November 19, 1993

The Impact of SCA depth Upon Position Resolution and dE/dx in the STAR TPC

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This note describes a study of the impact of SCA depth upon the drift direction position resolution (σ_z) and dE/dx resolution. The basic question addressed is the impact of using a 512 time bucket SCA, versus using a 1024 time bucket SCA, in a TPC filled with $He - C_2H_6$ and run with a drift field such that $v_d = 2.5\ cm/\mu sec$. The end result is that a 512 time bucket SCA negligibly impacts the drift direction position resolution while it degrades the dE/dx resolution by 0.8% out of a projected 6.7%.

I. Assumptions

There are two basic assumptions underlying the following calculations: i) the signal response in the longitudinal direction is well-described as a Gaussian whose width, σ_r, is the quadrature sum of the shaper response, σ_{shaper}, and the longitudinal diffusion, σ_L, and ii) the electronics signal-to-noise ratio is 20:1. For the $He - C_2H_6$ drift gas run with a drift field such that $v_d = 2.5\ cm/\mu sec$ and $\sigma_L = 160\ \mu m/\sqrt{cm}$ we have more than 5 time samples/hit over threshold with the 1024 time bucket SCA as is shown below. The signal is described as [Lyn78]

$$h(z) = h_0 exp(-z^2/2 * \sigma_r^2)$$

and in order to find the time span bracketing 15% of the maximum signal height (ie. 3 σ above noise), we solve for $z(h(z) = .15 * h_0)$ as

$$z/2 = \sqrt{(-2.ln(.15))} * \sigma_r = 1.95 * \sigma_r$$

hence,

$$\Delta_z = 3.9 * \sigma_r;$$

for sigma we need only use the longitudinal diffusion (assuming maximum drift) and shaper response in order to estimate σ_r as

$$\sqrt{\sigma_L^2 + \sigma_{shaper}^2}.$$

Since the electronics are specified to have 180 ns FWHM, this means that

$$\sigma_{shaper} = (180/2.36) = 76.3\ ns$$

which, when turned into a distance using v_d becomes

$$76.3\ ns * 25\ mm/\mu sec = 1.9\ mm.$$

Meanwhile, for the drift due to diffusion we have

$$160\ \mu m * \sqrt{200} = 2.3\ mm;$$

hence, $\sigma_r = 3\ mm$, and $\Delta_z = 11.7\ mm$. Since the distance corresponding to 5 time buckets is trivially

$$5 * 2000\ mm/1024 = 9.8\ mm$$

this is shown to be true. Thus, in the z dimension and 1024 time buckets it is possible to calculate the mean and sigma of all of the hit constituents in order to estimate z, σ_r for each hit [Pun86]. With a 512 time bucket SCA, we find that we will have 19.6 mm for 5 time buckets; in fact, in the above case of maximum longitudinal diffusion, the signal only covers 3.7 time buckets and most signals will of course cover fewer. Thus, with our slow cool gas and 512 time buckets we must use the same technique applied to the *transverse* (i.e. along the pad row dimension) case, employing our assumed Gaussian response function, in order to estimate z, σ_r

for each hit [Lyn78],[Pun86]. The validity of the assumed Gaussian signal response in the drift dimension could be questioned, but earlier studies of actual data for a tpc in a field-free environment support its validity [Ape92]. Also, note that in what follows we ignore any contriubtions due to dip angle.

II. Position Resolution

Thus, the calculation for σ_r follows as did that in STAR Note #137 [Wie93], i.e.

$$\sigma_r^2 = w^2 \frac{1}{ln(\frac{h_2^2}{h_1 h_3})}$$

where w = time bucket width; for the case of 512 time buckets, this time bucket width is 3.9 mm. We need merely determine h_1 and h_3 using the Gaussian in the following section. We assume that the peak of the signal falls at the end of a time bucket (and hence is seen by the SCA); thus the amplitudes at h_1 and h_3 are identical and dictated by the time bucket width to be

$$h_1 = h_0 exp(-3.9^2/2 * 3.^2) = 0.43 * h_0;$$

hence we find for the response width

$$\sigma_r = 3.9\ mm * \sqrt{\frac{1}{ln(\frac{1}{.43^2})}} = 3.0\ mm.$$

In order to go from this track-wise response width to a resolution function (for the three pad case) we recall that

