Track Fitting in the STAR Detector using the Kalman Filter Method

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Abstract

In a Monte Carlo study the *Kalman filter* method is implemented for the purpose of reconstructing charged tracks with the proposed STAR detector at the RHIC. As the mean momentum of tracks at central heavy ion collisions in the low rapidity region is below 1 GeV, multiple coulomb scattering has a strong effect on the flight path. The *Kalman filter* is a suited tool for the reconstruction of such tracks. Two tracking devices are in study for the STAR experiment: a Time Projection Chamber (TPC) and a Silicon Vertex Tracker (SVT). Results from a comparison of the reformance of the TPC alone and the TPC combined with the SVT are shown. The extrapolation accuracy to the vertex region (impact parameter resolution) and the momentum resolution are studied.

1 Introduction

For the purpose of the fitting of charged tracks the *Kalman Filter* method is studied in the context of the proposed STAR detector. In this article we describe first the properties of the *Kalman filter* method. We will then discuss the implementation to the Time Projection Chamber (TPC) and the Silicon Vertex Tracker (SVT) and compare obtained resolutions. Finally we will give a more detailed description of the used programs.

2 Kalman Filter

The *Kalman Filter*, a method derived from system theory, has been recently proposed as a novel approach to the reconstruction of charged tracks and of charged tracks [1].

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Concerning the reconstruction of charged tracks it combines the advantages of former methods, especially the track fit with the beakpoint method [2], and adds a new concept for identification of outliers and further advantages [8]. As it is not necessary to introduce additional parameters to describe multiple coulomb scattering, the concept for the reconstruction of a charged track within a detector is simplified.

The vertex fit by the mean of the *Kalman filter* is equivalent to former studied methods based on LSM estimators. Again an advantage is added by an easy and fast method of outlier identification, that allows to reconstruct a primary vertex also in the presence of a large number of tracks.

Both algorithm have been already implemented and studied in both fixed target [3, 4] and collider experiments [6, 7].

2.1 Track fitting

To reconstruct a charged track, usually a so-called *global* algorithem is used. A 5-dimensional parameter vector, at a certain surface within the detector is used to describe the track. Deviations of the track path due to multiple scattering can be described by the introduction of further parameters. These parameters add no additional degree of freedom to the fit, as they are supplemented by measuremts of the deviation with mean 0 and the usual error derived from the *Moliere* formula [2, 5]. The estimation of the parameters is usually derived by a Least Squares Method (LSM).

In contrary *local* methods, as the *Kalman filter* and others describe the path of the particle by a set of 5-dimensional parameter vectors at all suited surfaces within the detector. Generally one can distinguish *active* surfaces, as a silicon detector plaquette, and *passive* surfaces, as the beam pipe or a wall of a detector.

In the *Kalman filter* these parameter vectors $\mathbf{p}_i$ are linked by a set of so-called *system equations*:

$$\mathbf{p}_{i+1} = \mathbf{f}_i(\mathbf{p}_i) + \epsilon_i \tag{1}$$

with $i = 1 \ldots n$ denoting the various detector surfaces. The random quantity ϵ_i describes the *system noise*, given by multiple culomb scattering processes and has again an expectational value of 0 and a variance given by the *Moliere* formula.

A further set of equations, the *measurement* equations, connect these parameter vectors with the individual detector measurements $\mathbf{m}_i$:

$$\mathbf{m}_i = \mathbf{h}_i(\mathbf{p}_i) + \delta_i \tag{2}$$

The random quantitiy δ_i is describing the *measurement noise*, with an expectional value of 0 and a variance given by the detector resolution.

The fit itself is seperated into two stages. During the first stage the parameter vector $\mathbf{p}_i$ from a surface $i-1$ is propagated to surface i including the associated covariance matrix. An additional term describing multiple coulomb scattering within the transversed material is added to the covariance matrix.

This gives $\mathbf{p}_i^{i-1}$, the *predicted* value at surface i with the knowledge of $i-1$ detectors. In the next step, the so-called *filter* step, the measurement at surface i is added with a usual weighted mean. When the last detector, n, is reached, the full information of the setup is used and the calculated parameter vector is an optimal estimate of the track.

In the second stage, the *smoother* this information is propageted by mean of the *Kalman smoother* to the $n-1$ other suraces, giving $\mathbf{p}_i^n$, the parameter vector at all surfaces i, with the knowledge of all detectors.

