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Geometric Construction and Reconstruction of Particle Trajectories for p, η in Idealized STAR TPC-XTPC

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Abstract

A method is presented for exact reconstruction, as well as construction, of trajectories of relativistic particles in the complex of the STAR central time projection chamber (at RHIC) coupled with the STAR external time projection chamber, XTPC, for an idealized uniform magnetic field. The analysis is geometric and exact. This has also provided a means of testing the accuracy of a given mode of operating GEANT, and it is found that an increased-accuracy mode for GEANT is necessary for some of this work. Exact momentum and pseudorapidity of the particle can be deduced from the location and orientation, e.g., twist, of the track in the XTPC for this model B field. Two methods of distinguishing primary particles from secondaries are analyzed: (1) By taking cuts. (2) By comparing the reconstructed B-field length with the one from the primary vertex.

1 Introduction

The external time projection chamber (XTPC) for the STAR experiment at RHIC is necessary, since the central TPC only covers pseudorapidity $\eta < 2$, while the XTPC is to cover the particles with η from 2 to 4.5. A large fraction of the particles is expected to be in this range. Furthermore, physical models of these highly relativistic heavy-ion collisions differ significantly in the yields of various particles expected in this range, relative to those in central pseudorapidity. We have been working on a radial-drift XTPC for this purpose, and it is of considerable interest to ask what kind of tracks are to be expected. For this purpose, we employ a simple model of the magnetic field in the central TPC, and look for an equally simple way to analyze the particle trajectories throughout the system of TPC and XTPC. We intend to follow this, later, with analysis using realistic shapes of magnetic field. We want to lay a firm groundwork now, which we expect to be useful then.

We employ the following model for the magnetic field B. It is constant in magnitude and direction inside the solenoid of STAR's central TPC. This B field is of total length 2L; it is zero elsewhere. We take the z axis to be parallel to the B field, and through the point of origin of the particle. This is the model field currently used by GEANT, although GEANT is designed to also "swim" a particle through much more complicated field distributions. We treat this model problem exactly except to ignore multiple scattering and similar interactions of the particle from birth to detection in the XTPC. For the present, consider the point of origin of the particle to be at the primary vertex ($z = 0$, site of nucleus-nucleus collision inside the RHIC beam pipe). Initially the momentum vector is at the angle θ from the z axis. After the particle leaves the magnetic field at $z = L$ it drifts free for a distance whose z component is D until it reaches the first plane of the XTPC. We approximate the XTPC as being composed of two parallel plane (perpendicular to z) segments, with the first plane at $z = D$ and the last at $z = D + \delta$.

2 Construction of Trajectories

For all parts of the trajectory, inside the solenoid or outside, the z component and the transverse component of the particle's momentum remain constant. (Recall that the Lorentz force is perpendicular to the momentum vector, as well as to B). Thus, the angle between the momentum vector and the z axis is a constant, θ, for all parts of the trajectory, inside and outside the solenoid. Consequently, the intra-solenoid trajectory is a helix of constant pitch, and its projection onto the xy plane is a circle with radius of curvature,

$$\rho = \frac{p \sin\theta}{(0.3\ B\ q)}, \tag{1}$$

where the particle's charge is qe and the units are m, GeV/c and Tesla. The trajectory's projection onto any plane containing the z axis is a sinusoid with the same wavelength, λ, for all orientations of the plane, but with different phases for different planes. In particular, the projection onto the plane containing the z axis and the initial (at the source) momentum is a sinusoid of phase zero, and initial slope of $\tan\theta$. A property of a sinusoid of phase zero is that its tangent at argument zero has, at the quarter wavelength point, a height which is $\pi/2$ times its amplitude. Thus

$$\tan\theta = \frac{p_\perp}{p_z} = \frac{2\pi\rho}{\lambda} \tag{2}$$

So

$$\lambda = \left(\frac{2\pi}{0.3\ B\ q}\right) p\cos\theta \tag{3}$$

with $\cos\theta$ nearly unity for the desired particles, $\eta > 2(\theta < 15.4°)$. Consider the projection of the trajectory onto the xy plane, Fig. 1. The source has the same x, y as the center of the XTPC. The projected trajectory moves clockwise (counter clockwise) for positive (negative) charge, around the circle of radius ρ until it exits the B field at $z = L$ after the accumulated angle

$$\beta = \frac{2\pi L}{\lambda}. \tag{4}$$

Thereafter, it is a straight line, tangent to the circle at this exiting point, and with projected length $D \cdot \tan\theta$. At the end of this line it has reached the first plane of the XTPC. After traveling a further projected distance $\delta \cdot \tan\Theta$ it has reached the last plane of the XTPC.

In considering trajectories, note that the central TPC field cage has an outer radius R_0 (2 m) and inner radius R_i (0.5 m). We will often consider θ, rather than $\eta = -ln(\tan(\theta/2))$, since the former is more relevant to the geometry of trajectories.

We now have the ingredients needed to define exactly any uninterrupted trajectory within this model. The view in Fig. 1 tells the entire story. We only considered the projection onto the plane containing the initial momentum and the z axis in order to determine the wavelength λ, which in turn determines the angle β, different for each particle.

The question whether the trajectory crosses the first plane of the XTPC within the radius of the XTPC is dominated by the larger of ρ and $D\tan\theta$. The ratio

$$\mu = \frac{D \tan\theta}{\rho} \tag{5}$$

is $\propto D/(p\cos\theta)$, so is relatively independent of θ for desired (small) θ. Thus, the shape of Fig. 1 for given p and various desired θ remains roughly constant except for overall size. Large μ gives a picture in Fig. 1 that is mostly line; small μ gives mostly circle.

Regarding other numerical values, Ref. 1, page 2-2, lists $B = 0.25 - 0.5$ Tesla. Here, we use 0.50 Tesla. Currently, GEANT uses $L = 4.00$ m, as will we. The energy losses by ionization and radiation in the TPC gas, followed by laboratory air, are small enough to be corrected for in realistic cases.

An obvious visual characteristic of a "track" is the twist angle, τ, the angle between the direction of the "track" (point 2 to point 3 in Fig. 1) and the direction of the line from the center of the XTPC to the beginning (point 2) of the "track". We find

$$\tau = \frac{\pi}{2} - ATAN2(\mu + \sin\beta,\ 1 - \cos\beta) \tag{6}$$

Figure 2 shows this twist angle vs β for various μ. The diagonal line is the locus of maxima. Since the "track" has a direction, τ has an algebraic sign, with one sign for positive charge, and the other for negative.

Various calculated quantities are given in Tables 1 and 2. In both, angle θ ranges up to 14° in steps of 2°, then a coarser spread to 80°. Note that some of these larger-θ trajectories can also get into the XTPC. Table 1 has a wide range of momenta, while Table 2 has a range of smaller momenta, some of which could confuse the evaluation of θ and p, unless further steps are taken, as described below. Tabulated are: (A.) The radius ρ of the projected circle; (B.) The angle β around the circle; (C.) The twist angle τ; (D.) The ratio μ upon which the twist directly depends; (E.) The maximum excursion of the particle from the beam while in the central TPC: (F.) The distance R_x from the center of the XTPC to the beginning (point 2) of the "track"; (G.-J.) The x and y coordinates of the beginning and end (points 2 and 3) of the "track"; (K.) The wavelength λ of the projected (plane through beam) sinusoid; Three quantities defined in the Tables, viz., (L.) ϕ', (M.) $\Delta\phi'$, (N.) $\Delta\theta'$; and finally (O.) $\Delta\eta'$, the difference of pseudorapidity of the two lines from the vertex in the central TPC to the two ends of the "track".

One line drawn in the Tables for RTPCMAX shows the boundary R_0 for trajectories penetrating the outer wall of the field cage. The other line there for RTPCMAX shows the boundary R_i for particles to get into the field cage and thus have tracks in the central TPC which could be connected with XTPC "tracks". This would enable identification of some particles with smaller pseudorapidity that might otherwise confuse the assignments of momentum and θ. The line in the Tables for R_x indicates an arbitrary boundary of 2 m.

The quantities $\Delta\eta'$ and $\Delta\phi'$ have been discussed in Refs. 1 and 2. The Tables are listed in the detail shown because it permits easy cross-correlation of various quantities, and occupies much less space than the manifold of plots required for the same information.

A few plots, made from the Tables, illustrate some of the more important features. In plotting these Figs. 3-6, a trajectory is excluded if its radius in the central TPC exceeds 2.0 m or its $R_x > 2.0$ m.

Figure 3 shows $\Delta\eta'$ vs twist, τ. One sees a strong dependence on momentum and a weak dependence on θ for momenta above 0.5 GeV/c. Especially important is that for these larger momenta there is a very tight clustering around an ellipse-like locus. This will be important in identifying secondary particles (i.e., not from the primary vertex), discussed below. For the smaller momenta one sees the double-valued feature of Fig. 2. Some of these trajectories have $\beta > 360°$, but some do not, as seen in the Tables.

Figure 4 is $\Delta\eta'$ vs $\Delta\phi'$. This is the same type of plot as in Ref. 1, where the band-like feature was already identified as important in distinguishing secondaries from primaries. The general shape of Fig. 4 is similar to that of Fig. 3. In Figs. 3 and 4, large momentum (stiff) means twist, $\Delta\eta'$ and $\Delta\phi'$ all approach zero because β does.

The relative usefulness of twist compared to $\Delta\phi'$, Figs. 3 and 4, needs to be explored in more detail. The twist would be a direct result of a least-squares fit to a "track" line, and is easy to think about.

Figure 5 is "track" length vs twist, both being directly visible properties of the "track". For larger momenta it shows a grid-like distribution of points, somewhat like θ vs momentum. Figure 6, R_x vs twist, is similar to Fig. 5 in shape.

The Tables and Figs. 3, 5 and 6 illustrate the usefulness of the twist angle, τ, showing that it has a wide range of values for different momenta while being relatively insensitive to θ. Thus, τ alone gives a good sense of the momentum for many cases. The relative insensitivity of τ to θ at small θ arises from the insensitivity of $\cos\theta$ in β and the insensitivity of $\tan\theta/\sin\theta$ in μ.

3 Reconstruction of Trajectories

The only information available in the XTPC is the line segment labeled "track", Figs. 1 and 7, with length $\delta \cdot \tan\theta$, and with location and orientation shown there. This "track" length already gives the angle θ, the direction of the particle's momentum when it was born, since the length δ is known. It is emphasized that all of the discussion and constructions described in this section are in the xy plane.

The other property of the particle, the magnitude of its momentum, is determined by the location (distance R_x from the center of the XTPC) and orientation of the "track". To solve this geometry, extend the "track" upstream by the known ratio D/δ, reaching point 1 in Fig. 7. From the end of this extension draw the line c through the center of the XTPC. Then draw from the center of the XTPC the "impact parameter", b, of the projected track's extension - i.e., the perpendicular from the center of the XTPC to the indefinite extension of the track's projection. Then the radius of curvature is

$$\rho = \frac{c^2}{2b}, \tag{7}$$

from which we obtain the momentum p, knowing B and θ, Eq.(1). Further information is given by the angle

$$\beta = \pi - 2cos^{-1}\left(\frac{b}{c}\right). \tag{8}$$

Now we have all desired information about the particle, viz., the magnitude and direction (pseudorapidity) of its momentum at birth. We also have the sign of its charge, determined by whether the indefinite extension of the "track" passed to the right or to left of the center of the XTPC, or equivalently by the sign of the twist angle.

