STAR Note #204
IOA.311/94

PION THERMALISATION IN RELATIVISTIC HEAVY ION COLLISIONS

V. Emelyanov[a,b] and G. Pantis[a]

a) Division of Theoretical Physics, University of Ioannina,
GR 451 10, Ioannina, Greece

b) Department of High Energy Physics,
Moscow Physical Engineering Institute,
Kashirskoe shosse 31,
115409 Moscow, Russia

Abstract

We consider a model of thermalisation of pions due to elastic scattering and define the region of applicability of this model. In this region the chemical potential is positive. We analyse the absorptive and dispersive properties of pions during thermalisation, by calculating the pion collective potential. The pion gas can be thermally equilibrated but it will be out of chemical equilibrium. We define the degree of thermalisation near freezout at the level $\sim 70\%$ for $S+S$ and $\sim 90\%$ for $O+Au$ collisions at CERN. We find, that the effect of the chemical potential in pion spectra is stronger than the temperature effect at $p_\perp \leq 200$ MeV.

e-mail-G.Pantis: gpantis@cc.uoi.gr

1. Introduction.

A good deal of experimental data, obtained in the last few years at CERN and BNL give a strong evidence for the formation of dense nuclear matter in relativistic heavy ion collisions. However it is not clear a priori whether or not a local thermodynamic equilibrium in hadronic matter is established in these collisions. For this reason it seems resonable to consider models of preequilibrium states of hadronic matter, which describe the process of relaxation to equilibrium state.

Lately several attemps have been made to describe the kinetic process responsible for the equilibrium state, see for example the papers of Ref.[1]. In these papers, the initial conditions in heavy ion collisions are given by the structure functions of the colliding nuclei. The next important step to describe quantitatively the kinetic stage was made in Ref.[2, 3]. In the conventional kinetic theories, the evolution of the quark (antiquark) distribution function is described by the Boltzmann equation with the collision integral in the relaxation time approximation. The times of establishing local thermodynamic equilibrium in the quark - gluon system are estimated[4, 5] at the level $t \sim 1fm$, which are smaller than the characteristic times in heavy ion collisions. In this case there is a chance of formation of a quark-gluon plasma (QGP) at the initial temperature $T_i > T_c \sim 200MeV$.

A big progress in the description of preequilibrium states of hadronic matter has been achieved in recent works[6], which combine the kinetic approach with perturbative QCD. This so called parton cascade model predicts relaxation times $t \simeq 0.2 \div 0.3fm$, and the initial temperatures $T_i \simeq 500 \div 800MeV$ are greater, than in previous estimations[4, 5]. These results offer a more optimistic expectation for the observation of QGP formation in heavy ion collisions, i.e., QGP signals such as dilepton and photon emission from early stages of heavy ion collisions[7, 8, 9]. But this is only the one side of the thermalisation problem, which is connected with the evolution of quarks and gluons during the collision. The other one originates after the hadronization of quarks, antiquarks and gluons into hadrons. In thermalized QGP, this happens at the temperature $T \simeq T_c$, in non-thermalized QGP, at $t = t_0 < t_{AA}$ (t_{AA} - characteristic time of AA collision, $t_{AA} \simeq 10fm$). On the other hand, nuclear matter expands in the interval $t_0 < t < t_{AA}$ in the hadronic phase and thus the question arises about the equilibrium properties of the hadronic phase.

In Bjorken's picture of evolution, the equilibrium QGP transforms into equilibrium hadronic gas at $T = T_c$. But this is not a necessary condition and one can consider preequilibrium states of hadronic matter, relaxing to equilibrium ones. Under equilibrium properties we mean thermal and chemical equilibrium properties of hadronic matter.

It is well known, that pion changes its absortive and dispersive properties in nuclear medium. A collective potential is very sensitive to the details of pion interaction with the medium. In this paper we will also consider the pion collective potential at the preequilibrium state. The interest in the models of preequilibrium states of hadronic matter was stimulated by the low $p_\perp$ enhancement[10] in pion spectra, measured in the central region of $200A\,GeV$ nucleus-nucleus collisions at CERN. This deviation from a simple thermal distribution is surprising because pion is a Goldstone boson and soft pions should interact weakly and thus their production should be dynamically suppressed. The model[11, 12] of relaxation of pion gas was successfully applied in the explanation of low $p_\perp$ enhancement.

In this paper we wish to define the limits of applicability of this model, investigate the influence of the chemical potential on the pion spectra and make some suggestions of how to use the properties of transverse momentum spectra to test the picture of relaxation of a preequilibrium state of hadronic matter. In section 2 we give a short outline of our model of the preequilibrium state, in sect. 3 we consider absorptive and dispersive properties of pions during thermalisation and in sect. 4 we discuss our results. Finally, we give our conclusions in sect. 5.