It is important to mention that all these calculations are done in the context of a LSM. In the ideal case LSM is known to have very appealing properties, but its performace may be strongly degraded by the presence of wrong measurements, so-called outliers (e.g. wrongly associated hits, etc.).

During the smoother a χ^2-quantity for each measurment is calculated, that can be used to identify outliers. Note that this quantity has a significant advantage to a similar quantity, calculated during the *filter* steps, as now the full knowledge of all detectors is used.

Passive surfaces, as detector walls, can be trivially added as surfaces without measurements, where only the increase of the covarince matrix due to multiple scattering is important.

For a more detailed desciption of the algorithem see for example [9, 10].

3 Study for the STAR detector

The proposed STAR detector was simulated in a configuration consisting out of a Time Projection Chamber (TPC), starting at an radius of 50 cm and extending to 175 cm. The Silicon Vertex Tracker (SVT) consists out of 3 layers of silicon drift chambers at radii of 5, 8 and 11 cm. The beamtube, made out of Berillium, has an radius of 4 cm.

Tracks were generated within that configuration, including all physics effects as energy loss and multiple scattering using the GEANT Monte Carlo program. The simulated tracks were intersected the detector surfaces at various radii, corresponding to the senstive areas of the two detectors. Further detector resolution was simulated by adding gaussian error these space points. The assumed resolution for the TPC hits was 200 μm in x, y and z. The assumed resolution for the SVT was 25 μm.

The generated TPC hits were processed by the TPC pattern recognition. Tracks were provided as a string of hits and the track parameter information from the pattern recognition.

For the purpose of this study SVT pattern recognition was assumed to be perfect and generation information was used to link the hits to the reconstructed tracks.

For comparison also ideal tracks were studied, where the generation information was used to find hits. The final plots were actually produced by studying

this sample, due to the necessary high statistic.

3.1 Coordiante System

The general orientation of the coordinates system follows the usual conventions for tracking detectors in solenoidal fields, e.g. the z is paralel to the magnetic field axis.

As the track fitting within magnetic fields is a nonlinear problem, the choice of a suited coordinate system, that is well described by local expansion around some reference parameters, is important. This question is dicussed in detail in reference [2]. Solving the equation of motion within a magnetic fields leads to a set of 5 parameters at a given reference surface, in this case the radius R of a detector surface. Following parameterisation is used:

$(R\Phi)$ ... Radius times the anzimuthal angle of the position of track at the refence surface;

z ... The position of the track at the refernce surface in z;

θ ... The polar angle of the track at the reference surface;

ϕ ... The anzimuthal angle of the track at the reference surface;

(q/p) ... Charge (+1 or -1) divided by momentum.

This coordinates are used also by other collider experiments (e.g. DELPHI). Connected with these parameters is the choice of the track model. As the magnetic field is homogenous, it is in this case a helix. The tracking routines, that are used to propagate the parameters from one detector surface to next are described in appendix A.0.1. These routines calculate also the corresponding derivative matrices used to propagate the covariance matrix.

3.2 Track fitting in the STAR TPC

Input for the track fit are the associated hits from the pattern recognition. Together with these hits, a pattern recognition provides usually also a first estimate of the track parameters. Such information is not really necessary, but simplifies the fitting procedure, as the track model can be evaluated along this track.

It is necessary to define a direction for the various steps. As the track density at the outer wall of the TPC is assumed to be lower, the fit is initialised at the position of the hit with the largest distance from the vertex. During the first filter steps, enough information is accumulated to provide a stable extrapolation.

This property of the *Kalman filter* has been found to be a disadvantage, especially in application where the algorithm is also used for pattern recognition. It may then be necessary to try various alternatives, until a real track can be reconstructed. In an applictaion, where the *Kalman filter* is only used to fit the track, it is usually no problem.

For the evaluation of the track parameter a two iterations are performed. In each iteration filter steps are followed by the smoother. The track model is evaluated only during the filter steps in the first iteration. All other passes involve only matrix algebra.

Outliers are identified and remove during the first iteration. A very crude cut removes obvious outliers during the filter phase. Only in the smoother the full information of all hits is aviable and outliers are removed after a more severe cut.

If ouliers are identified and removed in the smoother, the optimal estimate of the track parameters is only aviable at the position of the first hit. The easiest way to provide this information also at the position of the hit nearest to the inner wall of the TPC, is to reevaluate the fit without further cuts.