The "impact parameter", b, is another obvious visual characteristic of the "track". Having ρ and β, the dependence of b on them is

$$b = 2\ \rho \sin^2\left(\frac{\beta}{2}\right) \tag{9}$$

If $\beta/2 << 1$, then

$$b \propto \frac{(\tan\theta)}{(p\cos\theta)} \tag{10}$$

Later will be discussed the use of the reconstructed L. Combining Eqs. (1), (3) and (4), we find

$$L = \frac{\beta\ \rho}{\tan\theta} \tag{11}$$

the length of B field required to produce the reconstructed β for the reconstructed ρ and θ in Eq. (11). Thus the arc length (on the circle) is $L \cdot \tan\theta$, analagous to the "track" length being $\delta \cdot \tan\theta$.

The full reconstruction, using Fig. 7 gives separately ρ and β. So μ is determined, and a single value of the otherwise double-valued τ is properly determined in Fig. 2. However, if only the twist were used for the reconstruction, then Fig. 2 would not give a unique solution.

It is attractive that all of this reconstruction can be made in the xy plane, as it reduces the processing time, and simplifies the understanding.

When $\beta > 360°$, the described construction of the "track" is correct. However, the reconstruction will assign the correct θ, but an incorrect L, by a multiple of λ. Thus, this "track" will be rejected by the L test, discussed below. For Table 1, all cases of $\beta > 360°$ are excluded by the particle hitting the periphery of the central TPC. In Table 2 the low momenta still survive this test, but often they can also be rejected by the larger dE/dx from these low momenta. Also in some cases, identified by cross referencing different parts of the Tables, connection of XTPC tracks with central TPC tracks will remove an ambiguity.

4 GEANT Trials

We have made comparisons of these geometrical results against GEANT calculations. Figures 8 and 9 show just two examples where GEANT was run in the standard-accuracy mode and with "no physics". The B field (shape and value) used in GEANT is the same as discussed here. The resulting GEANT x, y values of the two ends of the "track" were used in the present equations to reconstruct the trajectory and deduce the values of momentum and θ. The reconstructions pictured here, and other cases, agreed with the inputs to GEANT to better than 1% at low momenta, but for larger momenta the agreement deteriorated. The ratios listed in Figs. 8 and 9 show this. For the case of p_x, p_y, $p_z = 0.7, 0, 7$ the momenta disagreed by 44%. This arose from discrepancies up to ~5 mm in GEANT's x, y value of either end of a "track". Since the numerical accuracy of the present geometric calculations is at least 10^{-8}, this indicates our need to adjust the accuracy- control parameters input to GEANT for such runs. Of course, this also reflects the fact that the XTPC needs particularly good accuracy in its measured x, y coordinates in order to achieve good momentum and θ determinations. It further reflects the relative ease of geometrically constructing trajectories for this simple uniform B field, in contrast with GEANT's much more numerically-demanding job of "swimming" a particle through a realistic field.

Figure 10 shows "tracks" from a FRITIOF event propogated by GEANT out to the XTPC. Almost every "track" has eight dots (eight sections of XTPC); we have been discussing only the first and last dot, heretofor. The twist angle τ is illustrated there.

For investigating the reconstruction with a realistic shape of the B field, we intend to determine how well a lookup table for L (and a uniform field) will work for representing the effects of a realistic B field. To first order, one might expect such an effective L to depend on c, Fig. 7. The present tests assumed perfect spatial resolution in the XTPC, since our purpose is to test the geometrical ideas.

5 Secondary Particles

Secondary particles are those produced at some distance from the original vertex of the $A-A$ reaction because of a secondary interaction or decay. We will see that one can determine by two different methods whether a "track" is due to a primary or a secondary particle, using only information from the location and orientation of the "track".

Consider the first method. Figures 3 and 4 show two sets of points, each set clustering closely around an ellipse-like locus in the region of larger momenta and θ. In reality, multiple scattering and radiative loss would push each point randomly a bit away from the location shown here, but it is expected that a fairly narrow cut in either of these Figures for real trajectories would retain almost all of the desired trajectories and reject almost all of the unwanted trajectories. Once this selection of "tracks" has been made, the reconstruction represented by Fig. 7, will give exactly the momentum and θ of the particle for each actually primary "track" in the cut, in the limit of neglible multiple scattering and radiation effects.

We want to consider the reconstruction of a trajectory for a secondary, but first consider its construction. This would be the same as just described, but in general L would be different, and also the x, y projection of the secondary particle's origin would not be at the center of the XTPC. However, since the source of this secondary particle is at an unknown location, the amount of each of these two changes is unknown.

If one were not aware that this is a secondary particle (i.e., no cut used) one would use the afore-described reconstruction, and find an incorrect value of momentum that reconstructs the track back exactly to a vertex on the beam line, instead of to the proper secondary source. The reason is that the correct point of tangency to the correct circle can unambiguously be determined for any "track" by extending it by the ratio D/δ, as in Fig. 7. However, without assuming that the x, y projection of the trajectory goes through the projection of the beam line there is no way to choose the correct value of ρ, and thereby deduce the momentum. Note that in this process the correct value of θ would nevertheless be obtained, since the length of the "track" is still $\delta \cdot \tan\theta$.

However, this "track" of a secondary can be rejected by the fact that the reconstructed L, Eq. (11), would not be correct, e.g., 4.00 m in the present case. The reason is that the reconstructed β and/or ρ (arc length on the circle) are not correct in Eq. (11) for a secondary.

Consider an example of this which is easy to analyse. Postulate an unusual secondary source on the beam axis but at a value of L different from 4 m. Suppose that it has the same ρ and β as a primary with $L = 4$ m, but an arbitrarily different θ. Thus, the point 1 of tangency to the circle of Fig. 7 would be at the same x, y for both cases, but the point 3 and the "track" length would be different. Nevertheless, the reconstructed θ is correctly found in each case. Then Eq. (11) shows that the reconstructed L's would be in the ratio of the values of $\tan\theta$. The secondary with reconstructed $L \neq 4$ m would thereby be rejected.

As another example, consider a secondary source on the beam axis with the same ρ and β as a primary, but with $L \neq 4$ m. Equation (1) shows that they have the same p, and Eq. (2) or (3) shows they have the same λ. Equation (4) then shows that their values of β are in the ratio of their L's. They have different twists but the same lengths of "track". The reconstructed L's will be in the ratio of the reconstructed β's.

In Ref. 2 it is suggested that primaries could be distinguished from secondaries by "swimming" the particle back through the magnetic field to see whether the trajectory comes satisfactorally close to the primary vertex. This is the analogue of the test on L, just described.

By using the event simulator, FRITIOF, followed by GEANT, one finds that with the "physics off" about 4% of the events would lie outside a narrow strip at the bottom of Fig. 3a or 3b - i.e., they would lie in the upper portion of Fig. 3b. Similarly with "physics on" one finds that roughly half of all tracks in the described XTPC are secondaries. These secondaries are widely spread in a plot like Figs. 3 and 4, as seen in Ref. 1. The secondaries originating nearest to the primary vertex will be the most difficult to reject by the two methods just described. Many such secondaries originate in the beam pipe near the primary vertex. Taking the radius of this pipe to be $r = 5$ cm, we estimate geometrically that the change in R_x is $\delta \cdot r/(L + D) \approx 6$ mm, roughly two standard deviations of the spot size estimated for the radial-drift XTPC recently described, Ref. 3. This corresponds to $\Delta\eta' \approx \Delta\theta'/\sin\theta \approx 0.005$.

In summary, we have shown quantitatively two methods of rejecting secondaries: (1) The cut on an appropriately narrow region of Figs. 3 and 4. (2) The test of whether the reconstructed L, Eq. (11), matches the known L of the primary vertex sufficiently well. Both methods will best be used for the realistic case with multiple scattering.

In further summary, if x, y of the source is known, somehow, then reconstruction will correctly give p and θ. It will also give L (i.e., z of source) correctly, apart from a usually unimportant additive integer-multiple of λ. Thus, primaries are properly reconstructed. For secondaries, θ is correctly reconstructed. However, for secondaries, the x, y of the source is usually incorrectly assumed, but then such "tracks" are rejected by the two methods just described.

6 A Few Design Considerations

The unconsidered effects of multiple scattering and finite spatial resolution of the XTPC will give uncertainty to the length, location and orientation of the "track". We intend to use the present results to estimate the resulting uncertainty in reconstructed momentum and pseudorapidity for different designs. We also intend to investigate whether the required constant separation of dots in the "track" can be used to increase track-finding and reconstruction accuracy.

It is clear that larger δ provides less uncertainty from the XTPC's finite spatial resolution. This suggests splitting the XTPC into two segments, farther separated, to increase δ.

The several (e.g. eight here) hits per "track" should significantly reduce the uncertainty in dE/dx from Landau fluctuations.

A small D (hence small XTPC radius) permits more unwanted particles with $\eta < 2$ to reach the first section of the XTPC. In the limit of $D = 0$ all particles that reach the edge of the B field will enter the XTPC. On the other hand, for larger D, a large-θ trajectory will fortunately miss the XTPC because of large $D \cdot \tan\theta$. That can unfortunately be offset somewhat in the occasional case of a substantial ρ with β near 290° so the trajectory uses some of $D \cdot \tan\theta$ to get back closer to the beam axis. Such a large β requires $L/\lambda \approx 0.8$, meaning smaller $p\cos\theta$. This would mean smaller ρ, so the particle will unfortunately not strike the outer material of the solenoid. These features can be seen in the Tables. Design considerations need to be explored in detail to assure that the XTPC is not swamped by unwanted tracks.

The finite length of the beam-intersection diamond also affects the value of L for each event. However, the L of each event can be obtained from the vertex-locating detectors.

We are grateful to Lynn Wood for his help in some event simulations.

7 References

1. STAR Conceptual Design Report, June 15, 1992.

2. I. Stancu, STAR Workshop, Brookhaven National Laboratory, September, 1992.

3. J. Romero, D. Cebra and L. Wood, STAR Workshop, Feb. 4, 1994.

TABLE 1.

Angles = degrees, lengths = meters, momenta = GeV/c, B-field = Tesla
L, D, δ, B-field= 4, 2.15, .7, .5
Momenta= .5, 1, 1.5, 2.5, 4, 6, 10, 15, 20
Emission Theta = 2, 4, 6, 8, 10, 12, 14, 20, 30, 45, 60, 80

Tables: One momentum per column, one θ per row.