2. Description of the Model.

The model which we will use here, has been described previously[11, 12], and we shall only repeat here the main points as applied to the present work. We shall assume that pion matter, produced in the central region of heavy ion collisions, is not far from the state of local thermodynamic equilibrium. As is well known, $\pi\pi$ interactions at the center of mass energies $E_{\pi\pi} \leq 1GeV$ are predominantly elastic[13], thus the pion number is conserved during the expansion in AA collisions. In this case the relaxation to the equilibrium state is connected with elastic scattering. The pion collision frequency is density dependent[14, 15] and decreases as the density drops. In the Boltzmann framework, the phase space evolution of pion matter is defined by a distribution function $f(\mathbf{p}, \mathbf{r}, \mathbf{t})$. For the predominantly longitudinal expansion one can write the kinetic equation:

$$\Big(\frac{\partial}{\partial t} + \frac{p_z}{E_p}\frac{\partial}{\partial z}\Big) f(p_\perp, p_z, z, t) = \Big(\frac{\partial f}{\partial t}\Big)_{coll.}, \tag{1}$$

where $\left(\frac{\partial f}{\partial t}\right)_{coll.}$ is the rate of change of f due to the collisions.
In the relaxation - time approximation the collision term is

$$\Big(\frac{\partial f}{\partial t}\Big)_{coll.} = -\nu[f(\mathbf{p}, z, t) - f_{eq}(\mathbf{p}, z, t)], \tag{2}$$

where $f_{eq} = \{exp[(E_p - \mu)/T] - 1\}^{-1}$ is the local equilibrium distribution function with local temperature $T = T(t)$, chemical potential $\mu = \mu(t)$ and ν^{-1} is the relaxation time. From the analysis of $\pi\pi$ scattering at low and medium energies one can conclude[11, 12, 16], that $\nu^{-1} = \alpha^{-1}t$, summarizes the essential features. The parameter α is connected with the ratio of the scattering frequency to the expansion rate and is related to the final state pion rapidity density $\frac{dN}{dy}$ by

$$\alpha = \frac{\sigma_0}{\pi R_A^2}\frac{dN}{dy} \tag{3}$$

where $\sigma_0 \simeq 10mb$ is the elastic $\pi\pi$ cross section and R_A is the radius of the colliding nuclei.

The solution of (1) is the pion distribution function

$$f(p_\perp, p_z, t) = \Big(\frac{t_0}{t}\Big)^\alpha f_0(p_\perp, p_z\frac{t_0}{t}) + \alpha\int_{t_0}^{t}\frac{dt'}{t'}\Big(\frac{t'}{t}\Big)^\alpha f_{eq}(p_\perp, p_z\frac{t}{t'}, t'). \tag{4}$$

It contains the contributions from the initial state (at $t = t_0$) of pion gas (f_0) and the thermalized part f_{eq}, and thus it can be used to extract information about the partial thermalisation of pions. The time t_0 is the time, when pion -like excitations become well defined.

It depends on the geometry of collision, and we shall take[12] $t_0 \simeq 2fm$ and $3fm$ for $S+S$ and $O+Au$ respectively. Further it is convinient to introduce the time-dimensionless variable $s = t/t_0$ to describe the time evolution of the pion system.

Let's have at the initial state of nuclear collision $t = t_0$ the pion density n_0 and the energy density ϵ_0. The pion density n and energy density ϵ will be decreasing for $t > t_0$ due to the expansion of nuclear matter. The conservation of tensor energy - momentum during the expansion requires that

$$\begin{aligned} n(t) &= \frac{n_0 t_0}{t} \\ \epsilon(t) &= \epsilon_0 F(t) \end{aligned} \tag{5}$$

where $F(t)$ can be obtained as a solution of an integral equation[11].
For each time t we have two self - consistent equations, for the local chemical potential $\mu(t)$ and the local temperature $T(t)$.

$$\begin{aligned} \epsilon(t) &= \frac{g}{(2\pi)^3}\int \frac{\sqrt{p^2+m_\pi^2}\,d^3p}{e^{\frac{\sqrt{p^2+m_\pi^2}-\mu(t)}{T(t)}}-1} \\ n(t) &= \frac{g}{(2\pi)^3}\int \frac{d^3p}{e^{\frac{\sqrt{p^2+m_\pi^2}-\mu(t)}{T(t)}}-1} \end{aligned} \tag{6}$$

where $g = 3$ for the three - pion state and $c = 1$. Suppose that we have at the initial state some initial distribution function f_0 in eq.(4) and the initial values n_0 and ϵ_0. Using Eqs. (5,6) we can find $\mu(t)$ and $T(t)$ for any $t > t_0$ and define the distribution function $f(p_\perp, p_z, t)$.

3. The Pion Collective Potential at the Preequilibrium State.

In this section we are going to investigate the pion properties during the thermalisation process. For this purpose we calculate the momentum-dependent collective potential (both real and imaginary parts) for π-mesons, propagating in hadronic matter. In the central region of relativistic heavy ion collisions hadronic matter is predominantly consisting of pions. Thus we will consider the pion collective potential in pion medium[17, 18]. As we will see, in the collective potential there is a competition between the repulsive and attractive parts, which in some cases may even significantly compensate each other.

