

# Bose-Einstein correlations of charged kaons produced by $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions at the STAR experiment



ELTE  
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XV. WORKSHOP ON PARTICLE CORRELATIONS AND FEMTOSCOPY

**DÁNIEL KINCSES** FOR THE STAR COLLABORATION

EÖTVÖS UNIVERSITY, BUDAPEST

FACILITY FOR RARE ISOTOPE BEAMS,  
MICHIGAN STATE UNIVERSITY, JULY 2022



FRIB

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National Laboratory

# Part I: Introduction, motivation

- **Basic definitions of femtosopic correlation functions**
- **What is the shape of the source?**
- **Appearance of Lévy-type sources in heavy-ion collisions**

# Basic definitions of femtosopic correlation functions

- **Single particle distribution:**  $N_1(p) = \int dx S(x, p)$ 

phase-space density
- **Pair momentum distr.:**  $N_2(p_1, p_2) = \int dx_1 dx_2 S(x_1, p_1) S(x_2, p_2) |\psi(x_1, x_2)|^2$ 

Pair wave function, contains FSI

- **Correlation function:**  $C(p_1, p_2) = \frac{N_2(p_1, p_2)}{N_1(p_1) N_2(p_2)}$ 

relative coordinate

- **Pair source/spatial correlation:**  $D(r, K) = \int d^4 \rho S\left(\rho + \frac{r}{2}, K\right) S\left(\rho - \frac{r}{2}, K\right)$

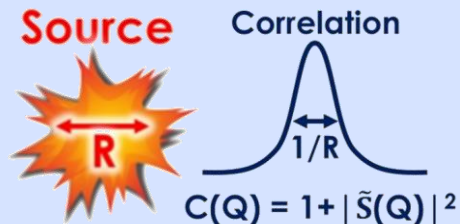
relative pair momentum

average pair momentum

Pair wave func., contains FSI

$$\psi_Q(r) = \frac{1}{\sqrt{2}} \frac{\Gamma(1+i\eta)}{e^{\pi\eta/2}} \{e^{iqr} F(-i\eta, 1, i(kr - qr)) + [r \leftrightarrow -r]\}$$

$$C(Q, K) = \frac{\int D(r, K) |\psi_Q(r)|^2 dr}{\int D(r, K) dr}$$



- **Experiments: measuring C(Q) to gain information about D(r)**

# What is the shape of the source?

## Gaussian & Lévy distributions in heavy ion physics

$$S(r, K) = \mathcal{L}(\alpha, R; r) = \frac{1}{(2\pi)^3} \int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$$

### • Experimental indications – power-law component in pion pair-source

$\alpha < 2$

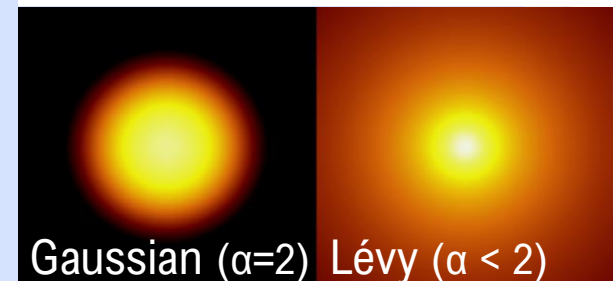
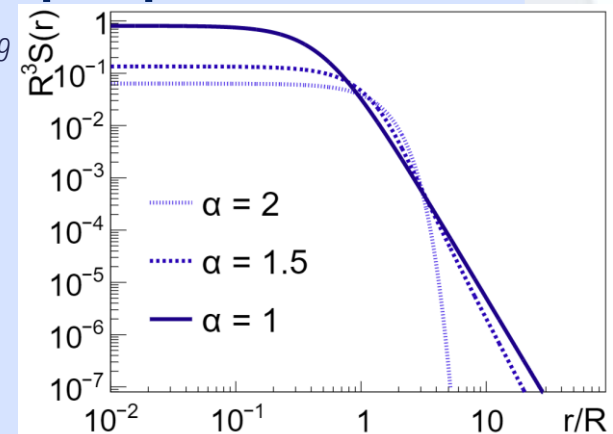
- STAR Au+Au@200 GeV *Phys.Part.Nucl. 51 (2020) 3, 267-269*
- PHENIX Au+Au@200 GeV *Phys.Rev. C97 (2018) no.6, 064911*
- NA61/SHINE Be+Be@150A GeV *Universe 5 (2019) 6, 154*
- CMS Pb+Pb@5.02 TeV *CMS-PAS-HIN-21-011*

### • Symmetric Lévy-stable distribution

- From generalized central limit theorem, power-law tail (if  $\alpha < 2$ )  $\sim r^{-(1+\alpha)}$
- $\alpha = 2$  Gaussian,  $\alpha = 1$  Cauchy
- Retains the same  $\alpha$  under convolution

$$S(r, K) = \mathcal{L}(\alpha, R; r) \Rightarrow D(r, K) = \mathcal{L}(\alpha, 2^{1/\alpha} R; r)$$

- $K$  dependence appear in source parameters



# Appearance of Lévy-type sources in heavy-ion collisions (Au+Au @ 200 GeV)

## • Possible (competing) reasons for the appearance of Lévy-type sources:

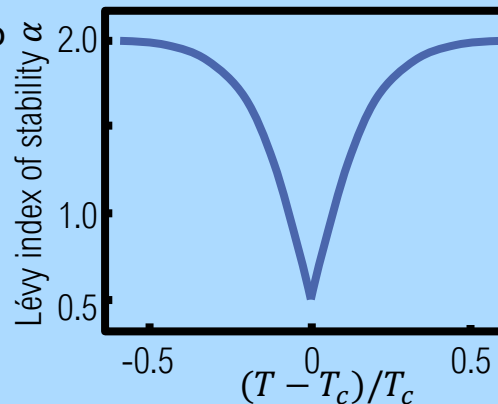
1. **Jet fragmentation**
2. **Proximity of the critical endpoint**
3. Event averaging (different shapes)?
4. Resonance decays?
5. Anomalous diffusion

### • Fractal phenomena in QCD jets

- Jets emitting jets, emitting jets, ...
- Csörgő, Hegyi, Novák, Zajc, *Acta Phys.Polon. B36*

### • 2nd order phase transitions: critical exponents

- Spatial correlations  $\sim r^{-d+2-\eta}$  at CEP
- Lévy source:  $\sim r^{-(1+\alpha)}$ ;  $\alpha \Leftrightarrow \eta?$
- QCD  $\leftrightarrow$  (random field) 3D Ising model
- $\eta = 0.50 \pm 0.05$
- Csörgő, Hegyi, Novák, Zajc, *AIP Conf.Proc. 828*;



