A Simulation of Hanbury-Brown-Twiss Experiments at STAR

STAR note #92

February 2, 1993

A. Dean Chacon

Single event HBT analyses

The HIJING event simulator predicts that a single Au + Au event at RHIC will produce about 1200-1300 negative pions with $|\eta| < 1.2$. A quick estimate of the number of pion pairs produced is about 750,000 to 880,000 pion pairs, which is an order of magnitude more pion pairs than have been used in prior HBT analyses, and this suggests that it should be possible to fit a correlation function to the data from a single event. However, a detailed examination of the distribution of the pion pairs in momentum difference (*q*) space, shows that most of the pion pairs have large momentum differences which are not useful for the HBT analysis. In this estimate, it was assumed that the coordinates of interest for the pion pair are $q_{||}$, the momentum difference parallel to the beam axis, $q_{\perp}$, the momentum difference perpendicular to the beam axis, and q_o, the energy difference. It was assumed that source sizes of 10 fm and larger are of interest, and it was also assumed that the lifetime would be 10 fm/c. With these assumptions the region of interest, taken to be the region in q-space out to the inflection point on the gaussian correlation function, is roughly given by $q_{||}$ <20 MeV/c, $q_{\perp}$ 20 MeV/c, and q_o < 20 MeV.

Ignoring the effect of the spectrometer's acceptance beyond requiring that $|\eta| <$ 1.2, and the Hanbury-Brown-Twiss effect (which will be at most a factor of two) by using uncorrelated data from HIJING, the number of pion pairs predicted to be within this momentum range is roughly 40 (or about 0.005%) for a central Au + Au event. Assuming 5 MeV bins for this range, the number of bins in this range is 64, leading to an average bin content of less than one. The HBT effect will, of course, increase this somewhat, but the correlation function will be poorly determined in any foreseeable single event HBT analysis using a three-dimensional correlation function. In an analysis using the FRITIOF event generator for central Au + Au events, and more closely simulating the STAR acceptance by using cut offs of $p_\perp > 150$ MeV/c, and $|y| < 1$, the number of pion pairs contributing to the HBT signal for a 10 fm source has been estimated to be about 25.[1] One can, of course, decrease the number of bins in the region of interest by decreasing the dimensionality of the correlation function. However, the lowest number of dimensions that makes physical sense is two (one spatial and one time), giving around 40 pairs in 16 bins, and an average bin content of two.

Ensemble HBT analyses

Assuming that the desired bin content for the analysis is around 10, then about 16 central Au + Au events should provide enough statistics to fit a correlation function assuming that there is no $p_\perp$ cut off. Analysis of the $p_\perp$ spectra for close pion pairs shows that a cut off of $p_\perp > 200$ MeV/c will eliminate about one-half of the pion pairs[2] so that, ignoring for the moment the spectrometer's two-track inefficiencies, it should be possible to fit a correlation function to 30-40 central Au + Au events, so there will be sufficient pion pairs in any reasonable data set to do an ensemble HBT experiment.

A simulation of an ensemble HBT Experiment at STAR

[1] R. Morse, STAR note #67, *Rate/Sensitivity Study for Two-Particle Correlations* at STAR.

[2] A particle with a $p\perp$ of 167 MeV/c will just reach the outer field cage and a $p\perp$ of 184 MeV/c will just reach the inner edge of the electromagnetic calorimeter (both in the 1992 design). The cut off of 200 MeV/c was chosen here to avoid particles that spiral in the TPC, which are not found well by the track finder, and may have to be cut from the analysis in the real experiment.