At leading order in the virial expansion, the mass operator of a pion with a four-momentum p moving in hadronic matter[17, 18] is given by

$$\sum(p) = \int \frac{d^3k}{(2\pi)^3 2w_k} f(\omega_k) T_{\pi\pi}(k,p) \tag{7}$$

where $\omega_k = \sqrt{m_\pi^2 + k^2}$ is the energy of the pion "medium" with four - momentum k; $T_{\pi\pi}(k,p) = T_{\pi\pi}(S)$ is the relevant $\pi\pi$ forward scattering amplitude and $S = (p+k)^2$ is the square of the centre-of-mass energy. Equation (7) was used in[17] to calculate the pion collective potential in equilibrium hadronic matter. In this case $f(\omega_k)$ is simply the Bose thermodynamic weight. To what extent is this assumption of thermal equilibrium of matter really important? The only relevant point is the fact that particles are attracted to each other, so the through going wave, changes its energy at a given momentum. Thus we can calculate the collective potential for non-equilibrium pion matter, substituting the distribution function of eq. (4) into eq. (7).

In [17] the forward scattering amplitude $T_{\pi\pi}(S)$ was taken as the leading resonance approximation i.e, the Breit-Winger formulae. This means that one selects only resonances in the direct channel, which are dominant in the scattering amplitudes and ignore other channels. As noted in[17], neglected amplitudes usually have a tendency to nearly cancel each other when the isospin averaging is made. However, in our case it is not sufficient to use the Breit-Wigner formulae for the forward scattering amplitude, because it ignores the finite pion chemical potential. This problem was solved in[19] using a generalized Bruckner-Hartree-Fock scheme, which incorporates higher order terms in the density, i.e. beyond the leading term in the virial expansion. In this approach the pion mass operator[19] is

$$\begin{aligned} \sum(\omega_p) &= \int \frac{d^3k}{(2\pi)^3 2\omega_k} f(\omega_k,\mu,T) \\ & \sum_{I,l}(-32\pi)\frac{2I+1}{3}\sqrt{\frac{S}{S-4m_\pi^2}}(2l+1)\frac{tg\delta_l^I(S)}{1-iLtg\delta_l^I(S)}, \end{aligned} \tag{8}$$

where $\omega_p = \sqrt{p^2+m_\pi^2}$

As it is shown in[18], it is sufficient to limit ourselves to three channels, namely $(I=0,l=0),(I=1,l=1)$ and $(I=2,l=0)$. Further, we will use in the following the parametrization of the phase shifts $\delta_l^I(S)$ given in[20]

$$\begin{aligned} \delta_0^0(S) &= \frac{\pi}{2} + arctan\Big(\frac{\sqrt{S}-m_\sigma}{\Gamma_\sigma/2}\Big) \\ \delta_1^1(S) &= \frac{\pi}{2} + arctan\Big(\frac{\sqrt{S}-m_\rho}{\Gamma_\rho/2}\Big) \\ \delta_0^2(S) &= -0.12\frac{q}{m_\pi} \end{aligned}$$

where $q=\frac{1}{2}\sqrt{S-4m_\pi^2}, \Gamma_\sigma = 2.06q, m_\sigma = 5.8m_\pi$,

$$\Gamma_\rho = 0.095q\Big(\frac{q/m_\pi}{1+(q/m_\rho)^2}\Big)^2, m_\rho = 5.53m_\pi \tag{9}$$

The phase shifts δ_0^0 and δ_0^2 are connected with s-wave scattering and the phase shift δ_1^1 -with p-wave scattering. The thermal factor L in eq. (8) is

$$L = 1 + \frac{T\sqrt{S}}{q|\mathbf{k}+\mathbf{p}|} ln\Big[\frac{1-exp(-B/T)}{1-exp(-D/T)}\Big], \tag{10}$$

where

$$B = \frac{\omega_p+\omega_k}{2} + \frac{q|\mathbf{k}+\mathbf{p}|}{\sqrt{S}} - \mu, \tag{11}$$

$$D = \frac{\omega_p + \omega_k}{2} - \frac{q|\mathbf{k} + \mathbf{p}|}{\sqrt{S}} - \mu. \quad (12)$$

As one can see from eq. (8), the pion mass operator depends on the temperature T and the chemical potential μ via the distribution function $f(\omega_k, \mu, T)$ and the thermal factor L.

The pion collective potential is defind in[17] by

$$V = \frac{\Sigma(\omega_p)}{2\omega_p}. \quad (13)$$

Physically,the real part of the collective potential ReV, is the difference between the pion energy in the medium and the "free space" at the same three-momentum **p**. The damping rate γ or the collision rate of the pions in the medium, is related to the imaginary part of the collective potential by

$$\gamma = -2ImV. \quad (14)$$

Using the pion distribution function eq.(4), we can calculate ReV and ImV from the initial state (at $t/t_o \sim 1$) up to the final state of pion matter in heavy ion collisions. The results of these calculations are represented in the next section.

4. Results and Discussion.

We have solved the system of the equations (5,6) numerically and we have observed three regions of solutions for different combinations of n_0, ϵ_0. These regions are shown in Fig.1:
a) solutions with negative μ (from the long - dashed line upwards)
b) solutions with positive μ (between long -dashed and solid line)
c) no solutions (below the solid line).

The long - dashed line corresponds to $\mu = 0$. But in the region, where $\mu < 0$ the pion temperature is $T > T_c \simeq 200MeV$. For such temperatures, pions probably cannot exist and go over to a strongly interacting pion state or quark - gluon plasma. In any case, the region with $\mu < 0$ is outside of the applicability of Eqs. (6), which is valid for non-interacting or weakly interacting pion gas.