It has been found necessary also to take multiple scattering effects within the gas volume of the TPC into account. Even if the radiation length of Argon is very large (about 1100 cm), the deviation of the track was noticable and had a surprising large effect on the χ^2 of the fit. This was especially noticed during the outlier identfication in the smoother. In real life, when the correct errors for the TPC hits are not known, but derived from the data, this effect could be compensated by a slight overestimation of the errors.

The final result is given by the estimate of the track parameters at the position of the innermost hit. This allows optimal extrapolation of the track into the vertex region. If it is necessary to extrapolate tracks also in the outer region of the detector (calorimeters) it is an advantage to use the estimation of the track at the position of the last hit. As the *Kalman filter* can follow the real path of a track, this would provide more precise information, than an extrapolation from the innermost hit.

3.3 Track fitting by adding the SVT hits

To add vertex detector hits, the track is then extrapolated to the SVT plaquettes. Multiple scattering is evaluated along the track and the covariance matrix accordingly modified (passive surfaces). Hits are than added by subsequent filter steps.

As in this case the generation information is used to link the hits, no outlier rejection is performed.

It has been found that not all extrapolation of TPC tracks have an intersection with the SVT or the beamtube. Only tracks with a low number of TPC hits or a large θ angle are affected. If those tracks have associated SVT hits, these hits cannot be added. Naturally this is only a problem, when these association are known *a priori*. In real life no hit association would have been possible.

4 Comparison of TPC alone and TPC combined with the SVT

This comparisons has been performed using 10 simulated central collisions of Au on Au. The p_t spectrum of those tracks within the fiducial volume is shown in figure 1. All considered tracks have common quality cuts - at least 40 hits within the TPC and three SVT hits. For this comparison the generation information was used to associate hits within the TPC and the SVT.

An important quantity is the momentum resolution obtained by a tracking detector. The STAR TPC by itself has already an excellent resolution, as shown in Figure 2. Adding of SVT hits improves this already excellent resolution mainly in the high momentum region.

The impact parameter reolution (accuray of extrapolation in the vertex region) on the other hand, is dramatically improved. Figure 3 shows the comparison of impact parameters with and without SVT. In Figure 4 the extraploation is shown as a function of the transverse momentum.

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A Short description of the routines

All programs were written in Fortran 77 using the codemanager PACHTY. The source can be found in `CAR` files - input files for PATCHY. You will further need the `CRA` steering files and usual `COM` files.

One part of the files are of generl use, e.g. the track model and *Kalman filter* matrix routines:

- `KTRACK.CAR` ... Track fit;
- `KVERTEX.CAR` ... Vertex fit;
- `HELIX.CAR` ... Tracking routines;

The second set consists out of the routines used to process the STAR events:

- `TPCFIT.CAR` ... TPC track fit;
- `SVTFIT.CAR` ... SVT track fit;
- `EVENT.CAR` ... The main event loop

For some of the routines are more detailed description is giving.

The track information is generally stored in an vector of length 23. The track is described by 6 entries for the track parameters - R, $R\Phi$, z, θ, ϕ, q/p - followed by the corresponding covariance matrix (15 entries), the χ^2 and the number degrees of freedom of the fit.

A.0.1 HELIX

This is the library to perform the calculations for the track model. As there is a homogenious magnetic field, this routines solve the intersection between the extrapolation of a helix and a cylinder with given radius.

```
SUBROUTINE VXBTRK(RIN,PIN,ROU,POU,IERR)
```

Input:

`RIN` ... First reference surface

`PIN(5)` ... Parameter vector at first reference surface

`ROU` ... Second reference surface

Output:

`POU(5)` ... Momentum vector at second reference surface

`IERR` ... Error Code

Purpose:
A track described by a 5-dimensional parameter vector `PIN` at reference `RIN` is extrapolated to the refernce surface `ROU` giving `POU` (see also section 3.1).

```
SUBROUTINE VXBTRD(RIN,PIN,ROU,POU,DER,IERR)
```

Input:

`RIN` ... First reference surface

`PIN(5)` ... Parameter vector at first reference surface

`ROU` ... Second reference surface

Output:

`POU(5)` ... Momentum vector at second reference surface

`DER(4,5)` ... Derivative matrix

`IERR` ... Error Code

Purpose:
A track described by a 5-dimensional parameter vector `PIN` at reference `RIN` is extrapolated to the refernce surface `ROU` giving `POU` (see also section 3.1). In addition also the corresponding derivative matrix (e.g. ∂`POU`/∂`PIN`). The trivial column, ∂`POU`/∂`PIN(5)`, `PIN(5)` beeing (q/p), is skipped.