A. ρ = Circle radius =

.1163	.2327	.3490	.5817	.9307	1.3960	2.3266	3.4899	4.6533
.2325	.4650	.6976	1.1626	1.8602	2.7903	4.6504	6.9756	9.3009
.3484	.6969	1.0453	1.7421	2.7874	4.1811	6.9686	10.4528	13.9371
.4639	.9278	1.3917	2.3196	3.7113	5.5669	9.2782	13.9173	18.5564
.5788	1.1577	1.7365	2.8941	4.6306	6.9459	11.5765	17.3648	23.1531
.6930	1.3861	2.0791	3.4652	5.5443	8.3165	13.8608	20.7912	27.7216
.8064	1.6128	2.4192	4.0320	6.4513	9.6769	16.1281	24.1922	32.2563
1.1401	2.2801	3.4202	5.7003	9.1205	13.6808	22.8013	34.2020	45.6027
1.6667	3.3333	5.0000	8.3333	13.3333	20.0000	33.3333	50.0000	66.6667
2.3570	4.7140	7.0711	11.7851	18.8562	28.2843	47.1405	70.7107	94.2809
2.8868	5.7735	8.6603	14.4338	23.0940	34.6410	57.7350	86.6025	115.4701
3.2827	6.5654	9.8481	16.4135	26.2615	39.3923	65.6539	98.4808	131.3077

B. β = Azimuthal angle traversed on circle =

68.797	34.398	22.932	13.759	8.600	5.733	3.440	2.293	1.720
68.923	34.461	22.974	13.785	8.615	5.744	3.446	2.297	1.723
69.134	34.567	23.045	13.827	8.642	5.761	3.457	2.304	1.728
69.431	34.715	23.144	13.886	8.679	5.786	3.472	2.314	1.736
69.816	34.908	23.272	13.963	8.727	5.818	3.491	2.327	1.745
70.291	35.145	23.430	14.058	8.786	5.858	3.515	2.343	1.757
70.860	35.430	23.620	14.172	8.857	5.905	3.543	2.362	1.771
73.167	36.584	24.389	14.633	9.146	6.097	3.658	2.439	1.829
79.391	39.696	26.464	15.878	9.924	6.616	3.970	2.646	1.985
97.234	48.617	32.411	19.447	12.154	8.103	4.862	3.241	2.431
137.510	68.755	45.837	27.502	17.189	11.459	6.875	4.584	3.438
395.944	197.972	131.981	79.189	49.493	32.995	19.797	13.198	9.899

C. Twist = Angle between $(X2, Y2)$-radius-vector and track =

22.028	11.145	7.445	4.472	2.796	1.864	1.119	.746	.559
22.067	11.165	7.459	4.480	2.801	1.868	1.121	.747	.560
22.132	11.199	7.482	4.494	2.810	1.873	1.124	.749	.562
22.224	11.247	7.514	4.513	2.822	1.881	1.129	.753	.564
22.343	11.309	7.555	4.538	2.837	1.892	1.135	.757	.568
22.490	11.385	7.607	4.569	2.857	1.905	1.143	.762	.571
22.666	11.476	7.668	4.606	2.880	1.920	1.152	.768	.576
23.376	11.847	7.917	4.756	2.974	1.983	1.190	.793	.595
25.279	12.845	8.587	5.160	3.226	2.151	1.291	.861	.645
30.595	15.692	10.506	6.317	3.951	2.635	1.581	1.054	.791
41.475	22.015	14.807	8.923	5.585	3.725	2.236	1.491	1.118
2.535	51.561	40.105	25.217	15.970	10.694	6.430	4.290	3.218

D. $\mu = D \times \tan(\theta)/\rho =$

.6454	.3227	.2151	.1291	.0807	.0538	.0323	.0215	.0161
.6466	.3233	.2155	.1293	.0808	.0539	.0323	.0216	.0162
.6486	.3243	.2162	.1297	.0811	.0540	.0324	.0216	.0162
.65I3	.3257	.2171	.1303	.0814	.0543	.0326	.0217	.0163
.6550	.3275	.2183	.1310	.0819	.0546	.0327	.0218	.0164
.6594	.3297	.2198	.1319	.0824	.0550	.0330	.0220	.0165
.6647	.3324	.2216	.1329	.0831	.0554	.0332	.0222	.0166
.6864	.3432	.2288	.1373	.0858	.0572	.0343	.0229	.0172
.7448	.3724	.2483	.1490	.0931	.0621	.0372	.0248	.0186
.9122	.4561	.3041	.1824	.1140	.0760	.0456	.0304	.0228
1.2900	.6450	.4300	.2580	.1613	.1075	.0645	.0430	.0323
3.7144	1.8572	1.2381	.7429	.4643	.3095	.1857	.1238	.0929

E. RTPCMAX = Max radial excursion (from beam) in main TPC =

.1314	.1376	.1388	.1393	.1396	.1396	.1397	.1397	.1397
.2631	.2755	.2778	.2790	.2794	.2796	.2797	.2797	.2797
.3954	.4141	.4176	.4194	.4200	.4202	.4204	.4204	.4204
.5284	.5536	.5583	.5608	.5616	.5619	.5621	.5621	.5621
.6625	.6944	.7005	.7036	.7046	.7050	.7052	.7053	.7053
.7979	.8370	.8443	.8481	.8494	.8499	.8501	.8502	.8502
.9350	.9815	.9903	.9948	.9963	.9969	.9972	.9972	.9973
1.3590	1.4313	1.4449	1.4519	1.4543	1.4552	1.4556	1.4558	1.4558
2.1290	2.2635	2.2889	2.3020	2.3065	2.3081	2.3089	2.3092	2.3093
3.5370	3.8811	3.9469	3.9808	3.9925	3.9967	3.9988	3.9995	3.9997
5.3811	6.5199	6.7449	6.8619	6.9023	6.9167	6.9240	6.9264	6.9272
6.5654	13.1308	17.9920	20.9222	21.9864	22.3730	22.5724	22.6350	22.6569

F. RX = XTPC-center to $(X2, Y2)$ =

.1980	.2105	.2129	.2141	.2145	.2146	.2147	.2147	.2148
.3963	.4215	.4262	.4287	.4295	.4298	.4300	.4300	.4300
.5954	.6334	.6406	.6443	.6456	.6460	.6463	.6463	.6464
.7956	.8468	.8565	.8615	.8632	.8638	.8641	.8642	.8643
.9973	1.0622	1.0745	1.0808	1.0830	1.0838	1.0842	1.0843	1.0844
1.2008	1.2801	1.2951	1.3029	1.3055	1.3065	1.3069	1.3071	1.3072
1.4065	1.5010	1.5189	1.5282	1.5313	1.5325	1.5330	1.5332	1.5333
2.0413	2.1881	2.2160	2.2303	2.2353	2.2370	2.2379	2.2382	2.2383
3.1844	3.4568	3.5088	3.5356	3.5448	3.5481	3.5498	3.5503	3.5505
5.2141	5.9071	6.0413	6.1107	6.1346	6.1432	6.1475	6.1489	6.1494
7.5727	9.8211	10.2777	10.5164	10.5990	10.6285	10.6436	10.6483	10.6500
14.1340	16.3550	25.5130	31.2982	33.4516	34.2398	34.6477	34.7757	34.8206

G. $X2$, first hit =

.1356	.1934	.2051	.2113	.2134	.2142	.2145	.2147	.2147
.2710	.3871	.4107	.4230	.4273	.4288	.4296	.4299	.4299
.4061	.5815	.6171	.6358	.6422	.6445	.6457	.6461	.6462
.5405	.7768	.8248	.8500	.8587	.8618	.8634	.8639	.8641
.6741	.9734	1.0343	1.0662	1.0773	1.0812	1.0833	1.0839	1.0841
.8066	1.1716	1.2460	1.2850	1.2985	1.3034	1.3058	1.3066	1.3069
.9376	1.3717	1.4604	1.5069	1.5230	1.5288	1.5317	1.5326	1.5330
1.3178	1.9873	2.1250	2.1973	2.2223	2.2312	2.2358	2.2373	2.2378
1.8667	3.0842	3.3394	3.4739	3.5206	3.5373	3.5459	3.5486	3.5495
2.0675	4.9583	5.6051	5.9510	6.0719	6.1152	6.1375	6.1444	6.1469
-.7961	6.7305	8.8070	9.9683	10.3823	10.5318	10.6087	10.6328	10.6413
11.7985	-13.6241	-.8352	18.4093	27.8874	31.6786	33.7090	34.3562	34.5842

H. $Y2$, first hit =

.1443	.0831	.0568	.0345	.0217	.0145	.0087	.0058	.0043
.2892	.1667	.1140	.0693	.0435	.0291	.0174	.0116	.0087
.4355	.2512	.1719	.1045	.0656	.0438	.0263	.0175	.0132
.5838	.3372	.2308	.1403	.0881	.0588	.0353	.0236	.0177
.7349	.4252	.2911	.1770	.1111	.0742	.0446	.0297	.0223
.8895	.5158	.3532	.2148	.1349	.0901	.0541	.0361	.0271
1.0484	.6094	.4175	.2540	.1595	.1065	.0640	.0426	.0320
1.5589	.9156	.6283	.3826	.2403	.1605	.0964	.0643	.0482
2.5799	1.5613	1.0771	.6576	.4134	.2762	.1659	.1106	.0830
4.7867	3.2108	2.2539	1.3881	.8753	.5854	.3518	.2347	.1760
7.5308	7.1523	5.2980	3.3507	2.1320	1.4303	.8610	.5746	.4311
7.7824	9.0482	25.4994	25.3116	18.4744	12.9935	8.0101	5.3852	4.0508

I. $X3$, last hit=

.1445	.2136	.2276	.2350	.2376	.2385	.2389	.2391	.2391
.2886	.4275	.4558	.4706	.4757	.4775	.4785	.4788	.4789
.4323	.6420	.6848	.7072	.7150	.7177	.7192	.7196	.7198
.5751	.8576	.9153	.9455	.9560	.9597	.9616	.9622	.9624
.7167	1.0746	1.1477	1.1860	1.1993	1.2040	1.2065	1.2072	1.2075
.8567	1.2932	1.3826	1.4294	1.4456	1.4514	1.4543	1.4553	1.4556
.9948	1.5140	1.6204	1.6761	1.6955	1.7024	1.7059	1.7070	1.7074
1.3916	2.1919	2.3571	2.4438	2.4738	2.4846	2.4901	2.4918	2.4924
1.9411	3.3951	3.7012	3.8626	3.9187	3.9388	3.9491	3.9523	3.9534
1.9794	5.4211	6.1961	6.6111	6.7562	6.8082	6.8349	6.8433	6.8462
-1.6901	7.1699	9.6517	11.0437	11.5406	11.7201	11.8124	11.8414	11.8515
15.0125	-17.4003	-3.4906	19.1539	30.4660	35.0082	37.4443	38.2213	38.4950

J. $Y3$, last hit =

.1670	.0969	.0664	.0404	.0253	.0169	.0102	.0068	.0051
.3349	.1944	.1331	.0810	.0508	.0340	.0204	.0136	.0102
.5042	.2930	.2007	.1221	.0767	.0512	.0307	.0205	.0154
.6759	.3933	.2694	.1639	.1029	.0687	.0413	.0275	.0206
.8508	.4959	.3398	.2068	.1299	.0867	.0521	.0347	.0260
1.0296	.6014	.4123	.2509	.1576	.1052	.0632	.0421	.0316
1.2133	.7106	.4874	.2967	.1863	.1245	.0747	.0498	.0374
1.8028	1.0675	.7336	.4470	.2808	.1876	.1127	.0751	.0563
2.9772	1.8195	1.2572	.7681	.4831	.3228	.1939	.1293	.0970
5.4811	3.7360	2.6291	1.6212	1.0227	.6841	.4111	.2742	.2057
8.3497	8.2823	6.1677	3.9106	2.4902	1.6712	1.0061	.6715	.5038
10.1127	7.8233	28.4504	29.2110	21.4929	15.1554	9.3546	6.2916	4.7332

