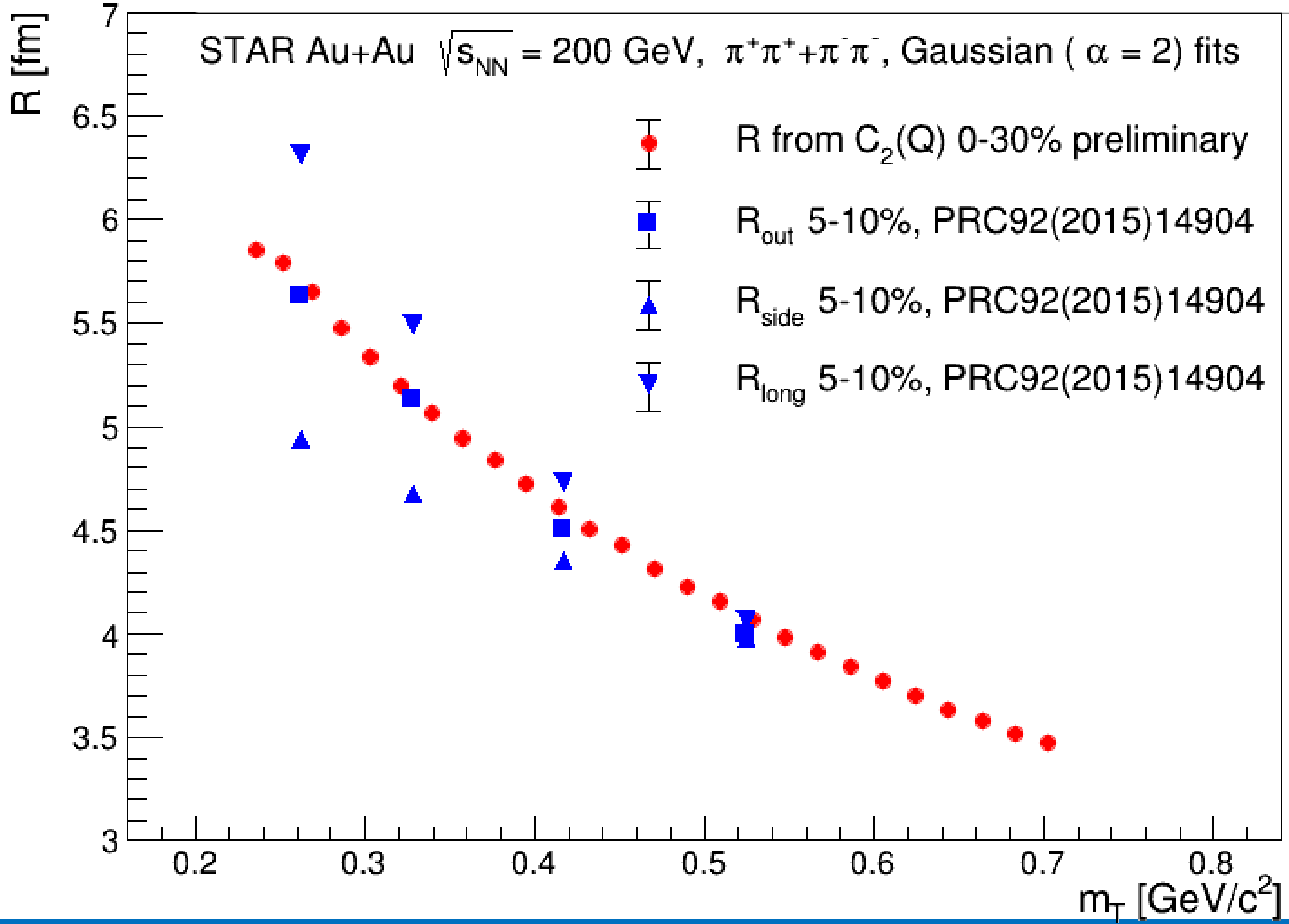
- **GOAL: DETAILED SHAPE ANALYSIS, EXTRACTING THE LEVY EXPONENT ALPHA**
- COMPARE GAUSSIAN FITS (FIXED ALPHA = 2) TO PREVIOUS PUBLISHED RESULTS
- INVESTIGATE LEVY FITS AND BEYOND