In the region between the long - dashed ($\mu = 0$) and the solid line there is a short - dashed line, which corresponds to $T = T_c$. Between the two dashed lines the solutions have $\mu > 0$, but $T > T_c$. In this region we cannot again use eq.(6). Therefore we have only the region between the short - dashed line ($T = T_c$) and the solid line, where we can use eq.(6) for weakly interacting pion gas. In this region $T < T_c$, but the chemical potential is positive.

Physically, positive μ means that in the initial state the number of pions, generated by hadron dynamics, is larger than that corresponding to chemical equilibrium. Thus, we see, that the pion system can be thermally equilibrated due to elastic collisions, but it will be out of chemical equilibrium, if the rate of inelastic reactions is not high enough.

Let us now consider the influence of positive chemical potential on the energy and particle density. Assuming ideal gas behaviour, one can deduce the temperature of the system as the ratio of energy density to particle density. Indeed, for the energy density one has $\epsilon = 42.9gT^4$ GeV/fm^3, $g = 3$ while $n = 15.9gT^3fm^{-3}$. Then $\frac{\epsilon}{n} = 2.7T$, and for $T = 0.2GeV$ we have $\epsilon = 0.54n$. This line lies below the short-dashed line ($T = T_c$) of Fig. 1. We can thus conclude that for fixed particle density the states of pions with positive chemical potential have larger energy density.

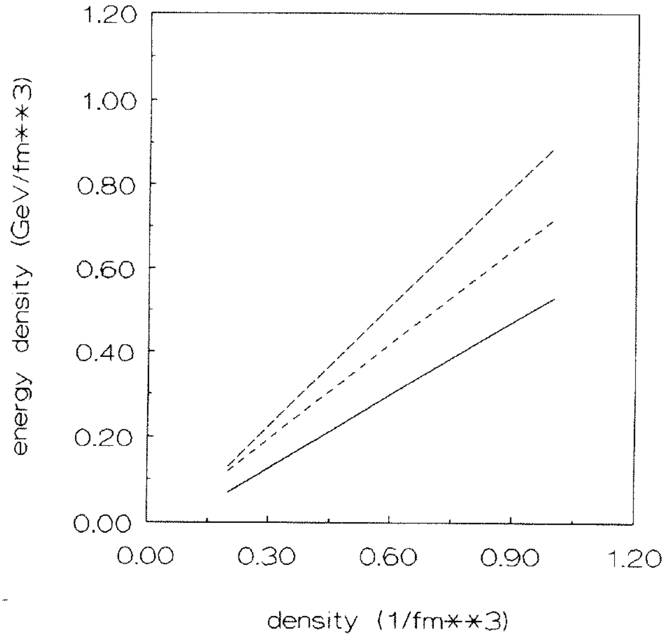

Figure 1. Different regions of solutions of the system of E qs.(5,6). The long-dashed line corresponds to $\mu = 0$, the short-dashed line to $T = T_c$. No physical solutions exist below the solid line.

As we expect in heavy ion collisions, the temperature of pions will be $T \sim 100 \div 200 MeV$. If the pion gas is not far from equilibrium, then $n_o \sim 0.5 \div 1.0 fm^{-3}$ and $\epsilon_0 \sim 0.2 \div 0.5 \frac{GeV}{fm^3}$. These intervals of n_0 and ϵ_0 correspond to $\mu > 0$, and $T < T_c$ on Fig. 1. If at the initial state $\mu > 0$, then the chemical potential will be positive up to the final state. The evolution of pion gas is defined by the parameter α in eq. (3). For NA35 conditions[10] the rapidity density for negative particles $\frac{dN}{dy} \simeq 38$ for $O + Au$ and $\frac{dN}{dy} \simeq 27$ for $S + S$, so that the parameter α will be 4.3 and 1.8 respectively. For RHIC conditions ($Au + Au$ or $Pb + Pb$ collisions) the parameter $\alpha \simeq 4 \div 5$.

Now we must define the decay time of pion gas. The freezout takes place when the evolution becomes three-dimensional[12], $t_F \simeq R_A/v$ (R_A is the projectile radius, $v \sim 1/\sqrt{3}$ is the average pion velocity). So, for O + Au and S + S collisions we have $t_F \sim 6 \div 7 fm$, for Au + Au and Pb + Pb collisions - $t_F \simeq 10 \div 11 fm$. For the dimensionless time variable $s = t/t_0$ freezout corresponds to $s \simeq s_F \simeq 2 \div 2.5$ for O + Au and S + S and $s = s_F \simeq 3 \div 3.5$ for Au + Au and Pb + Pb collisions.

As an example, we show in Fig. 2. the s-dependence of the temperature T (solid line) and the chemical potential μ for $n_0 = 0.8 fm^{-3}$, $\epsilon_0 = 0.4 \frac{GeV}{fm^3}$ and $\alpha = 3$. In this region $T \sim s^{-1/3}$ corresponds to the hydrodynamic behaviour and the chemical potential is a slow decreasing function of s. If the initial chemical potential is fixed, then pions cool faster, if α becomes larger[16].

