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Sizing of TPC/Magnet Iron Support Structures

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Abstract

The STAR Time Projection Chamber (TPC) is held in place by four supports which are bolted to the magnet iron ring. The primary parts of these supports are a beam, flange, beam-to-flange weld and bolts. This note describes calculations made to select the size of these support parts. The TPC is suspended off the magnet return iron ring using a modified kinematic configuration with two points hanging from a wire rope, one hanging from a flexural hinge and one supported by a stiff post. All the supports have a *y* adjustment, the post has an *x* and *z* and the flexural joint has an *x* adjustment available. The estimated weight of the TPC plus the Time of Flight (TOF) is 34,000 lb. The weight to be carried by any support is assumed to be one forth of this amount. The rectangular 6061-T6 aluminum beam is 11.5 x 4.25 with a 0.5 in wall. This beam is attached to a 3 in thick, 18 in high flange plate using a 0.5-in weld. The welded subassembly must be heat treated to regain the T6 strength. The plate is held to the magnet iron ring by eight 1.25 in SAE grade 5 bolts. The minimum safety factor for the normal loading condition was 4 and with the addition of a 0.1g side load, the smallest safety factor was 3.

Introduction

The TPC/magnet iron supports must maintain the TPC position relative to the magnet to within ± 0.5 mm. They must hold the TPC in position but not produce any large stresses on its structure which could cause local departures from the required nominal position. As shown in Figure 1, a modified kinematic four-point support concept is used. With this type of support torsion can be applied to the large cylindrical TPC structure. By proper adjustment in *y,* this torsion load can be reduced to zero. Two of the TPC suspension points are wire ropes, shown in Figure 2, which eliminate any restraining forces in *x* and *z*. One of the supports is a flexural hinge, shown in Figure 3. that allows movement in *z* but resists movement in *x*.

The thickness of this hinge is selected such that its moment of inertia in the *z* direction is small compared to that of the stiff post support. The rigid post support, shown in Figure 3, locks the TPC in the *x-z* plane at one point, the flexural hinge locates the opposite end in *x* and all supports are adjustable in *y*.

The weight of the TPC and TOF is estimated to be a maximum of 34,000 lb as designed, therefore, each of the supports will have 8,500 lb applied when adjusted properly. The weight during installation (without the readout cards, power cables, Inner Field Cage (IFC), Silicon Vertex Tracker (SVT) and the TOF) is estimated to be 20,000 lb. It is possible during installation for any single support to carry half of the weight. A minimum SF of 4 was determined for the baseline support design at the operating load which gives a SF of 3.4 for the worst case during initial installation. If the TOF is removed with the TPC, the estimated weight is 30,000 lb. For this case, one support could have 15,000 lb applied giving a minimum SF of 2.3. A method of precluding this case is discussed. For the 33-in long aluminum 11.5 x 4.25 x 0.5 in wan rectangular beam, the deflection is 0.045 in after installation and with all the TPC components attached. After the beam was selected, the flange, beam-to-flange weld and bolts were sized. The minimum SF for these components is at least 4 for the operational load.

This note will provide information on the calculations used to size the components. An appendix is included which details the calculations and values used.

TPC and TOF weight

Table 1 shows the estimated operational weight of the TPC.

Table 1
Estimated operational weight of the TPC

Item	Total weight,lb	Total weight,kg
Sectors	9100	4100
OFC	7500	3400
Wheel/supports	3200	1500
IFC	80	36
Cooling	100	45
Laser	500	230
SVT	460	210
TOF	10000	4500
Subtotal	30900	14000
Add 10%	3100	1400
Total	34000	15400

The installation weight is less than this due to the absence of the readout cards, power cables, IFC, SVT and TOF. The estimated weight at installation is about 20,000 lb. If the TOF were to be attached to the TPC prior to installation, the weight would increase to 30,000 lb.

Safety factor

The guidelines for design of STAR mechanical structures specify that the SF should be at least 3 on the yield strength and if human safety is an issue, then 5 should be used.[1] An operating SF of 4 was used initially for designing the TPC supports. The high value was selected since the actual weight is not well known, the equipment is expensive and a failure would have serious effects on the program. The yield strength used in Table 2 is defined by reference 2 as "At least 99% of the population of values is expected to equal or exceed the ..(yield strength).. with a confidence of 95 percent."

The operating weight on each support is 34,000/4 = 8,500 lb. During installation, it is possible for half of the installation weight, or 10,000 lb, to be placed on any one support. If the TPC were installed with the TOF already in place on the Outer Field Cage, the load at installation could be as high as 30,000 lb, therefore, a 15,000 lb load could be placed on any one support. The SF for this case would still be > 2. A method of ensuring that this high loading does not occur will be discussed later.

Description of supports

The TPC supports use a modified kinematic mount arrangement which reduces stresses in the main structure. A sketch of this arrangement is shown in Figure 1. A support, shown in Figure 5, consist of a flange plate bolted to the magnet return iron ring and a beam welded to this plate which spans the distance from the iron ring to the TPC. Adjustment mechanisms are attached to the TPC end of the beam. Two of the supports use wire ropes, one has a flexural hinge and one a rigid post. These configurations are shown on Figures 2 through 4, respectively. The wire rope will not support any *x* or *z* forces and is adjustable in *y* only. There can be no *x* or *z* stresses generated due to forces in these directions at this support.

The flexural hinge will not support any significant force in the *z* direction or resist a small rotation about its vertical axis. It has adjustments for *x* and *y*. The stiff rod support has *x, y*, and *z* adjustments.

During installation, the deflection of the beam will be monitored by use of a pointer or a dial gage as the *y* adjustment is made. A base with a smaller beam can be attached to the flange and a dial gage can indicate the deflection of the main support beam. A load-versus-deflection test will be conducted on each support prior to its first use. During installation, the load on each support can be controlled by observing the deflection as *y* adjustments are made.

Sizing of support components

Beam

As illustrated by Figure 5, the support beam has a span of about 33 in. The operating force is 8,500 lb which gives a maximum moment of 280,000 lb-in. The limits on the beam size are about 5 in width and about 12 in height. The beam will be fabricated from plates and its configuration win be a rectangular shape. In order to keep the wall thickness small and minimize weight, the overall height and width should be close to the maximum allowable; therefore, a 11.5-in high 4.25-in wide beam was selected. Using the relationships:

$$\sigma = \frac{Mc}{I} = \frac{M}{S}$$

where: M = Bending moment

I = Moment of inertia of rectangular beam
c = Distance to farthest fiber
$S = \frac{I}{c} =$ Section modulus

and:

$$y = \frac{Fl^3}{3El}$$

where: Force on cantilevered beam

l= Beam length
E= Modulus of elasticity

and assuming a SF of 4, the following table was generated.