K. λ = wavelength of sinusoid =

20.931	41.862	62.794	104.656	167.450	251.174	418.624	627.936	837.248
20.893	41.786	62.679	104.465	167.143	250.715	417.859	626.788	835.717
20.829	41.658	62.488	104.146	166.634	249.951	416.584	624.877	833.169
20.740	41.480	62.220	103.701	165.921	248.882	414.803	622.204	829.605
20.626	41.252	61.877	103.129	165.006	247.509	412.515	618.773	825.031
20.486	40.973	61.459	102.431	163.890	245.835	409.726	614.588	819.451
20.322	40.644	60.965	101.609	162.575	243.862	406.437	609.655	812.873
19.681	39.362	59.043	98.404	157.447	236.171	393.618	590.426	787.235
18.138	36.276	54.414	90.690	145.104	217.656	362.760	544.140	725.520
14.810	29.619	44.429	74.048	118.477	177.715	296.192	444.288	592.384
10.472	20.944	31.416	52.360	83.776	125.664	209.440	314.159	418.879
3.637	7.274	10.911	18.184	29.095	43.643	72.738	109.106	145.475

L. $\phi' =$ Angle ATAN2 $(Y2, X2) =$

46.769	23.254	15.487	9.287	5.804	3.869	2.321	1.547	1.161
46.856	23.296	15.515	9.304	5.814	3.876	2.325	1.550	1.163
47.002	23.368	15.563	9.333	5.832	3.888	2.333	1.555	1.166
47.207	23.469	15.630	9.373	5.857	3.904	2.343	1.562	1.171
47.473	23.599	15.717	9.425	5.890	3.926	2.356	1.570	1.178
47.801	23.760	15.824	9.489	5.930	3.953	2.372	1.581	1.186
48.194	23.953	15.952	9.566	5.978	3.985	2.391	1.594	1.195
49.791	24.737	16.472	9.878	6.172	4.115	2.469	1.646	1.234
54.112	26.851	17.877	10.719	6.698	4.465	2.679	1.786	1.339
66.639	32.925	21.906	13.130	8.203	5.468	3.281	2.187	1.640
96.034	46.740	31.030	18.579	11.604	7.734	4.640	3.093	2.320
33.409	146.411	91.876	53.971	33.523	22.302	13.367	8.908	6.681

M. $\delta\phi' =$ Angle ATAN2 $(Y3, X3) -$ ATAN2 $(Y2, X2) =$

2.379	1.154	.765	.458	.286	.191	.114	.076	.057
2.384	1.157	.767	.459	.286	.191	.115	.076	.057
2.392	1.160	.769	.460	.287	.192	.115	.077	.057
2.403	1.165	.773	.462	.289	.192	.115	.077	.058
2.417	1.172	.777	.465	.290	.193	.116	.077	.058
2.435	1.180	.782	.468	.292	.195	.117	.078	.058
2.457	1.190	.789	.472	.295	.196	.118	.078	.059
2.544	1.229	.815	.487	.304	.203	.122	.081	.061
2.783	1.337	.885	.529	.330	.220	.132	.088	.066
3.505	1.648	1.087	.648	.404	.269	.162	.108	.081
5.409	2.378	1.550	.920	.573	.381	.229	.152	.114
.556	9.380	5.119	2.775	1.679	1.106	.660	.439	.329

N. $\delta\theta' =$ Angle [ATAN2 $(\sqrt{(X3^2 + Y3^2)}, L + D + \delta) -$ ATAN2 $(RX, L + D)] =$

.002634	.000608	.000266	.000095	.000037	.000016	.000006	.000003	.000001
.005278	.001219	.000534	.000191	.000074	.000033	.000012	.000005	.000003
.007946	.001832	.000802	.000287	.000112	.000050	.000018	.000008	.000004
.010648	.002452	.001073	.000383	.000149	.000066	.000024	.000011	.000006
.013398	.003078	.001346	.000481	.000187	.000083	.000030	.000013	.000007
.016209	.003714	.001624	.000580	.000226	.000100	.000036	.000016	.000009
.019094	.004361	.001906	.000680	.000265	.000118	.000042	.000019	.000011
.028373	.006396	.002788	.000993	.000387	.000172	.000062	.000027	.000015
.047349	.010280	.004448	.001579	.000614	.000272	.000098	.000044	.000024
.097047	.018602	.007856	.002755	.001066	.000472	.000170	.000075	.000042
.278862	.036744	.014431	.004874	.001862	.000821	.000294	.000131	.000073
2.786474	.857170	.112447	.021766	.006883	.002838	.000983	.000432	.000242

O. $\delta\eta' = -\eta($ ATAN2 $(\sqrt{(X3^2+Y3^2)}, L+D+\delta)) + \eta($ ATAN2 $(\sqrt{(X2^2+Y2^2)}, L+D)) =$

.001427	.000310	.000134	.000048	.000019	.000008	.000003	.000001	.000001
.001431	.000311	.000135	.000048	.000019	.000008	.000003	.000001	.000001
.001438	.000312	.000135	.000048	.000019	.000008	.000003	.000001	.000001
.001448	.000314	.000136	.000048	.000019	.000008	.000003	.000001	.000001
.001460	.000316	.000137	.000048	.000019	.000008	.000003	.000001	.000001
.001475	.000318	.000138	.000049	.000019	.000008	.000003	.000001	.000001
.001494	.000321	.000139	.000049	.000019	.000008	.000003	.000001	.000001
.001571	.000333	.000144	.000051	.000020	.000009	.000003	.000001	.000001
.001796	.000366	.000157	.000055	.000021	.000010	.000003	.000002	.000001
.002617	.000469	.000196	.000068	.000026	.000012	.000004	.000002	.000001
.006258	.000757	.000294	.000099	.000038	.000017	.000006	.000003	.000001
.052505	.015939	.002018	.000387	.000122	.000050	.000017	.000008	.000004

TABLE 2.

Angles = degrees, lengths = meters, momenta = GeV/c, B-field = Tesla.
L, D, δ, B-field = 4, 2.15, .7, .5
Momenta = .05, .1, .15, .2, .3, .4, .55, .7, .85
Emission Theta = 2, 4, 6, 8, 10, 12, 14, 20, 30, 45, 60, 80

Tables: One momentum per column, one θ per row.

A. ρ = Circle radius =

.0116	.0233	.0349	.0465	.0698	.0931	.1280	.1629	.1978
.0233	.0465	.0698	.0930	.1395	.1860	.2558	.3255	.3953
.0348	.0697	.1045	.1394	.2091	.2787	.3833	.4878	.5923
.0464	.0928	.1392	.1856	.2783	.3711	.5103	.6495	.7886
.0579	.1158	.1736	.2315	.3473	.4631	.6367	.8104	.9840
.0693	.1386	.2079	.2772	.4158	.5544	.7623	.9703	1.1782
.0806	.1613	.2419	.3226	.4838	.6451	.8870	1.1290	1.3709
.1140	.2280	.3420	.4560	.6840	.9121	1.2541	1.5961	1.9381
.1667	.3333	.5000	.6667	1.0000	1.3333	1.8333	2.3333	2.8333
.2357	.4714	.7071	.9428	1.4142	1.8856	2.5927	3.2998	4.0069
.2887	.5774	.8660	1.1547	1.7321	2.3094	3.1754	4.0415	4.9075
.3283	.6565	.9848	1.3131	1.9696	2.6262	3.6110	4.5958	5.5806

B. β = Azimuthal angle traversed on circle =

687.968	343.984	229.323	171.992	114.661	85.996	62.543	49.141	40.469
689.228	344.614	229.743	172.307	114.871	86.154	62.657	49.231	40.543
691.337	345.668	230.446	172.834	115.223	86.417	62.849	49.381	40.667
694.306	347.153	231.435	173.577	115.718	86.788	63.119	49.593	40.842
698.156	349.078	232.719	174.539	116.359	87.269	63.469	49.868	41.068
702.910	351.455	234.303	175.727	117.152	87.864	63.901	50.208	41.348
708.598	354.299	236.199	177.149	118.100	88.575	64.418	50.614	41.682
731.675	365.837	243.892	182.919	121.946	91.459	66.516	52.262	43.040
793.914	396.957	264.638	198.478	132.319	99.239	72.174	56.708	46.701
972.342	486.171	324.114	243.085	162.057	121.543	88.395	69.453	57.197
1375.099	687.549	458.366	343.775	229.183	171.887	125.009	98.221	80.888
3959.439	1979.719	1319.813	989.860	659.906	494.930	359.949	282.817	232.908

C. Twist = Angle between $(X2, Y2)$-radius-vector and track =

1.472	.754	49.860	48.630	35.534	27.273	20.082	15.858	13.093
1.355	.692	49.781	48.681	35.591	27.320	20.118	15.887	13.116
1.169	.595	49.645	48.766	35.688	27.399	20.178	I5.934	13.156
.932	.473	49.445	48.884	35.823	27.510	20.262	16.002	13.212
.666	.336	49.170	49.034	35.998	27.654	20.371	16.089	13.284
.402	.202	48.807	49.215	36.214	27.831	20.506	16.197	13.374
.175	.088	48.336	49.426	36.471	28.043	20.667	16.326	13.481
.168	.084	46.013	50.203	37.503	28.900	21.320	16.848	13.915
4.914	2.660	36.329	51.588	40.190	31.178	23.071	18.251	15.082
9.065	16.501	4.422	46.289	46.864	37.396	27.989	22.231	18.405
2.749	1.513	12.220	.775	49.886	48.613	38.312	30.882	25.733
.000	6.145	7.434	6.898	5.380	17.686	.000	24.878	49.128

D. $\mu = D \times \tan(\theta)/\rho =$

6.4539	3.2270	2.1513	1.6135	1.0757	.8067	.5867	.4610	.3796
6.4658	3.2329	2.1553	1.6164	1.0776	.8082	.5878	.4618	.3803
6.4855	3.2428	2.1618	1.6214	1.0809	.8107	.5896	.4633	.3815
6.5134	3.2567	2.1711	1.6283	1.0856	.8142	.5921	.4652	.3831
6.5495	3.2748	2.1832	1.6374	1.0916	.8187	.5954	.4678	.3853
6.5941	3.2970	2.1980	1.6485	1.0990	.8243	.5995	.4710	.3879
6.6475	3.3237	2.2158	1.6619	1.1079	.8309	.6043	.4748	.3910
6.8639	3.4320	2.2880	1.7160	1.1440	.8580	.6240	.4903	.4038
7.4478	3.7239	2.4826	1.8620	1.2413	.9310	.6771	.5320	.4381
9.1217	4.5608	3.0406	2.2804	1.5203	1.1402	.8292	.6515	.5366
12.9000	6.4500	4.3000	3.2250	2.1500	1.6125	1.1727	.9214	.7588
37.1441	18.5720	12.3814	9.2860	6.1907	4.6430	3.3767	2.6531	2.1849