**Not relevant for Au+Au @ 200 GeV**

# Appearance of Lévy-type sources in heavy-ion collisions (Au+Au @ 200 GeV)

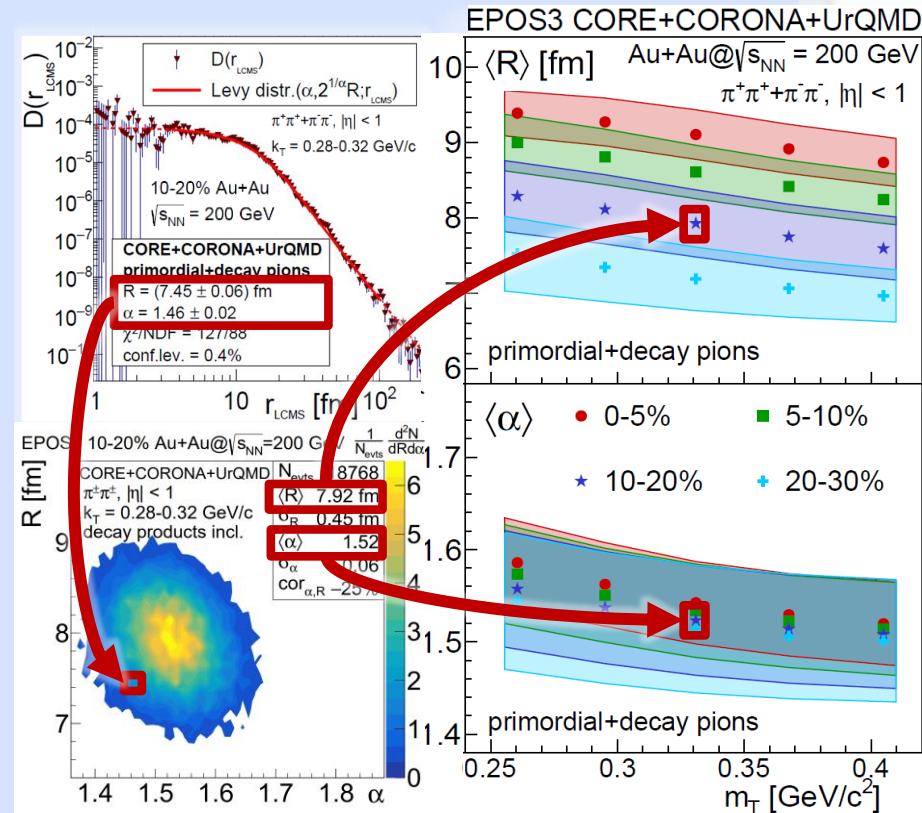
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1. Jet fragmentation
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## EPOS 200 GeV Au+Au collisions: Event-by-event non-Gaussianity!!!

- Single-event Lévy fits  $\rightarrow$  good description
- power-law tail strongly affected by rescattering, decays;  $2 > \alpha_{EPOS} > \alpha_{exp}$
- Lévy shape not from event averaging!

*D. Kincses, M. Stefaniak, M. Csanád, Entropy 24 (2022) 3, 308*



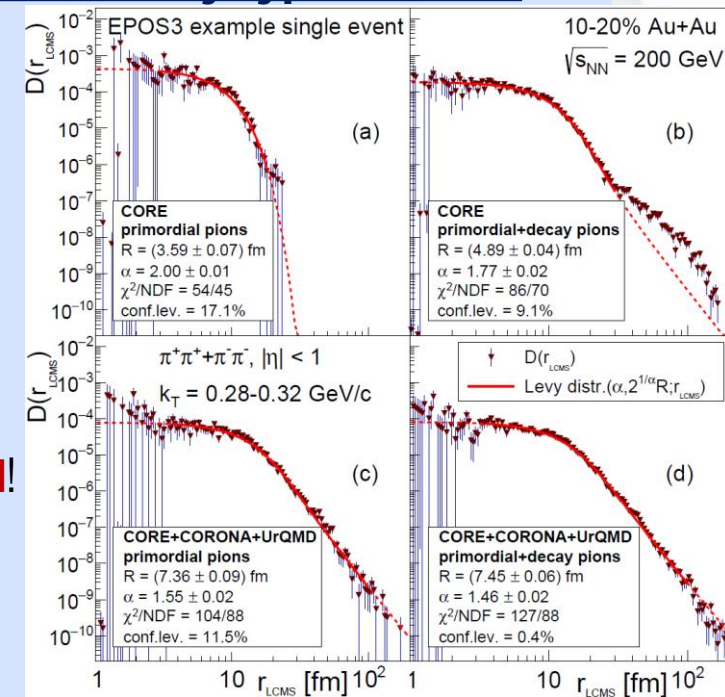
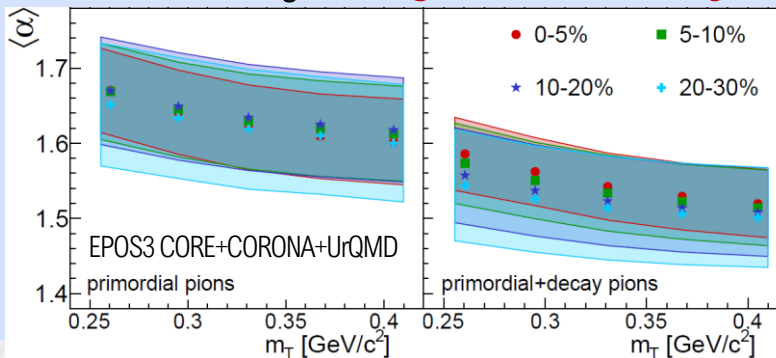
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## Decay pions create power-law-like structures

- Without rescattering not perfectly Lévy
- With rescattering: **stronger tail with decays included!**

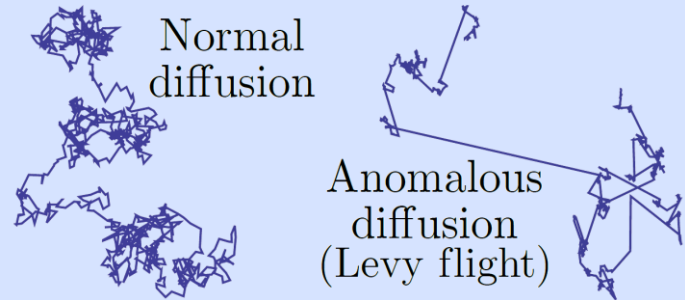


*D. Kincses, M. Stefaniak, M. Csanád, Entropy 24 (2022) 3, 308*

# Appearance of Lévy-type sources in heavy-ion collisions (Au+Au @ 200 GeV)

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1. Jet fragmentation
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5. **Anomalous diffusion**



## • Elastic rescattering of hadrons

- Expanding hadron gas  $\rightarrow$  time dependent increasing mean free path
- Hadronic Resonance Cascade (HRC) model
  - $\alpha$  depends on total inelastic cross-section
  - $\alpha_{\pi}^{HRC} > \alpha_K^{HRC}$  (smaller c.s.  $\rightarrow$  larger m.f.p.)