Table 2
Material and possible beam wall thickness
Load = 8500 lb, Beam length = 33 in
Height 11.5 in, Width = 4.25 in

Material	Yield stress kpsi[2]	Allowed stress, kpsi	Wall thickness, in	Deflection, in	Weight per in, lb	Beam weight, lb
1025 stl	36	9.0	0.40	0.019	3.6	118
304 ss	40	10.0	0.35	0.021	3.1	103
2024-T4	47	11.8	0.29	0.074	0.9	30
5052-H36	29	7.3	0.51	0.046	1.6	52
6061-T4	16	4.0	1.18	0.025	3.6	120
6061-T6	36	9.0	0.40	0.057	1.2	41
7075-T6	70	17.5	0.25	0.110	0.8	25

A 11.5 x 4.25 in beam with a 0.5-in wall made of 6061-T6 is chosen as the baseline design. This material is readily available and is easily welded. With the 0.5-in wall, the deflection is reduced to 0.045 in and the beam weight increases to 51 lb.

The vertical shear stress in the beam is:

$$\tau_V = \frac{F}{A} = \frac{8500}{0.5 * 2 * (11.50 + 4.25)} = 540 \text{ psi}$$

With the thicker wall, the maximum bending stress is reduced to 7,200 psi and if the shear stress is neglected, a SF of 5 for the beam is obtained.

Beam to flange weld

The 0.5-in thick beam is welded to the flange. The weld configuration shown in Figure 6. There are three components of stress in this weld. There is the shear stress due to the force acting downward, the shear stress due to a torque about the *x-x* axis and a tensile stress due to a moment about the *z-z* axis. For a weld, the area is calculated using the throat length.[3] The shear stress in the weld due to the downward force is:

$$\tau_{\mathrm{V}} = \tau_{\mathrm{zy}} = \frac{F}{A} = \frac{8500}{12.2} = 700 \text{ psi}$$

The second shear stress is due to a 190,000 in-lb moment about the *x-x* axis. The unit second polar moment of area for the weld is:[3]

$$\mathrm{J_u} = \frac{(l+b)^3}{6} = 850 \text{ in}^3$$

The second polar moment of area of the weld of size *h* is:

$$J = J_u \, 0.707 \, h = 300 \text{ in}^4$$

The maximum shear stress due to the moment around the *x-x* axis is:

$$\tau_{yx} = \frac{Mr}{J} = \frac{M * \left(b^2 + l^2\right)^{0.5} * 0.5}{J}$$

$$= 4{,}100 \text{ psi}$$

The bending stress is calculated using the unit second moment of the weld area.

$$I_u = \frac{l^2}{6}(3b + l) = 630 \text{ in}^3$$

The second moment of area for the weld is:

$$I = I_u 0.707h = 220 \text{ in}^4$$

The bending stress due to the moment about the *z-z* axis is:

$$\sigma_z = \frac{\text{Mc}}{I} = \frac{\text{M} * l * 0.5}{I}$$

=5,500 psi

These three stresses act concurrently at the corner of the weld. For this triaxial case of stress:[3]

$$\sigma^3 - \sigma_x \sigma^2 - (\tau_{zy}^2 + \tau_{xy}^2)\,\sigma + \sigma_x \tau_{xy}^2 = 0$$

By plotting this equation and using Excel Solver, the principle stresses were determined to be:

σ_1 = 5,700 psi
σ_2 = 3,900 psi
σ_3 = -4,100 psi

Since the material is ductile, the predicted stress is compared to the von Mises stress to determine the SF. Yielding is predicted to occur when σ' > yield strength of the material.

$$\sigma' = \left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2}\right]^{1/2}$$

= 9,000 psi

For the 6061-T6 selected, the allowed tensile stress for the as-welded condition is 0.4σy or 14,400 psi. In order to keep the SF at about 4, the part must be heat treated after welding to regain the T6 condition.

Support flange-magnet iron ring

The beam is welded to a flange that is bolted to the magnet iron ring. Due to the solenoid and iron ring design, this flange can only be bolted on one side of the beam. This arrangement is shown in Figure 7. The height of the flange plate is greater than that of the beam, but must be less than about 24 in. This is to ensure there is no interference with the installation or removal of the calorimeter. If the height is 18 in and the thickness is 3 in, then the inertia about the *x-x axis* is 1,500 in.^{4} There are bending moments about the *x-x and* the *z-z* axes. These are 213,000 lb-in and 238,000 lb-in, respectively. The moment about the *x-x* axis will cause a bending stress in the plate of 1,300 psi. The moment about the *z-z* axis is a polar moment as it is trying to twist the plate and tear it. The maximum torsional stress in a rectangular section is:[3]

$$\tau_{\max} = \frac{M}{wt^2}\left(3 + 1.8\frac{t}{w}\right)$$ where:

τ_{max} = Maximum torsional stress

M = torque
w = flange height
t = flange thickness

For the case chosen, with M = 238,000 lb-in, w = 18 in and t = 3 in, the maximum torsional stress is about 4,900 psi. The location of this stress is at the midpoint of the 18-in high side. This location is where the bending stress is zero. The allowed stress due to shear is normally about 60% of the tensile strength. For the 6061-T6, this value is 21,600 psi giving a SF of 4.4. The stress due to the vertical shear is:

$$\tau_{\mathrm{V}} = \frac{F}{A} = \frac{8500}{18 \times 3} = 160 \text{ psi}$$

This was neglected in the analysis.

Bolts-flange to magnet end ring

The baseline design for the supports allow the installation and removal of the TOF and the EMC without removing the TPC. This places a limit of about 18 in on the allowed height of the flange plate. If the usual guidelines for bolt locations from an edge and each other are used, there is only enough room for four 1.25-in diameter bolts along the 18-in side. Two columns of bolts, as shown in Figure 8, are required to keep the stress low enough to maintain a SF of 4. It is assumed that a grade 5 bolt will be used that has a yield of 80,000 psi. The bolts are subjected to a vertical shear, a shear due to rotation about the *x-x* axis and a tensile load due to bending about the *z-z* axis. The vertical shear stress is:

$$\tau_{xy} = \frac{F}{A} = 1{,}100 \text{ psi}$$

The moment about the *x-x* axis must account for the slightly longer lever arm due to the location of the bolt centroid. The bolt centroid is about 7.2 in from the beam weld center. The moment about the *x-x* axis is 250,000 in-lb. Taking moments about the centroid, the force in the outer bolts was calculated to be 7,000 lb giving a shear of:

$$\tau_{yz} = \frac{F}{A} = 7{,}200 \text{ psi}$$

The force on the outer bolts was calculated to be 6,800 lb due to the moment about the *z-z* axis. This gives a tensile stress of:

$$\sigma_x = \frac{F}{A} = 7{,}000 \text{ psi}$$

Again, a triaxial state of stress exists. By using the same methods described above, the principle stresses were determined to be:

$\sigma_1 = 7{,}900$ psi
$\sigma_2 = 6{,}300$ psi
$\sigma_3 = -7{,}200$ psi

The von Mises stress was calculated to be:

$$\sigma' = 14{,}000 \text{ psi}$$

which gives a SF of about 5.7 for the bolts.

