A Model for Evaluating the Hit Resolving Abilities of a VTX style micro-TPC in a High Track Density Environment

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Abstract

A simple model has been constructed in Mathcad to study the hit resolving capability of a VTX style micro-TPC[1]. This model has been used to study a micro-TPC suitable for using as a vertex detector in STAR. There are parameters in the model for electron diffusion, drift velocity, TPC geometry, amplifier shaping time, read out anode length and pitch. Hit resolving power is studied as a function of these parameters to help in optimizing the design. Example applications for other TPC designs are also presented.

Introduction

We have developed a model for studying the ability of a VTX style micro-TPC to resolve single hits (track points) without confusion from nearby tracks. Micro TPCs are a promising detector for studying very high track density heavy ion collisions such as will be encountered at LHC and RHIC. In these collider experiments it is desirable to track as close to the collision point as possible to resolve vertices from short lived particles. These experiments are particularly challenging because the collisions are in the lab frame where there is no Lorenz boost to $c\tau$ and because the particle momenta are low coulomb multiple scattering can be a serious limitation. This multiple scattering problem makes the micro-TPC, with its very low mass, an attractive solution for the inner region near the vertex, because added material in this region compromises the momentum resolution of the outer detectors. The ideal detector would allow tracking to start at the vertex, but a detector's ability to resolve close tracks sets a limit on how close to the vertex a detector can operate effectively in this very high track density region. There are a number of design trade-offs in a micro-TPC and this model allows us to quickly explore a variety of design options. In the end, a more detailed Monte Carlo study is required, but this model is necessary to do the initial design tests and optimization. Below we develop the model and demonstrate how it works for an example design. The model is then used to explore hit-resolving sensitivity to the various parameters. We then show results from the model for a variety of design cases, including the main STAR TPC.

We have chosen to write this document completely in Mathcad which has caused some cosmetic flaws in the presentation, but the resulting reduction in writing time plus having a document which is an active program more than makes up for the cosmetic shortcomings. The Mathcad version will be available on the web for those interested in using the program.

Description of the Model

The model is for a tracking TPC of the following design[1]: As shown in figure 1 the TPC has a conventional cylindrical drift volume with the field cage split in the center with a central membrane so that the electrons left along the tracks drift to each end where they are detected by reading out the anodes of Micro-Strip Gas Chambers (MSGC).[2] The MSGCs have short, radially directed anodes and are arranged in concentric rings. The active volumes of the detector are the thin walled cylinders which project onto the MSGC rings. A hit is generated at every point that the tracks cross one of these active volumes. This model determines the fraction of these hits that are isolated and cleanly resolved. The model is developed in the

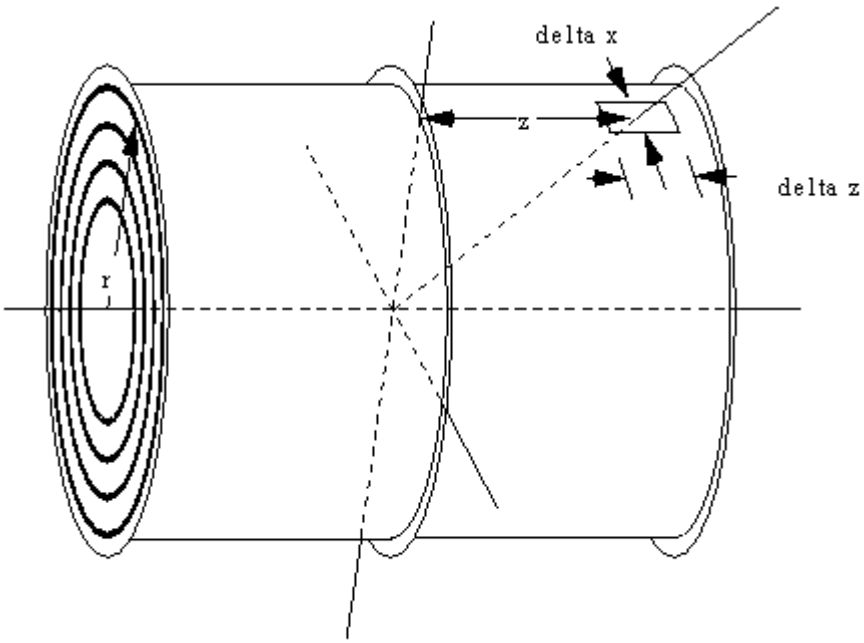


Figure 1. Schematic outline of the geometry of the TPC showing one of the cylindrical shell active volumes that is being modeled.

following manner: There is an area around every hit or track crossing point on the active cylinder, inside of which another hit cannot be resolved from the first. This is the hit resolving area (HRA) and it is determined by a variety of detector parameters which will be discussed in detail shortly. The fraction of hits that are cleanly resolved (FHR) is given by Poisson statistics (see appendix 1) as FHR = $\exp(-\rho hit * HRA)$, where ρhit is the area density of track crossings on the active cylinder. As will be shown, ρhit is determined using the assumption that $dN/d\eta$ (the particle pseudo rapidity density) is constant. The two hit resolving area is determined from drift length, electron diffusion constants, amplifier shaping time and drift velocity, anode length and the dip angle projection on to the time axis (z), and anode pitch. We have neglected resolution broadening due to the track crossing angle (α) relative to the anodes to avoid the complications of a momentum dependence. For the small radii considered for the micro-TPC and the low magnetic field this simplification does not cause a significant error. This omission is more serious when using the model for the full sized STAR TPC.

The model:

Determine area track density on the active cylinder

$$\eta = -\ln\left(\tan\left(\frac{1}{2}\cdot\operatorname{atan}\left(\frac{r}{z}\right)\right)\right)$$ pseudo rapidity

$$\frac{d\eta}{dz} = \frac{1}{2}\cdot\frac{r}{\left[\cos\left(\frac{1}{2}\cdot\operatorname{atan}\left(\frac{r}{z}\right)\right)\cdot\left[\left(z^2+r^2\right)\cdot\sin\left(\frac{1}{2}\cdot\operatorname{atan}\left(\frac{r}{z}\right)\right)\right]\right]}$$

$$\rho hit = \frac{\frac{dN}{d\eta}\cdot\frac{d\eta}{dz}}{2\cdot\pi\cdot r} = \frac{\frac{dN}{d\eta}}{2\cdot\pi\cdot r}\cdot\left[\frac{1}{2}\cdot\frac{r}{\left[\cos\left(\frac{1}{2}\cdot\operatorname{atan}\left(\frac{r}{z}\right)\right)\cdot\left[\left(z^2+r^2\right)\cdot\sin\left(\frac{1}{2}\cdot\operatorname{atan}\left(\frac{r}{z}\right)\right)\right]\right]}\right]$$ hit density

We use a value of 1000 particles per unit pseudo rapidity for central Au + Au collisions at RHIC. As shown in appendix 2 the hit density formula compares well with a Au + Au FRITIOF event[3] when $dN/d\eta$ is set equal to 750 particles per unit pseudo rapidity.

$$dNd\eta := 1000$$

$$\rho hit(r, z) := \frac{dNd\eta}{2\cdot\pi\cdot r}\cdot\left[\frac{1}{2}\cdot\frac{r}{\left[\cos\left(\frac{1}{2}\cdot\operatorname{atan}\left(\frac{r}{z}\right)\right)\cdot\left[\left(z^2+r^2\right)\cdot\sin\left(\frac{1}{2}\cdot\operatorname{atan}\left(\frac{r}{z}\right)\right)\right]\right]}\right]$$

$\rho hit(5\cdot cm, 10\ cm) = 2.847\cdot cm^{-2}$ an example for a radius of 6 cm and a z distance of 15 cm from the vertex.