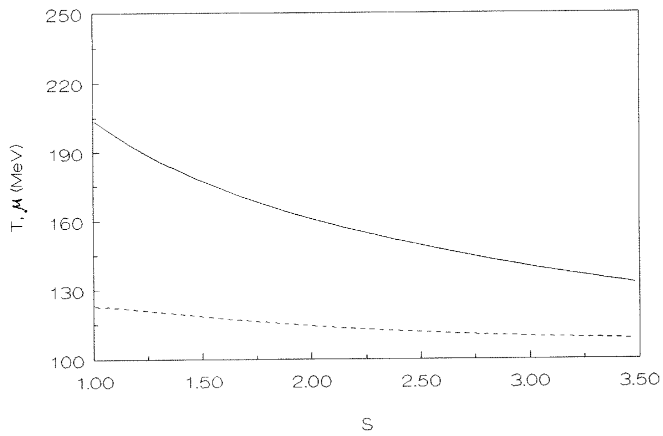

Figure 2. Temperature T (solid line) and chemical potential (dashed line) as a function of the time parameter $s = t/t_0$.

Let us now discuss the modificaton of the pion propagation properties during the pion thermalisation process. As we have already noted, using the pion distribution function of eq.(4), one can calculate the real and imaginary part of the collective potential in eq. (13) for any initial energy ϵ_0 and particle density n_0 for the "physical" region of Fig.1. We can also calculate ReV and ImV for any $s = t/t_0$, starting from the initial state up to the decay of pion matter into free-streaming pions.

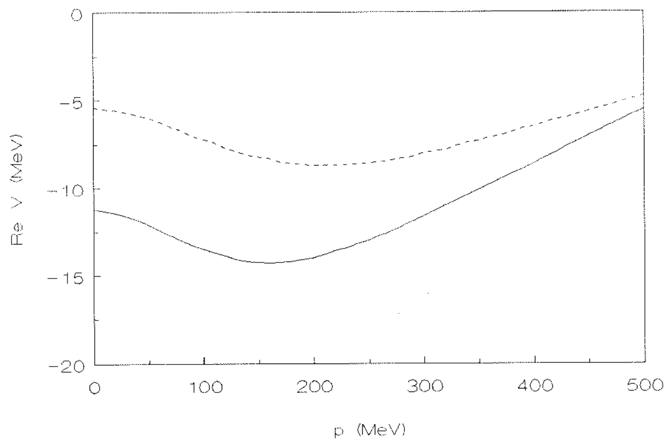

Figure 3. Real part of the pion collective potential as a function of pion momentum. Solid line corresponds to $s = 1.1$, the dashed to $s = 3.5$.

As an example, we show in Fig.3 for $n_0 = 0.5 fm^{-3}$, $\epsilon_0 = 0.25 \frac{GeV}{fm^3}$, $\alpha = 4.2$ the real part of the pion collective potential as a function of pion momentum for $s = 1.1$ (close to the initial state) and $s = 3.5$ (near freezout). The solid line corresponds to $s = 1.1$, the dashed line to $s = 3.5$. The initial (at $s = 1$) temperature and the chemical potential are $T_0 = 175 MeV$ and $\mu_0 = 115 MeV$ respectively. As we can see from Fig.3, the real part ReV is small with respect to the "free" pion energy at the same three-momentum. This reflects the Goldstone nature of pions-which should interact only weakly at small p. Thus, for such a rather dilute matter, the pions are good quasiparticles. But if the initial temperature $T_0 \geq 200MeV$ then ReV becomes comparable with the "free" pion energy and the quasiparticle approximation becomes questionnable. The real part of the collective potential changes sign at $\sqrt{S} = m_\rho$, as there is a strong compensation between repulsive and attractive interactions of pions colliding with S above and below m_ρ^2.

We have also investigated the effect of the positive chemical potential on the real and imaginary parts of the collective potential. If we use in Eqs. (7) or (8) the Bose thermodynamic weight instead of the distribution function $f(\omega_k, \mu, T)$ and neglect the chemical potential dependence of the $\pi\pi$ forward scattering amplitude, we reproduce ReV, calculated by E. Shuryak[17] for the case $\mu = 0$. The dependence of ReV on chemical potential is rather weak: ReV is changed by less than a factor of 2, if we go from $\mu = 0$ to finite μ in wide range of initial values of $n_0 \sim 0.5 \div 1 fm^{-3}$ and $\epsilon_0 \sim 0.25 \div 0.5 \frac{GeV}{fm^3}$. We show in Fig.4 the damping rate as a function of the pion momentum for two values of $s = 1.1$ (solid line) and $s = 3.5$ (dashed line). The initial n_0, ϵ_0 are the same, as in Fig.3. Remember, that the damping rate γ is connected with the pion mean collision time $\tau = \frac{1}{\gamma}$. As one can see from Fig.4, the damping rate varies between 10 MeV and 60MeV for these initial n_0, ϵ_0. Thus the mean collision time varies between 3fm and 20fm, which is of the order of the size of the hot system. At the moment $\frac{t}{t_0} \simeq 3.5$ pions have $\tau > 10fm$ i.e. pions propagate without collisions. If the initial temperature $T_0 \geq 200MeV$, the damping rate becomes larger than $100MeV$. In this case the mean collision time $\tau < 2fm$ and again the quasiparticle approximation becomes questionnable.