$$\sigma_{z-resolution} = \sqrt{{\sigma_0}^2 + {\sigma_L}^2/(N_0)}$$

where σ_0 for the three pad case (using the track-wise response width) has been shown to be [Lyn78]

$$\sigma_0 = \delta h_0 (\frac{\sigma_r}{w})^2 \sqrt{(\frac{2\bar{x} - \frac{w}{2}}{h_2})^2 + (\frac{\bar{x}}{h_3})^2 + (\frac{\bar{x} - w}{h_1})^2}$$

and, N_0 represents the primary ionisation liberated along the pad length, and $\frac{-w}{2} < \bar{x} < \frac{w}{2}$. Inserting the constants appropriate to our situation (ie. as above we assume $h_1 = h_3 = 0.43 h_0$, $w = 3.9\ mm$ $\bar{x} = 0$, and $\delta h_0 = 0.05 h_0$) we find that

$$\sigma_0 = 0.300\ mm.$$

Meanwhile, assuming that $N_0 = 60\ \frac{pairs}{cm} * 1.2\ cm$ (the mean of the values for He and C_2H_6) implies that

$$\sigma_L^2/(N_0) = (2.3\ mm)^2/(60 * 1.2) = 0.073\ mm^2$$

and hence

$$\sigma_{z-resolution} = \sqrt{(0.300\ mm)^2 + (0.073\ mm^2)} = 0.404\ mm.$$

Note that in the case of the 1024 time bucket SCA, this position resolution calculation would be modified in that $h_1 = h_3 = 0.81 h_0$, $w = 1.95\ mm$ leading to

$$\sigma_{z-resolution} = 0.418\ mm.$$

III. dE/dx Resolution

In order to determine the impact upon dE/dx, we remember that essentially

$$\sigma_{dE/dx} = \sigma_{dE/dx/sample} / \sqrt{N_{sample}}$$

because dE/dx is in practice measured as the integral of the observed charge, not from an assumed functional form. In this case, the *phase* of the pulse with respect to the time bins matters. Since the SCA records the amplitude seen at the end of the time bucket, our integral is sensitive to the fluctuations, governed by σ_r, of the signal peak relative to the bucket edge. We can perform three short calculations in order to examine the fluctuations in the charge integral and hence the dE/dx capability. First we integrate over the assumed Gaussian functional form in order to fix the normalisation; this comes out to be

$$A = \sqrt{2\pi}\sigma_r h_0 = 298 h_0$$

where we have put in the value of σ_r in ns from

$$\sigma_r = \sqrt{\sigma^2_{shaper} + \sigma_L}$$

with σ_{shaper}=76 ns and

$$\sigma_L = 2.3\ mm/25\ mm/\mu sec = 92\ ns.$$

Second, if we discretise the integration over the Gaussian shape into seven steps (as dictated by the signal response), we find that the area will be equal to the time bucket size times the value of the function at each of the seven points