$HRA = \Delta x\cdot\Delta z$ hit resolving area on the active cylinder

without consideration for anode pitch limitations we define:

$\Delta x = 3\cdot\sigma trans\cdot\sqrt{zmax - z}$ from transverse diffusion

some additional unit definitions

$ns := 10^{-9}\cdot sec$

$\mu s := 10^{-6}\cdot sec$

$\mu m := 10^{-6}\cdot m$

$$\Delta z = zproj + 3\cdot\sqrt{\sigma long^2\cdot(zmax - z) + \left(\frac{\tau shaper\ v}{2.35}\right)^2}$$

anode projection on to the z axis, longitudinal diffusion plus shaping time broadening

$$\Delta z = \frac{z\cdot w}{r} + 3\cdot\sqrt{\sigma long^2\cdot(zmax - z) + \left(\frac{\tau shaper\ v}{2.35}\right)^2}$$

where

$\tau shaper := 200\cdot ns$ shaping time FWHM

$v := 0.275\cdot\frac{cm}{\mu s}$ drift velocity[4]

$w := 3\cdot mm$ anode length (the thickness of the active cylinder wall)

$zmax := 14.5\cdot cm$ maximum drift length, the distance from the central membrane to the MSGC readout end plate

$\sigma long := 70\cdot\frac{\mu m}{\sqrt{cm}}$ longitudinal electron diffusion parameter[4]

$\sigma trans := 70\cdot\frac{\mu m}{\sqrt{cm}}$ transverse electron diffusion parameter

$pitch := 200\cdot\mu m$ anode pitch

Now, imposing some limitations due to the anode pitch. We set a lower limit on the two track resolution in the x or $r\phi$ direction as two times the anode separation and, when transverse diffusion sets the limit, we set the resolution to be 3 times the transverse diffusion sigma plus the separation between two anodes.

$$\Delta x(z) := if\left(3\cdot\sigma trans\cdot\sqrt{zmax - z} < pitch,\ pitch,\ 3\cdot\sigma trans\cdot\sqrt{zmax - z}\right) + pitch$$

We do not impose a similar limitation on the z resolution due to the time sampling frequency because compared to to other limitations the time sampling can always be easily made more than adequate.

$$\Delta z(r, z) := \frac{z\cdot w}{r} + 3\cdot\sqrt{\sigma long^2\cdot(zmax - z) + \left(\frac{\tau shaper\cdot v}{2.35}\right)^2}$$

$HRA(r, z) := \Delta x(z)\cdot\Delta z(r, z)$ hit resolving area

$HRA(6\cdot cm, 14\cdot cm) = 0.031 \cdot cm^2$ example value for r = 6 cm and z = 14 cm

$a(r, z) := \rho hit(r, z) \cdot HRA(r, z)$ filling fraction

$FHR(r, z) := \exp(-a(r, z))$ Fraction of Hits Resolved, the justification for this expression is given in appendix 1

$FHR(6\cdot cm, 14\cdot cm) = 0.95$ example case: 95% of the hits are resolved for tracks passing through a 6 cm radius cylinder at 14 cm in z from the vertex

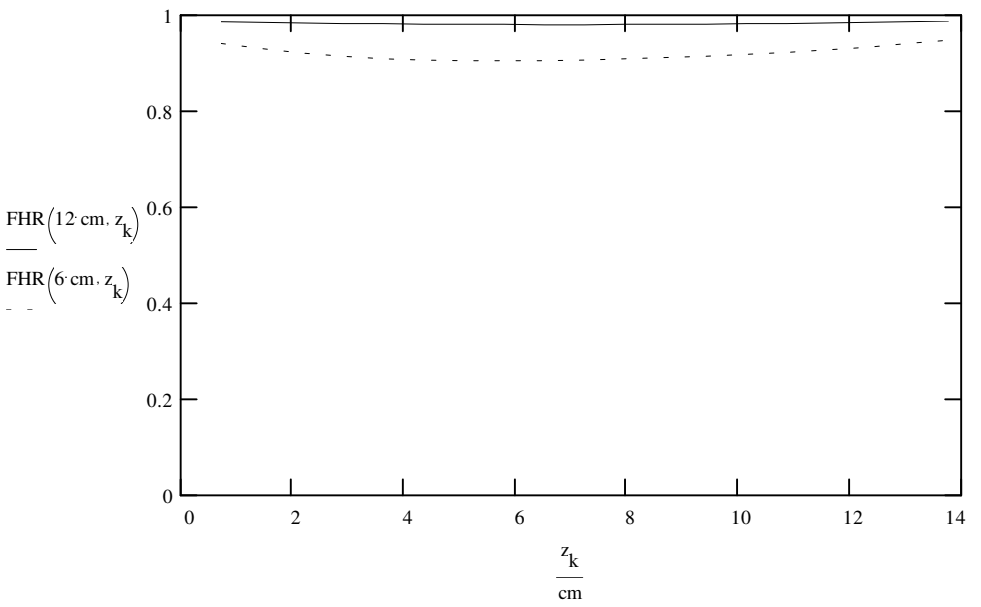
CASE 1:

The parameters have been set for the proposed VTX, micro-TPC. The following graph shows the fraction of hits resolved for the inner cylinder at 6 cm radius and the outer cylinder at 12 cm radius as a function of the track crossing distance (z) from the vertex. This is the expected performance for a Au + Au central collision at RHIC,

$kmax := 20$

$k := 1 .. kmax - 1$

$z_k := \frac{k \cdot zmax}{kmax}$



The minimum fraction of hits resolved on the inner cylinder is 91 % and the minimum on the outer cylinder is 98%.

$l := 0 .. \ kmax - 2$

$fhr_l := FHR\left(6 \cdot cm, z_{l+1}\right) \qquad min(fhr) = 0.9$

$fhr_l := FHR\left(12 \cdot cm, z_{l+1}\right) \qquad min(fhr) = 0.98$

In the above example we wrote down several equations leading to the expression for FHR. We have combined these equations into a single function of all the parameters for examining the remaining cases. The function is rather long and unreadable so we have used the Mathcad trick of pushing the function out of sight to the right of the page.