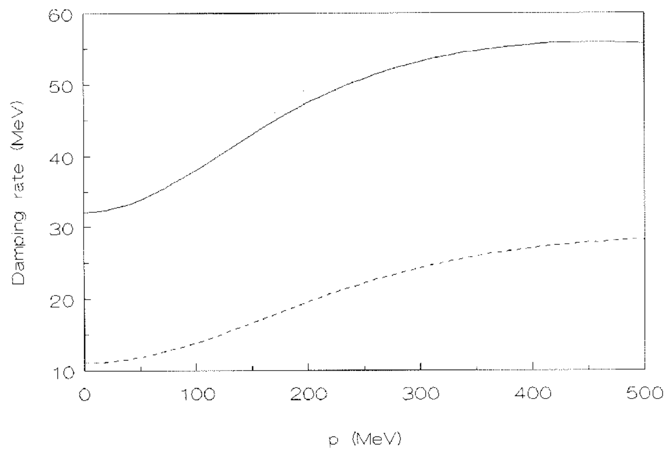

Figure 4. Damping rate as a function of pion momentum for $s = 1.1$ (solid line) and $s = 3.5$ (dashed line).

The damping rate is more sensitive to the chemical potential, than to the real part of the collective potential. Indeed, if we compare our results of the damping rate calculations with those of[17] we observe a factor 3-4 enhancement for pion momenta $p > 200MeV$. But one can find an especially big difference of a factor 5 at low momentum $p < 200MeV$. This difference is connected with the positive chemical potential $\mu \sim 100MeV$. If the chemical

potential $\mu = 0$, for "soft" pions with $p \to 0$, the mean collision time $\tau \to \infty$. The finite chemical potential leads to times $\tau \simeq 10 \div 20 fm$, a value which is greater, than the size of the colliding nuclei.
Using the data from Fig.4, we can estimate the mean damping rate

$$\bar{\gamma} = \frac{\int d^3p f(\omega_p)\gamma(p)}{\int d^3p f(\omega_p)} \tag{15}$$

and its dependence on the temperature during the thermalisation process. As is well known[18], in chiral perturbation theory $\bar{\gamma} \sim T^5$. The significant contribution from ρ-resonance modifies this leading-order of the chiral perturbation theory and for the temperatures $T \geq 150 MeV$ the mean damping rate $\bar{\gamma} \sim T^4$[18]. In addition, the positive chemical potential, suppress the temperature dependence of the mean damping rate. We obtained $\bar{\gamma} \sim T^3$ during the thermalisation.

In Fig. 5 we show the contributions of s-wave (dashed line) and p-wave (solid line) to the real part of the collective potential for $\frac{t}{t_0} = 1.1$ and $n_0 = 0.5 fm^{-3}, \epsilon_0 = 0.25 \frac{GeV}{fm^3}$. The sum of the two curves in Fig.5 gives the solid line in Fig.3. The s-wave and p-wave constributions correspond to phase shifts δ_0^0, δ_0^2 and δ_1^1, respectevely. The s-wave scattering gives positive contributions to ReV, which means that there is an attractive interaction between pions. But this interaction is weaker than the repulsion generated by the p-wave scattering. As a result, there is a compensation between the repulsive and attractive interactions of pions (solid line in Fig.3).

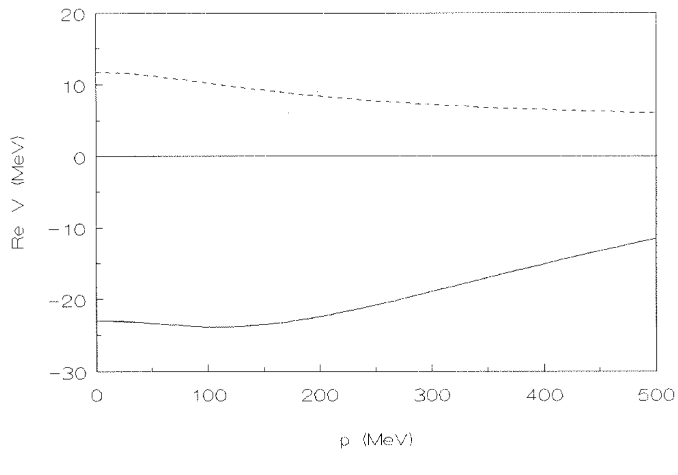

Figure 5. The contributions of s-wave (dashed line) and p-waves (solid line) to the real part of the pion collective potential.