Introduction

The goal of this simulation was to show that it is possible to do a pion-pion HBT experiment using the STAR detector. In the spirit of much of STAR, the scope of the project has been chosen to fit within the applicable constraints; Here the constraint of computing power was the limiting one. The strongest concern in a HBT experiment is that the track-finding software has sufficient two-track resolution to resolve the correlation function from the largest source expected (note that smaller sources produce wider correlation functions and are, therefore, easier to measure). The two-track resolution is different from the single track resolution in that it is harder to determine if a hit is due to one particle or two, than it is to determine the center of an isolated hit. The limitations of computing power forced several choices to be made in the single-pion phase space that reduced the accuracy in the determination of the number of events required. These were necessary to allow the correlation function fits necessary to determine the source-size resolution to be completed in a reasonable amount of computer time.

The Model

To first order, the two-track resolution is dominated by the effect of the two pion tracks that are physically close to each other and so the rest of the event, which is farther away from the pions, can be ignored. So, to speed up the simulations, only the pions were considered.

It was decided to limit the number of pions to two per event. Generating many pions per event poses two problems: First, the generation time for an event using the current program goes roughly as $N \cdot 2^N$ (where N is the number of pions per event) and it is not practical to generate 1200-1300 correlated pions; Second, the currently available correlation function fitting routines are not capable of fitting large numbers of pions in an event.[3] Also, the momentum range of the pions was restricted to 1/8 of 4π by generating the pions so that $p_x \geq 0$, p_y, ≥ 0, and $p_z \geq 0$. The restriction on the momentum of the individual pions makes it more likely that the pion pair has a small momentum difference. It also makes it more likely that a pion pair generated from event mixing (event mixing was used to form the reference sample) has a small momentum difference and reduces the statistical fluctuations in the reference sample.

The emission spectra for the pions (in absence of the correlation) is an approximation of the spectra seen in the HIJING model. The $p_\perp$ distribution was taken to be exponential, with pions being generated in the range of 200 MeV/c to 2,000 MeV/c. The slope of the exponential was taken from a fit to HIJING data in the same region giving $N(p_\perp) \propto \exp(-p_\perp/283 \text{ MeV/c})$. The distribution in rapidity was taken to be flat, and rapidities in the range of -2 to 2 were generated. This should give pions in a larger range of $p_\perp$ and rapidity than the TPC's acceptance.

The source size simulated was chosen to be about 10 fm. There are few predictions of the source size, and so it was decided to use the largest source size that back-of-the-envelope calculations indicate would be resolvable. The parametrization of the correlation function was chosen to be

$$C\,(q_\perp, q_{||}, q_o) = 1 + \exp\{(- q_\perp^2 R_\perp^2 - q_{||}^2 R_{||}^2 - q_o^2 \tau^2)/2\}, \qquad (1.1)$$

where $q_\perp$, $q_{||}$ are projections of the relative two-pion momentum $\mathbf{q} = (\mathbf{P_1} - \mathbf{P_2})$ perpendicular and parallel to the beam axis, and q_o is the energy difference $|E_1, -E_2|$, which is the usual parametrization for Bevalac energies. Here the source parameters were taken to be $\lambda = 1.0$, $\tau = 9.0$ fm/c, $R_\perp$ 10.0 fm, and $R_{||} = 12.0$ fm. Pion-pion and pion-nucleus Coulomb effects were ignored.

Two pion events were generated with the above single pion momentum distribution. The events were then accepted or rejected to give the desired correlation

---

[3] W. A.Zajc,Phys. Rev. D. **35**, 3396(1987)

function using the Metropolis algorithm. The correlation function was evaluated from the permanent of the two-particle correlation functions as given by W. Zajc.[4] In principle more that two pions per event can be generated and given a known correlation function with these codes, but for the reasons given only two pions were used. Forty 150,000 event samples were generated. The data from the event generator was fit directly to verify the operation of the event generator, these results will be given below.

