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TPC Central Membrane Deflection Due to Gas Flow and Electrostatic Forces

by

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Abstract

The high voltage central membrane (CM) in the center of the TPC must maintain its position in z to within ± 0.5 mm. This note estimates the deflection in this membrane due to gas flow from one chamber to the other and also compares the displacement electrostatic force with the resisting tension force. The gas flow is through an annular area between the membrane and the outer and inner field cages. It was assumed that all the flow during operations was input into one chamber and was than removed out the opposite chamber. The flow was 0.01 m^3/s (one TPC change per 1.4 hr) and the annular gap was 0.015 m. Assuming an idealized, incompressible gas flow through a sluice gate of height 0.015 m, the pressure drop was estimated to be 0.0014 Pa (2.0 x 10^{-7} psi). For a stretched 1.5 m wide, 0.0254 mm (0.001 in) thick Mylar diaphragm with an allowable membrane principle stress of 14 MPa (2000 psi) due to tension, a gas pressure of 0.45 Pa (6.4 x 10^{-5} psi) is required to deflect the center of the membrane 0.5 mm. This is about 321 times higher than estimated for the pressure drop due to flow across the gap. This estimated pressure will increase due to the effects of real flow, but possibly only by a factor of 10. These results indicate that pressure differences in the two TPC tracking volumes will not deflect the central membrane more than that specified.

The electrostatic force on the membrane due to a small displacement was estimated and it was found that the resisting tension force was about 24,000 times the electrostatic force, therefore, the central membrane is stable for small displacements in z.

Introduction

During the TPC Preliminary Design Review, some concern was expressed about central membrane deflection due to potential unequal gas flows into the two tracking volumes. If these flows are not equal, differential pressure will develop and gas will pass through the annular gaps between the CM and the field cages. This pressure difference will cause a deflection of the CM. This memo attempts to estimate the maximum pressure drop expected and to show that, for the worst case, the CM will not deflect more than the 0.5 mm allowable.

It was assumed that *all* the supply from the gas system is fed into only one of the chambers, passes by the CM and exits out the other chamber. This case is shown in Figure 1. The "x" in the manifold pipes indicate closed pipes. A gap of 0.015 m was assumed to exist between the CM and the inner and outer field cages. A sluice-gate model was assumed for gas flow through the gaps to retain the effect of the field cage walls. Since the maximum flow rate is known, the pressure drop across the gap can be calculated.

The CM was assumed to be a stretched cylindrical diaphragm which should be a better model than a sphere. Using the relationship between internal pressure and surface tension, the diameter of the cylinder, and the deflection could then be estimated. Using these same relationships, assuming a deflection and a tension, the allowed pressure difference for a 0.5-mm deflection can be found.

The electrostatic force was estimated by assuming a small displacement and calculating the net force due to this displacement. A sketch of this configuration is shown in Figure 2. Since the displacement was much smaller than the distance to ground, a simplification was made that related an electrostatic pressure to the voltage, displacement and the distance to ground. When compared to the restoring pressure due to membrane tension, it was found that this restoring pressure was about 24,000 times higher than the electrostatic case.

Appendix A contains detailed calculations and results discussed in this note.

Central Membrane Configuration

The CM is a large, conductive diaphragm that clamps the electric field at z = 0 to the maximum value. For the present TPC baseline this is -30 kV. The length of each tracking chamber is 2.1 m. The CM is attached to the OFC and IFC by hangers that provide a means to apply tension at many points on the outside and inside diameters. This configuration is based upon the ALEPH TPC design. This tension can be adjusted to a value that removes all the ripples in the thin membrane. The tensile strength of Mylar is about 55 MPa (8000 psi)[1]. If a 0.0254-mm (0.001-in) thick sheet is used, the allowable tension with a safety factor of 4, will be about 350 N/m (2 lb/in). This is reduced by $\sqrt{2}$ since the tension may be applied in two directions.