Now we shall return to the problem of thermalisation of pion gas during the expansion. The thermalisation is characterised by the initial pion distribution function at $t \simeq t_0$ or $s \simeq 1$. We have used an isotropic pion distribution function near t_0:

$$f_0 = \{exp\Big(\frac{\sqrt{p_\perp^2 + p_z^2 + m_\pi^2} - \mu_0}{T_0}\Big) - 1\}^{-1} \tag{16}$$

where $\mu = \mu(t_0), T_0 = T(t_0)$. We have selected this form of distribution function, because pion spectra in proton -proton collisions are usually fitted by this function.
Let us compare the contributions of the two terns in the pion distribution function of eq. (4). The first term corresponds to the non-equilibrium contribution, the second to the equilibrium contribution. One can define a ratio

$$B = \frac{\alpha \int_{t_0}^{t} \frac{dt'}{t'} (\frac{t'}{t})^{\alpha} f_{eq}}{(\frac{t}{t_0})^{\alpha} f_0 + \alpha \int_{t_0}^{t} \frac{dt'}{t'} (\frac{t'}{t})^{\alpha} f_{eq}}. \tag{17}$$

The value B shows the degree of thermalisation of pion gas (formally for asymptotic $t >> t_0$ $B \simeq 100\%$, but for real heavy ion collisions $t > t_0$ $B < 100\%$). The degree of thermalisation B crucially depends on the parameter α and slightly on pion the transverse momentum $p_\perp$. For the initial values $n_0 = 0.8 fm^{-3}, \epsilon_0 = 0.45 \frac{GeV}{fm^3}, p_\perp = 100 MeV$ and rapidity $y = 0$ we show on Fig.6 the dependence of the degree of thermalisation on $s = t/t_0$. The solid line corresponds to $\alpha = 1$, the short - dashed line to $\alpha = 3$, and the long - dashed line $\alpha = 4.5$. Thus, for fixed s, the larger α, the higher the degree of thermalisation of the pion gas.

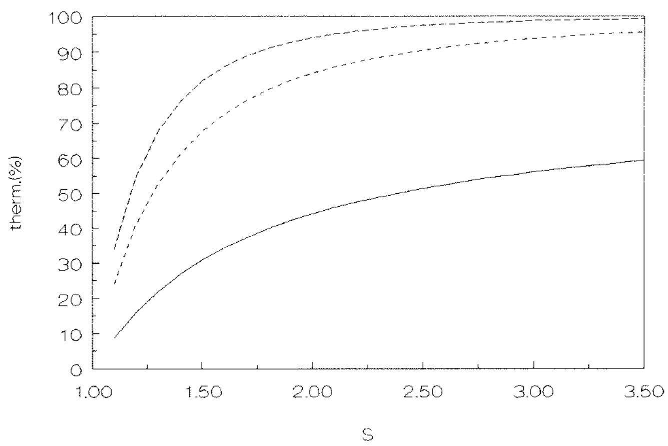

Figure 6. The degree of thermalisation as a function of s. Solid line for $\alpha = 1$, short-dashed line for $\alpha = 3$, long-dashed line for $\alpha = 4.5$.

As we noted before, $\alpha = 1.8$ for $S + S$ collisions at CERN and $\alpha = 4.3$ for $O + Au$. The freezout time for these collisions is $s \simeq 2 \div 2.5$. For these parameters we expect the degree of thermalisation to be around $\simeq 70\%$ for S+S collisions and $\sim 90\%$ for O+Au collisions. For RHIC conditions, $\alpha \simeq 4 \div 5$ and in this case we have a pion gas with a degree of thermalisation at the level of 70%, already at $s \simeq 1.5$. Near freezout the degree of thermalisation is near 100%.

Thus we have the following picture of pion evolution in heavy ion collisions: at the initial state pions form chemically and thermally a non - equilibrium system. But for $\alpha \simeq 4 \div 5$ during the expansion, thermal equilibrium is quickly establisted, while chemically, the pion gas is in a non-equilibrium state with positive chemical potential.

How can one check this picture of pion evolution in heavy ion collisions? Using the pion distribution function of eq. (4), we have calculated the pion distributions over transverse momentum $p_\perp$ and rapidity y:

$$\frac{dN}{Ndp_\perp dy} = \frac{3p_\perp m_\perp coshy}{4\pi^2 n(t_F)(0.197)^3} f(p_\perp, m_\perp shy, t_F), \tag{18}$$

where $m_\perp = \sqrt{p_\perp^2 + m_\pi^2}$ - transverse mass, $n(t_F)$ - pion density at $t = t_F$ and $\frac{dN}{Ndp_\perp dy}$ has a dimension GeV^{-1}. We can compare the spectra $\frac{dN}{Ndp_\perp dy}$ of eq. (18) with the pion spectra measured by NA35 collaboration and predict the pion spectra for future experiments at RHIC. In order to relate the spectra of eq. (18) with central region ($y = 0$), to those measured by the NA35 collaboration, we follow the prescription of Ref.[17] and integrate over y.

We have performed calculations for different combinations of initial n_0, ϵ_0. In Fig. 7 we compare our calculations for $n_0 = 0.5 fm^{-3}, \epsilon_0 = 0.24\frac{GeV}{fm^3}, s = s_F = 2.5, \alpha = 4.3$ with O+Au data[10]. As we can see from eq. (5), the density of pion gas near freezout is $n_F = 0.2 fm^{-3}$. If we take the decay volume, defined by the $\pi\pi$ interferometry measurment[18], then we can expect ~ 130 pions in the central region of O+Au collisions. Assuming isotopic invariance, 1/3 of these pions must be negative. This number is close to the density of negative pions in the central region $\frac{dN}{dy} = 38$, measured by the NA35 collaboration. One can suggest a test for the above - mentioned picture of nuclear collisions. We found that the chemical potential is positive and around $\mu \sim 100 MeV$. It is easy to see that for $p_\perp >> \mu$, the effect of the chemical potential is very small in the pion distribution function. The region, where the chemical potential may be important, is $p_\perp \simeq \mu$. Suppose we fix the initial pion density n_0. Then the elastic collisions conserve the total number of pions N, but the pion density, of course, will be lower at the final state $n_F = n_0 \frac{t}{t_F}$. It is important to note that n_F can be extracted from experimental data, using total pion multiplicity and decay volume, measured by the method of pion interferometry. So, we can connect n_0 with measured data.