Spectrometer Simulation

The data from the event generator were then passed to the spectrometer simulator, which was based on the GEANT package from CERN with subroutines defining the STAR geometry written by W. Christie and R. Morse. The space points corresponding to the tracks were then passed to the fast simulator for the TPC, written by P. Jones, which converted the space points to simulated hits in the TPC.[5]

Track Finding

The track finder used in this work was adapted for the STAR TPC by I. Sakrejda *et al*. The default form of the track finder was used without special tuning to improve the two-track resolution in these events. The only additional cut was that it was required that forty hits be present in a chain for the chain to be considered a valid track. The default value for this cut is five, which sometimes results in more tracks being output than input.

Correlation Function Fitting

---

[4] W. A. Zajc, Phys. Rev. D **35**, 3396 (1987). In the particle case, the permanent reduces to the usual correlation function.

[5] P. Jones and I. Sakrejda, Star note #56, *The STAR FaST TPC Simulation (a User's Guide).*

The fitting routines were taken from A. Chacon *et al.*[6] The routines were written for a small acceptance spectrometer, when they were used for the large acceptance spectrometer some changes were made. The most important change is related to the way the residual correlation correction is made. The code uses one pion of each pair to make a data set of single pions with a momentum spectrum that approximates the momentum spectrum of the pions in the absence of the HBT effect. These pions are then used to calculate weights so that a histogram of pion pairs, with the weights, gives the correct momentum difference spectrum for pairs in the absence of any HBT effects. This calculation uses an iteration procedure where the weights are used to calculate a better approximation to the weights. The code was modified so that if a pion's weight had more than a 30% statistical uncertainty, then the pion was not included in the calculation of the weights in the next iteration. This was done since the TPC's acceptance includes regions of phase space where pion pairs with low momentum difference are unlikely. The weights associated with such pion momenta are large and uncertain, and this causes large fluctuations in some of the calculated weights.[7] This effect can also be reduced by relaxing the q space cuts at the expense of processing time and storage space.

Results

Forty 150,000 event samples were generated. To reduce the disk space required for the output files, the pion pairs were cut so that only those events with $q_\perp \leq 100$ MeV, $q_{||} \leq 100$ MeV, and $q_0 \leq 100$ MeV/c were output. The total number of events tried was 6,000,000, and the number of events output was 218,490.

To verify the proper operation of the event generator and the fitting routines, a subset of the pion pairs was fit. The subset of the data was determined by making cuts

[6] A.D. Chacon et al., Phys. Rev. C **43**, 2670 (1991).

[7] The weights are large because the algorithm is attempting to correct for pion pairs lost due to the q space cuts. They are uncertain because so few point pairs pass the cuts.

such that the pairs had $q_\perp \leq 80$ MeV/c, $q_{||} \leq 80$ MeV/c, and $q_o \leq 80$ MeV. The cut was made to suppress pion pairs that are on the tail on the correlation function. The first 103,655 pion pairs were used, resulting in 60,000 pion pairs that passed all the cuts. The generation of the correlation function and the fit took about 30 hours of CPU on the SGI computer at Brookhaven. The result of this fit is given in Table 1.1. Within statistical uncertainty, the parameters extracted from the fit agree with the input.

Seventeen of the forty data sets were run through the whole simulation (this took about 1 day on the SGI for 3 files). The number of events processed was 88,854, and in those data files 112,628 pion pairs were found. Although only one pion pair per event was input, it was possible for the track finder to find more pions than input. The pair generating program used all possible combinations of pions to generate pairs, and that is why the average number of pion pairs per event is more than one. These data were then cut to eliminate particles with $|\,p\,| \leq 150$ MeV/c, .