Sketches obtained from ALEPH personnel indicated that a 20-mm gap existed between the CM and the field cages in their design. For the present calculations, a 15-mm gap is assumed. The area of the gaps is then:

$$A_g = 0.24 \text{ m}^2$$

Maximum Pressure Drop Allowed

For a cylindrical bubble:

$$P_c = \frac{2\ T}{2\ r}$$

where:

r = radius of cylinder

The cylindrical shape is close to that existing in the TPC. The maximum tension allowed based upon the tensile strength of Mylar is:

$$T = 251 \text{ N/m}^2 \ (1.4 \text{ lb/in})$$

The allowed deflection of the CM is 0.5 mm. This is indicated as h on Figures 2 and 3. With h = 0.5 mm and c, the segment cord = 1.5 m, the radius of the cylinder is 563 m. The pressure required to deflect the membrane 0.5 mm was found to be:

$$P_c = 0.45 \text{ Pa } (6.4 \times 10^{-5} \text{ psi})$$

This is the maximum allowed pressure difference between the two chambers.

Estimated pressure drop across chambers

The maximum flow rate into the TPC during operation is 1 change of gas every 1.4 hours.[2] This is about 0.01 m^3/s. The area of the tracking chamber, $A_c = 11.8\ m^2$. For an ideal, incompressible fluid compared in the chamber and the gap:

$$y_c + \frac{v_c^2}{2g} = y_g + \frac{v_g^2}{2g}$$

where:

y = head
v = velocity

The average gas velocity in the tracking chamber is small compared to that in the gap. The estimated pressure difference was:

$$\Delta y = 9 \times 10^{-5} \text{ m } (3.5 \times 10^{-3} \text{ in}) \text{ of argon}$$

The pressure due to this height of argon is about 0.0014 Pa (2.1×10^{-7} psi). For the idealized case, the estimated pressure is about 620 times that allowed. There is a deviation from idealized flow due to viscosity and orifice flow effects. For a sluice gate this is accounted for by:

$$\Delta y \propto \left(\frac{1}{K_s}\right)2$$

where:

K_s = sluice coefficient[3]

For free flow, K_s is between 0.55 and 0.60, but for submerged flow the value is reduced.[3] From the results above, K_s can be as low at about 0.04 and still the pressure will not exceed the maximum allowable. Since this pressure is due to assuming all the gas flow through the gap, the results are still conservative.

Pressure drop in supply and return manifold pipes

The effective conductance of the gaps was estimated to be 7.6×10^8 l/s. The conductance of a 100-mm diameter manifold pipe, 6 m long was estimated to be 1.3×10^6 l/s.[4] With all the gas flowing in only one branch of the manifold, the pressure drop is about 0.078 Pa (1.1×10^{-4} psi). If the diameter of the pipes varies by 0.51 mm (0.02"), the conductance will only change by 2%. The effect of a small length change is even less. This implies that the input gas flow should split evenly with 50% of the total going into each chamber and very little will flow through the gap at the CM.

Electrostatic force due to displacement in z

If there is a displacement h in the z location of the CM, the electric field is:

$$E = \frac{V}{L \pm h}$$

The field increases on the membrane side that is closer to ground. The force on the charge, therefore, on the membrane is:

$$F = \varepsilon_o E^2 A \qquad \text{where:}$$

ε_o = permeability of vacuum
A = CM area

The net pressure is:

$$P = (P1 - P2) = \varepsilon_o (E_1^2 - E_2^2)$$

Noting that h << l:

$$P \cong \varepsilon_0 V^2 \frac{4\,h}{L^3}$$

With $\varepsilon_o = 8.85 \times 10^{-12}$ coul/V m, V = 100 kV, and L = 2.1 m:

$$P_{electrostatic} \cong 0.038 \text{ h N/m}^2$$

The resisting pressure, P_c due to the tension in the Mylar is:

$$P_c \cong \frac{4\,(2\,T)\,h}{c^2} = \frac{8\,T\;h}{c^2}$$

$$Pc = 892 \text{ h N/m}^2$$

For stability:

$$\frac{P_c}{P_{electrostatic}} > 1$$

$$\frac{P_c}{P_{electrostatic}} \cong 20{,}000$$ therefore, the membrane is stable.

Conclusions

If the gas flow into the TPC is held to 0.01 m^3/s, then the deflection of the central membrane will not exceed the maximum allowable. There is a considerable safety factor for this deflection if the gap between the CM and the field cages is at least 0.015 m. The expected pipe diameter variations should have little effect on the distribution of gas flows into each chamber.