$$h_1 = h_0 exp(-(f\Delta t - 3\Delta t)/2\sigma_r^2)$$

$$h_2 = h_0 exp(-(f\Delta t - 2\Delta t)/2\sigma_r^2)$$

$$h_3 = h_0 exp(-(f\Delta t - \Delta t)/2\sigma_r^2)$$

$$h_4 = h_0 exp(-(f\Delta t)/2\sigma_r^2)$$

$$h_5 = h_0 exp(-(f\Delta t + \Delta t)/2\sigma_r^2)$$

$$h_6 = h_0 exp(-(f\Delta t + 2\Delta t)/2\sigma_r^2)$$

$$h_7 = h_0 exp(-(f\Delta t + 3\Delta t)/2\sigma_r^2)$$

where f denotes the fraction of a signal width displacement from the signal peak being *in phase*. Assuming that our signal is in phase, we see that for 1024 time buckets the error on the area is on the order of 1.8%. Mapping out the dependence of this error in the area estimate as a function of *phase*, we find from figure 1 that the variation is on the order of .7%. Finally, we can perform a similar exercise for the cases in which we have 512 and 256 time buckets; these are presented in figures 2 and 3. For the 512 time bucket case, we see that the error in the charge estimation varies from about 3.2% for the in phase case to 7.8% for the out of phase case, or a 4.6% variaion. Likewise, the variation in the area estimate for the 256 time bucket case is on the order of 11%; note that the gross mismatch betwwen the time bucket width and the signal width leads to pathological fluctuations in the area integral. In order to get a feeling for how this translates into error on the dE/dx calculation, we can plot these quantities normalised to the number of samples left over after truncation (ie. let the maximum of 45 samples become 32 when we keep 70% of the samples); this is presented in figures 4 and 5. Figure 5 shows us that the error on the dE/dx estimate varies from about 0.12% for the 1024 time bucket case to 2.0% in the 256 time bucket case.

IV. References

[Lyn78] G. Lynch, TPC-LBL-78-17, (1978).

[Pun86] T. Pun and D. Schlatter, Coordinates Finding in the Time Projection Chamber: Algorithms and Resolution, ALEPH-TPC-86-68, (1986).

[Ape92] H. Apelshäuser, Systematische Untersuchungen zur Rekonstruktionseffizienz und -genauigkeit einer Spurendriftkammer, unpublished thesis, University of Frankfurt, (1992).

[Wie93] H. Wieman and I. Sakrejda, Position Resolution from Noise and Gain Variation, STAR Note # 137, (1993).

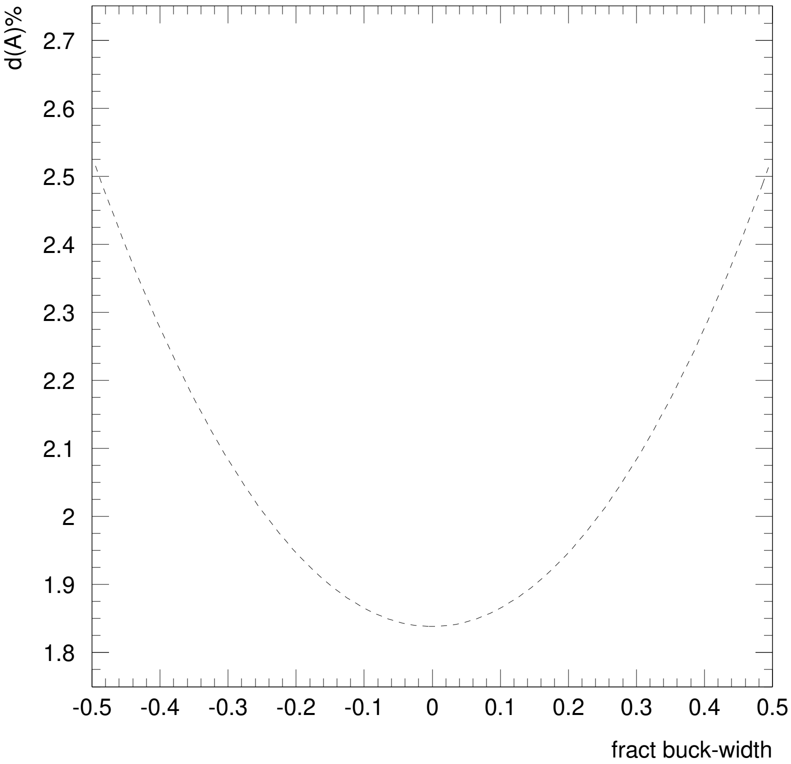

Fig. 1. Map of error in the pulse area estimate versus pulse arrival time in SCA time bucket for the case of a 1024 bucket SCA.