*Csanád, Csörgő, Nagy,  
Braz.J.Phys. 37 (2007) 1002;*

*T. J. Humanic, Int. J. Mod. Phys. E 15, 197 (2006)*

## • Kaon vs. pion measurements can test the anom.diff. picture

# Motivation for Lévy femtoscopy with kaons!

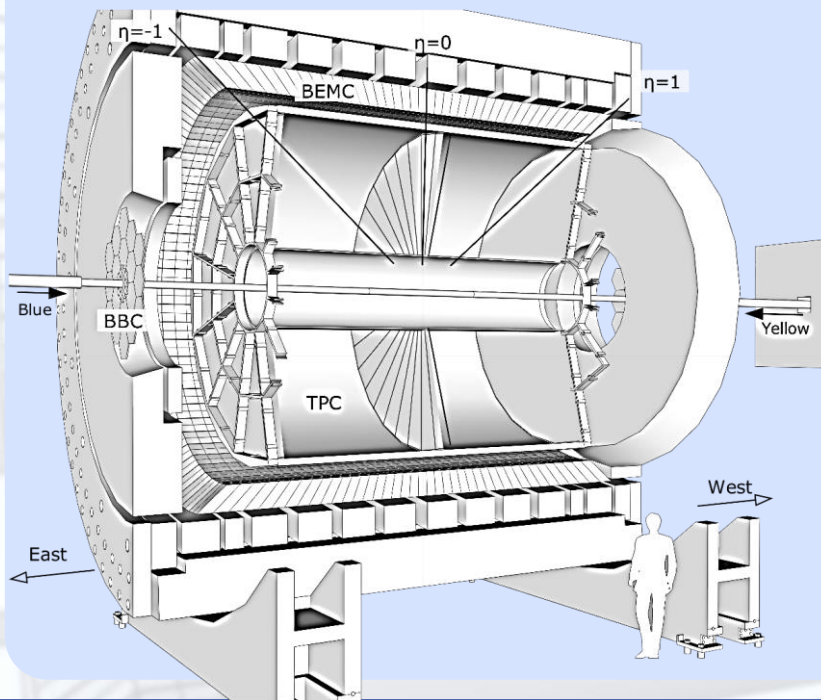


# Part II: Measurement and fitting of correlation functions, preliminary results

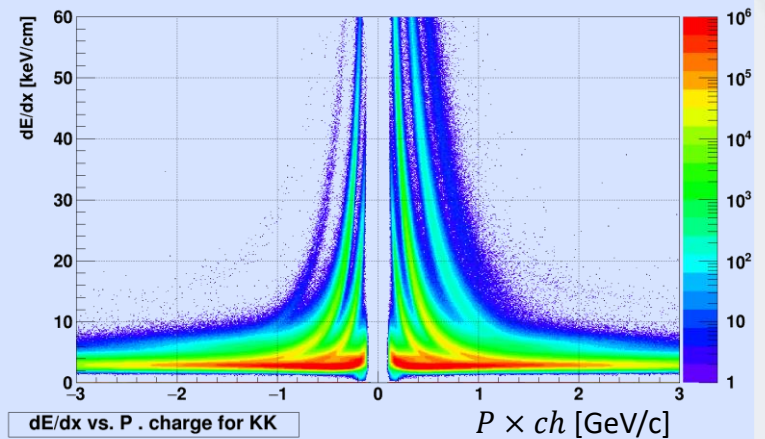
- **The STAR detector**
- **Measurement of the kaon-kaon correlation functions**
- **An example fit for 0-30% 200 GeV Au+Au collisions**
- **Average transverse mass dependence of the Lévy source parameters**

# The STAR detector

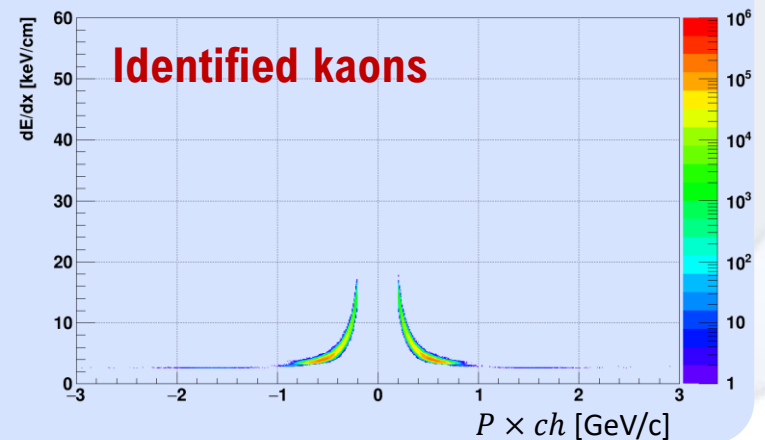
- **Detectors used for the analysis:**
  - BBC, ZDC, VPD: centrality, vertex position
  - TPC: tracking, dE/dx PID



dE/dx vs. P . charge

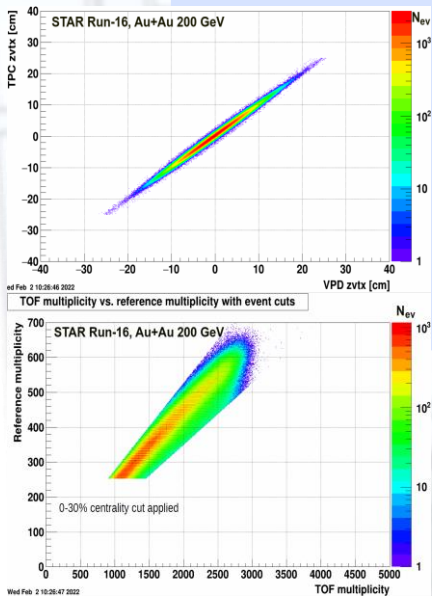
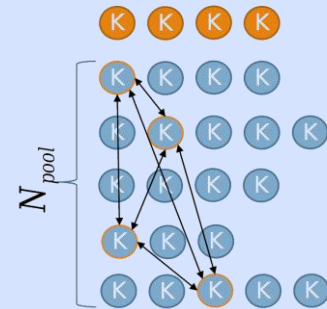


dE/dx vs. P . charge for KK

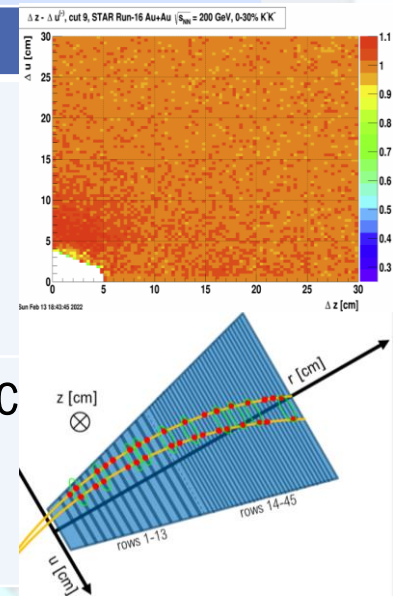


# Measurement of the $K^\pm K^\pm$ correlation functions

- Analyzing data from **200 GeV Au+Au, Run-16**
- Measurements of **1D two-kaon HBT correlation functions**
- Event mixing** is done similarly to *Phys.Rev. C97 (2018) no.6, 064911*
  - 2 cm wide z vertex bins, 5% wide centrality bins
  - $C(Q) = A(Q) / B(Q)$