Parameter	$R_\perp$(fm)	$R_{||}$ (fm)	τ(fm/c)		χ^2NDF
Input	10.0	12.0	9.0	1.0	-
Fit w/o spectrometer	10.3±0.5	11.9±0.8	8.9±1.6	1.02±0.08	2850/2720
Fit w/spectrometer	9.9±0.8	12.8 ± 1.4	12.7±2.4	1.0±0.12	2513/2441

Table 1.1: Fits to simulated data, without (w/o) and with (w/) the spectrometer simulation.

since no pions were generated with $p_\perp < 200$ MeV/c.[8] The number of pairs passing this cut was 71,205. A subset of this data was used to make the fit. This subset had cuts such that the pairs had $q_\perp \leq 80$ MeV/c, $q_{||} \leq 80$ MeV/c, and $q_o \leq 80$ MeV, as before. An additional cut was made requiring one of $q_{out} \geq 5$ MeV/c, $q_{side} \geq 5$ MeV/c, $q_{||} \geq 5$ MeV/c, or $q_o \geq 5$ MeV, where q_{out} is the component of $\mathbf{q}$ parallel to $\langle \mathbf{p} \rangle_\perp = [(\mathbf{p}_1 + \mathbf{p}_2)/2]_\perp$ and q_{side}, is the

[8] The cut was made to remove secondary particles since particle identification in the track finder was not yet implemented. The cut was made at 150 MeV/c to allow for poorly reconstructed primary tracks.

component of **q** perpendicular to $\langle \mathbf{p} \rangle_\perp$ and $q_{||}$. The cut at 5 MeV was made to suppress a spike at very low momentum differences that is believed to be due to tracks being split by the track finder, and returned as two tracks at approximately the same momentum. The results of this fit as given in Table 1.1. It can be seen that the extracted parameters agree with the input.

The estimate of the number of events needed to produce the number of pairs used is made as: The correlation function fitting routine (STOUT3D) used 30,000 pairs of the 56,991 pairs it read. The routine (TRANSFORM) that generated the pairs to be fit read 88,854 (pion pair) events, and produced 71,205 pairs. Thus 71,116 (pion pair) events would be required to produce 30,000 fitted pairs. The event generator (BLACKTHORN) tried 6 x 10^6 pion pairs in 1/8 of 4π to produce 218,490 pion pairs for input to TRANSFORM. Thus 1.952 x 10^6 pion pairs in 1/8 of 4π are required to produce 30,000 fitted pion pairs. In a 4π detector, approximately 1/8 of the pion pairs will be within 1/8 of 4π of each other. Thus approximately 15.6 x 10^6 pion pairs in 4π are required to produce 30,000 fitted pion pairs. The event generator HIJING produces about 10^3 pions for a RHIC central Au + Au event. Thus about 32 events are required to produce 30,000 fitted pion pairs. So the statistical uncertainty quoted in Table 1.1 could be reached with approximately 30-40 Au + Au events at RHIC, using the STAR detector as modelled in 1992 .

Discussion

Effects on the number of events required

The effect of the omission of the remainder of the event other than the pions will increase the number of events required over the estimate, since some pions will be lost in track finding when their tracks get confused with tracks from other particles in the event. This will increase the number of events required somewhat, but it should not effect the maximum source size detectable, since this is determined by the pion-pion track confusion effects.

The effect of the omission of 7/8 of the detector increases the number of the events required by approximately eight (assuming similar pion densities in the eight sections). Using exactly eight ignores the fact that in a 4π event, there will be some close pairs where one of the pions is inside the 1/8th considered, and the other is outside. This effect is limited to pions near the boundary (within 10 MeV/c or so of $p_x = 0$ or $p_y = 0$ or $p_z = 0$), and so this effect is believed to be small.

The restriction of the number of pions to two reduces the number of pion pairs required in the estimate since this sample is more likely to have pairs at low momentum differences than if there are multiple pions in the event (assuming that in each event all possible combinations of pion pairs are used to generate the correlation function). This effect is partially compensated for by restricting the pion momenta to 1/8 of 4π , but is believed to give a factor two uncertainty in the number of events required.[9] Assuming a worst case, the number of events required should be revised upwards to about 100.

Effects on the computer codes

The codes used to fit the correlation function in the real data will have to be more sophisticated than the current codes. One effect not explored by this simulation, is that if the pion density is high enough, then three (and higher) pion correlation effects cannot be ignored, and the current fitting algorithm will not give the correct source-size parameters.