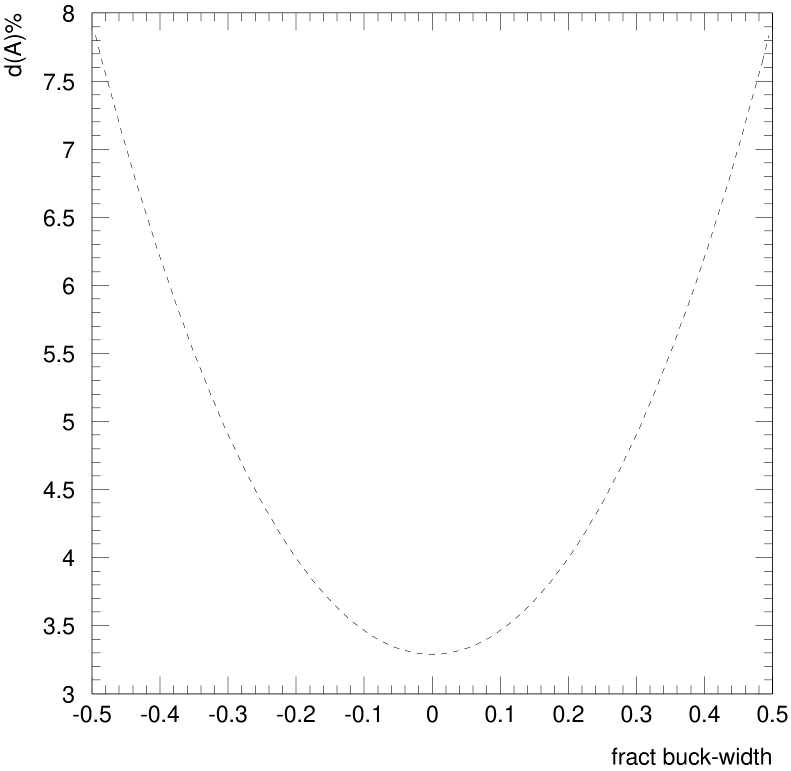

Fig. 2. Map of error in the pulse area estimate versus pulse arrival time in SCA time bucket for the case of a 512 bucket SCA.

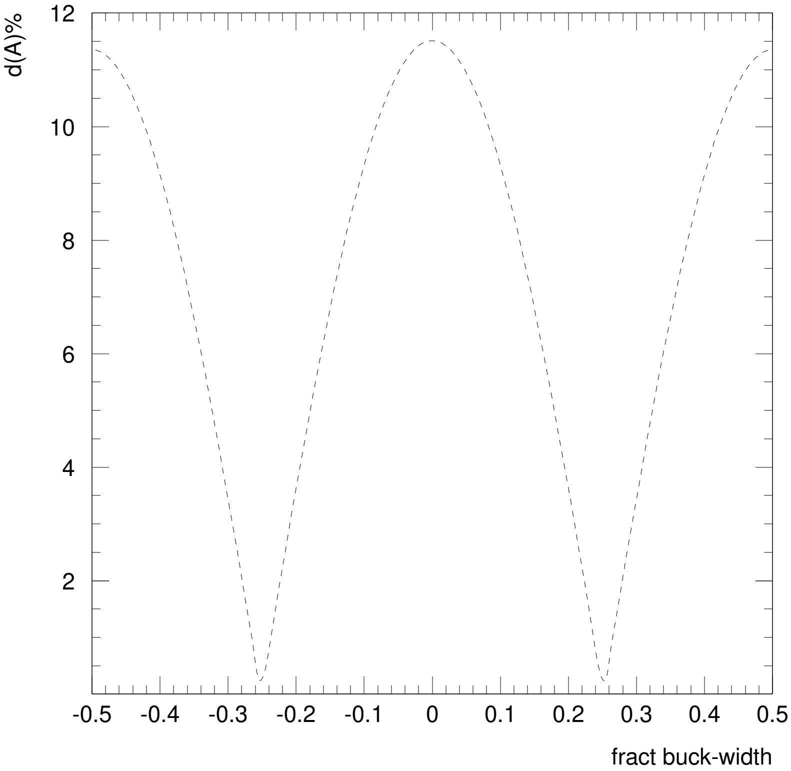

Fig. 3. Map of error in the pulse area estimate versus pulse arrival time in SCA time bucket for the case of a 256 bucket SCA.