The net electrostatic pressure due to the maximum allowed membrane deflection is resisted by a pressure due to tension that is many times that required for equilibrium.

References

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2. Etkin, A., Private communication, Feb. 2, 1993.

3. Dautherty, R. L., and Franzini, J. B., "Fluid Mechanics With Engineering Applications", McGraw-Hill Co., N. Y., 6 th Edition, 1965, P. 375.

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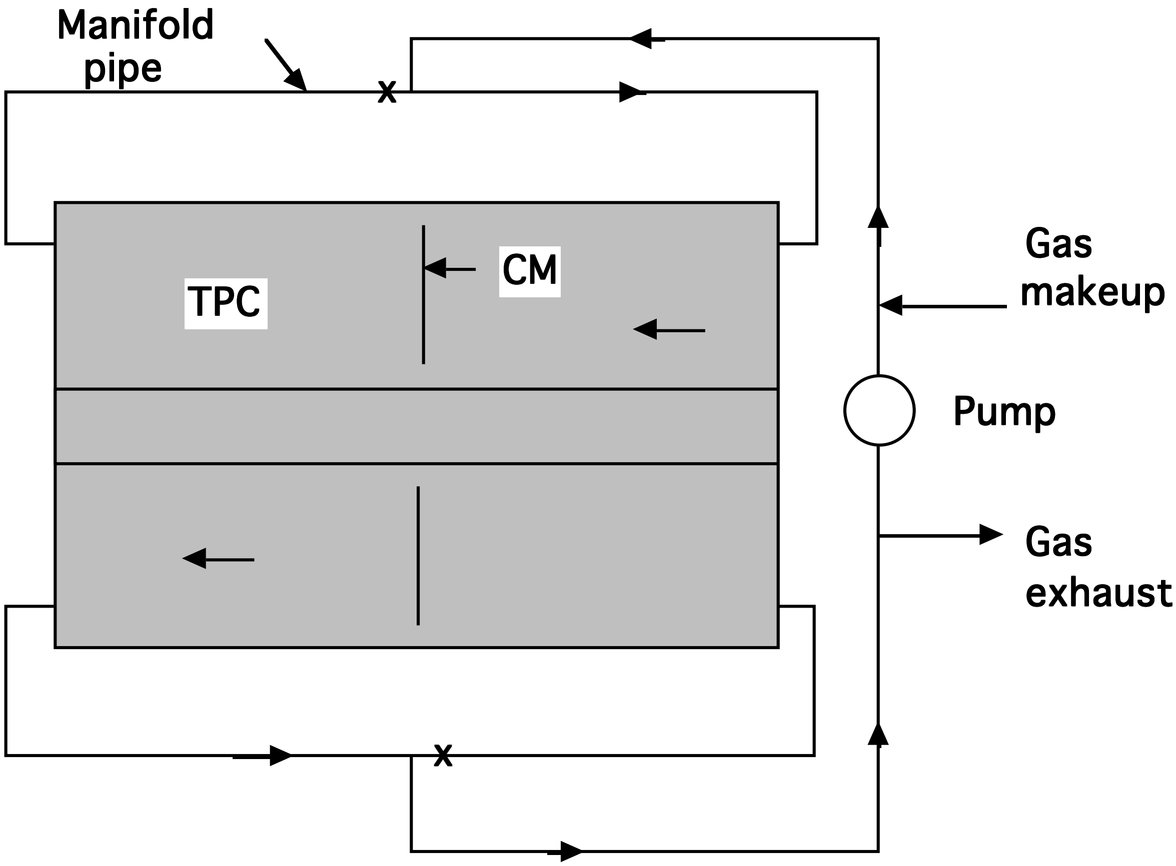

Figure 1. Pictorial of TPC showing gas flow paths.