The bolts will be attached to the magnet iron ring. This iron has a yield strength of 25,000 psi.[4] The force required to pullout a bolt is estimated by:[5]

$$F_{pullout} = \frac{\pi D_m F_s L}{3}$$

where: D_m = mean bolt diameter
F_s = shear yield stress
L = thread engagement

For our case, and assuming a thread engagement of 2D:

$$F_{pullout} = \frac{\pi 1.1 * 0.6 * 25{,}000 * 2 * 1.25}{3} = 43{,}000 \text{ lb}$$

The force on the outer bolts was estimated to be 6,800 lb which gives a SF of 6.3.

Side loading due to earthquake or sudden movements

Although there is not a specific earthquake requirement at BNL, there will be side loading on the supports due to moving the detector. It is assumed that these side movements win cause a loading on the supports equal to 0.1 g. If the load is in the x direction, the force will be distributed between the flexural and rigid supports. If the load is in the z direction, the load will be resisted by the rigid post support only. The load will be 0. 1 *34,000 = 3,400 lb directed along the z direction. This load will cause a moment to appear about the y-y axis, a shear stress along the z-z axis, and a compressive stress in the beam. The area of the beam is 16 in^2 and the area of the weld is 9.1 in ^{2}.

The shear stress in the weld is $\frac{3400}{9.1} = 370$ psi. Figure 9 shows the force resolved into two components, a compressive force and a moment force. The compressive force is 2,300 lb and the moment force is 2,500 lb. The compressive stress in the beam is $\frac{2300}{16} = 140$ psi.

The maximum bending moment resisted by the beam is 2,500*33 = 83,000 lb-in. The moment of inertia of the beam for the narrow section is 44 in^4. The maximum bending stress is 4,000 psi. This stress will be added to the normal operating bending stress due to the downward force. The total maximum bending stress in the beam will be 7,200 + 4,000 = 11,200 psi which gives a SF Of 3.2.

The weld will also experience an increase in the bending stress and in shear along the *z* direction. The shear stress increase (370 psi) can be neglected. The bending stress will be reduced slightly by the compressive stress, but this effect can also be neglected. The bending stress in the weld due to the 0.1-g load is:

$$\sigma_z = 1,400 \text{ psi}$$

This stress must be added to the 7,100 psi bending stress due to the downward force. Solving again for the three principle stresses:

$$\sigma_1 = 7,300 \text{ psi}$$
$$\sigma_2 = 4,900 \text{ psi}$$
$$\sigma_3 = -5,300 \text{ psi}$$

The von Mises stress is 11,600 psi which gives a SF of 3.1 for the 0.1-g loading case.

Summary

The final estimated safety factors for the three major support components are shown on Table 3. The SF values are based upon the normal operating load.

Table 3
Safety factors with operating load

Component	Von Mises stress, psi	Safety factor
Beam	7,200	5
Weld	9,000	4 *
Flange	4,900	4.4
Bolts	14,000	5.7

*Assumes post weld heat treat.

The minimum estimated SF is 4 for operating loads. The envisioned absolute worst case is applying 15,000 lb to one support. This is possible if the TPC is removed with the TOF attached. This is a load increase of about 76%. The minimum SF would still be > 2. The use of a dial gage during installation to control the *y* adjustments will preclude the application of such a large load on any one support. Each support will be proof tested at 1.5 times the operating load. During this test, a load-vs-deflection table can be generated for use during *y* adjustments.

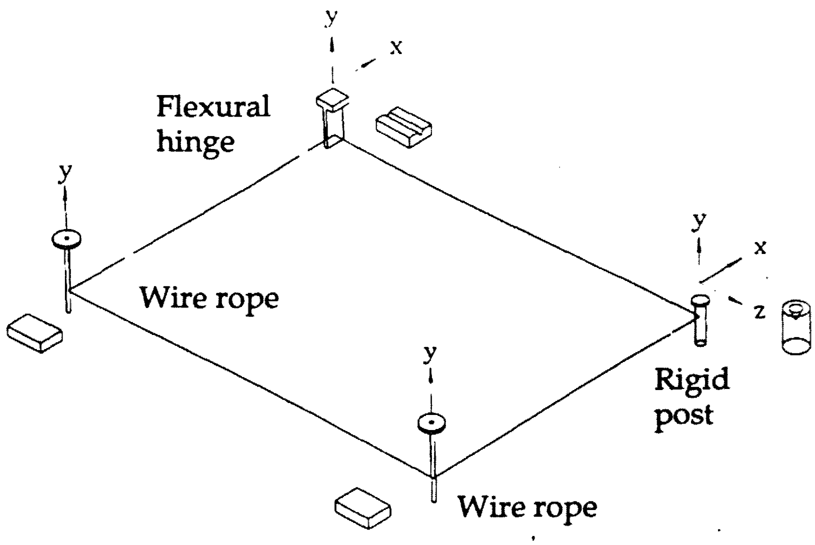


Figure 1. Sketch showing modified kinematic TPC mounts.