Gaussian (α = 2) fits – example fit



- **Gaussian fits: fixed α = 2**
  - ROOT Minuit2Minimizer framework
  - χ^2/NDF ~ 2500-3000/270
  - Fits are statistically unacceptable
  - R is compatible with previously measured 3D Gaussian radii ($R_{out}, R_{side}, R_{long}$)

Gaussian (α = 2) fits – comparing with published results



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  - Fits are statistically unacceptable
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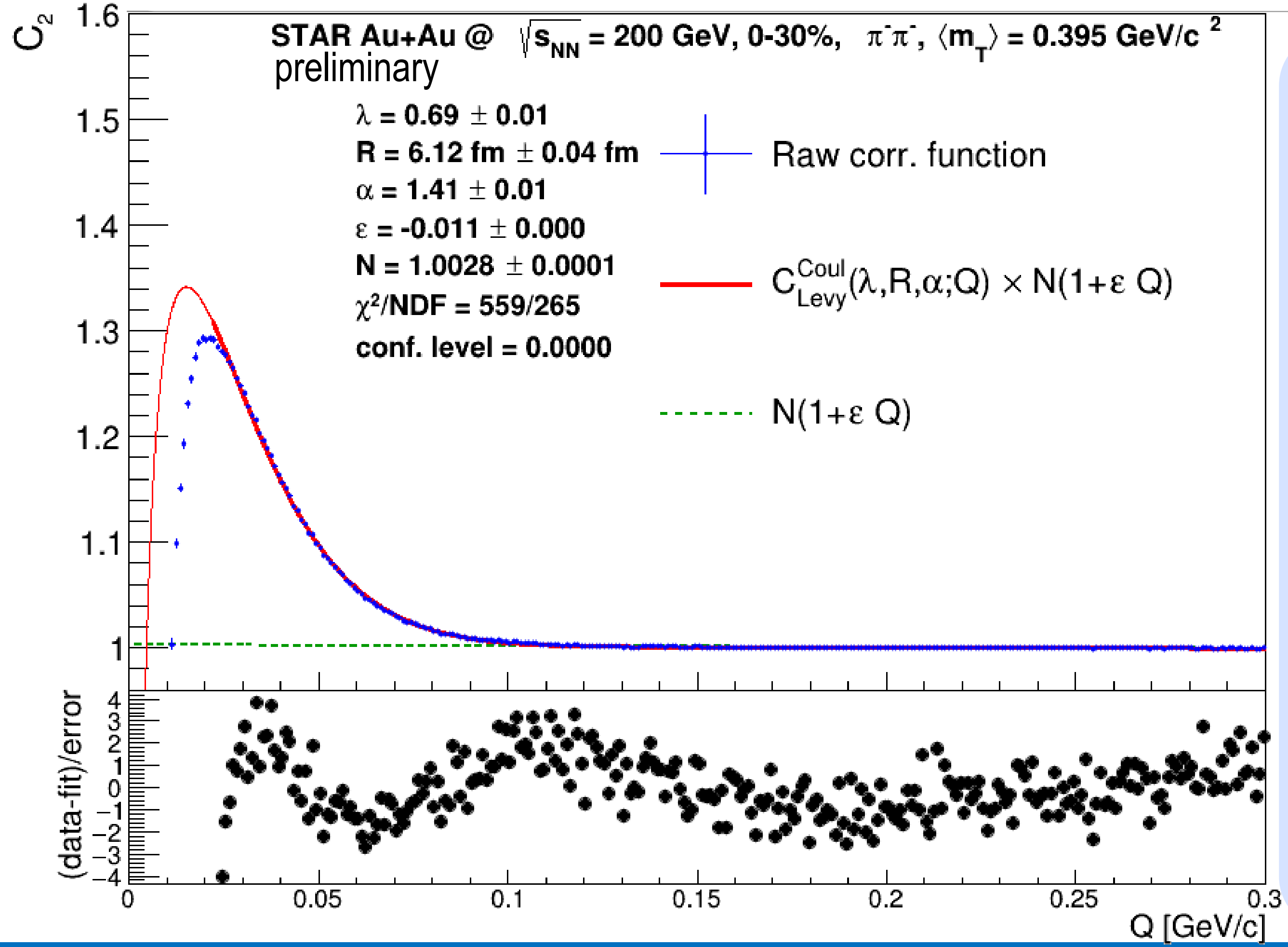
Levy fits (free α) – example fit