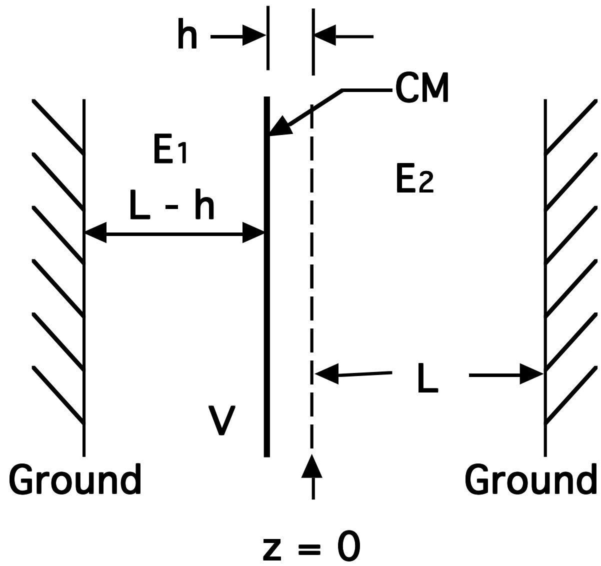

Figure 2. Schematic for calculating electrostatic force.

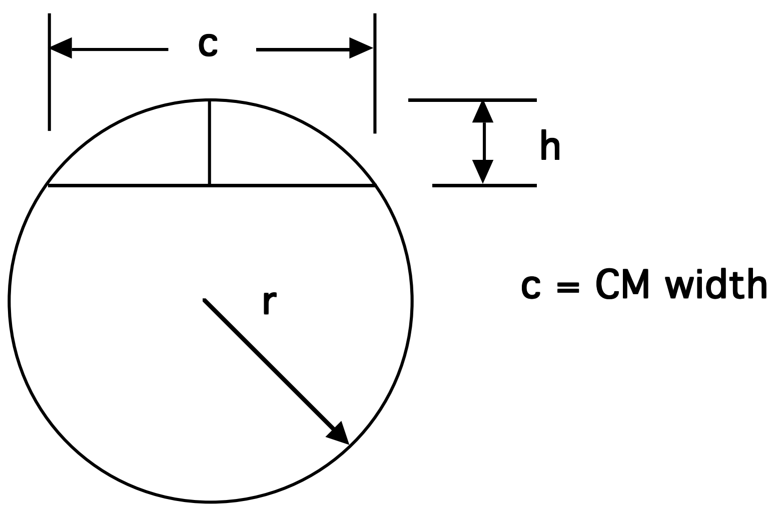

Figure 3. Cord c, and sagitta h, for cylinder and sphere.

TPC Central Membrane deflection due to gas flow and electrostatic forces

Appendix

Central Membrane Configuration

Gap area

Sketches obtained from ALEPH personnel indicate that a 20-mm gap exists between the CM and the field cages in their design. For these calculations, a 15-mm gap is assumed. The area of the gaps is then:

$$A_g = \pi\,(D_o + D_i)\,w \qquad \text{where:} \qquad 1)$$

A_g = gap area
D = diameter
w= gap width

$$A_g = \pi\,(4 + 1)\,0.015 = 0.24\ m^2$$

Maximum pressure allowed on central membrane

For a spherical bubble:

$$P_s = \frac{4\,T}{2\,r} \qquad \text{where:} \qquad 2)$$

P_s = internal pressure
T = tension
r = radius of sphere

For a cylindrical bubble:

$$P_c = \frac{2\,T}{2\,r} \qquad \text{where:} \qquad 3)$$

P_c = internal pressure
r = radius of cylinder

$$P_c = \frac{P_s}{2} \qquad 4)$$

Maximum allowed tension in the Mylar

The tensile strength of Mylar is about 55 MPa (8000 psi)[1]. If a safety factor of 4 is used, the strength is limited to 14 MPa (2000 psi). If the tension is applied to two orthogonal axes, the allowed stress in one direction is $\frac{S}{\sqrt{2}}$.

$$T = \frac{S}{1.414} t \qquad \text{where:} \qquad 5)$$

T = tension
S = maximum principle stress
t = Mylar thickness

$$T = \frac{14 \times 10^6}{1.414}\ 25.4 \times 10^{-6} = 251\ N/m^2\ (1.4\ lb/in)$$