Event cuts	Single track cuts	Pair cuts
Vertex position $ v_z  < 30$ cm $ v_z^{vpd} - v_z^{TPC}  < 3$ cm $v_r < 2$ cm	PID $N_\sigma(K) < 1$ , $N_\sigma(\pi, p, e) > 3$ Number of TPC hits $N_{hits} > 18$	Splitting level SL < 0.6 Frac. of merged hits FMH < 0.001 <i>J. Adams et al. (STAR Coll.),                      Phys. Rev. C 71, 044906 (2005)</i>
Charged particle multiplicity cuts 0-30% centrality	$0.2 \text{ GeV}/c < p_T < 4.0 \text{ GeV}/c$ Dist. of Closest Appr. DCA < 3 cm	Geometric cuts on TPC local coordinates $\langle \Delta u \rangle^{pad \text{ rows}}$ , $\langle \Delta z \rangle^{pad \text{ rows}}$



# Fitting process with Lévy parametrization

- Lévy parametrization without final state effects:**

LCMS three-momentum difference

$$Q = |\mathbf{q}_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{long,LCMS}^2}$$

$$C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^\alpha}$$

Intercept parameter (correlation strength)      Lévy exponent      Lévy scale parameter

- Bowler-Sinyukov procedure:**

$$C(Q) = (1 - \lambda + \lambda \cdot \underbrace{K(Q; \alpha, R)}_{\text{Coulomb correction}} \cdot \underbrace{(1 + e^{-|RQ|^\alpha})}_{\text{Possible linear background (usually negligible)}}) \cdot N \cdot (1 + \epsilon Q)$$

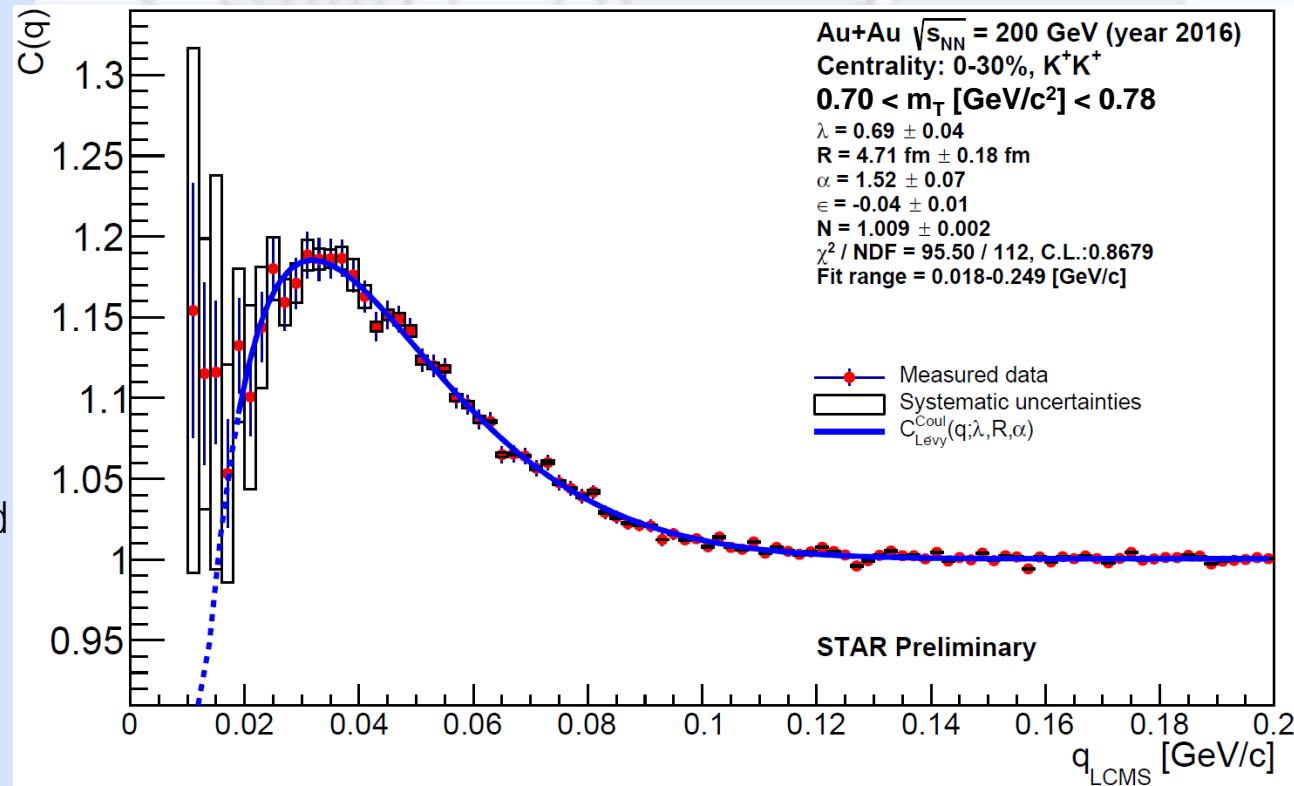
- Coulomb-correction:**

$$K(Q; \alpha, R) = \frac{\int D(\mathbf{r}) |\psi^{Coul}(\mathbf{r})|^2 d\mathbf{r}}{\int D(\mathbf{r}) |\psi^{(0)}(\mathbf{r})|^2 d\mathbf{r}} \rightarrow \text{calculated numerically}$$

Spatial correlations      Two-particle wave function (with Coulomb interaction)      Two-particle wave function (plane wave)

# An example $K^+K^+$ Lévy fit

- 0-30% Au+Au @ 200 GeV
- $K^+K^+$  correlation function
- $k_T = 0.5 \sqrt{K_x^2 + K_y^2}$
- $m_T = \sqrt{k_T^2 + m_K^2}$
- $0.5 \text{ GeV}/c < k_T < 0.6 \text{ GeV}/c$
- $0.70 \text{ GeV}/c^2 < m_T < 0.78 \text{ GeV}/c^2$
- Confidence level 87%
- Pair cut syst. uncertainties illustrated with boxes (not incl. in fit)
- **Good description over the whole  $q_{LCMS}$  range**
- **Lévy exponent  $\alpha = 1.52$ , far from Gaussian ( $\alpha = 2$ )**



# Average transverse mass dependence of the Lévy source parameters

- Weak  $m_T$  dependence, large systematic uncertainties
- $\lambda$  values close to unity (within systematic uncertainties)