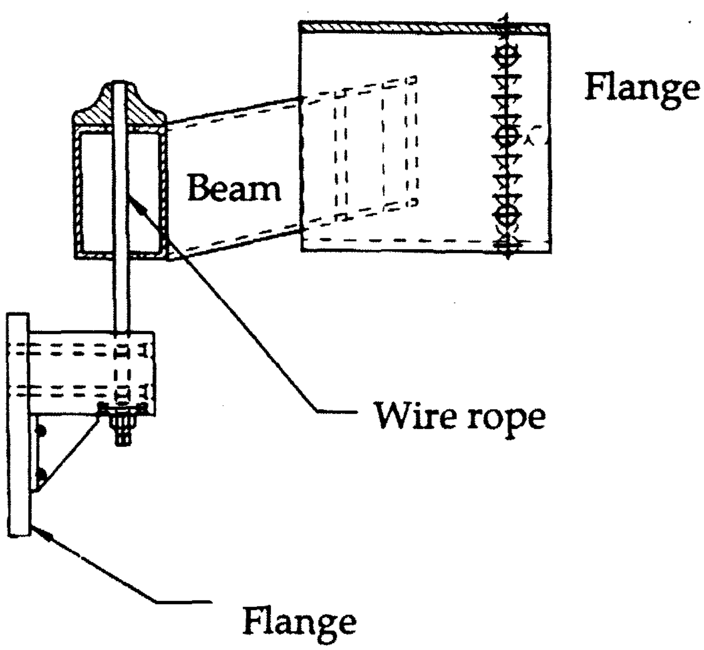


Figure 2. TPC wire rope support to locate *y* at two points.

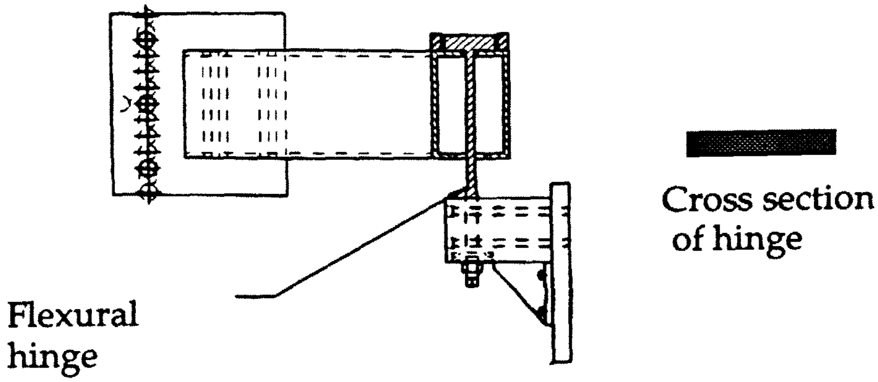


Figure 3. TPC 1-D flexural support to locate *x* and *y* at one point.

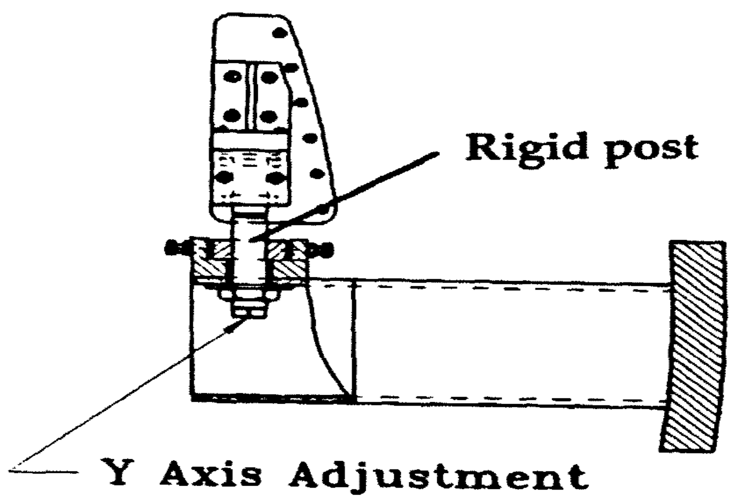


Figure 4. TPC rigid cone support to locate *x*, *y* and *z*.

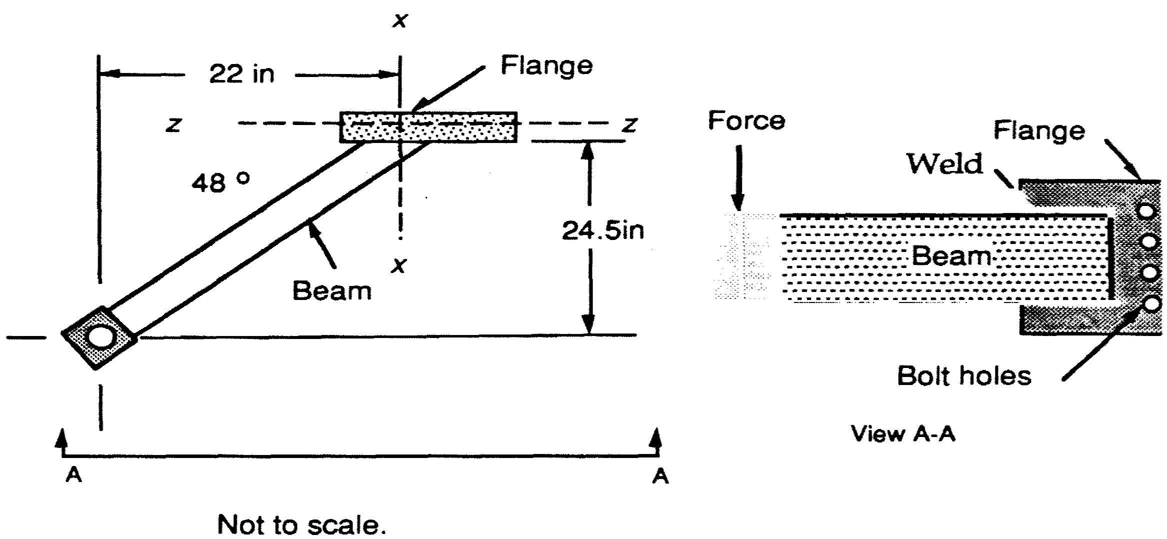


Figure 5. Basic TPC support.

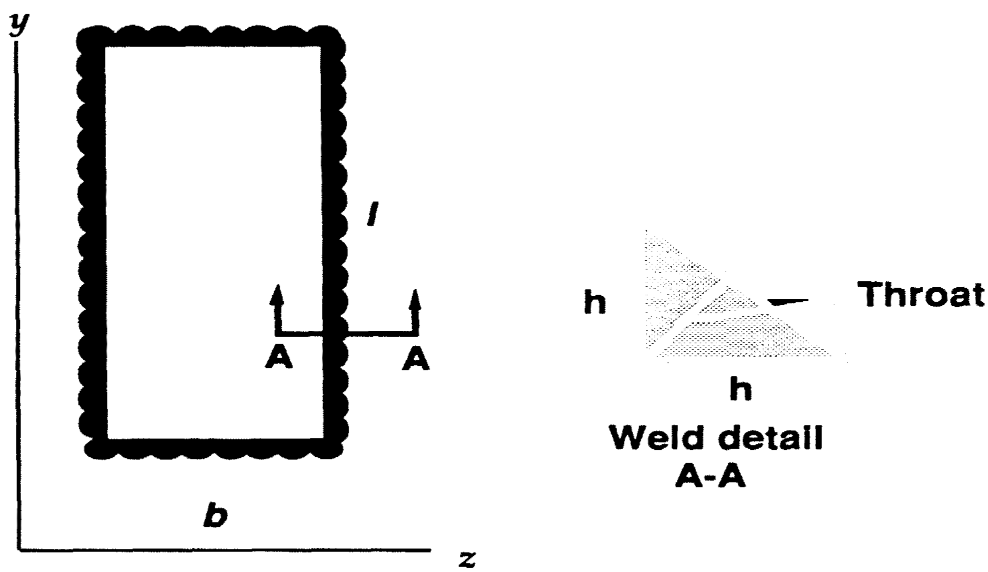


Figure 6. The configuration of the beam-to-flange plate weld.