A.1 TPCFIT

This are the routines, used to reconstruct track within the TPC. It is using KTRACK.CAR to fit the tracks and HELIX.CAR to calculate the track model. Outlier recjection is integrated (see also section 3.2.

```
SUBROUTINE TPCINI(CHTOP,OUTFIL,OUTSMO)
```

Input:

CHTOP ... RZ File for Ntuples; if ' ', no Ntuples are filled;

OUTFIL ... Outlier Cut during Filter

OUTSMO ... Outlier Cut during Smoother

Purpose:
Initialisation of the TPC track fit; once at the start of the program; Ntuples are initialised

```
SUBROUTINE TPCFIT(NEVT,NTRK,TRKGEN,TRKREF,NTPC,TPC,TPCCH2,TRKTPC,IRET)
```

Input:

NEVT ... Event Number

NTRK ... Track Number

TRKGEN(6) ... Track generation parameter at position of first hit (R, $R\Phi$, z, θ, ϕ, q/p);

TRKREF(6) ... Reference track at position of last hit;

NTPC ... Number of TPC hits

TPC(6,NTPC ... TPC hits (R, $R\Phi$, Z, var($R\Phi$), cov($R\Phi, Z$), var(Z));

Output:

TPCCH2(NTPC) ... χ^2-contribution of corresponding hit; for Ouliers -1;

TRKTPC(23) ... Reconstructed Track;

IRET ... Return Code;

Purpose:
The given hits within the TPC are fitted.

```
SUBROUTINE TPCEND
```

Purpose:
Termination of the TPC track fit; once at the end of the program; Ntuples are flushed.

A.2 SVTFIT

This are the routines, used to reconstruct track within the SVT. It is using `KTRACK.CAR` to fit the tracks and `HELIX.CAR` to calculate the track model. This routines need beside the information concerning the SVT hits also an extrapolation from the TPC, together with the corresponding covariance matrix (see also section 3.3.

```
SUBROUTINE SVTINI(CHTOP)
```

Input:

`CHTOP` ... RZ File for Ntuples; if ' ', no Ntuples are filled;

Purpose:
Initialisation of the SVT track fit; once at the start of the program; Ntuples are initialised

```
SUBROUTINE SVTFIT(NEVT,NTRK,TRKGEN,TRKTPC,SVT,TRKSVT,IRET)
```

Input:

`NEVT` ... Event Number

`NTRK` ... Track Number

`TRKGEN(6)` ... Track generation parameter at position of the first SVT hit (R, $R\Phi$, z, θ, ϕ, q/p);

`TRKTPC(23)` ... Track, as reconstructed by TPCFIT;

`SVT(6,3` ... SVT hits (R, $R\Phi$, Z, var($R\Phi$), cov($R\Phi, Z$), var(Z)); one entry per layer; If no hit is found in layer i, `SVT(1,`i`)` has to be 0;

Output:

`TRKSVT(23)` ... Reconstructed Track;

`IRET` ... Return Code;

Purpose:
The given hits are fitted with the TPC track.

```
SUBROUTINE SVTEND
```