- **Levy fits: free α**
  - χ^2/NDF ~ 400-900/270
  - χ^2 drops by a factor of 3-5
  - Q ≳ 25 MeV can be described much better
  - Low Q behavior is not described by the fits (similar for all cuts, also for q_{inv})

Summary, outlook

- Ongoing two-pion HBT correlation analysis of data from Run-10, 200 GeV Au+Au
- Current status:
  - Lévy fits:
    χ^2/NDF ~ 1.5-3, low Q behavior is not clear, fits cannot describe the data at Q $\lesssim$ 25 MeV
  - Possible improvement (work in progress): Lévy-expansion fits
- Outlook:
  - Inclusion of **systematic uncertainties** in the fits,
    trying **different expansion methods**,
    investigating the details of $\boldsymbol{m_T}$, **centrality and** $\sqrt{\boldsymbol{s_{NN}}}$ **dependence**

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Thank you for your attention!

Backup slides

What shape could describe the data well?

- Issues with the simple Levy fit:
  - $\chi^2/\mathrm{NDF} \sim$ 1.5-3 (even with ignoring low Q)
  - Very low Q range cannot be described
  - Strong dependence on $\boldsymbol{Q_{min}}$

$$t = QR, \qquad \mu_{n,\alpha} = \frac{1}{\alpha}\Gamma\left(\frac{n+1}{\alpha}\right)$$

$$\mathrm{L_n}(\mathrm{t}|\alpha) = \det\begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \cdots & \mu_{n,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & & \mu_{n+1,\alpha} \\ \vdots & & \ddots & \vdots \\ \mu_{n-1,\alpha} & \mu_{n,\alpha} & \cdots & \mu_{2n-1,\alpha} \\ 1 & t & & t^n \end{pmatrix}$$

- Possible improvement (work in progress): Lévy-expansion:
  - $C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^\alpha} \cdot \left(1 + \sum_{\mathrm{n}=1}^{\infty} c_\mathrm{n} L_\mathrm{n}\right)$
  - $\boldsymbol{C(Q) = \left(1 - \lambda + \lambda \cdot K(Q;\alpha,R) \cdot \left(1 + e^{-|RQ|^\alpha} \cdot \left(1 + \sum_{n=1}^{\infty} c_n L_n\right)\right)\right) \cdot N \cdot (1 + \varepsilon Q)}$

M. B. De Kock, H. C. Eggers, T. Csörgő, PoS WPCF 2011 (2011) 033.

Parametrizing the Coulomb-correction, using an empirical formula

- **Parametrized Coulomb-correction:**

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$$F(q) = \frac{1}{1+\exp\left(\frac{q-q_0}{D_q}\right)}$$