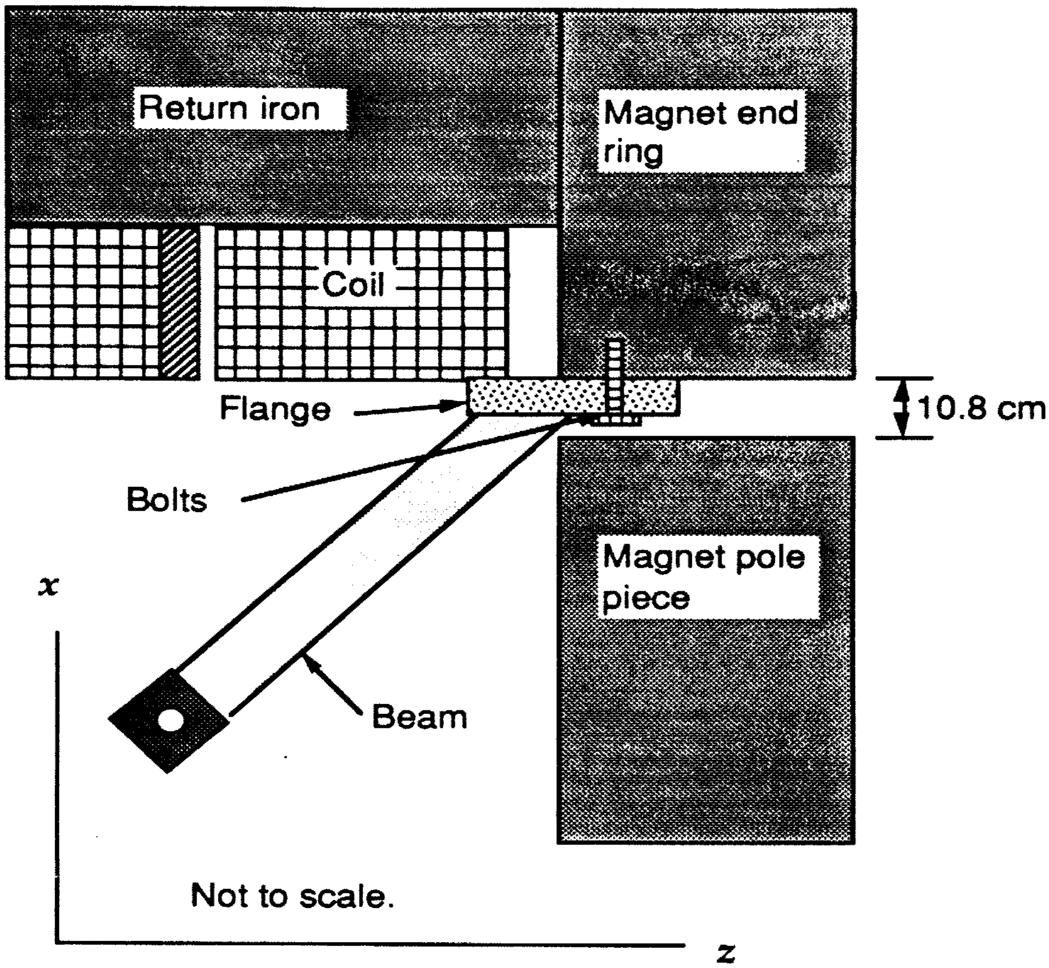


Figure 7. Plan view of the support-magnet iron ring configuration.