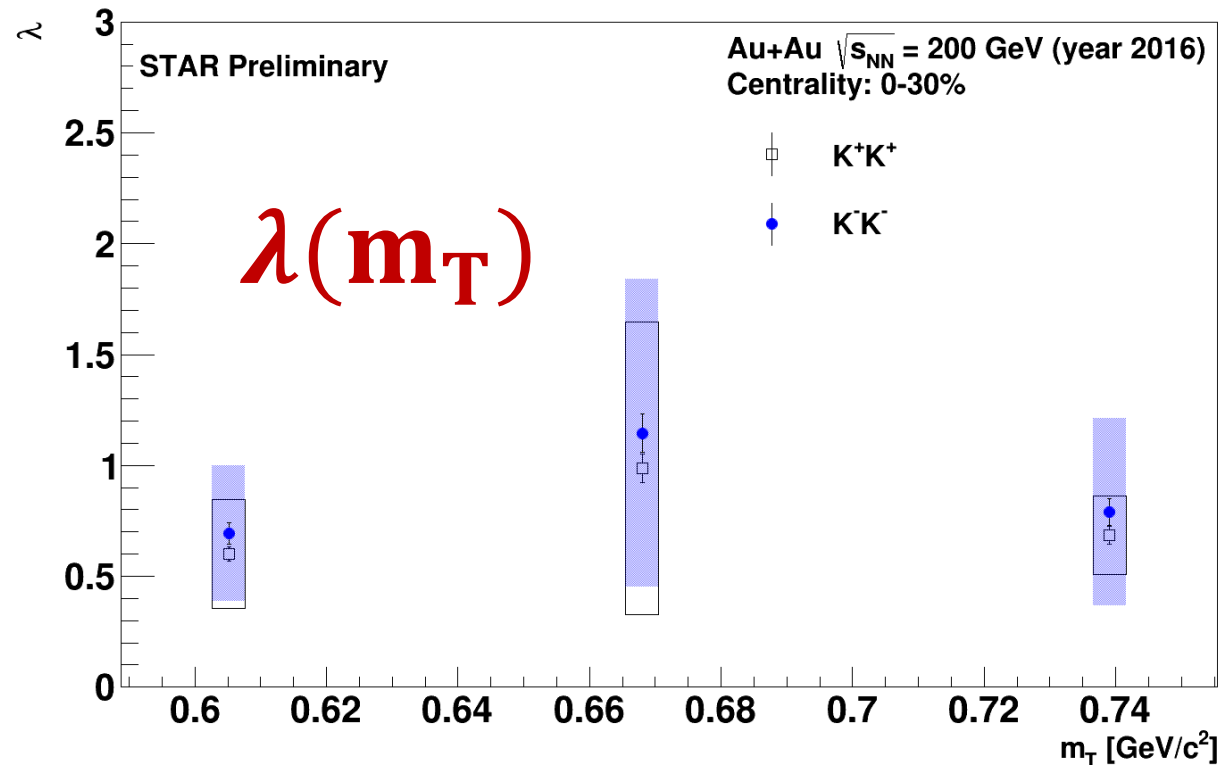
- Core-Halo model:

$$\lambda = \frac{N_{Core}}{N_{Core} + N_{Halo}} \approx 1$$

Small frac. of decay kaons

*Core – prim. + short-lived decays*

*Halo – long-lived decays*



# Average transverse mass dependence of the Lévy source parameters

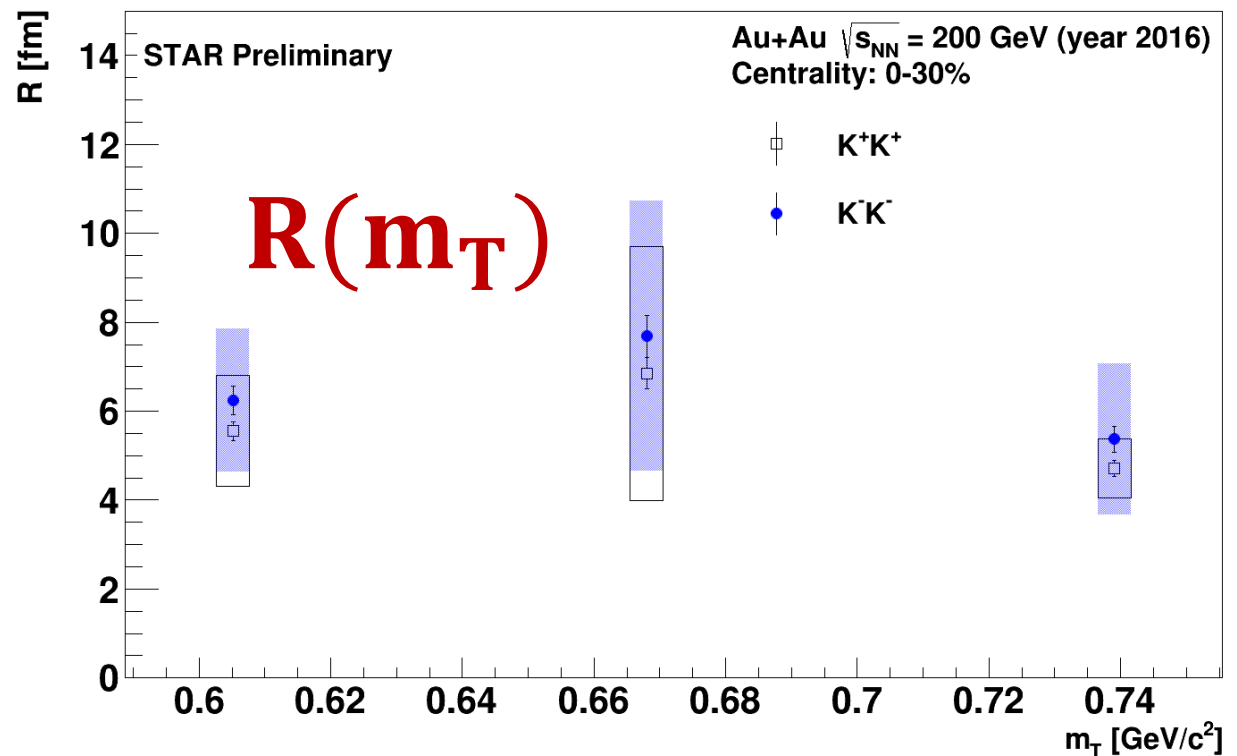
- Weak  $m_T$  dependence, large systematic uncertainties

- $R$  values close to PHENIX pion results

$$R_{\pi}^{exp} \simeq 7 \text{ fm} - 5 \text{ fm}$$
$$m_T = 0.6 - 0.7 \text{ GeV}/c^2$$

(*Phys.Rev. C97 (2018) no.6, 064911*)

- Not incompatible with usual decreasing behavior (hydro expansion)