$$K_{Gamow}(q) = \frac{2\pi\eta}{e^{2\pi\eta}-1}$$

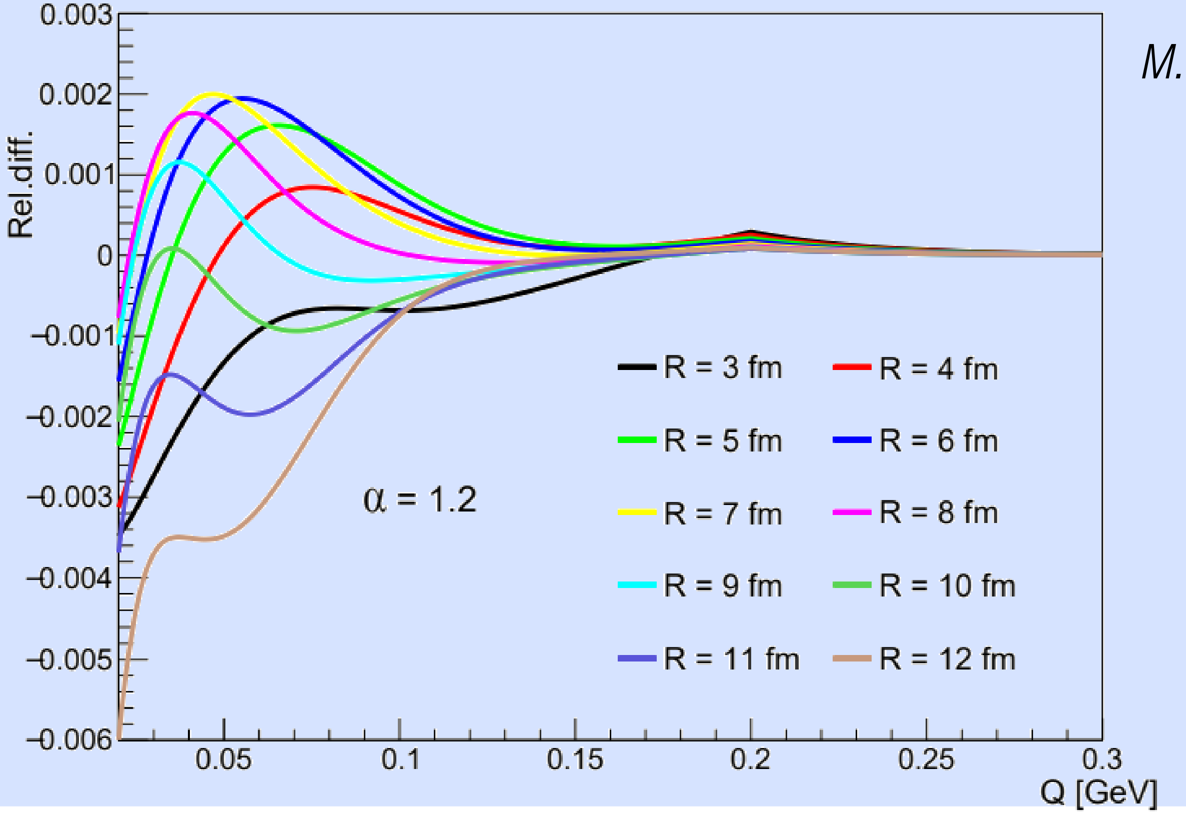
$$E(q) = 1 + A(\alpha,R)\exp(-B(\alpha,R)q)$$

$$K(q;\alpha,R)^{-1} = F(q)\cdot K_{Gamow}^{-1}(q)\cdot K_{mod}^{-1}(q;\alpha,R) + \big(1-F(q)\big)\cdot E(q)$$

$$K_{mod}^{-1}(q;\alpha,R) = 1 + \frac{A(\alpha,R)\frac{\alpha_{EM}\pi m_\pi R}{\alpha\hbar c}}{1 + B(\alpha,R)\frac{qR}{\alpha\hbar c} + C(\alpha,R)\left(\frac{qR}{\alpha\hbar c}\right)^2 + D(\alpha,R)\left(\frac{qR}{\alpha\hbar c}\right)^4}$$

Parametrizing the Coulomb-correction, using an empirical formula

- <u>**Relative deviation of the parametrization from the numerically calculated lookup table:**</u>