CASE 2:

Try a longer TPC, this will increase the diffusion, but it gives better η coverage

$zmax := 60\ cm$

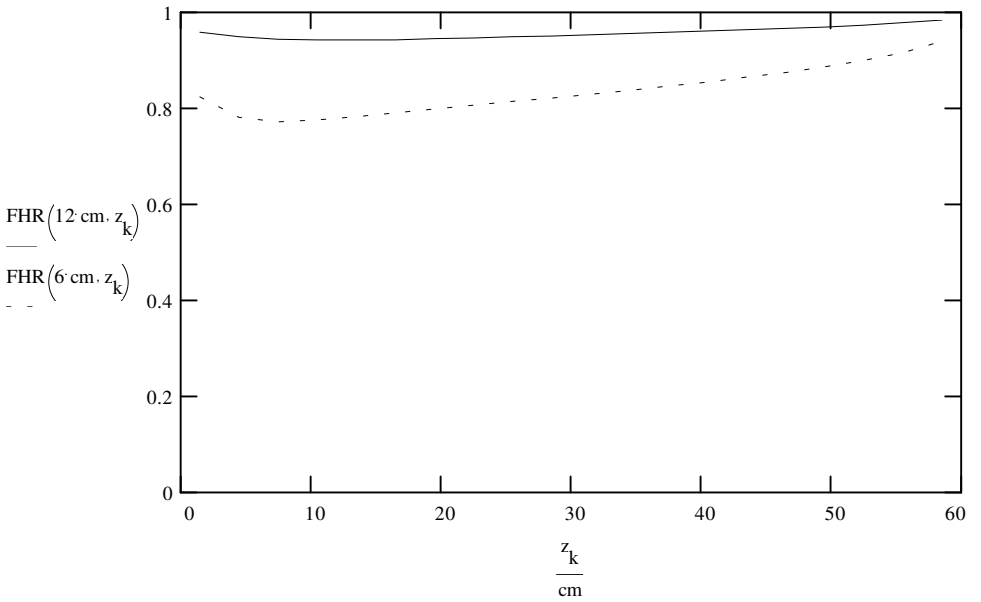
$FHR(r, z) := FHRT(r, z, dNd\eta, \sigma trans, \sigma long, zmax, w, pitch, v, \tau shaper)$

$kmax := 20$

$k := 0 .. kmax - 1$

$z_k := \frac{(k + 0.5) \cdot zmax}{kmax}$ $\quad fhr_k := FHR(12\ cm, z_k)$ $\quad min(fhr) = 0.94$ case 1

$fhr_k := FHR(6 \cdot cm, z_k)$ $\quad min(fhr) = 0.77$ 91%



In the above example we show the fraction of hits resolved for a larger zmax, TPC length. In the following we determine the dependence of FHR on each of the TPC parameters. In each case we scan in z and find the lowest FHR as a function of the parameter examined.

FHR dependence on TPC length:

In this first example zmax, the TPC length is varied over a range of 10 to 60 cm. This is for a cylinder radius of 6 cm. All of the other parameters are the same as used in the case 1 VTX test.

$r := 6 \cdot cm$

$kmax := 20$

$k := 0 .. kmax - 1$

$jmax := 20$

$j := 0 .. jmax$

$zm0 := 10 \cdot cm$

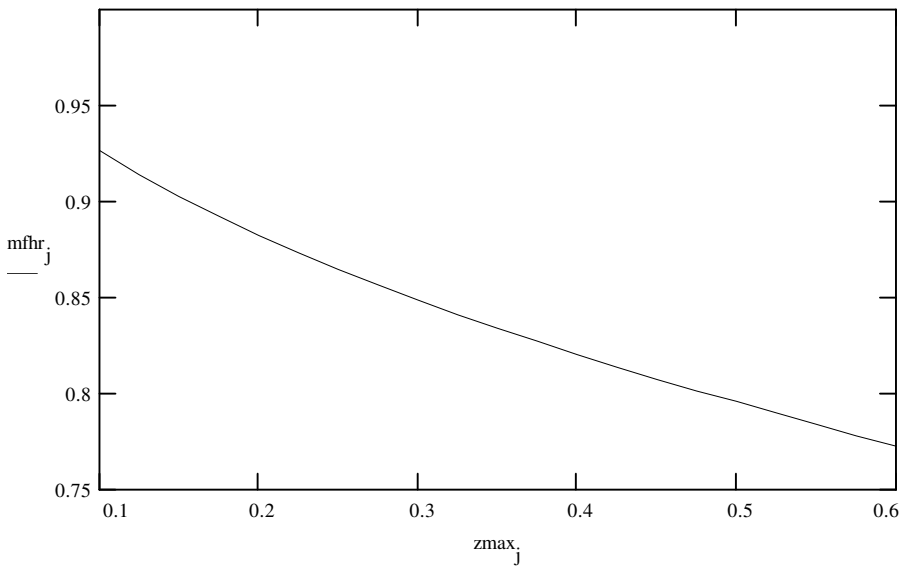
$zmf := 60\ cm$

$zmax_j := \frac{zmf - zm0}{jmax} \cdot j + zm0$

$z_{k,j} := \frac{(k + 0.5) \cdot zmax_j}{kmax}$

$fhr_{k,j} := FHRT(r, z_{k,j}, dNd\eta, \sigma trans, \sigma long, zmax_j, w, pitch, v, \tau shaper)$

$mfhr_j := \min(fhr^{\langle j \rangle})$



$max(mfhr) = 0.93$

$min(mfhr) = 0.77$

FRH as a function of the anode length, w:

$z := 0$ redefine z (necessary to clear definition for Mathcad)

$zmax := 14.5 \cdot cm$

$w0 := 1 \cdot mm$

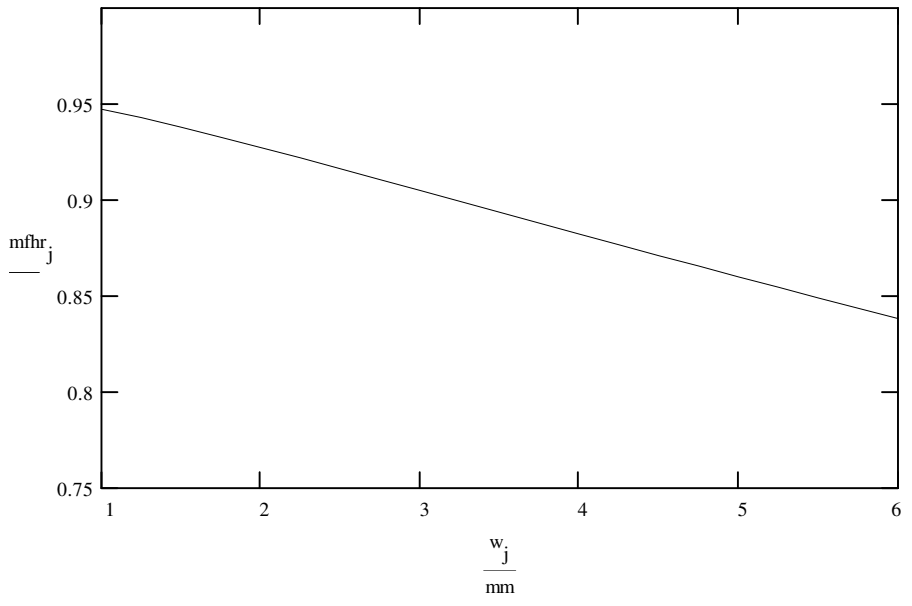
$wf := 6 \cdot mm$

$w_j := \frac{wf - w0}{jmax} \cdot j + w0$

$z_k := \frac{(k + 0.5) \cdot zmax}{kmax}$

$fhr_{k,j} := FHRT(r, z_k, dNd\eta, \sigma trans, \sigma long, zmax, w_j, pitch, v, \tau shaper)$