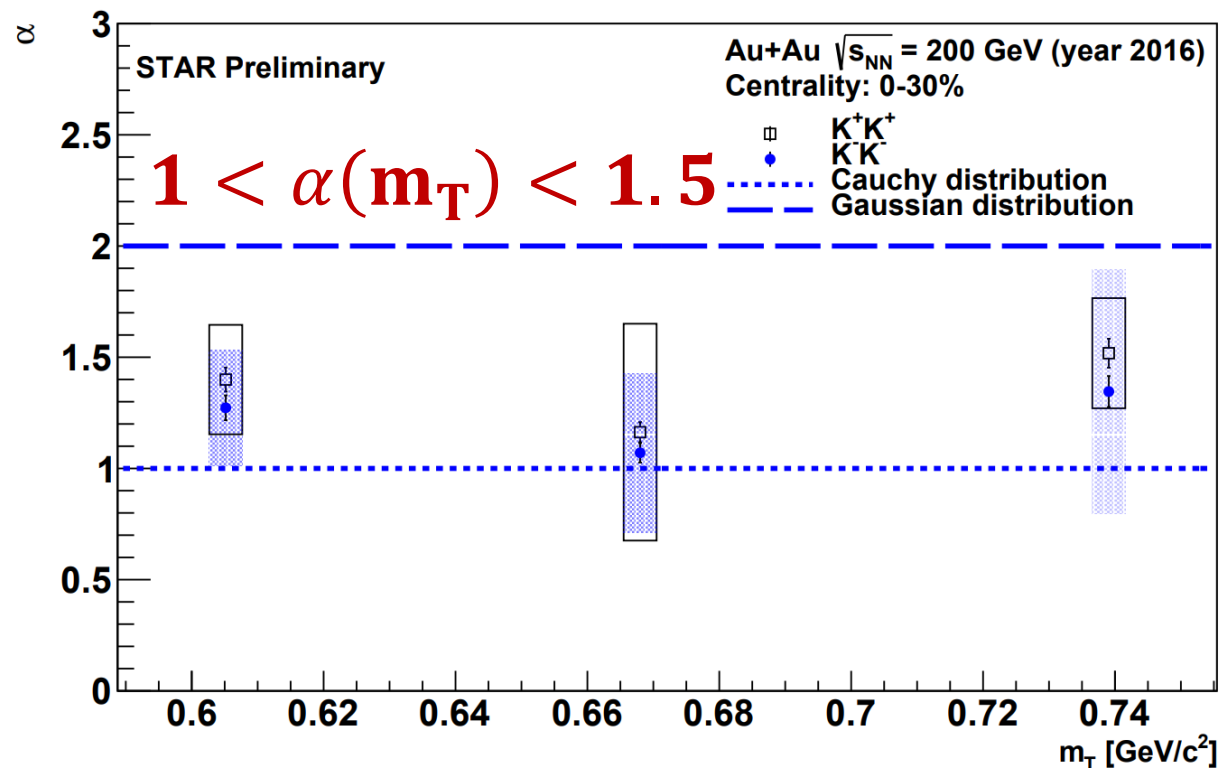


# Average transverse mass dependence of the Lévy source parameters

- Weak  $m_T$  dependence, large systematic uncertainties
- $\alpha$  values close to PHENIX pion results  
 $\alpha_\pi^{exp} \simeq 1.2$   
*(Phys.Rev. C97 (2018) no.6, 064911)*

$$\alpha_K^{exp} \simeq \alpha_\pi^{exp} \stackrel{?}{\Leftrightarrow} \alpha_\pi^{HRC} > \alpha_K^{HRC}$$

- **Preliminary results indicate non-Gaussian kaon pair-source function**





# Summary, outlook



Thank you for  
your attention!

- Preliminary results for identical charged kaon correlations indicate Lévy sources for central Au+Au collisions @ 200 GeV
- Lévy exponent  $\alpha$  comparable to pion results  $\alpha_K^{exp} \simeq \alpha_\pi^{exp} \stackrel{?}{\Leftrightarrow} \alpha_\pi^{HRC} > \alpha_K^{HRC}$
- Next step: more thorough systematic uncertainty analysis, centrality dependence,  $\sqrt{s_{NN}}$  dependence

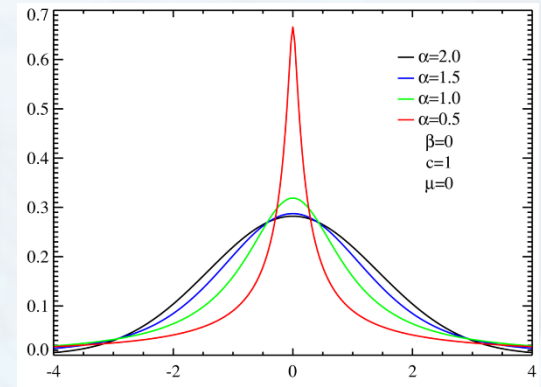
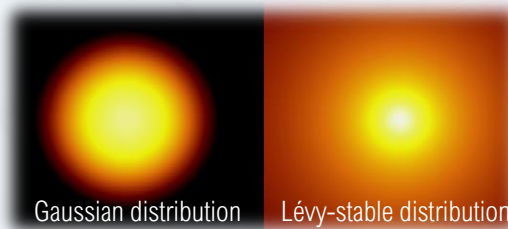
# BACKUP – Properties of univariate stable distributions

- **Univariate stable distribution:**  $f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi(q) e^{-ixq} dq,$

where the characteristic function:

$$\Phi = \begin{cases} \tan\left(\frac{\pi\alpha}{2}\right), & \alpha \neq 1 \\ -\frac{2}{\pi} \log|q|, & \alpha = 1 \end{cases}$$

- $\varphi(q; \alpha, \beta, R, \mu) = \exp(iq\mu - |qR|^\alpha (1 - i\beta \operatorname{sgn}(q)\Phi))$
- $\alpha$ : index of stability
- $\beta$ : skewness, symmetric if  $\beta = 0$
- $R$ : scale parameter
- $\mu$ : location, equals the median, if  $\alpha > 1: \mu = \text{mean}$