E. RTPCMAX = Max radial excursion (from beam) in main TPC =

.0233	.0465	.0698	.0928	.1175	.1269	.1329	.1354	.1368
.0465	.0930	.1395	.1856	.2352	.2541	.2660	.2712	.2739
.0697	.1394	.2091	.2782	.3531	.3817	.3997	.4075	.4116
.0928	.1856	.2783	.3705	.4714	.5099	.5342	.5448	.5503
.1158	.2315	.3473	.4625	.5902	.6391	.6698	.6833	.6903
.1386	.2772	.4158	.5540	.7097	.7693	.8068	.8233	.8319
.1613	.3226	.4838	.6449	.8299	.9009	.9456	.9652	.9755
.2280	.4560	.6840	.9121	1.1962	1.3062	1.3755	1.4059	1.4219
.3333	.6667	1.0000	1.3333	1.8293	2.0314	2.1597	2.2163	2.2460
.4714	.9428	1.4142	1.8856	2.7938	3.2911	3.6149	3.7596	3.8360
.5774	1.1547	1.7321	2.3094	3.4641	4.6072	5.6335	6.1105	6.3670
.6565	1.3131	1.9696	2.6262	3.9392	5.2523	7.2219	9.1915	11.1612

F. RX = XTPC-center to $(X2, Y2)$ =

.0689	.0687	.0754	.1234	.1702	.1889	.2008	.2061	.2089
.1385	.1380	.1504	.2466	.3405	.3781	.4021	.4126	.4182
.2093	.2087	.2245	.3692	.5111	.5679	.6041	.6201	.6285
.2821	.2815	.2974	.4911	.6819	.7585	.8073	.8288	.8402
.3576	.3572	.3685	.6119	.8532	.9502	1.0121	1.0394	1.0537
.4366	.4364	.4375	.7312	1.0250	1.1433	1.2188	1.2522	1.2698
.5201	.5200	.5040	.8488	1.1974	1.3381	1.4280	1.4678	1.4887
.8056	.8057	.6845	1.1863	1.7181	1.9353	2.0747	2.1365	2.1690
1.4066	1.4433	.9229	1.6578	2.5929	2.9891	3.2462	3.3608	3.4212
1.9498	2.6392	1.7407	1.8947	3.7817	4.7290	5.3698	5.6607	5.8l54
3.4665	3.4153	4.6869	3.4016	3.7452	6.1255	8.0607	8.9998	9.5129
12.1900	12.2669	11.4382	10.9595	10.5323	14.7496	12.1900	8.5009	11.8309

G. $X2$, first hit =

.0575	.0657	-.0754	-.0679	.0321	.0981	.1482	.1723	.1855
.1173	.1326	-.1504	-.1365	.0633	.1957	.2963	.3447	.3712
.1816	.2017	-.2245	-.2068	.0928	.2923	.4442	.5174	.5574
.2522	.2740	-.2972	-.2795	.1197	.3875	.5918	.6904	.7443
.3303	.3503	-.3678	-.3553	.1429	.4806	.7390	.8639	.9323
.4164	.4313	-.4355	-.4351	.1615	.5711	.8857	1.0380	1.1214
.5095	.5174	-.4992	-.5194	.1743	.6583	1.0316	1.2127	1.3120
.7894	.8017	-.6515	-.8047	.1664	.8918	1.4620	1.7412	1.8947
.5041	1.1923	-.6138	-1.3886	-.0963	1.1167	2.1253	2.6318	2.9133
-.8768	-.8884	1.3274	-1.8139	-1.6098	.4823	2.6519	3.8445	4.5327
1.3182	2.8326	.3150	3.2529	-3.7449	-3.3607	.4644	3.4674	5.4353
12.1895	-12.1899	-6.9823	-1.3429	4.3720	-6.7521	12.1900	-1.7763	-11.8051

H. $Y2$, first hit =

-.0381	-.0198	.0007	.1031	.1672	.1615	.1356	.1131	.0960
-.0736	-.0382	.0001	.2053	.3346	.3235	.2718	.2268	.1926
-.1041	-.0538	-.0031	.3058	.5026	.4869	.4094	.3418	.2903
-.1264	-.0649	-.0103	.4038	.6714	.6520	.5491	.4586	.3896
-.1369	-.0697	-.0228	.4981	.8412	.8197	.6915	.5779	.4912
-.1312	-.0664	-.0419	.5877	1.0122	.9904	.8374	.7004	.5956
-.1044	-.0525	-.0689	.6714	1.1846	1.1650	.9875	.8269	.7035
.1607	.0808	-.2102	.8716	1.7100	1.7176	1.4721	1.2381	1.0557
1.3132	.8133	-.6891	.9055	2.5911	2.7726	2.4538	2.0901	1.7936
-1.7415	2.4852	-1.1261	-.5475	3.4220	4.7044	4.6692	4.1549	3.6433
-3.206l	-1.9080	4.6763	-.9945	.0459	5.1212	8.0473	8.3050	7.8072
-.1194	1.3728	-9.0597	-10.8769	-9.5820	13.1133	-.0109	-8.3132	-.7800

I. $X3$, last hit =

.0782	.0892	-.0913	-.0921	.0219	.0998	.1594	.1883	.2041
.1593	.1798	-.1820	-.1850	.0428	.1990	.3187	.3767	.4084
.2461	.2730	-.2713	-.2798	.0615	.2969	.4777	.5653	.6132
.3408	.3699	-.3585	-.3773	.0770	.3930	.6363	.7542	.8188
.4449	.4715	-.4426	-.4782	.0881	.4865	.7941	.9435	1.0253
.5587	.5785	-.5223	-.5834	.0936	.5766	.9511	1.1332	1.2331
.6806	.6910	-.5963	-.6937	.0921	.6626	1.1069	1.3235	1.4423
1.0389	1.0551	-.7636	-1.0592	.0316	.8853	1.5636	1.8971	2.0809
.6161	1.5l53	-.6516	-1.7719	-.3684	1.0518	2.2490	2.8536	3.1905
-1.0891	-1.3015	1.8945	-2.1308	-2.2757	.1161	2.6715	4.0902	4.9120
1.8325	3.8558	.1386	4.4171	-4.5374	-4.5610	-.2311	3.2940	5.6273
16.1592	-16.1597	-8.9785	-1.3526	6.3514	-9.5558	16.1599	-.8956	-14.1994

J. $Y3$, last hit =

-.0510	-.0266	-.0178	.1065	.1894	.1858	.1573	.1316	.1119
-.0987	-.0512	-.0373	.2119	.3790	.3724	.3153	.2639	.2244
-.1394	-.0720	-.0599	.3150	.5691	.5603	.4749	.3976	.3382
-.1691	-.0867	-.0873	.4148	.7600	.7502	.6368	.5335	.4540
-.1828	-.0931	-.1210	.5098	.9518	.9430	.8019	.6723	.5723
-.1750	-.0885	-.1627	.5988	1.1446	1.1391	.9710	.8148	.6939
-.1389	-.0698	-.2140	.6801	1.3386	1.3394	1.1449	.9618	.8196
.2123	.1067	-.4389	.8586	1.9262	1.9723	1.7057	1.4395	1.2296
1.7015	1.0562	-1.0915	.7774	2.8899	3.1715	2.8385	2.4280	2.0877
-2.4085	3.0503	-1.5364	-1.1717	3.6376	5.3010	5.3690	4.8104	4.2317
-4.3040	-2.5586	5.8758	-1.3333	-.8716	5.2923	9.0404	9.5050	9.0044
-.1583	1.3922	-12.4913	-14.8468	-13.0233	15.9239	-.0144	-12.1842	-3.9467

K. λ = Wavelength of sinusoid =

2.093	4.186	6.279	8.372	12.559	16.745	23.024	29.304	35.583
2.089	4.179	6.268	8.357	12.536	16.714	22.982	29.250	35.518
2.083	4.166	6.249	8.332	12.498	16.663	22.912	29.161	35.410
2.074	4.148	6.222	8.296	12.444	16.592	22.814	29.036	35.258
2.063	4.125	6.188	8.250	12.375	16.501	22.688	28.876	35.064
2.049	4.097	6.146	8.195	12.292	16.389	22.535	28.681	34.827
2.032	4.064	6.097	8.129	12.193	16.257	22.354	28.451	34.547
1.968	3.936	5.904	7.872	11.809	15.745	21.649	27.553	33.457
1.814	3.628	5.441	7.255	10.883	14.510	19.952	25.393	30.835
1.481	2.962	4.443	5.924	8.886	11.848	16.291	20.733	25.176
1.047	2.094	3.142	4.189	6.283	8.378	11.519	14.661	17.802
.364	.727	1.091	1.455	2.182	2.910	4.001	5.092	6.183

L. $\phi' =$ Angle ATAN2 $(Y2, X2) =$

-33.504	-16.769	179.463	123.362	79.128	58.723	42.461	33.283	27.376
-32.126	-16.078	179.962	123.626	79.280	58.834	42.539	33.344	27.427
-29.832	-14.927	-179.199	124.068	79.535	59.018	42.671	33.447	27.511
-26.625	-13.320	-178.009	124.693	79.895	59.279	42.857	33.592	27.630
-22.510	-11.258	-176.451	125.505	80.361	59.616	43.097	33.779	27.784
-17.492	-8.747	-174.504	126.512	80.938	60.033	43.395	34.011	27.974
-11.578	-5.789	-172.137	127.724	81.628	60.532	43.751	34.289	28.201
11.507	5.753	-162.122	132.715	84.442	62.560	45.196	35.415	29.125
69.000	34.297	-131.691	146.891	92.129	68.062	49.103	38.457	31.619
-116.724	109.670	-40.308	-163.203	115.193	84.147	60.405	47.222	38.791
-67.650	-33.963	86.147	-17.000	179.297	123.274	86.697	67.339	55.155
-.561	173.575	-127.621	-97.038	-65.474	117.244	-.051	-102.061	-176.220

M. $\delta\phi' =$ Angle ATAN2 $(Y3, X3) -$ ATAN2 $(Y2, X2) =$

.385	.198	-348.416	7.488	4.274	3.044	2.148	1.666	1.363
.354	.181	-348.395	7.510	4.284	3.050	2.152	1.669	1.366
.304	.155	11.640	7.545	4.300	3.061	2.159	1.675	1.370
.241	.122	11.688	7.596	4.323	3.076	2.169	1.682	1.376
.171	.086	11.745	7.662	4.353	3.095	2.182	1.692	1.384
.102	.051	11.809	7.744	4.390	3.119	2.198	1.704	1.394
.044	.022	11.876	7.843	4.435	3.148	2.217	1.718	1.405
.040	.020	12.013	8.254	4.618	3.265	2.294	1.777	1.453
1.096	.582	10.856	9.419	5.136	3.590	2.506	1.936	1.580
2.391	3.437	1.268	12.010	6.837	4.599	3.140	2.404	1.954
.712	.396	2.502	.204	-348.423	7.481	4.768	3.547	2.842
.000	1.501	1.914	1.833	1.472	3.723	.000	7.857	11.753