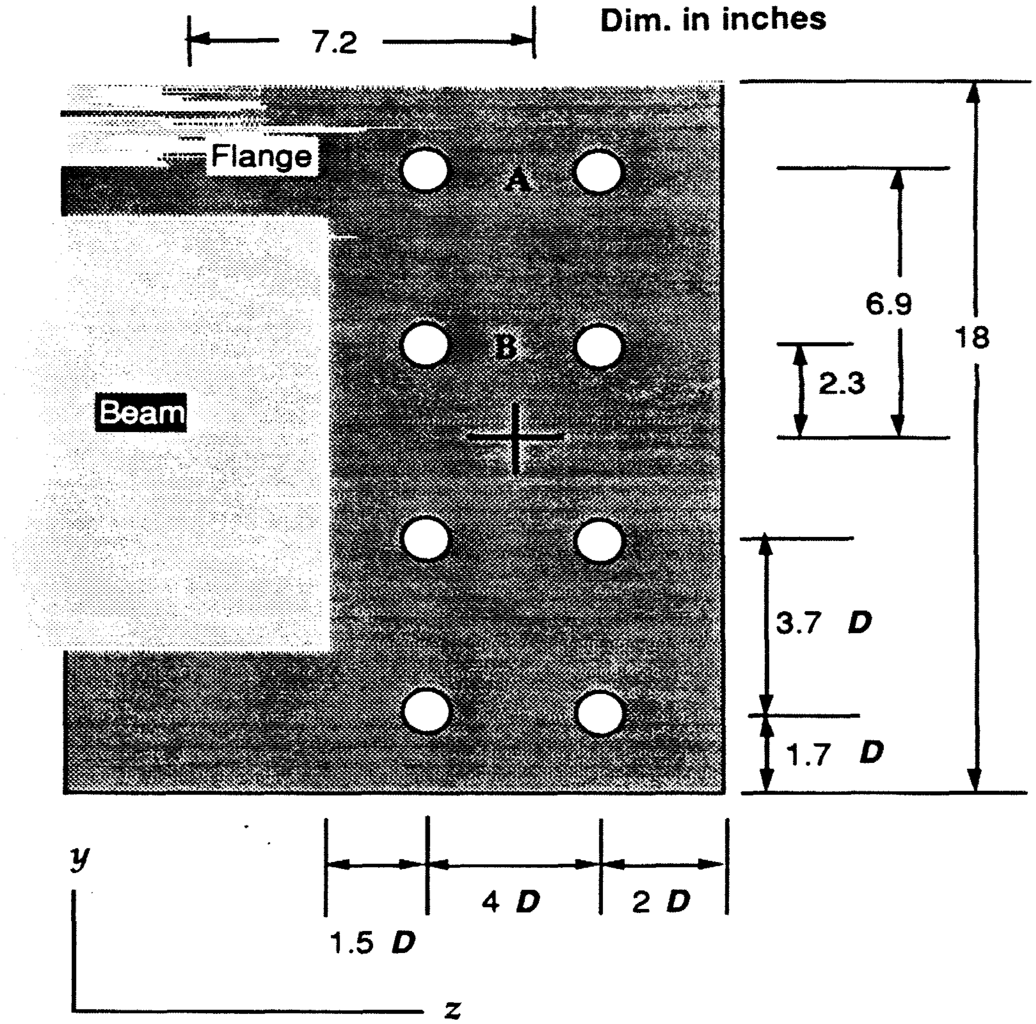


Figure 8. Configuration of the bolt pattern for the flange plate.

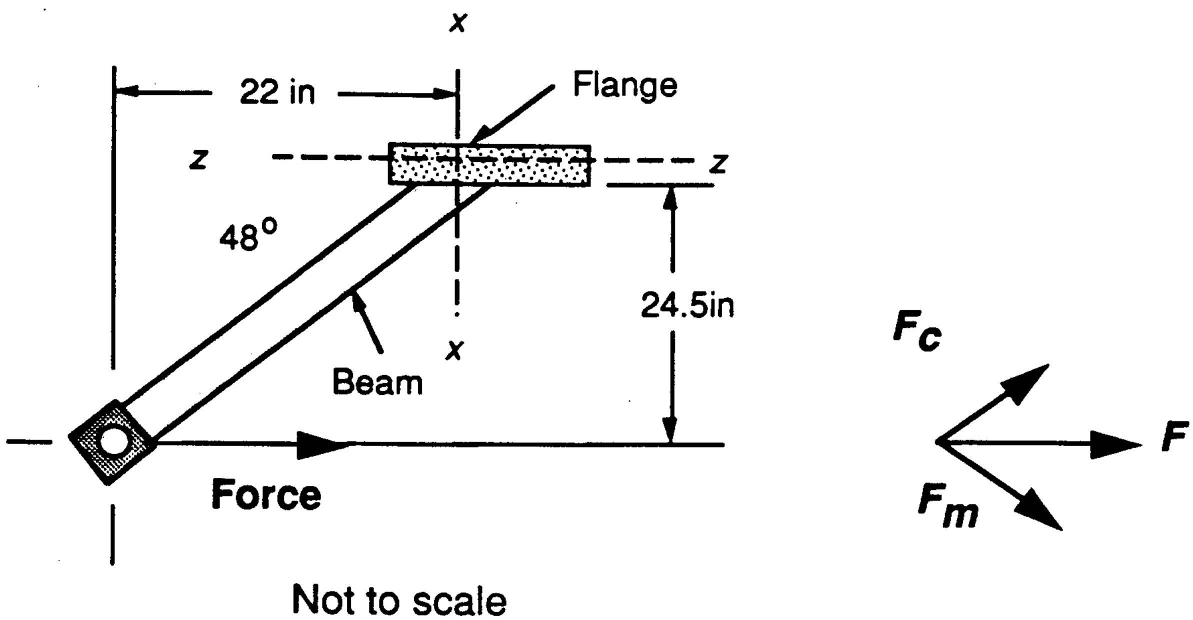


Figure 9. The loading due to an 0.1 g force along *z*. F_c is the compressive component and F_m is the moment component of the force.

References

1 Edwards, W., STAR Mechanical Design Standards and Guidelines", STAR Note 105, April 27,1993.

2 MIL-HDBK-5C, "Metallic Materials and Elements for Aerospace Vehicle Structures", Naval Publications and Forms Center, 5801 Tabor Ave, Philadelphia, Pennsylvania 19120.

3. Shigley, J. E. and Mischke, C. R., <u>Mechanical Engineering Design</u> 5th ed. (McGraw-Hill Book Company**, New York,** 1989), p. 55.

4 Brown, R., Private communication, May, 1993.

5. Barrett, R. T., "Fastener Design Manual", 1990.

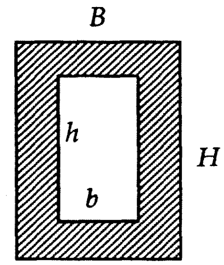
6. Young, W. C., <u>Roark's Formulas for Stress and Strain</u>, McGraw-Hill Book Company, New York, 1989.

Sizing of TPC/Magnet Iron Support Structures

Sizing of support components

Beam

It is assumed that a rectangular beam with a constant wall thickness will be used. This will give good bending and torsion stiffness. For a rectangular beam:[6]



$$I = \frac{BH^3 - bh^3}{12}$$

$$\sigma = \frac{M_c}{I} = \frac{M}{S}$$

where: M= Bending moment
I = Moment of inertia
c = Distance to farthest fiber
$s = \frac{I}{c}$ = Section modulus

and:

$$y = \frac{Fl^3}{3El}$$

where: F= Force on cantilevered beam
I = Beam length
E= Modulus of elasticity

The available area allows a 4.25 x 11 .5 in rectangular beam to be used . Table 2 was generated using the above equations and a SF = 4. The material selected was 6061-T6 aluminum with a yield strength of 36 kpsi. Although Table 2 indicates that a 0.4-in wan will suffice, a standard plate thickness of 0.5-in was selected. As shown in Figure 5, the beam length is about 33 in.