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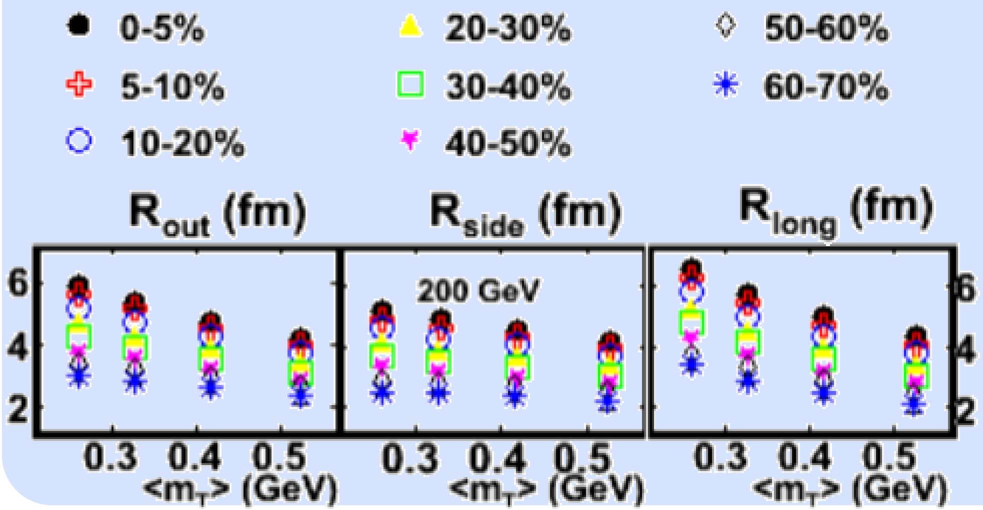
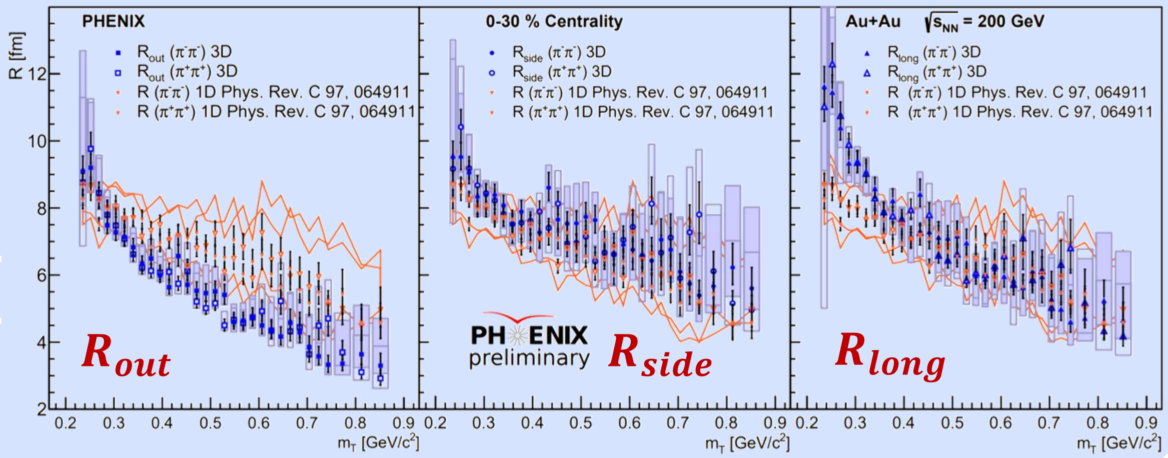
Kinematic variables of the correlation function

- Info from the tracks: E, p_x, p_y, p_z
- $\boldsymbol{q_{inv}} = \sqrt{(E_1 - E_2)^2 - \left((p_{1,x} - p_{2,x})^2 + (p_{1,y} - p_{2,y})^2 + (p_{1,y} - p_{2,y})^2\right)} = \sqrt{\boldsymbol{q^2_{0,LCMS} - Q^2_{LCMS}}}$
- $q_{0,LCMS} = \frac{(E_1^2 - E_2^2) - (p_{1,z}^2 - p_{2,z}^2)}{\sqrt{(E_1 + E_2)^2 - (p_{1,z} + p_{2,z})^2}}$
- $q_T = \sqrt{(p_{1,x} - p_{2,x})^2 + (p_{1,y} - p_{2,y})^2}$, $q_{z,LCMS} = \frac{2(p_{1,z}E_2 - p_{2,z}E_1)}{\sqrt{(E_1 + E_2)^2 - (p_{1,z} + p_{2,z})^2}}$
- $\boldsymbol{Q_{LCMS}} = \sqrt{\boldsymbol{q_T^2 + q_{z,LCMS}^2}}$