N. $\delta\theta' =$ Angle [ATAN2 $(\sqrt{(X3^2 + Y3^2)}, L + D + \delta) -$ ATAN2 $(RX, L + D)] =$

.138765	.139055	.075860	.027675	.008767	.004361	.002137	.001276	.000851
.277324	.277835	.152862	.055673	.017597	.008745	.004283	.002557	.001704
.415437	.416051	.232185	.084330	.026555	.013175	.006446	.003847	.002563
.552809	.553390	.315076	.114000	.035707	.017676	.008636	.005150	.003430
.689064	.689501	.402891	.145074	.045128	.022274	.010861	.006470	.004307
.823751	.823996	.497131	.177992	.054897	.026996	.013131	.007814	.005199
.956359	.956442	.599484	.213258	.065103	.031873	.015458	.009187	.006108
1.336729	1.336617	.975557	.339707	.099438	.047801	.022902	.013536	.008973
1.915133	1.883608	1.979121	.687849	.179456	.081896	.037897	.022049	.014494
3.510512	2.607257	3.796620	2.420918	.475137	.183125	.075533	.041707	.026663
4.921256	4.995569	3.319471	5.015820	2.659586	.679852	.199572	.094142	.055344
3.799917	3.731052	4.262874	4.621607	4.978664	2.387547	3.799917	6.605121	2.537760

O. $\delta\eta' = -\eta(\text{ ATAN2 }(\sqrt{(X3^2+Y3^2)}, L+D+\delta)) + \eta(\text{ ATAN2 }(\sqrt{(X2^2+Y2^2)}, L+D)) =$

.195643	.196703	.102548	.023791	.005516	.002476	.001142	.000665	.000437
.194752	.195683	.103582	.023967	.005540	.002484	.001145	.000666	.000438
.193250	.193989	.105334	.024264	.005581	.002498	.001150	.000669	.000440
.191114	.191632	.107841	.024691	.005639	.002517	.001157	.000673	.000442
.188324	.188632	.111162	.025256	.005715	.002543	.001167	.000677	.000445
.184872	.185014	.115372	.025974	.005811	.002575	.001178	.000683	.000449
.180775	.180815	.120565	.026863	.005928	.002613	.001192	.000690	.000453
.165319	.165285	.143225	.030832	.006430	.002776	.001250	.000720	.000471
.139948	.134688	.209523	.045130	.008032	.003265	.001416	.000802	.000520
.185458	.109707	.218754	.134519	.015726	.005232	.002003	.001074	.000677
.162993	.167001	.092171	.168102	.086037	.016715	.004375	.001989	.001150
.073120	.071730	.082902	.090557	.098273	.044773	.073120	.136950	.049369

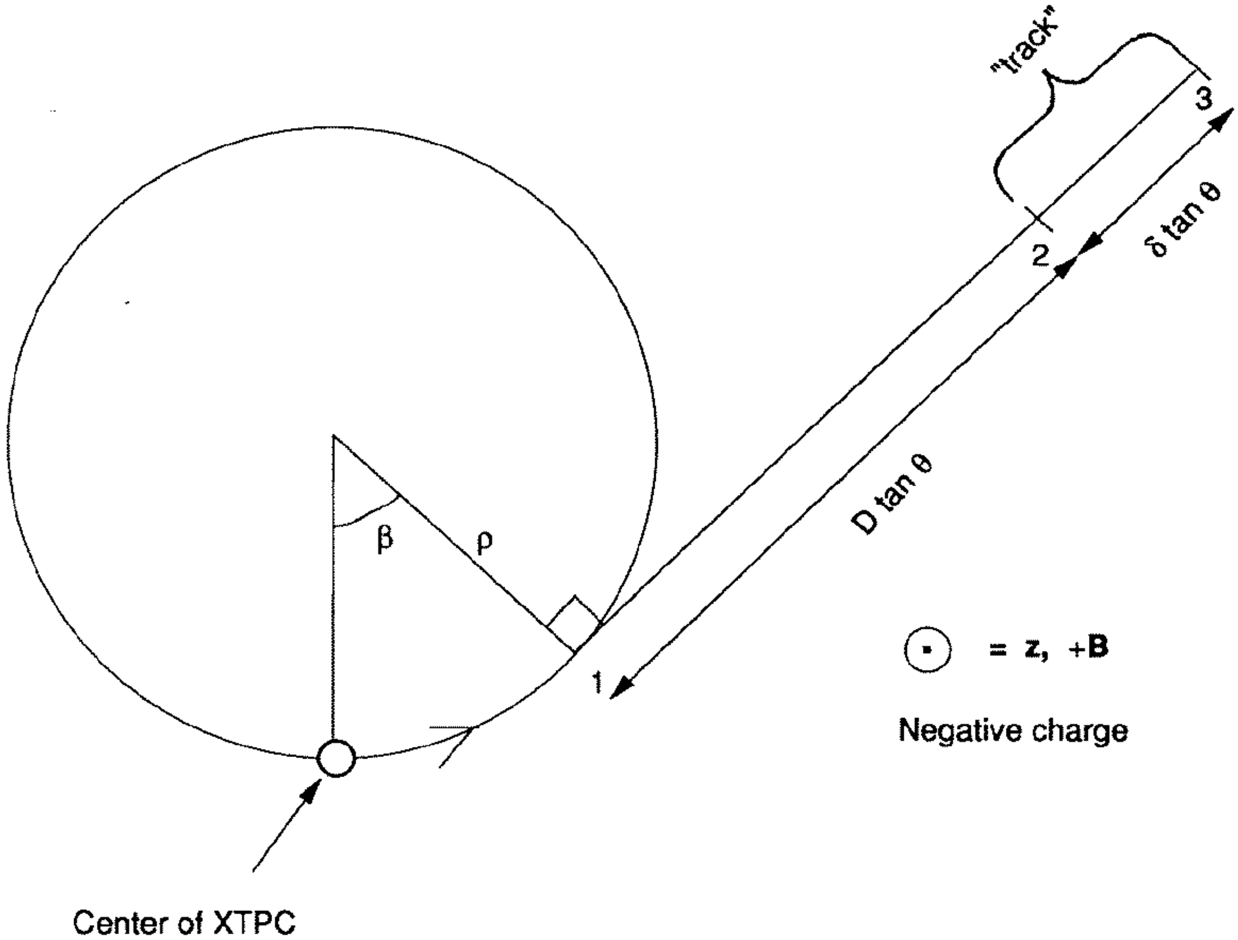


Figure 1: $x - y$ projection of a trajectory.

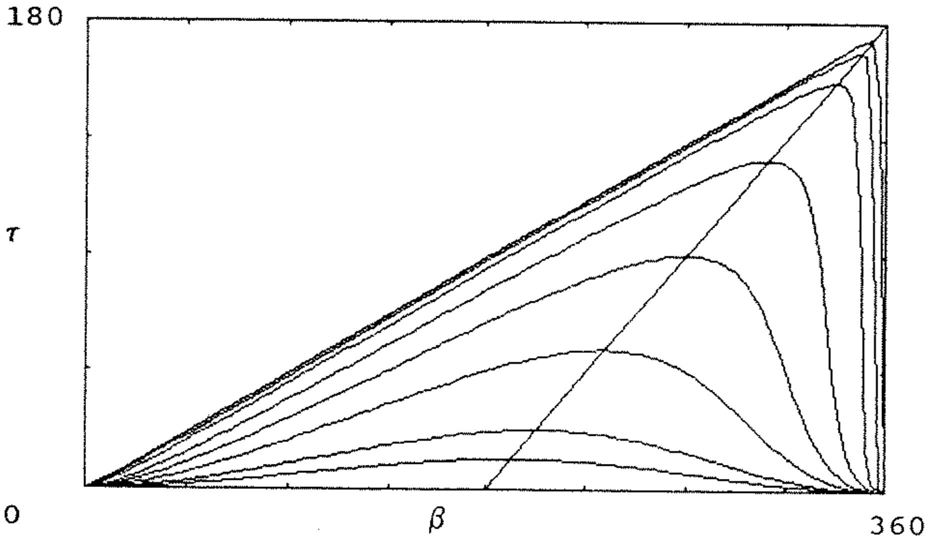
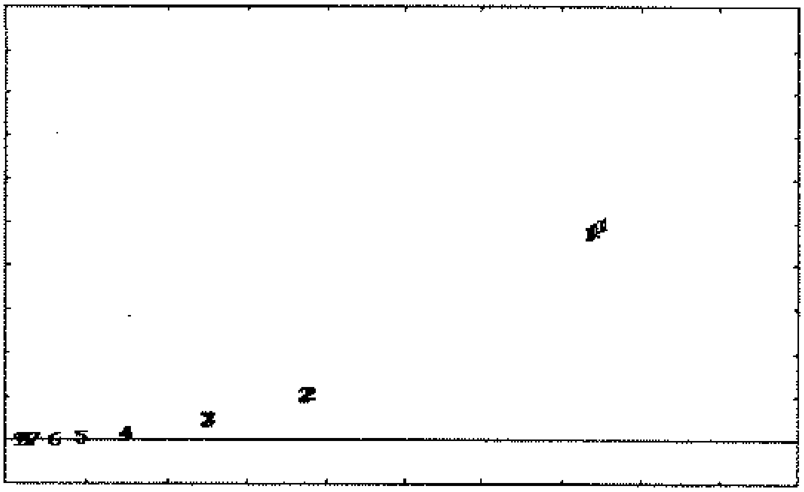


Figure 2: Twist, 0 to 180 degrees, vs β, 0 to 360 degrees. Curves from top to bottom are $\mu = 0.05$, 0.1, 0.2, 0.5, 1, 2, 5, 10.

ALL ANGLES = DEGREES, LENGTHS = METERS, MOMENTA = GEV/C, B_FIELD = TESLA.
L, D, DELTA, B_FIELD= 4 2.15 .7 .5
MOMENTA =
.5 1 1.5 2.5 4 6 10 15 20
EMISSION THETA =
2 4 6 8 10 12 14 20 30 45 60 80
PLOT IS DELTA_ETAP, -.0003 TO .003 , vs TWIST, 0 TO 30 DEGREES.



MOMENTA =
.05 .1 .15 .2 .3 .4 .55 .7 .85
EMISSION THETA =
2 4 6 8 10 12 14 20 30 45 60 80
PLOT IS DELTA_ETAP, -.03 TO .3 , vs TWIST, 0 TO 60 DEGREES.