$mfhr_j := \min(fhr^{\langle j \rangle})$



$\max(mfhr) = 0.95$

$\min(mfhr) = 0.84$

FRH as a function of the anode pitch:

$w := 3 \cdot mm$

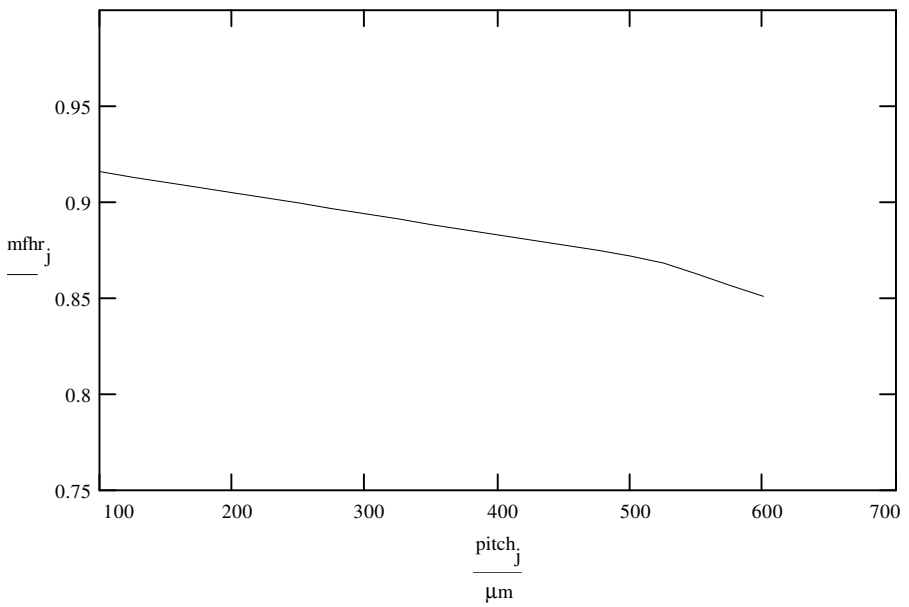
$pitch0 := 100 \cdot \mu m$

$pitchf := 600 \cdot \mu m$

$pitch_j := \frac{pitchf - pitch0}{jmax} \cdot j + pitch0$

$fhr_{k,j} := FHRT(r, z_k, dNd\eta, \sigma trans, \sigma long, zmax, w, pitch_j, v, \tau shaper)$

$mfhr_j := \min(fhr^{\langle j \rangle})$



$\max(mfhr) = 0.92$

$\min(mfhr) = 0.85$

FRH as a function of drift velocity:

$pitch := 200 \cdot \mu m$

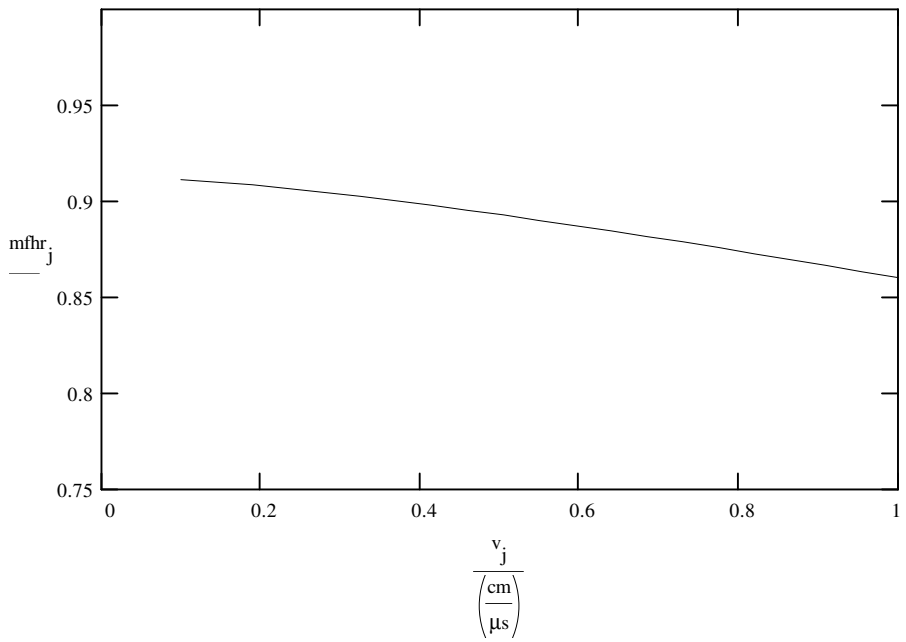
$v0 := 0.1 \cdot \frac{cm}{\mu s}$

$vf := 1.0 \cdot \frac{cm}{\mu s}$

$v_j := \frac{vf - v0}{jmax} \cdot j + v0$

$fhr_{k,j} := FHRT\left(r, z_k, dN d\eta, \sigma trans, \sigma long, zmax, w, pitch, v_j, \tau shaper\right)$

$mfhr_j := \min\left(fhr^{\langle j \rangle}\right)$



$\max(mfhr) = 0.91$

$\min(mfhr) = 0.86$

FRH as a function of shaping time:

$v := 0.275 \cdot \frac{cm}{\mu s}$

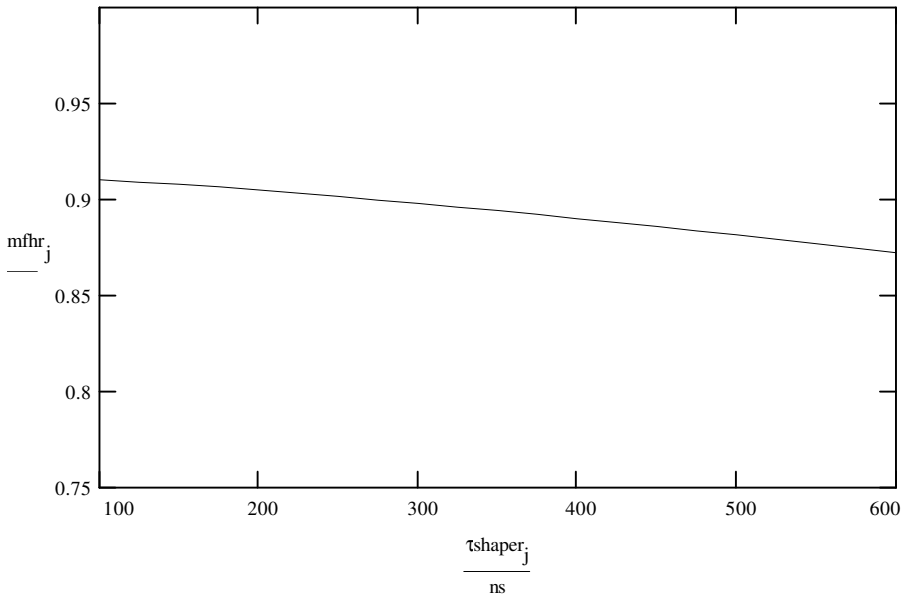
$\tau shaper0 := 100 \cdot ns$

$\tau shaperf := 600 \cdot ns$

$\tau shaper_j := \frac{\tau shaperf - \tau shaper0}{jmax} \cdot j + \tau shaper0$

$fhr_{k,j} := FHRT\left(r, z_k, dN d\eta, \sigma trans, \sigma long, zmax, w, pitch, v, \tau shaper_j\right)$

$mfhr_j := \min\left(fhr^{\langle j \rangle}\right)$



$\max(mfhr) = 0.91$

$\min(mfhr) = 0.87$

FRH as a function of electron diffusion:

$\tau shaper := 200 \cdot ns$

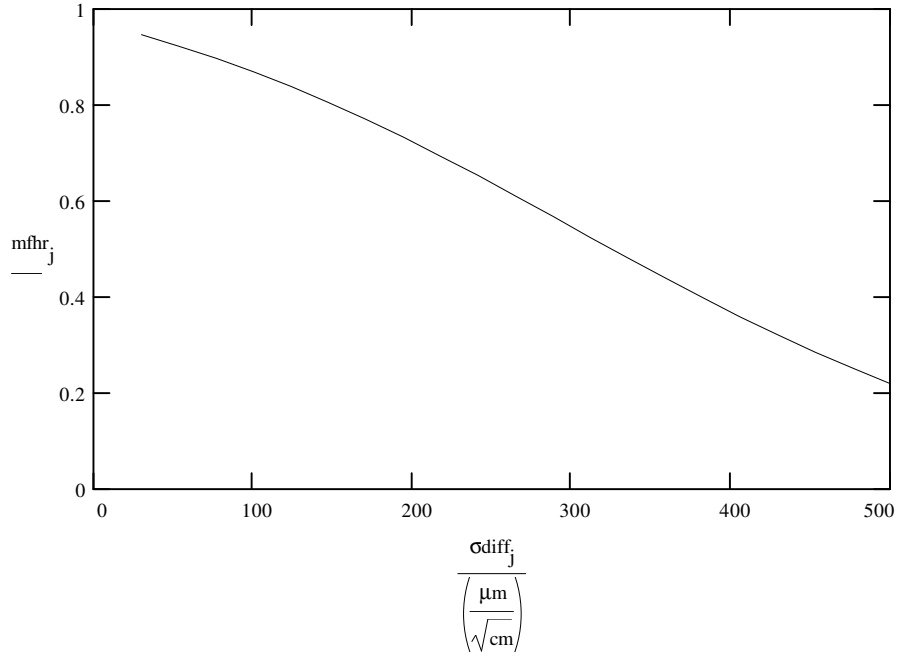
$\sigma 0 := 30 \cdot \frac{\mu m}{\sqrt{cm}}$

$\sigma f := 500 \cdot \frac{\mu m}{\sqrt{cm}}$

$\sigma diff_j := \frac{\sigma f - \sigma 0}{jmax} \cdot j + \sigma 0$

$fhr_{k,j} := FHRT\left(r, z_k, dN d\eta, \sigma diff_j, \sigma diff_j, zmax, w, pitch, v, \tau shaper\right)$

$mfhr_j := \min\left(fhr^{\langle j \rangle}\right)$



$\max(mfhr) = 0.95$

$\min(mfhr) = 0.22$

FRH as a function of active cylinder radius:

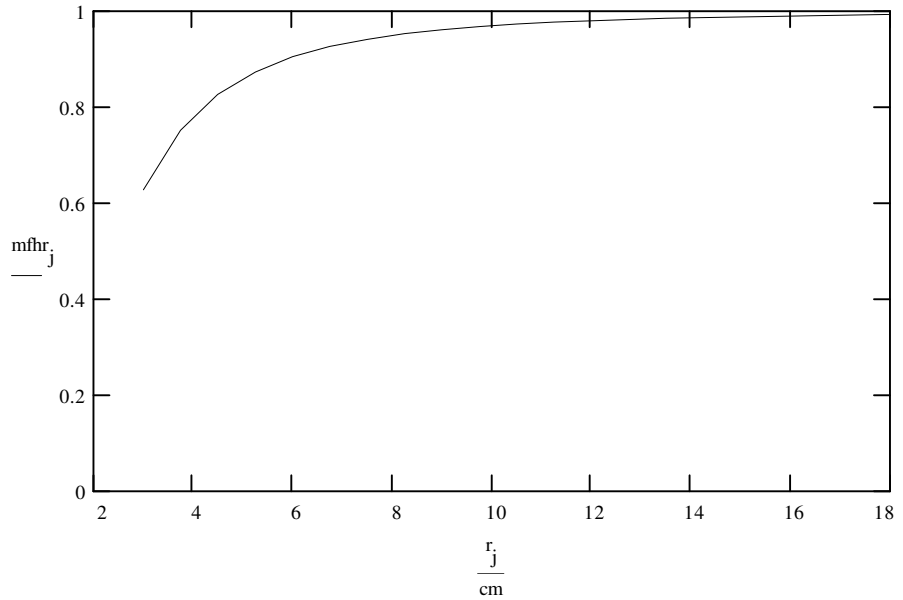
$r0 := 3 \cdot cm$

$rf := 18 \cdot cm$

$r_j := \frac{rf - r0}{jmax} \cdot j + r0$

$fhr_{k,j} := FHRT\left(r_j, z_k, dNd\eta, \sigma trans, \sigma long, zmax, w, pitch, v, \tau shaper\right)$

$mfhr_j := \min\left(fhr^{\langle j \rangle}\right)$



$\max(mfhr) = 0.99$

$\min(mfhr) = 0.63$

$fhr := 0$

$r := 6 \cdot cm$

In summary, the parameter study of the VTX geometry shows that the hit resolving performance is only slowly dependent on the various parameters, except for the electron diffusion and the cylinder radius. Clearly it is important in this application to control the diffusion as much as possible. In this case we have assumed that the longitudinal and transverse diffusion parameters track together, which is valid for the cooler gases. The diffusion term in this case comes in as a square while the other parameters have a more linear effect. None of the other parameters, taken individually, have a dramatic effect so we are free to make some adjustments for other optimizations. The strong dependence of FRH on radius sets hard limits on the minimum radius that can be tracked with this geometry and technology. Only a reduced diffusion will allow a significant improvement in the minimum radius that can be tracked. At this time we do not know of any diffusion values better than the DME values we have been using for this exercise.