References

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Figures

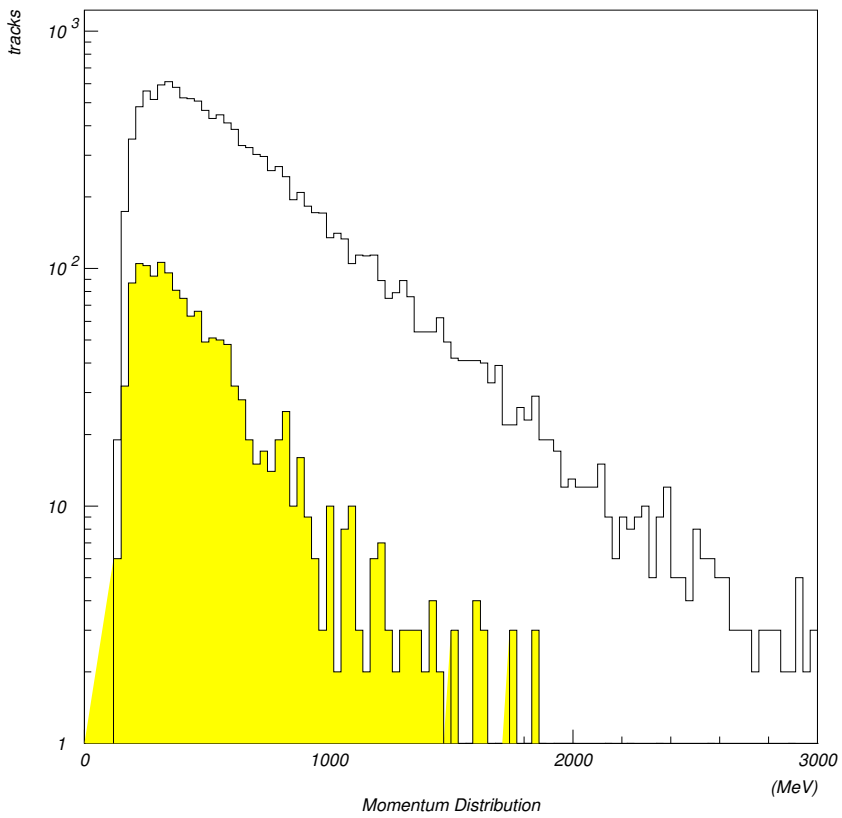

Figure 1: P_t spectrum of tracks at central Au - Au collision within the fiducial volume. All tracks have at least 40 hits in the TPC and 3 SVT hits.

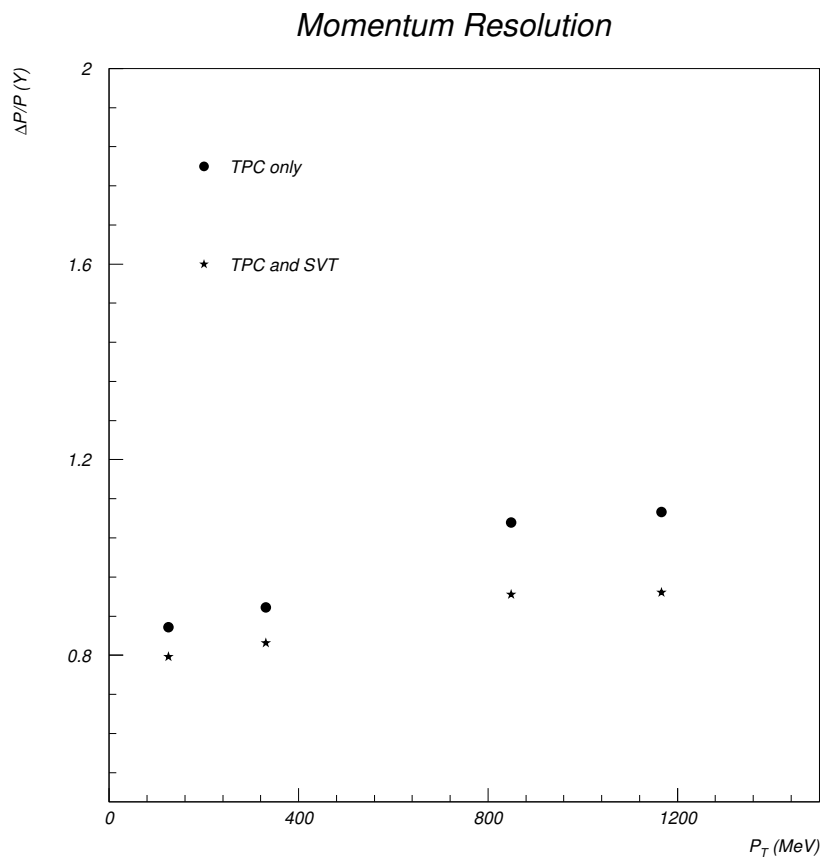

Figure 2: Momentum resolution for tracks reconstructed by TPC hits alone and by TPC hits and SVT hits. The same track sample has been used to derive this numbers.