Minimum cylinder diameter

The allowed deflection of the CM is 0.5 mm. For a cylindrical segment: (Refer to Figure 2)

$$c = 2\,[h\,(2r - h\,)]^{0.5} \qquad \text{where:} \qquad 6)$$

c = cord of segment
h = sagitta (deflection)
2r = diameter of cylinder

The segment cord is the width of the CM which is the difference between the outer and inner radii = 2 - 0.5 = 1.5 m. Solving for 2r, i. e. the diameter of the cylinder:

$$2r = \frac{1}{h}\left(\frac{c}{2}\right)^2 + h \qquad 7)$$

$$2r = \frac{1}{0.0005}\left(\frac{1.5}{2}\right)^2 + 0.0005 = 1125\ \text{m or } r = 563\ \text{m}$$

Maximum allowed pressure difference

Using 3):

$$P_c = \frac{2\,T}{2\,r} = \frac{2\,(251)}{1125} = 0.45 \text{ Pa } (6.4 \times 10^{-5} \text{ psi})$$

This is the maximum allowed pressure difference between the two chambers.

Estimated pressure drop across chambers

The maximum flow rate into the TPC during operation is 1 change of gas every 1.4 hours. The TPC volume is about 50 m^3.

$$q = \frac{50}{(1.4)(3600)} = 9.9 \times 10^{-3} \text{ m}^3/\text{s, say } 0.01 \text{ m}^3/\text{s}$$

$$A_c = \frac{\pi}{4}\left(D_o^2 - D_i^2\right) \qquad \text{where:} \qquad 8)$$

A_c = chamber area
D_o = OFC diameter
D_i = IFC diameter

$$A_c = \frac{\pi}{4}\left(4^2 - 1^2\right) = 11.8 \text{ m}^2$$

For ideal, incompressible fluid flow at the chamber an in the gap;

$$y_c + \frac{v_c^2}{2g} = y_g + \frac{v_g^2}{2g} \qquad \text{where:} \qquad 9)$$

y = pressure head
v = velocity

$$y_c - y_g = \frac{v_g^2}{2g} - \frac{v_c^2}{2g} = \Delta y \qquad 10)$$

The average gas velocity in the tracking chamber is:

$$q = Av = A_c\, v_c = A_g\, v_g \qquad 11)$$

$$v_c = \frac{q}{A_c} = v_g \frac{A_g}{A_c}$$

where:

q = flowrate, m^3/s
v_c = velocity in chamber, m/s

$$\Delta y = \frac{v_g^2}{2g}\left(1 - \left(\frac{A_g}{A_c}\right)^2\right) \cong \frac{v_g^2}{2g} \qquad 12)$$

$$v_g = \frac{0.01}{0.24} = 4.2 \times 10^{-2} \text{ m/s}$$

$$\Delta y = \frac{(4.2 \times 10^{-2})^2}{2 \;\; 9.8} = 9 \times 10^{-5} \text{ m } (3.5 \times 10^{-3} \text{ in}) \text{ of argon}$$

$$\rho_{ar} = 1.6 \text{ kg/m}^3$$

$$\Delta P = \rho\, g\;\; \Delta y = (1.6)(9.8)(9 \times 10^{-5}) = 0.0014 \text{ Pa}$$

$$1 \text{ Pa} = 1.45 \times 10^{-4} \text{ psi};\ \Delta P = 2.0 \times 10^{-7} \text{ psi}$$

$$\frac{P_{allowed}}{\Delta P} = \frac{0.45}{0.0014} = 321$$

For the idealized case, the estimated pressure is about 321 times that allowed. For a sluice gate:

$$\Delta y \propto \left(\frac{1}{K_s}\right)2 \qquad 13)$$

where:

K_s = sluice coefficient

Ks usually have a value of 0.55 to 0.60 for free flow, but is reduced when the flow downstream is submerged.[3]

$$\text{If } \frac{P_{allowed}}{\Delta P} = 1 \text{ then } (\frac{1}{Ks})2 = 321 \text{ or:}$$

$$K_s = 0.06 \text{ (minimum)}$$

Effective conductance of the gaps

Using; $q = C\ \Delta P$ where: 14)

C = conductance, l/s

ΔP = pressure drop, torr

q = flowrate, torr l/s

1 psi = 51.7 torr

$$\Delta P = 2.0 \times 10^{-7} \text{ psi} = 1.0 \times 10^{-5} \text{ torr}$$

$$q = 0.01\ (1000)\ (760) = 7600 \text{ torr l/s}$$

$$C_{eff} = \frac{7600}{1.0 \times 10^{-5}} = 7.6 \times 10^{8} \text{ l/s}$$

Pressure drop in supply and return pipes

The conductance of a pipe with viscous flow is:

$$C_p = 2.27 \times 10^{-2} \frac{D^4}{\eta L} P_{av}\ ^4$$ where: 15)