In the following we show results for some other TPC configurations.

CASE 3:

Find FHR for the full sized STAR TPC inner sector and P10 gas

$zmax := 210 \cdot cm$

$\sigma trans := 167 \cdot \frac{\mu m}{\sqrt{cm}}$ $\sigma long := 330 \cdot \frac{\mu m}{\sqrt{cm}}$

$v := 5.5 \cdot \frac{cm}{\mu s}$

$w := 1.2 \cdot cm$

$pitch := 3.35 \cdot mm$

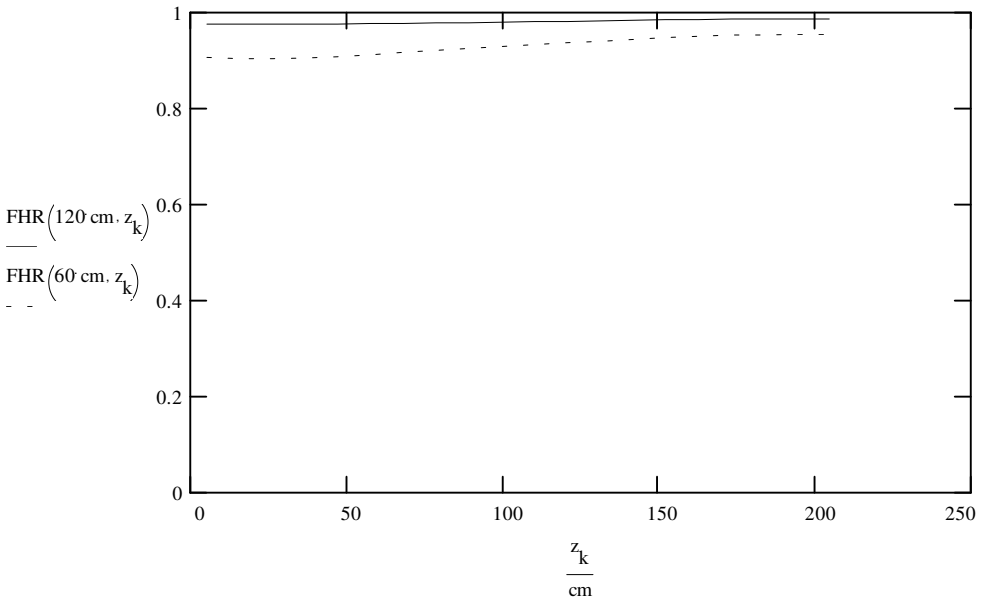
$FHR(r, z) := FHRT(r, z, dNd\eta, \sigma trans, \sigma long, zmax, w, pitch, v, \tau shaper)$

$kmax := 20$

$k := 0 .. kmax - 1$

$z_k := \frac{(k + 0.5) \cdot zmax}{kmax}$ $fhr_k := FHR(120\ cm, z_k)$ $\min(fhr) = 0.98$

$fhr_k := FHR(60\ cm, z_k)$ $\min(fhr) = 0.9$



To make a direct comparison with the fast simulator results we must average FHR weighted by the hit density from η = 0 to 1,

$\eta = -\ln\left(\tan\left(\frac{1}{2}\cdot\operatorname{atan}\left(\frac{r}{z}\right)\right)\right)$ pseudo rapidity

$zl(r,\eta) := \frac{r}{\tan(2\cdot\operatorname{atan}(\exp(-\eta)))}$ $zl(60\cdot cm, 1) = 71\cdot cm$

$zl(120\cdot cm, 1) = 141\cdot cm$

$\eta := 1$

$$Ave_FHR(r) := \frac{\sum_k if\left(z_k < zl(r,\eta), \rho hit\left(r, z_k\right), 0\cdot cm^{-2}\right)\cdot FHR\left(r, z_k\right)}{\sum_k if\left(z_k < zl(r,\eta), \rho hit\left(r, z_k\right), 0\cdot cm^{-2}\right)}$$

model result	Monte Carlo average from the FST, fast simulator.[3]
$Ave_FHR(60\ cm) = 0.91$	87%
$Ave_FHR(120\cdot cm) = 0.98$	96%

CASE 4:

Find FHR for the full sized STAR TPC outer sector and P10 gas

$zmax := 210\cdot cm$

$\sigma trans := 167\cdot\frac{\mu m}{\sqrt{cm}}$ $\sigma long := 330\cdot\frac{\mu m}{\sqrt{cm}}$

$v := 5.5\cdot\frac{cm}{\mu s}$

$w := 2.0\cdot cm$

$pitch := 6.7\cdot mm$

$FHR(r,z) := FHRT(r, z, dNd\eta, \sigma trans, \sigma long, zmax, w, pitch, v, \tau shaper)$

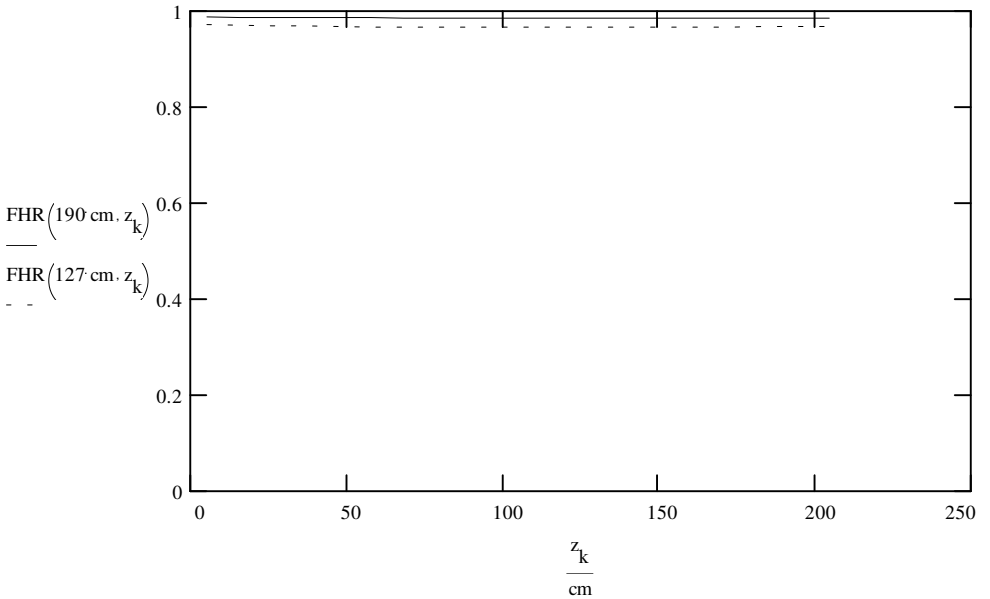
$kmax := 20$

$k := 0..kmax - 1$

$z_k := \frac{(k + 0.5)\cdot zmax}{kmax}$ $fhr_k := FHR\left(127\ cm, z_k\right)$ $\min(fhr) = 0.97$