The retention of the 7/8 of the spectrometer ignored in the simulation will increase the number of close pairs per event by a factor of about eight (assuming one pion pair per segment of the detector), reducing the number of events required by a factor of eight. Coupled with the factor of eight more data taken (per event), the number of tracks in a data

[9] The HIJING events have about 1% of the pion pairs passing $q_{\perp} \leq 100$ MeV/c, $q_{¿} \leq 100$ MeV/c, and $q_o \leq 100$ MeV/c, which is approximately the same as for the event generator used here, when corrected to 4π. The details of the distribution below 100 MeV are not known.

set will be approximately the same as in the simulation. But, if one makes pion pairs (as the current codes do), then the number of pairs per event [N(N - 1)/2] will change from 1 to 120 while the number of ***useful*** pairs is changing from 1 to 8 (approximately). So, the fraction of useful data decreases by a factor of 8/120 = 1/15 as the acceptance increases and the storage space required for the fitted pion pairs will increase by an order of magnitude. At the same time, keeping the full spectrometer will increase the number of pion momenta used to form the reference sample by 120, and will increase the number of pion pairs in the reference sample to be histogrammed by a factor of 14,400. If the number of events is decreased by a factor of eight to allow for the increase in close pairs, the number of pion pairs in the reference sample to be considered is still a factor of 225 above what was used for the partial detector, resulting in an increase of two orders of magnitude in processing time required. While these calculations assumed only one pion pair in 1/8 of the detector, the number of pions in the Au + Au events is much higher than this, and the data will be even more biased away from useful (low momentum difference) data. So the requirements for processing power and storage are likely to be more severe than this simple estimate.

If one is using event mixing to form the reference sample then one should keep in mind that cutting the data to favor small $\boldsymbol{q}$ in the fitted sample, while keeping the full acceptance of the spectrometer, favors statistics (at small $\boldsymbol{q}$) in the fitted data over the statistics (at small $\boldsymbol{q}$) in the reference sample. It may be possible to do this to the point that the currently implemented fitting program will fail since this program assumes that the statistical fluctuations in the reference sample are small.

In this simulation, the cuts made in $\boldsymbol{q}$ space were partially offset by the cuts made in the single particle phase space. It is believed that imposing the $\boldsymbol{q}$ space cut without imposing the phase space cut will result in poorly determined correlation function parameters. So, using current algorithms, if it is desired to use the full spectrometer, the $\boldsymbol{q}$ space cuts will have to be relaxed and the above estimates will apply. However, it certainly is possible to divide the spectrometer up into segments, and regard each of the segments as an independent measurement of the source size (and event mix only within a segment) to bias the data towards small **q**, as was done in the simulation.

Conclusions

Single event HBT analyses are not possible except for one-dimensional analyses in $Q_{invariant}$.[10] Within the assumptions of these simulations (that $q_{||}$, $q_{\perp}$ and q_{o} are suitable variables for the analysis, and Coulomb effects can be ignored), and using the 1992 version of the STAR TPC track finder, about 10 fm source sizes (and about 10 fm/c lifetimes) can be measured using the STAR TPC, and there are enough pion pairs in a data set of 100 central Au + Au events at RHIC to measure the correlation function parameters to an accuracy of about 10%. Such a data set taken using the full spectrometer will require an order of magnitude more computing power and two orders of magnitude more memory to process than the current data set, assuming that similar codes to the current are used without any special modifications. However, by the time **STAR** takes such a data set, it is very possible that computing hardware will have advanced that far. Tests (either Monte Carlo or theoretical calculations) will have to be run to see if the current two-particle correlation functions are adequate to fit the data sets, or if the correlation functions to be fit will have to be expanded to include three or higher particle effects.

[10] P. Jacobs, in STAR note #61 *Rate Estimates for HBT Analysis at STAR*, shows that a $Q_{invariant}$ single event analysis is possible.