Kinematic variables of the correlation function

- $q_{inv} = |q_{PCMS}|$, in PCMS the source is not spherically symmetric (but in LCMS it is, approximately, see figs.)
- Measurements in Q_{LCMS} gives similar radii magnitude as three dimensional ($q_{out}, q_{side}, q_{long}$) meas.
- To compare λ and R with 3D (LCMS) results, |q| in the same frame (LCMS) should be used

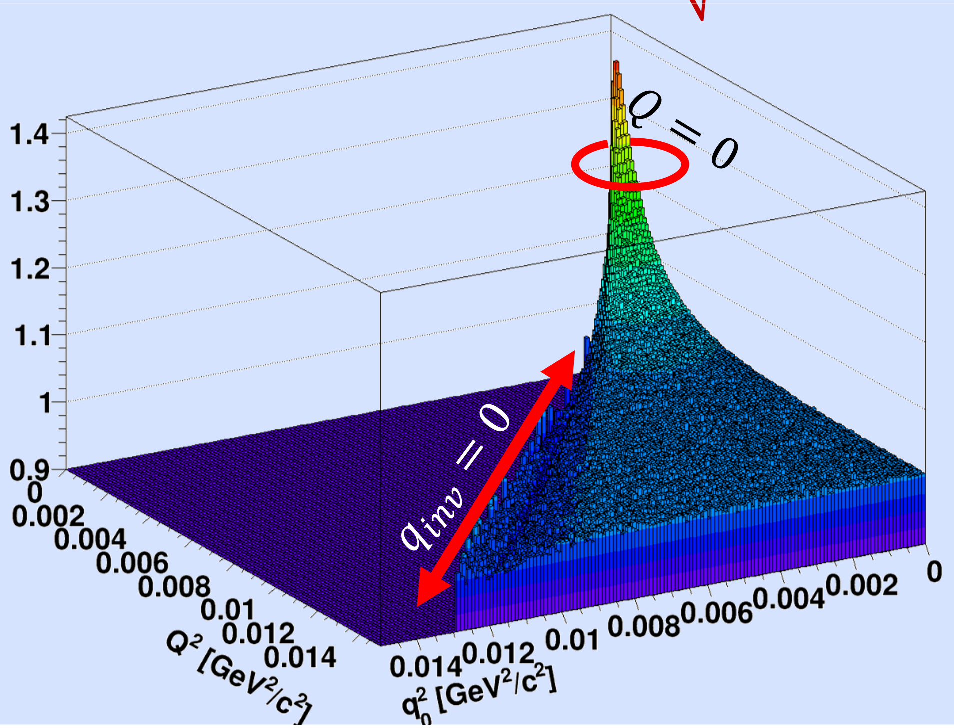


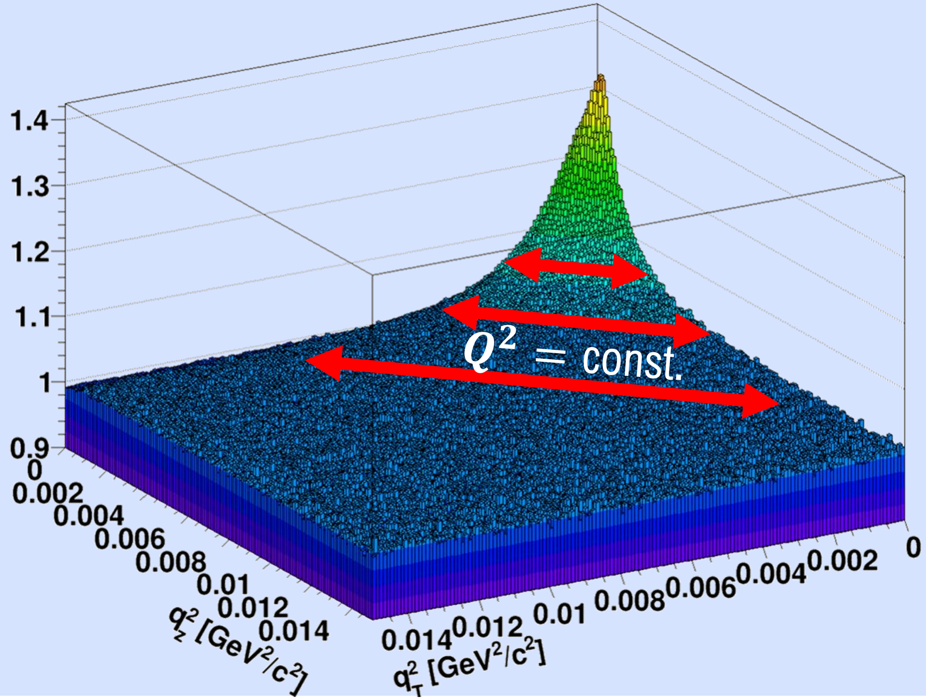


Kinematic variables of the correlation function