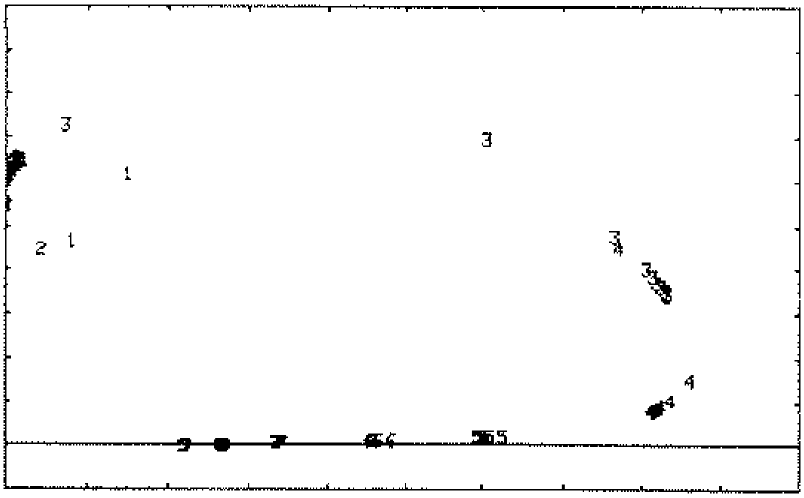
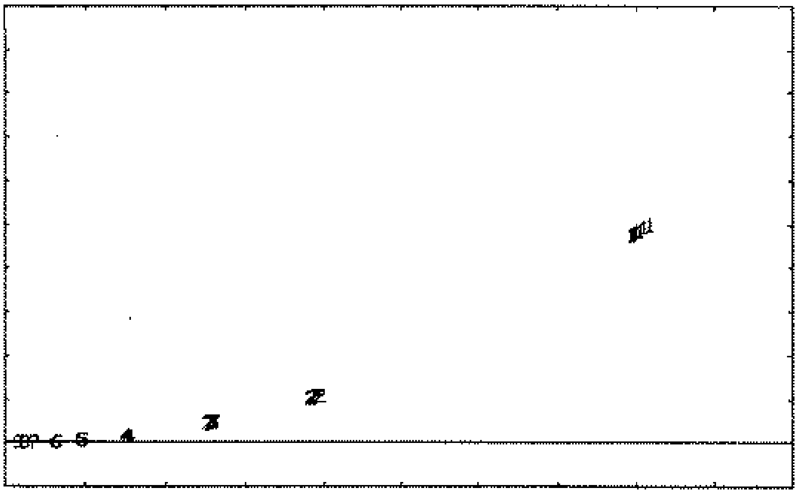


Figure 3: Point labels indicate momenta in list, lowest to highest. (Momentum, θ) points are unplotted for too large RX or RTPCMAX. No acceptable point is off scale.

```
ALL ANGLES = DEGREES, LENGTHS = METERS, MOMENTA = GEV/C, B_FIELD = TESLA.
L, D, DELTA, B_FIELD= 4  2.15  .7  .5
MOMENTA =
 .5  1  1.5  2.5  4  6  10  15  20
EMISSION THETA =
 2  4  6  8  10  12  14  20  30  45  60  80
PLOT IS DELTA_ETAP, -.0003  TO  .003 , vs DELTA_PHIP,  0  TO  3  DEGREES.
```



```
MOMENTA =
 .05  .1  .15  .2  .3  .4  .55  .7  .85
EMISSION THETA =
 2  4  6  8  10  12  14  20  30  45  60  80
PLOT IS DELTA_ETAP, -.03  TO .3 , vs DELTA_PHIP,  0  TO  15  DEGREES.
```

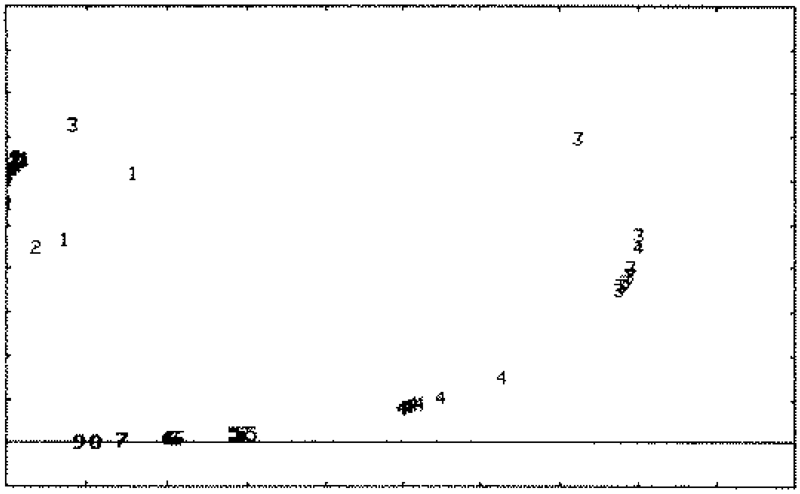
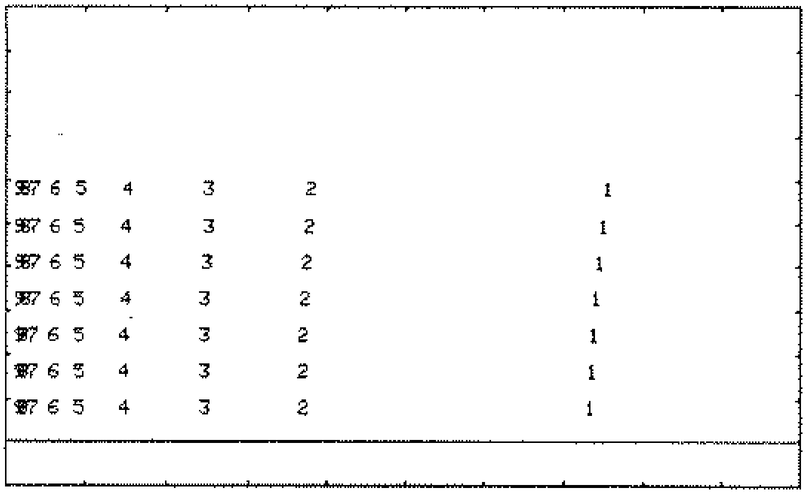


Figure 4: Point labels indicate momenta in list, lowest to highest. (Momentum, θ) points are unplotted for too large RX or RTPCMAX. No acceptable point is off scale.

```
ALL ANGLES = DEGREES, LENGTHS = METERS, MOMENTA = GEV/C, B_FIELD = TESLA.
L, D, DELTA, B_FIELD= 4  2.15  .7  .5
MOMENTA =
 .5  1  1.5  2.5  4  6  10  15  20
EMISSION THETA =
 2  4  6  8  10  12  14  20  30  45  60  80
PLOT IS DELTA*TAN(THETA), -.03  TO  .3 , vs TWIST,  0  TO  30  DEGREES.
```



```
MOMENTA =
 .05  .1  .15  .2  .3  .4  .55  .7  .85
EMISSION THETA =
 2  4  6  8  10  12  14  20  30  45  60  80
PLOT IS DELTA*TAN(THETA), -.1  TO  1 , vs TWIST,  0  TO  60  DEGREES.
```

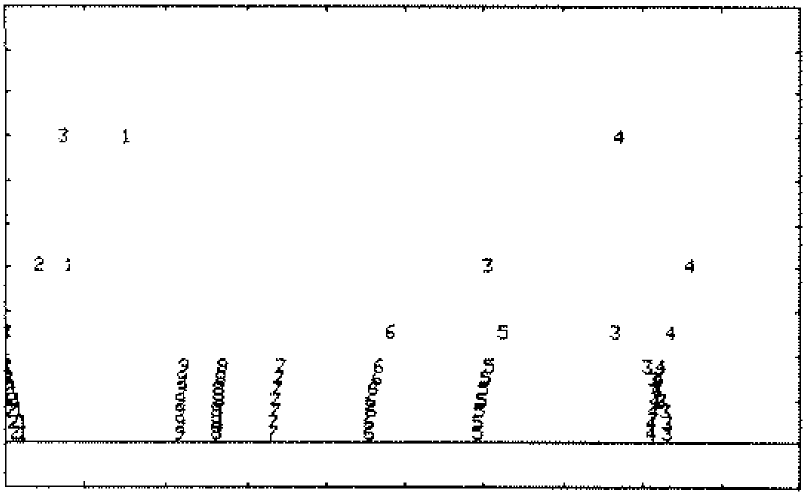
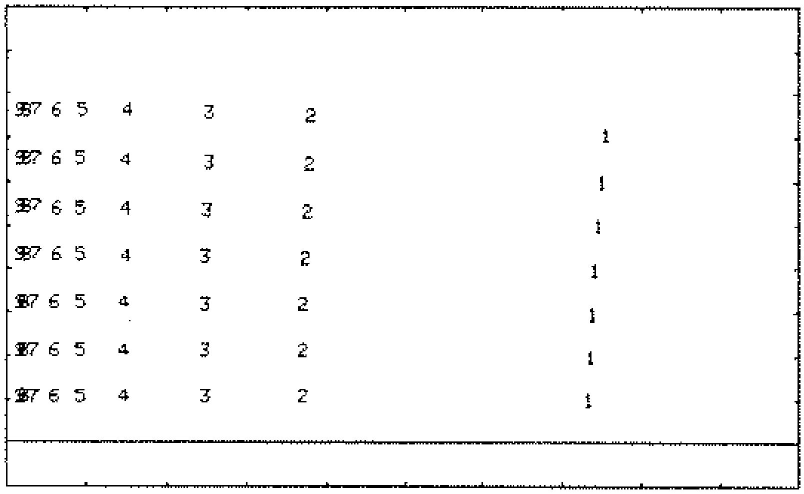


Figure 5: Point labels indicate momenta in list, lowest to highest. (Momentum, θ) points are unplotted for too large RX or RTPCMAX. No acceptable point is off scale.

```
ALL ANGLES = DEGREES, LENGTHS = METERS, MOMENTA = GEV/C, B_FIELD = TESLA.
L, D, DELTA, B_FIELD= 4  2.15  .7  .5
MOMENTA =
 .5  1  1.5  2.5  4  6  10  15  20
EMISSION THETA =
 2  4  6  8  10  12  14  20  30  45  60  80
PLOT IS RX, -.2  TO  2 , vs TWIST,  0  TO  30  DEGREES.
```



```
MOMENTA =
 .05  .1  .15  .2  .3  .4  .55  .7  .85
EMISSION THETA =
 2  4  6  8  10  12  14  20  30  45  60  80
PLOT IS RX, -.3  TO  3 , vs TWIST,  0  TO  60  DEGREES.
```

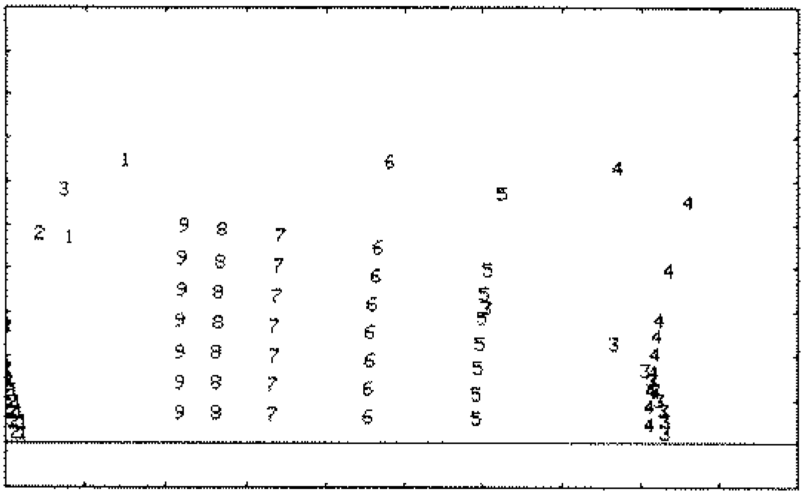


Figure 6: Point labels indicate momenta in list, lowest to highest. (Momentum, θ) points are unplotted for too large RX or RTPCMAX. No acceptable point is off scale.