$fhr_k := FHR\left(190\ cm, z_k\right)$ $\min(fhr) = 0.99$

Outer sector STAR TPC



$$\text{Ave_FHR}(r) := \frac{\sum_k \text{if}\left(z_k < zl(r,\eta), \rho\text{hit}\left(r, z_k\right), 0\cdot\text{cm}^{-2}\right)\cdot \text{FHR}\left(r, z_k\right)}{\sum_k \text{if}\left(z_k < zl(r,\eta), \rho\text{hit}\left(r, z_k\right), 0\cdot\text{cm}^{-2}\right)}$$

model result	Monte Carlo average from the FST, fast simulator.[5]
$\text{Ave_FHR}(127\cdot\text{cm}) = 0.97$	94%
$\text{Ave_FHR}(190\cdot\text{cm}) = 0.99$	98%

The above results for the STAR TPC are shown compared with the Monte Carlo studies with the fast simulator[5]. For the inner sector, inner row the model gives 91 % average FHR while the fast simulator gives an average of only 87 %. A similar discrepancy is observed for other rows of the TPC. We assume that this is because this analytical model does not include tan(α) degradation of the two track resolution. This effect is not as important for the VTX geometry.

CASE 5:

Test of VTX at LHC particle densities.

$dNd\eta := 3000$

$zmax := zl(24\ cm, 1)$ $\quad zmax = 28 \cdot cm$ Set outer cylinder at 24 cm radius and choose a drift length that will give coverage out to η = 1.

Set the rest of the parameters back to the original VTX values with DME gas.

$\tau shaper := 200 \cdot ns$

$v := 0.275 \cdot \frac{cm}{\mu s}$

$w := 3 \cdot mm$

$pitch := 200 \cdot \mu m$

$\sigma long := 70 \cdot \frac{\mu m}{\sqrt{cm}}$

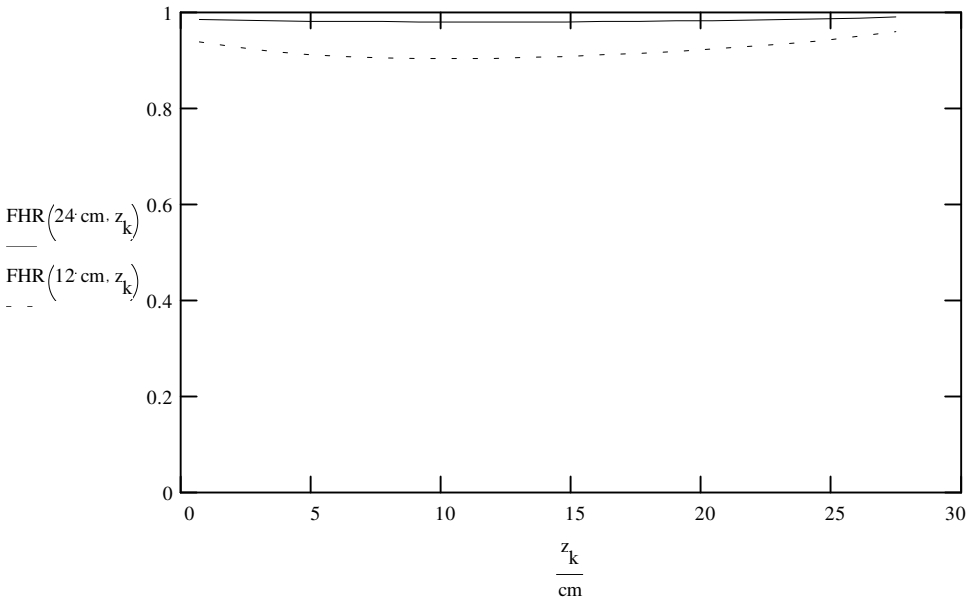
$\sigma trans := 70 \cdot \frac{\mu m}{\sqrt{cm}}$

$FHR(r, z) := FHRT(r, z, dNd\eta, \sigma trans, \sigma long, zmax, w, pitch, v, \tau shaper)$

$z_k := \frac{(k + 0.5) \cdot zmax}{kmax}$ $\quad fhr_k := FHR(24\ cm, z_k)$ $\quad min(fhr) = 0.98$

$fhr_k := FHR(12\ cm, z_k)$ $\quad min(fhr) = 0.9$

VTX at LHC



This shows that the VTX with an inner radius of 12 cm and an outer radius of 24 cm will work quite well at LHC particle densities of 3000 tracks per unit pseudo rapidity . The pseudo rapidity coverage is up to $\eta = 1$.

Conclusion

This study of the hit resolving capability of the VTX geometry show that the micro-TPC will work quite effectively as a vertex detector in the track densities expected at RHIC with central Au + Au collisions. These results are in rough agreement with earlier Monte Carlo studies with a fast simulator.[6] This study provides additional information showing that the parameters chosen are safe. The design has a margin of safety and, in fact, it may be desirable to increase the TPC length if DME can be shown to support longer drift lengths. The model also shows reasonable agreement with fast simulator studies of the STAR TPC.

It has also been shown with this model that the VTX micro-TPC concept will work very well at LHC track densities with an inner radius of 12 cm.

Acknowledgments

We would like to thank Howard Matis for helpful discussions on hit resolution statistics.

Appendix 1

Statistics for Determining the Fraction of Hits Resolved (FHR)

In the following exercise we review the statistics for track pile up. We designate a hit as a point where a track crosses an active detector surface and we determine the probability that the hit is resolved without confusion from a near by hit. Think of the surface as divided up into area elements where the area of these elements is defined by the two track resolution of the detector, namely, if two or more hits share the area element they will not be resolved. We refer to this area element as the hit resolution area (HRA). Consider a limited region on the detector surface where the hit density can be treated as a constant. Then each HRA element in this region samples the hit density. This is the standard situation where Poisson statistics applies. The probability of finding m hits in a HRA area sample is given by the Poisson distribution:

$$P(m,a)=\frac{a^m\cdot\exp(-a)}{m!}$$

Where "a" is a constant determined by the area hit density. (These are the same statistics that apply for the more familiar question concerning the probability that m cosmic rays are detected in a specified time interval.[7])

So, in our limited region on the detector surface where both the hit density and the HRA can be treated as constant

$$Nsamples\cdot\sum_{m=1}^{\infty}P(m,a)\cdot m=Nhits \qquad (1)$$ where

$Nsamples=\frac{Aregion}{HRA}$ is the number of samples in the region

$Aregion$ is the area of the region

HRA is the local hit resolving area of the detector

$Nhits=Aregion\cdot\rho hit$ is the number of hits in the region and

ρhit is the local area hit density.