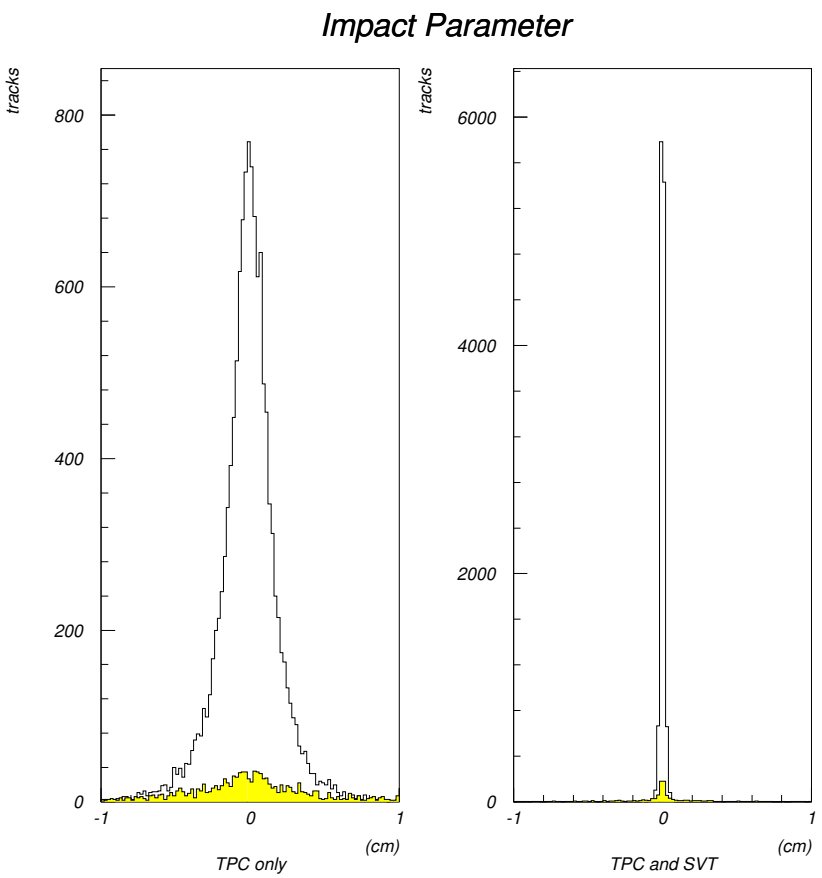

Figure 3: Impact parameter distribution of reconstructed primary tracks. For comparison the impact parameter distribution of secondary tracks is shown in the shaded area.

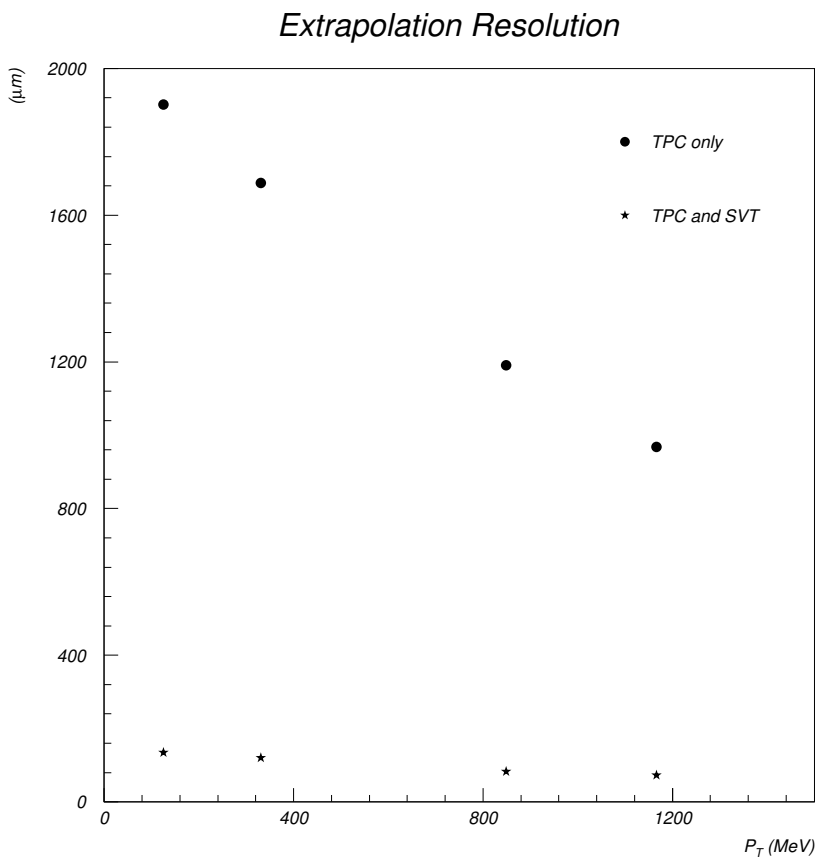

Figure 4: Extrapolation resolution to the vertex region for tracks reconstructed by TPC hits alone and by TPC hits and SVT hits. The same track sample has been used to derive this numbers.