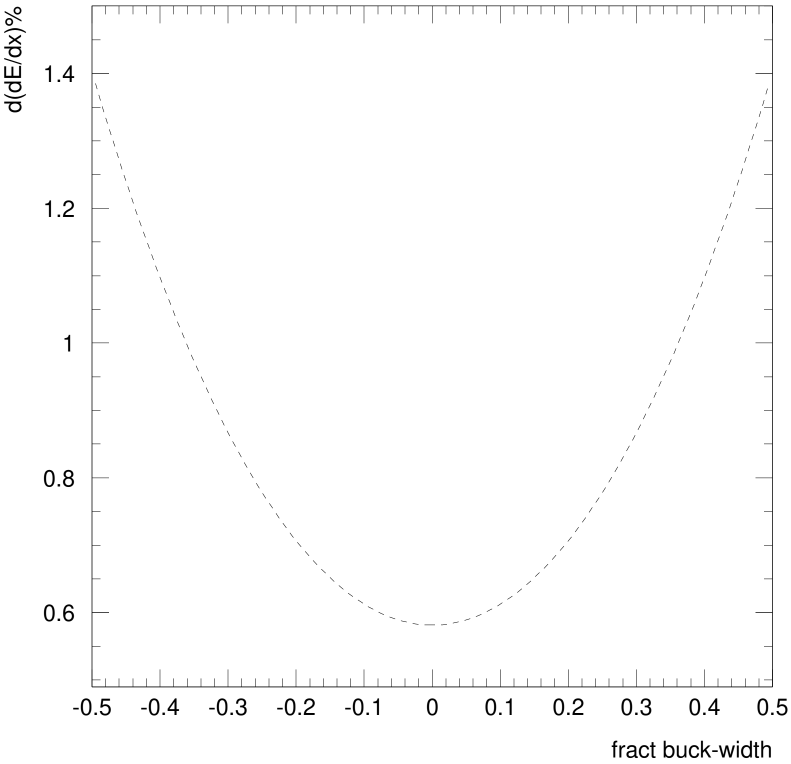

Fig. 4. Error in the track dE/dx estimate versus pulse arrival time in SCA time buckets for the case of a 512 bucket SCA.

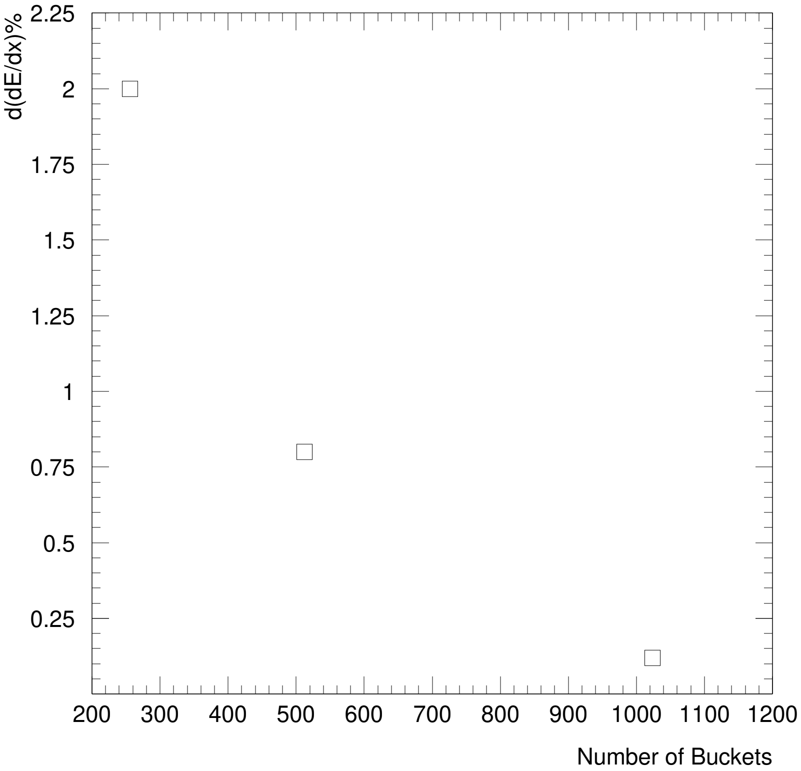

Fig. 5. Map of width of the distribution of the error in track dE/dx estimate versus pulse arrival time in SCA time bucket for the cases of a 256,512, and 1024 time bucket SCA.