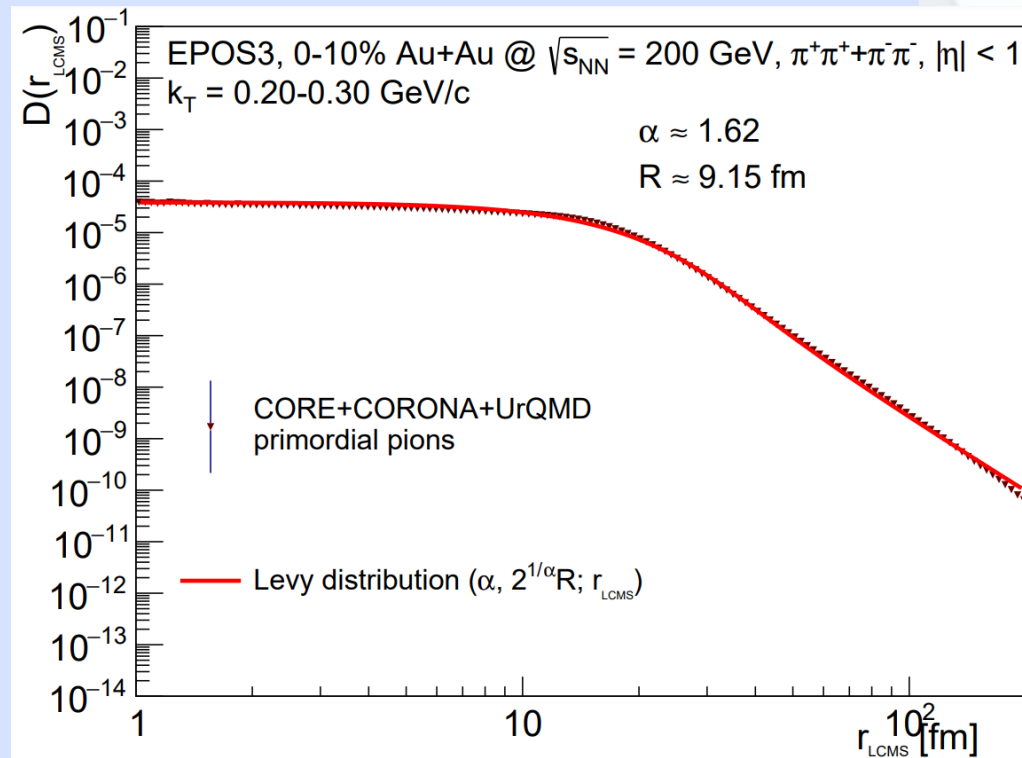
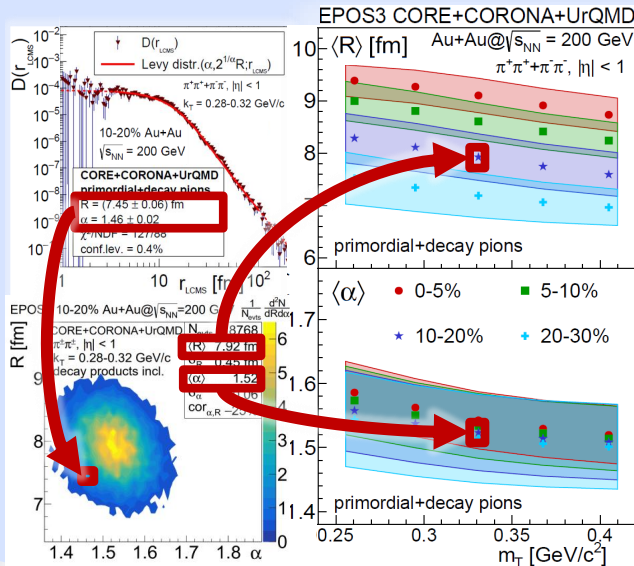
- **Important characteristics of stable distributions:**

- The distributions retain the same  $\alpha$  and  $\beta$  under convolution of random variables
- Any moment greater than  $\alpha$  isn't defined

# BACKUP – EPOS event-averaged two-pion source distribution

Entropy 24 (2022) 3, 308

- Event-averaged source not perfectly Lévy
- Nevertheless, very similar parameters
- Event averaged:  $\alpha \approx 1.62$ ,  $R \approx 9.15$  fm
- Event-by-event:  $\alpha \approx 1.66$ ,  $R \approx 8.96$  fm
- More reasonable approach for kaons

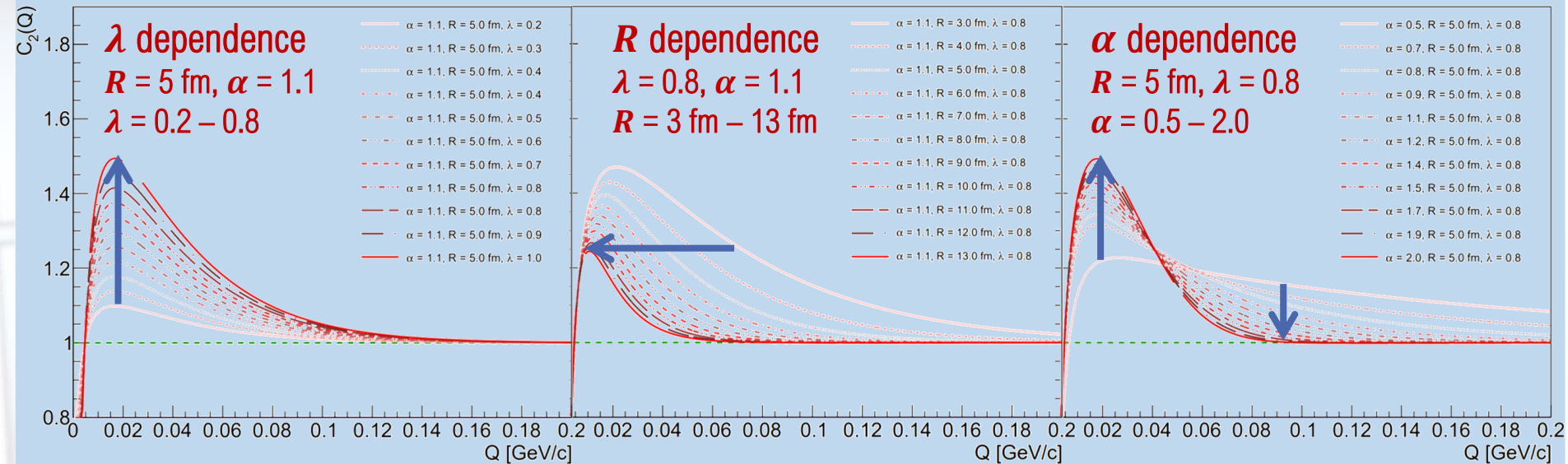


# BACKUP - Shape of the correlation functions with Coulomb effect included

Correlation function

Correlation function

Correlation function



$$C(Q) = 1 - \lambda + \lambda \cdot K(Q; \alpha, R) \cdot (1 + e^{-|RQ|^\alpha})$$

# BACKUP – Kinematic variables of the correlation function

- Usually used one-dimensional variable (with the Bertsch-Pratt variables in the LCMS frame):

$$q_{inv} = \sqrt{(1 - \beta_T^2)q_{out}^2 + q_{side}^2 + q_{long}^2}, \quad \beta_T = 2k_T/(E_1 + E_2)$$

- If  $\beta_T$  is close to 1 (intermediate-high  $k_T$ )  $q_{inv}$  can be small even if  $q_{out}$  is not
- Radius extracted from  $q_{inv}$  dependent two pion HBT correlations (in Au+Au) overestimates the 3D LCMS ( $R_{out}, R_{side}, R_{long}$ ) results (see e.g. the thesis of A. Enokizono)
- Another approach: **LCMS three-momentum difference:** <http://inspirehep.net/record/673843/>

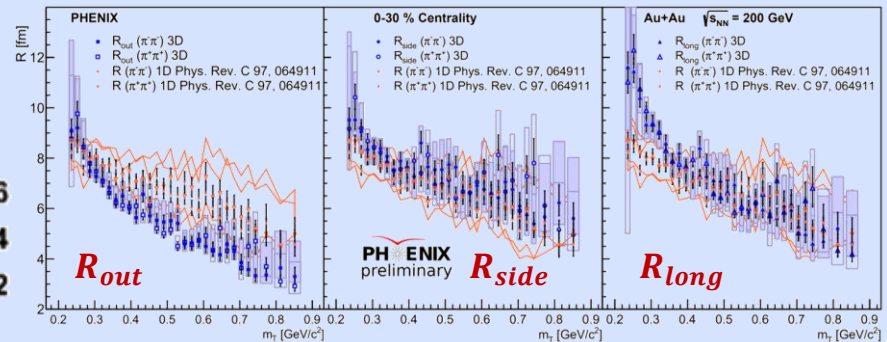
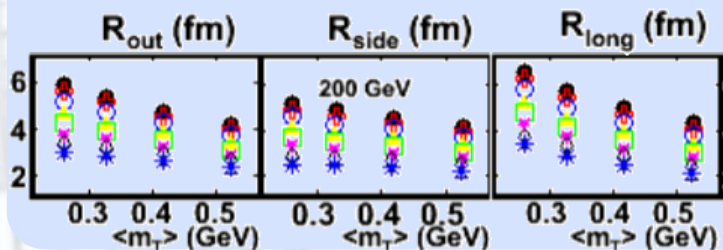
$$Q = |q_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{long,LCMS}^2}$$