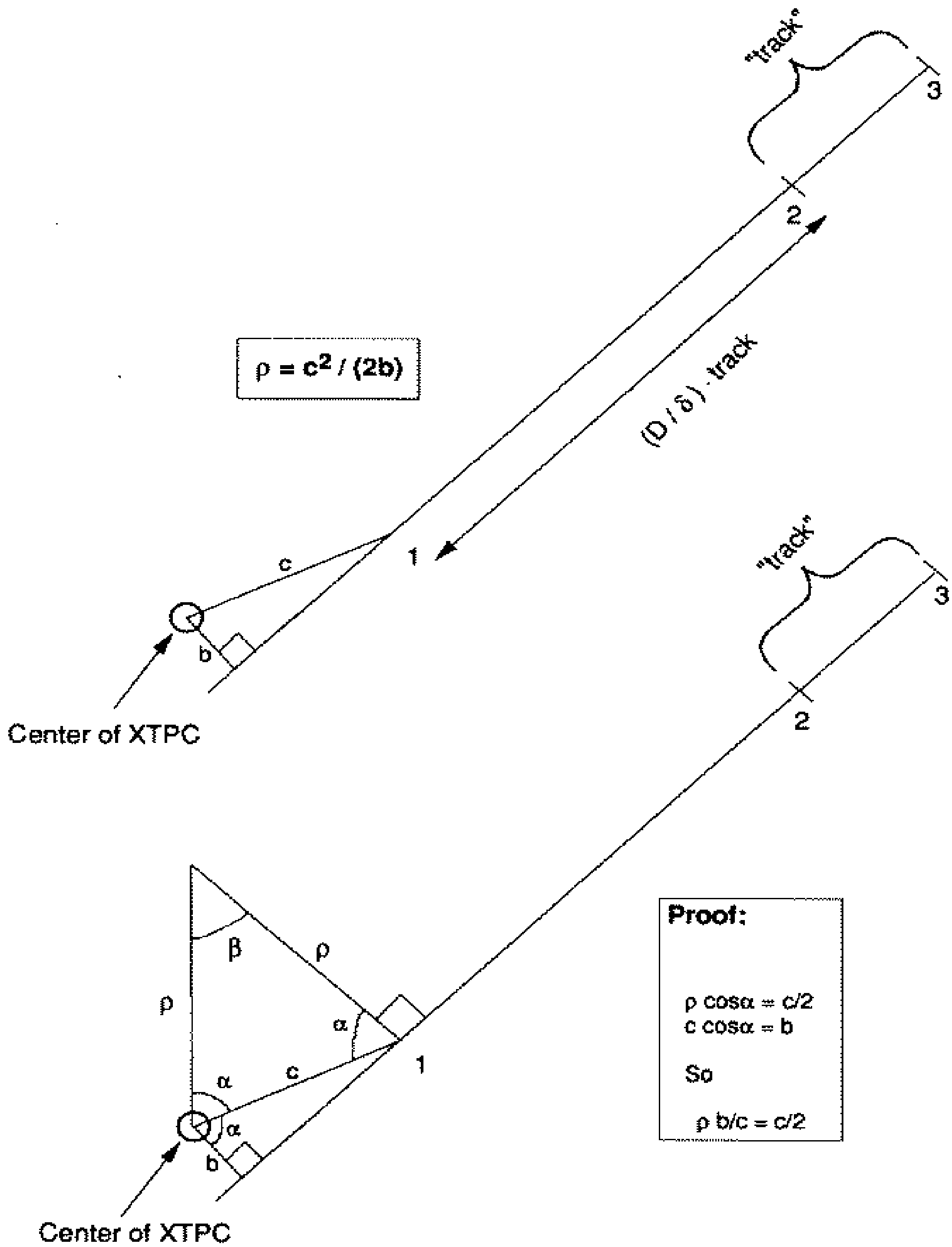


Figure 7: Reconstruction in $x - y$ plane from the "track".

```
INPUTS L+D, L, D, B_FIELD_GT(TESLA), DELTA =  6.150  4.000  2.150   .500   .700
GEANT'S TRACK, PARTID, Q, /PX, PY, PZ, /X2, Y2, /X3, Y3 (X AND Y IN m) =
 83  2  1 / 0 -.048  1.463 /-.0546 -.1923 /-.0637 -.2134
GEANT'S P, THETA, ETA, R_XTPC2, TRACK_LENGTH =  1.464  1.879  4.110  .200  .023
RHO, LAMBDA, BETA CONSTRUCTED FROM GEANT =     .320     61.282     23.498
RECONSTRUCTED b, c, RHO, ALPHA =     .0260    .1302    .3260  78.4771
RATIOS (RECONSTRUCTED/GEANT) FOR P_PERP, THETA, ETA, BETA, RHO =
  1.018787   1.000530    .999871    .980762   1.018787
```

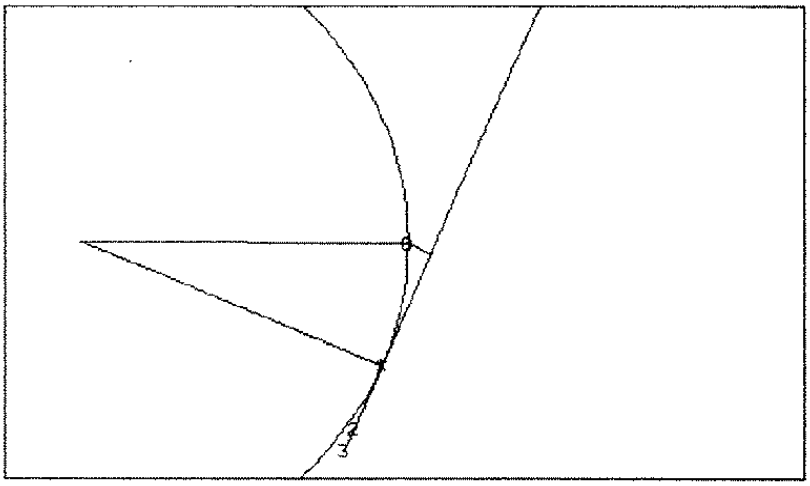


Figure 8: Center of XTPC is at '0'; frame is .8 meters long (horizontally).

```
INPUTS L+D, L, D, B_FIELD_GT(TESLA), DELTA =  6.150  4.000  2.150   .500   .700
GEANT'S TRACK, PARTID, Q, /PX, PY, PZ, /X2, Y2, /X3, Y3 (X AND Y IN m) =
 32  9 -1 /-.016  .192  .744 /-.904  1.2411 /-1.045  1.355
GEANT'S P, THETA, ETA, R_XTPC2, TRACK_LENGTH =   .769 14.518  2.061 1.535  .181
RHO, LAMBDA, BETA CONSTRUCTED FROM GEANT =    1.284     31.165     46.206
RECONSTRUCTED b, c, RHO, ALPHA =     .3974   1.0080   1.2785  66.7825
RATIOS (RECONSTRUCTED/GEANT) FOR P_PERP, THETA, ETA, BETA, RHO =
   .995382    .999926   1.000036   1.004949    .995382
```

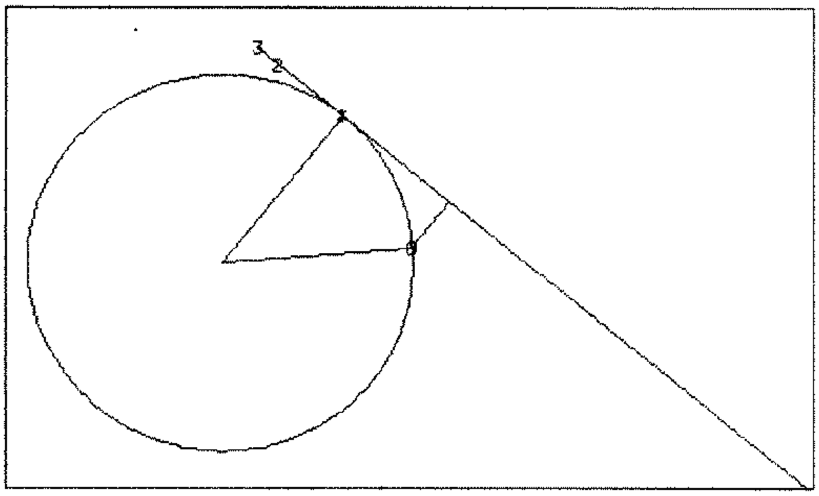


Figure 9: Center of XTPC is at '0'; frame is 5.4 meters long (horizontally).

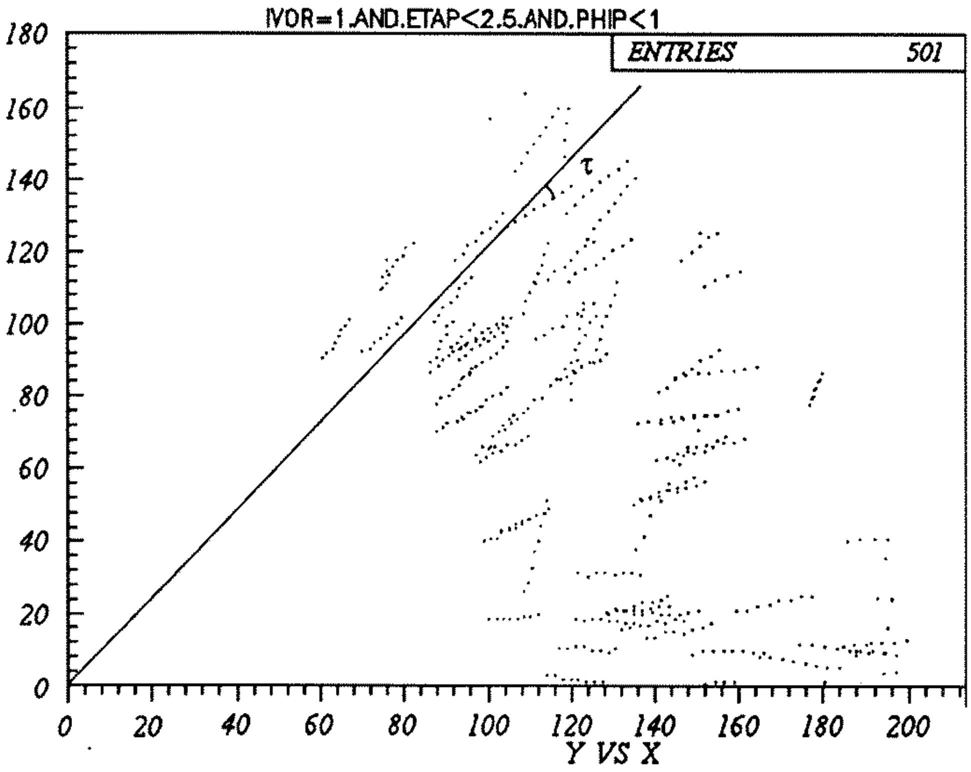


Figure 10: Sample tracks in the XTPC. Twist angle is illustrated.