Then equation 1. becomes:

$$\sum_{m=1}^{\infty}P(m,a)\cdot m=\sum_{m=1}^{\infty}\frac{a^m\cdot\exp(-a)}{m!}\cdot m=\rho hit\cdot HRA$$

Evaluating the sum and solving for "a" gives:

$a = \rho hit \cdot HRA$

Now the fraction of hits resolved is given by the probability of finding one hit in a HRA area element divided by the total average number of hits in the element.

$$FHR = \frac{P(1, a) \cdot 1}{\sum_{m=0}^{\infty} P(m, a) \cdot m} = \frac{a \cdot \exp(-a)}{\sum_{m=1}^{\infty} \frac{a^m \cdot \exp(-a)}{m!} \cdot m}$$

evaluating the sum gives

$FHR = \exp(-a) = \exp(-\rho hit\ HRA)$ <<<<<<<<<<<<<<<<<<<<<<

Monte Carlo check of the formula

$Nsamples := 10000$ each bin of a histogram is a sample

$\Delta := \frac{1}{Nsamples}$ $i := 0 .. Nsamples$ $bin_i := i \cdot \Delta$ boundary of histogram bins, dividing up the region from 0-1

$k := 0 .. Nsamples - 1$ index for each histogram bin

$Nhits := 40000$ number of hits distributed over the region 0-1

$j := 0 .. Nhits - 1$ hit index

$xhit_j := rnd(1)$ location of each hit

$a = \rho hit \cdot HRA = \frac{Nhits}{1} \cdot \Delta = \frac{Nhits}{Nsamples}$ $a := \frac{Nhits}{Nsamples}$

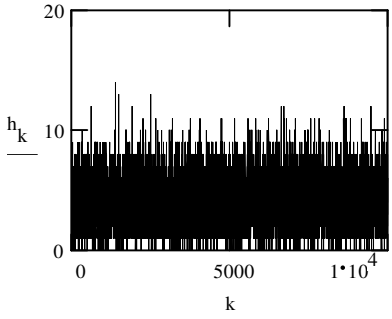
$h := hist(bin, xhit)$ histogram the hit distribution

$$FRH := \frac{\sum_k \text{if}\left(h_k = 1, 1, 0\right)}{Nhits}$$

sum all of the histogram bins with one and only one hit and divide by the total number of hits to get the fraction of resolved hits.

Compare with the derived expression

FRH = 0.01768 $\exp(-a)$ = 0.01832 checks



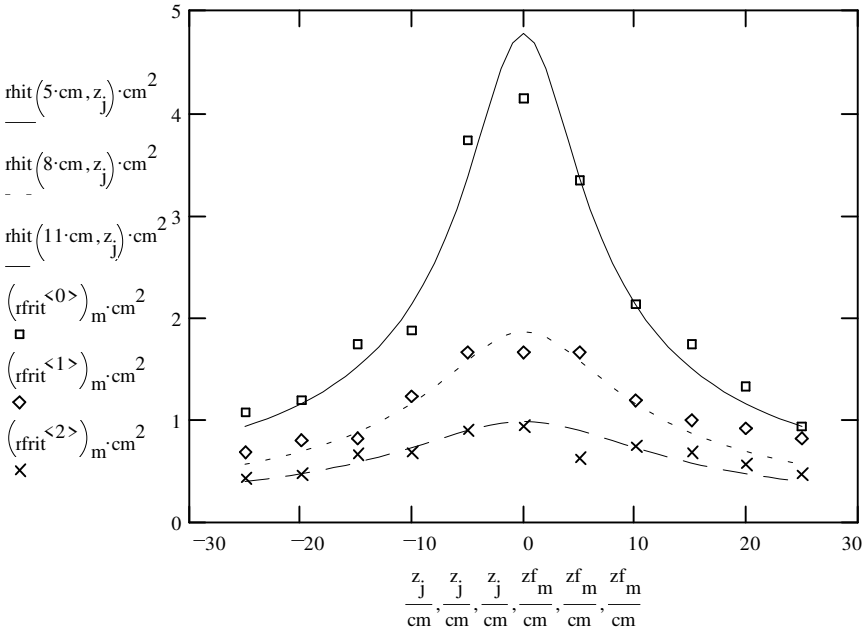
Appendix 2

Comparison of the constant dN/dη hit density formula with a Au + Au FRITIOF event at the RHIC energy of 100 GeV*A on 100 GeV*A

$$dNdh := 750$$

$$rhit(r,z) := \frac{dNdh}{2\cdot p\cdot r}\cdot\left[\frac{1}{2}\cdot\frac{r}{\left[\cos\left(\frac{1}{2}\cdot\operatorname{atan}\left(\frac{r}{|z|}\right)\right)\cdot\left[\left(z^2+r^2\right)\cdot\sin\left(\frac{1}{2}\cdot\operatorname{atan}\left(\frac{r}{|z|}\right)\right)\right]\right]}\right]$$

Comparison of FRITIOF dN/dArea values with the dN/dArea formula obtained assuming a constant dN/dη .



The hit density (hits/cm^2) is shown for cylinders of radii: 5, 8 and 11 cm as a function z (cm) along the axis of the cylinder. The data points are FRITIOF values extracted from ref. 2. The value of dN/dη = 750 was chosen to give a good fit to the eye.

References:

1. H. Wieman, S. Margetis, "The VTX micro TPC", Presented at the SVT working group, STAR Collaboration meeting, Aug. 1994.

2. W.G. Gong, H. Wieman, J.W. Harris, J.T. Mitchell, W.S. Hong, V. Perez-Mendez, "Microstrip Gas Chambers on Glass and Ceramic Substrates", IEEE Transactions on Nuclear Science, V0l. 41 No. 4, (1994) 890

3. S. Margetis, W. Christie and G. Odyniec, "Some Considerations About the SVT Geometry", STAR Note # 69, June 1992.

4. J. Va'vra, P. Coyle, J. Kadyk and J. Wise, "Measurement of electron drift parameters for helium and CF4-based gases", NIM **A324** (1993) 113

5. STAR Collaboration, "STAR Project Conceptual Design Report Update", Jan. 1993, see figure 4C-3b, p15. The fraction of resolved hits is 1 - the fraction of merged hits in the FST simulation.

6. Jaw-Luen Tang, S. Margetis, H. Wieman, W. Gong, S. Paganis and C. Byrd,"A Fast Simulator for VTX, a microTPC for STAR", STAR Note # 199, March 1994.

7. F. Mathews and R.L. Walker,"Mathematical Methods of Physics", W.A. Benjamin, Inc., New York, Amsterdam, 1965.