# BACKUP – Kinematic variables of the correlation function

- Info from the tracks:  $E, p_x, p_y, p_z$
- $q_{inv} = \sqrt{(E_1 - E_2)^2 - \left( (p_{1,x} - p_{2,x})^2 + (p_{1,y} - p_{2,y})^2 + (p_{1,z} - p_{2,z})^2 \right)} = \sqrt{q_{0,LCMS}^2 - Q_{LCMS}^2}$
- $q_{0,LCMS} = \frac{(E_1^2 - E_2^2) - (p_{1,z}^2 - p_{2,z}^2)}{\sqrt{(E_1 + E_2)^2 - (p_{1,z} + p_{2,z})^2}}$
- $q_T = \sqrt{(p_{1,x} - p_{2,x})^2 + (p_{1,y} - p_{2,y})^2}$ ,  $q_{z,LCMS} = \frac{2(p_{1,z}E_2 - p_{2,z}E_1)}{\sqrt{(E_1 + E_2)^2 - (p_{1,z} + p_{2,z})^2}}$
- $Q_{LCMS} = \sqrt{q_T^2 + q_{z,LCMS}^2}$

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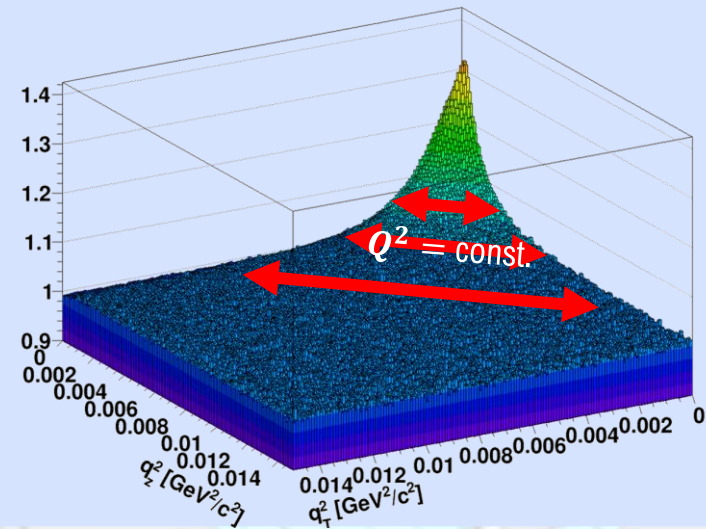
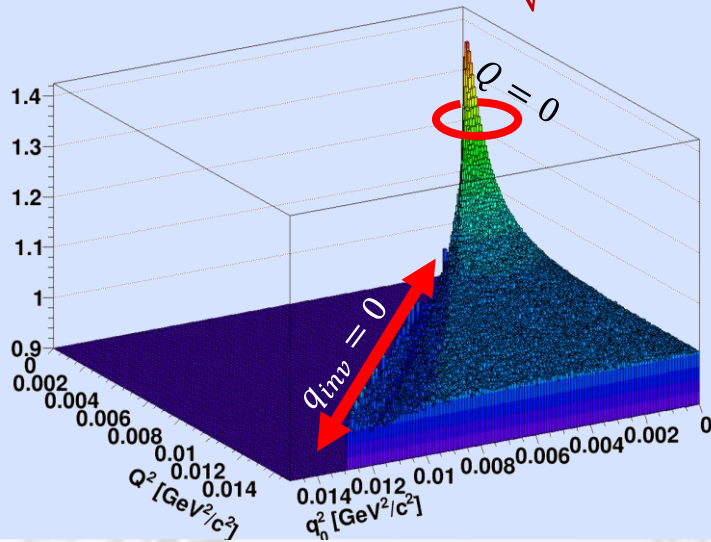
- $q_{inv} = |q_{PCMS}|$ , in PCMS the source is not spherically symmetric (but in LCMS it is, approximately, see figs.)
- Measurements in  $Q_{LCMS}$  gives similar radii magnitude as three dimensional ( $q_{out}, q_{side}, q_{long}$ ) meas.
- To compare  $\lambda$  and  $R$  with 3D (LCMS) results,  $|q|$  in the same frame (LCMS) should be used



# BACKUP – Kinematic variables of the correlation function

- A more appropriate one-dimensional variable: LCMS three-momentum difference

$$Q = |\mathbf{q}_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{long,LCMS}^2}$$

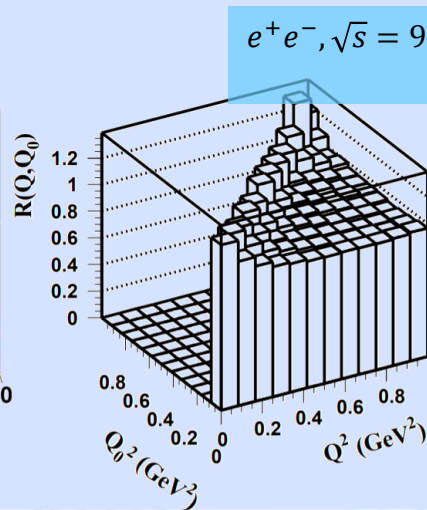
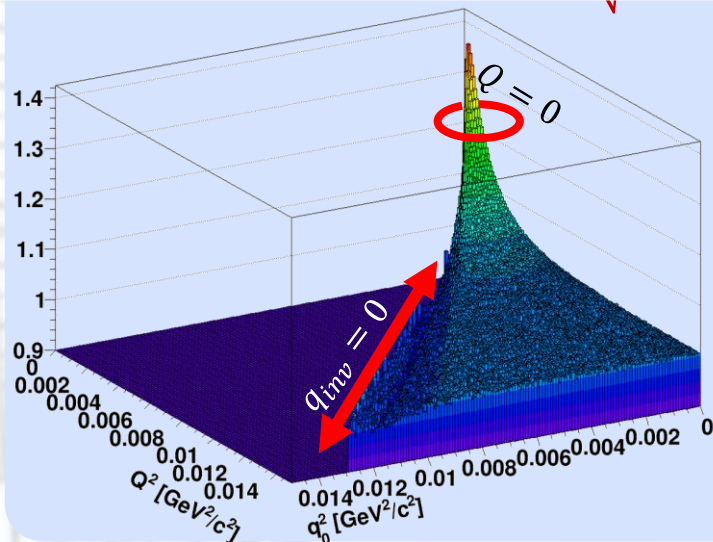




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$e^+e^-, \sqrt{s} = 91 \text{ GeV}, \pi\pi$  correlations