With H = 11.5, B = 4.25 and wall thickness = 0.5 in:

$$I = \frac{4.25 * 11.5^3 - 3.25 * 10.5^3}{12} = 225 \text{ in}^4$$

The bending stress is:

$$\sigma = \frac{Mc}{I} = \frac{8{,}500 * 33}{225} * \frac{11.5}{2} = 7{,}200 \text{ psi}$$

The shear stress is:

$$\tau = \frac{F}{A} = \frac{8{,}500}{2*(11.5+4.25)*0.5} = 540 \text{ psi}$$

If the small shear stress is neglected, there is a SF of 5 on the beam. The deflection of an aluminum beam is:

$$y = \frac{Fl^3}{3EI} = \frac{8{,}500*33^3}{3*1*10^7*225} = 0.045 \text{ in}$$

Beam to flange weld

A 0.5-in weld will be assumed and the magnitude of the various types of stress will be calculated. For this assumption, the value of h shown in Figure 6 is 0.5 in. The height of the weld, l is 11.5 in and the width b is 4.25/sin 48° = 5.72 in. The total weld area is:

$$A_{weld} = 0.707 * 0.5 * 2 * (11.5 + 5.72) = 12.2 \text{ in}^2$$

The shear stress in the weld due to the force acting downward is:

$$\tau_V = \tau_{zy} = \frac{F}{A} = \frac{8{,}500}{12.2} = 700 \text{ psi}$$

The moment at the weld can be broken down into two components about the x and z axes. The moments are 190,000 and 210,000 lb-in, respectively. The moment about the x axis will produce a shear stress. The unit second polar moment of area is:[3]

$$J_u = \frac{(l+b)^3}{6} = \frac{(11.5+5.72)^3}{6} = 850$$

The second polar moment of area of the weld of size h is:

$$J = J_u\, 0.707h = 850 *(0.707*0.5) = 300$$

The maximum shear stress due to the moment around the x-x axis is:[3]

$$\tau_{yx} = \frac{Mr}{J} = \frac{M*\left(b^2+l^2\right)^{0.5}*0.5}{J}$$

$$= \frac{190{,}000*\left(5.72^2+11.5^2\right)^{0.5}*0.5}{300} = 4{,}100 \text{ psi}$$

The moment about the z-z axis will produce a bending stress in the weld. The unit second moment of area for the weld is:[3]

$$I_u = \frac{l^2}{6}(3b + l) = \frac{11.5^2}{6}(3 * 5.72 + 11.5) = 630$$

The second moment of area for the weld is:

$$I = I_u \ 0.707h = 630*(0.707*0.5) = 220$$

The bending stress due to the moment about the *z-z* axis is:

$$\sigma_z = \frac{Mc}{I} = \frac{M * l * 0.5}{I}$$

$$= \frac{210{,}000 * 11.5 * 0.5}{220} = 5{,}500$$

The weld is subjected to the following stresses:

τ_{zy} = 700 psi
τ_{yx} = 4,100 psi
σ_z = 5,500 psi

At the comers of the weld these three stresses act concurrently in three directions, therefore, the principle stresses must be determined. For the triaxial case above:

$$\sigma^3 - \sigma_z \sigma^2 - (\tau_{zy}^2 + \tau_{xy}^2)\,\sigma + \sigma_z \ \tau_{xy}^2 = 0$$
$$\sigma^3 - 5{,}500\ \sigma^2 - (700^2 + 4,\ 100^2)\ \sigma + 5{,}500*4{,}100^2 = 0$$
$$\sigma^3 - 5{,}500\ \sigma^2 - 17{,}300{,}000\ \sigma + 0.92e^{11} = 0$$

There are three solutions to this equation. These were found by plotting the function and using Excel Solver. The solutions are:

σ_1 = 5,700 psi
σ_2 = 3,900 psi
σ_3 = -4,100 psi

Using the von Mises theory that failure occurs when $\sigma' > \sigma$ tensile:

$$\sigma' = \left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2}\right]^{1/2}$$

$$= \left[(5,700 - 3,900)^2 + (3,900 - (-4,100))^2 + (5,700 - (-4,100))^2\right]^{1/2}$$

$$= 9,000 \text{ psi}$$

The 0.5-in weld will give a SF of 4 on the parent material strength. For 6061-T6, the allowed tensile stress for the as-welded condition is *0.4*σ*y* or 14,400 psi. In order to keep the SF at about 4, the part must be heat treated after welding to regain the T6 condition.

Support flange-magnet iron ring

The space available for this flange is 4.25 in (10.8 cm). Preliminary calculations indicated that a 3-in thick flange would be required. The height of the flange is required to be 18 in due to the number and size of bolts needed. The stress due to the vertical shear is:

$$\frac{F}{A} = \frac{8500}{18*3} = 160 \text{ psi or very small.}$$

There is a bending moment about the *x-x* axis for this plate. The level arm is > 22in and is assumed to be 25 in. The moment is 8,500*25=213,000 in-lb. The moment of inertial for a rectangular section is:

$$I = \frac{bh^3}{12}$$

For the plate selected:

$$I = \frac{3*18^3}{12} = 1,500 \text{ in}^4$$

The resulting stress is:

$$\sigma = \frac{213000*9}{1500} = 1,300 \text{ psi}$$

There is also a moment about the *z-z* axis applied to the rectangular plate, It's value is 8500*28 = 238,000 in-lb. An estimation for the maximum torsional stress in a rectangular section is:[3]

$$\tau_{\max} = \frac{M}{wt^2}\left(3 + 1.8\frac{t}{w}\right)$$ where: τ_{max} = Maximum torsional stress

M = torque
w = flange height
t = flange thickness

$$\tau_{\max} = \frac{238{,}000}{18 * 3^2}\left(3 + 1.8\frac{3}{18}\right) = 4{,}900 \text{ psi}$$

The location of this maximum shear stress is at the midpoints of the 18-in side. This is where the bending stress is Ó. The allowed stress due to shear is normally about 60% of the allowed tensile stress. For the 6061-T6, this value would be about 21,600 psi giving a SF of 4.4.

Bolts-flange to magnet end ring

The recommended bolt position is 2 diameters from any edge and 4 diameters between bolts. In 18 in, this allows four 1-in bolts. Preliminary calculations indicated that four 1-in diameter bolts arranged in a vertical line would experience large stresses. For this reason, eight 1.25-in bolts were selected arranged as shown in Figure 8. The bolt edge distance and spacing are slightly smaller in the vertical direction than recommended. For a 1.25-in bolt, the tensile stress area is 0.969 in^2. If a SAE grade 5 bolt is specified, the minimum proof strength is about 70 kpsi and the minimum yield strength is about 80 kpsi. Since the load is not fluctuating, the yield strength will be used for the following calculations.

The bolts will be subjected to a vertical shear, a shear due to rotation about the *x-x* axis and a tensile load due to bending about the *z-z* axis. The vertical shear stress is:

$$\tau_{xy} = \frac{F}{A} = \frac{8500}{8 * 0.969} = 1{,}100 \text{ psi}$$

The centroid of the bolts is at the center of the pattern and about 7.2 in from the center of the beam weld along *z*. This increases the length of the lever arm about the *x-x* axis to 22 + 7.2 = 29.2 in. The bolts shown in Figure 8 are lettered for convenience.