- A more appropriate one-dimensional variable: **LCMS three-momentum difference**

$$Q = |\boldsymbol{q}_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{long,LCMS}^2}$$

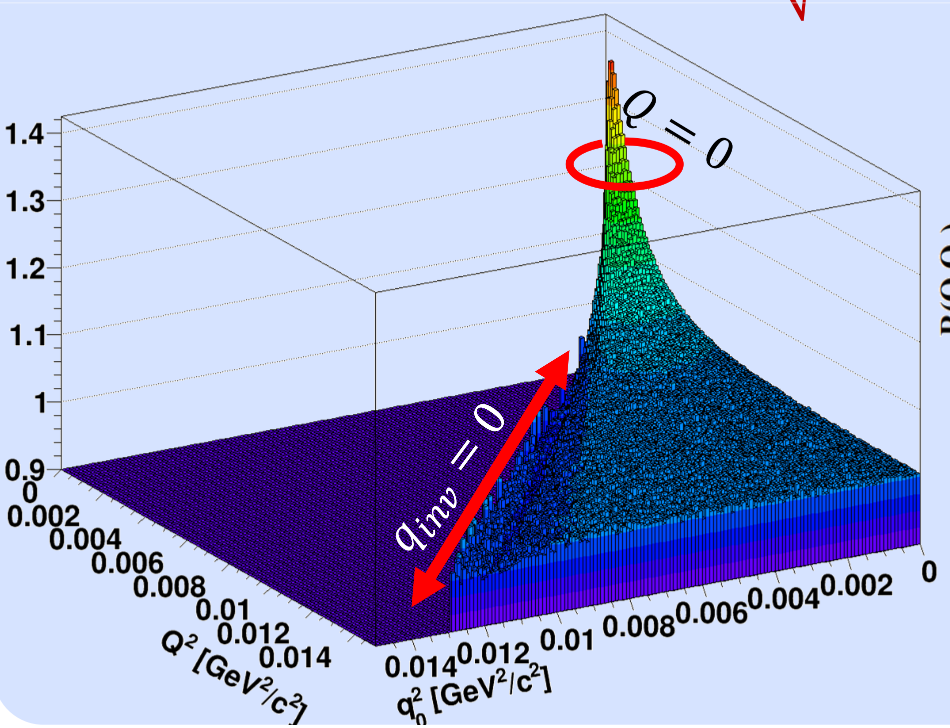




Kinematic variables of the correlation function

- A more appropriate one-dimensional variable: **LCMS three-momentum difference**

$$Q = |\boldsymbol{q}_{LCMS}| = \sqrt{(p_{1x}-p_{2x})^2 + (p_{1y}-p_{2y})^2 + q^2_{long,LCMS}}$$



$e^+e^-, \sqrt{s} = 91$ GeV, $\pi\pi$ correlations