C_p = conductance, l/s

D = Pipe diameter, cm

η = Viscosity, poise

L = Length, cm

P_{av} = Avg. pressure, torr

Assuming a 10-cm diameter pipe that goes along 1/2 of the iron, down the iron and back into the TPC, L is about = 363 + 100 + 5.25 + 66 = 534 cm, say 600 cm. (Refer to Integration drawings "Interface Envelope", dated 3/93 by Doug Fritz).

$$C_p = 2.27 \times 10^{-2} \frac{10^4}{(2.29 \times 10^{-4})(600)} 760 = 1.3 \times 10^6 \ \mathrm{l/s}$$

$$q = C\,\Delta P; \ \ \Delta P = \frac{q}{C} = \frac{7600}{1.3 \times 10^6} = 5.8 \times 10^{-3} \text{ torr}$$

$$1 \text{ torr} = 133 \text{ Pa}$$

$$\Delta P = 0.078 \text{ Pa} = 1.1 \times 10^{-4} \text{ psi}$$

The pressure drop down the pipe is much greater than through the central membrane. For an error in the diameter of the pipe:

$$\frac{\Delta C}{\Delta D} = 4\,K\,D^3 \qquad \text{where:}$$

K = constant

$$\frac{\Delta C}{C} = 4\frac{\Delta D}{D} \qquad 16)$$

If the error in the pipe diameter is 0.51 mm (0.020"); then the change in the conductance is:

$$\frac{\Delta C}{C} = 4\frac{0.05}{10} = 0.02 = 2\%$$

Note that the effective conductance of the gaps is about 580 times that of the pipe.

Electrostatic force due to displacement in z

Consider the left side of the CM,:

$F_1 = E_1 Q_1$ where: 17)

F_1 = force due to charges
E_1 = electric field
Q_1 = total charge

$Q_1 = \sigma_1 A$ where: 18)

σ_1 = surface charge/area

Substitute into 17):

$$F_1 = E_1 \sigma_1 A \qquad 19)$$

Using Gauss' law for a box at one side of the CM surface:

$\varepsilon_0 E_1 A = \sigma_1 A$ where: 20)

ε_o = permeability of vacuum

Combining 19) and 20):

$$F_1 = E_1 \varepsilon_0 E_1 A = \varepsilon_0 E_1^2 A \qquad 21)$$

$\frac{F_1}{A} = P_1 = \varepsilon_0 E_1^2$ where: 22)

P_1 = pressure due to electrostatics

The net pressure on the membrane using 22) is:

$$P = (P_1 - P_2) = \varepsilon_0 (E_1^2 - E_2^2) \qquad 23)$$

$$E_1 = \frac{V}{L + h} \quad \text{and } E_2 = \frac{V}{L - h} \qquad 24)$$

Squaring these terms and noting that h << L gives:

$$E_1^2 - E_2^2 \cong V^2 \frac{4\,L\,h}{L^4 - 4\,L^2\,h^2} \cong V^2 \frac{4\,h}{L^3} \tag{25}$$

$$P \cong \varepsilon_0\, V^2 \frac{4\,h}{L^3} \tag{26}$$

The net pressure is toward the closest ground

With: $\varepsilon_0 = 8.85 \times 10^{-12}$ coul/V m
$L = 2.1$ m
$V = 100$ kV

$P = 0.038\,h\ \ N/m^2$

For a cylinder with a deflection h, and combining 3) and 7):

$$P_c = \frac{2\,T}{\frac{1}{h}\left(\frac{c}{2}\right)^2 + h} \tag{27}$$

For h << c:

$$P_c \cong \frac{4\,(2\,T)\,h}{c^2} = \frac{8\,T\ h}{c^2} \tag{28}$$

Using the same values as above:

$$Pc \cong \frac{8\,(251)\,h}{1.5^2} = 892\ \ h$$

Therefore:

$$P_c \cong 24{,}000 \text{ times } P_{electrostatic}$$