The moment about the centroid and the *x-x* axis is 8500*29.2 250,000 in-lb. The distance from the bolts to the centroid is:

$$l_A = (6.9^2 + 2.5^2)^{0.5} = 7.33$$

$$l_B = (2.3^2 + 2.5^2)^{0.5} = 3.40$$

Taking the moment about the centroid:

$$250{,}000 = 4 * F_A * + 4 l_A * F_B * l_B$$

$$F_A = \frac{7.33}{3.40} F_B = 2.16 F_B$$

250,000 = 4 * 2.16F$_B$ * 7.33 + 4 * F_B 3.40

250, 000 = F_B * 76.9

F$_B$ = 3,300 lb and F$_A$ = 7, 000 lb

The shear stress in bolt A is $\frac{7000}{0.969}$ = 7,200 psi. The tensile stress in the bolts is found by using the moment about the *z-z* axis.

8,500 * 24.5 = 4 * F$_A$ * 6.9 + 4 * F_B * 2.3

$$F_A = \frac{6.9}{2.3} F_B = 3F_B$$

210,000 = F$_B$ * 92

F_B = 2,300 lb and F$_A$ = 6,800 lb

The tensile stress in bolt A is $\frac{6800}{0.969}$ = 7,000 psi. This is a triaxial state of stress with:

τ_{xy} = 1,100 psi
τ_{zy} = 7,200 psi
σ_x = 7,000 psi

For this triaxal case:

$$\sigma^3 - \sigma_x \sigma^2 - (\tau_{zy}{}^2 + \tau_{xy}{}^2)\,\sigma + \sigma_x \tau_{zy}{}^2 = 0$$
$$\sigma^3 - 7{,}000\,\sigma^2 - (7{,}200^2 + 1{,}100^2)\,\sigma + 7{,}000 * 7{,}200^2 = 0$$
$$\sigma^3 - 7{,}000\,\sigma^2 - 53{,}100{,}000\,\sigma + 3.63e^{11} = 0$$

There are three solutions to this equation. These were found by plotting the function and using Excel Solver. The solutions are:

σ_1 = 7,900 psi
σ_2 = 6,300 psi
σ_3 = -7,200 psi

Again, using the von Mises theory that failure occurs when σ' > σ tensile:

$$\sigma' = \left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2}\right]^{1/2}$$

$$= \left[(7{,}900 - 6{,}300)^2 + \ (6{,}300 - (-7{,}200))^2 + \ (7{,}900 - (-7{,}200))^2 \right]^{1/2}$$

$$= 14{,}000 \text{ psi}$$

The safety factor for this condition is: $\frac{80000}{14000} = 5.7$.

The pullout force for a bolt or a threaded bolt hole is:[5]

$$F_{pullout} = \frac{\pi D_m F_s L}{3}$$

where: D_m = mean bolt diameter
F_s = shear yield stress
L = thread engagement

The shear yield stress for steel is about 0.6 of the tensile yield stress. The material used for the magnet end ring is 1006-1008 low carbon steel with the suggested mechanical properties of yield strength 25 kpsi and the tensile strength 35 kpsi.[4] For our case and assuming a thread engagement of 2*D:

$$F_{pullout} = \frac{\pi 1.1 * 0.6 * 25{,}000 * 2 * 1.25}{3} = 43{,}000 \ \text{lb}$$

The force on bolt A was estimated to be 6,800 lb. This is a SF of about 6.3.

Side loading due to earthquake or sudden movements

For a 0.1 g movement, the worst-case load win be 0.1*34,000 = 3,400 lb directed along the *z* direction. This load will cause a moment to appear about the *y-y* axis, a shear stress along the *z-z* axis, and a compressive stress in the beam. The area of the beam is 16 in^2 and the area of the weld is 9.1 in^2. The shear stress in the weld is $\frac{3400}{9.1}$ psi. Figure 9 shows the force resolved into two components, a compressive force and a moment force. The compressive force is, F_c = 3,400*cos(48) = 2,300 lb and the moment force is 2,500 lb. The compression stress in the beam is $\frac{2300}{16} = 140 \ \text{psi}$.

The maximum bending moment resisted by the beam is 2,500*33 = 83,000 lb-in. The moment of inertia of the beam for the narrow section is:

$$I = \frac{BH^3 - bh^3}{12}$$

$$I = \frac{11.5 * 4.25^3 - 10.5 * 3.25^3}{12}$$

$$I = 44 \ \text{in}^4$$

The bending stress is:

$$\sigma = \frac{Mc}{I}$$

$$\sigma = \frac{83{,}000 * 4.25 * 0.5}{44}$$

$$\sigma = 4{,}000 \ \ \text{psi}$$

It was calculated earlier that the bending stress on the beam due to the downward force was 7,200 psi. The 4,000 psi will be added to this stress on the 4.25-in side. The total stress will be 11,200 psi which is a SF of > 3 on the yield.

The weld will also experience an increase in the bending stress and in shear along the z direction. The shear stress increase (370 psi) can be neglected. The bending stress will be reduced slightly by the compressive stress, but this effect can also be neglected. The bending stress in the weld due to the 0.1-g load is:

$$\sigma_z = \frac{Mc}{I}$$

$$\sigma = \frac{2{,}500 * 33 * 5.72 * 0.5}{170}$$

$$\sigma = 4{,}000 \ \ \text{psi}$$

This must be added to the 7,100 psi calculated for the weld due to the downward force. The three stresses in the weld become:

τ_{zy} = 930 psi
τ_{yx} = 5,300 psi
σ_z = 6,900 psi

For this triaxial case:

$$\sigma^3 - \sigma_z \sigma^2 - (\tau_{zy}^2 + \tau_{xy}^2)\,\sigma + \sigma_z \tau_{xy}^2 = 0$$

$$\sigma^3 - 6{,}900\, \sigma^2 - (930^2 + 5{,}300^2)\,\sigma + 6{,}900 * 5{,}300^2 = 0$$

$$\sigma^3 - 6{,}900\, \sigma^2 - 29{,}000{,}000\, \sigma + 1.9e^{11} = 0$$

The principle stresses are:

σ_1 = 7,300 psi
σ_2 = 4,900 psi
σ_3= -5,300 psi

Calculating σ':

$$\sigma' = \left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2}\right]^{1/2}$$

$$= \left[(7{,}300 - 4{,}900)^2 + (4{,}900 - (-5{,}300))^2 + (7{,}300 - (-5{,}300))^2\right]^{1/2}$$

= 11,600 psi

This is a SF of 3.1.