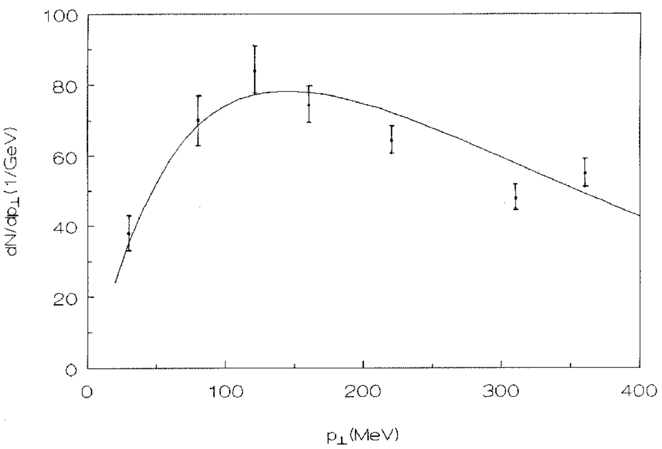

Figure 7. Comparision of the NA35 $O + Au$ pion spectrum with our calculations (solid line).

It is well known, that the energy density ϵ_0 is connected to the transverse energy E_T in heavy ion collisions. We calculated the pion spectrum for fixed $n_0 = 0.5 fm^{-3}$ and different

$\epsilon_0(0.23; 0.25; 0.27 \frac{GeV}{fm^3})$ from the physical region of Fig. 1. The results are shown in Fig. 8. The solid line corresponds to $\epsilon_0 = 0.23 \frac{GeV}{fm^3}$, the short -dashed line to $0.25 \frac{GeV}{fm^3}$ and the long - dashed line to $0.27 \frac{GeV}{fm^3}$. One can conclude from Fig. 8, that the effect of chemical potential is small already for $p_\perp > 200 MeV$, but for $p_\perp \leq 200 MeV$ the chemical potential effect is much stronger than the temperature effect. Indeed, suppose for a moment, that the chemical potential $\mu = 0$. In this case, if the pion gas is thermalised, the higher ϵ_0 the higher the temperature T and the solid line on Fig.8 will be lower than any other line for any $p_\perp$.

It will be interesting to check this behaviour of pion spectra at low $p_\perp$ in future RHIC experiments. For a quantitative comparison with experimental data it is necessary to take into account the transverse expansion of pion gas, the interaction into the final state and resonance decays [12]. The important question is connected however, with the contribution of inelastic channels to particle and energy densities and pion gas evolution. This problem is now under investigation.

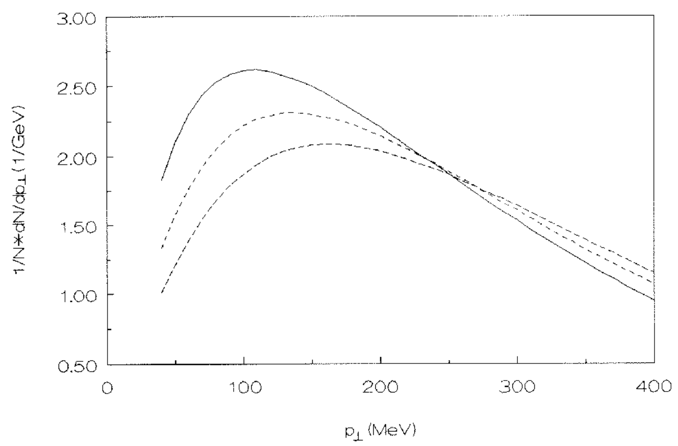

Figure 8. The pion spectrum for fixed $n_0 = 0.5 fm^{-3}$ and different ϵ_0 (solid line $0.23 \frac{GeV}{fm^3}$, short-dashed line $0.25 \frac{GeV}{fm^3}$, long-dashed line $0.27 \frac{GeV}{fm^3}$).

5. Conclusions.

It is not clear wheather the first generation of heavy ion experiments in CERN has been able to produce quark-gluon plasma. But from the observed pion multiplicity of the order of a few hundreds, it is clear that these collisions are able to produce macroscopic drops of hadronic matter mainly made of pions. Thus, the question arises about equilibrium properties of a pion phase. One of the models of the pion thermalisation, based on elastic scattering, was investigated in this paper.

We have defined the region of initial particle and energy densities, for which the model of pion gas with elastic scattering can be applied to heavy ion collisions. In this region the chemical potential is positive, i.e. in the initial state the number of pions, generated by hadron dynamics is larger that that, corresponding to the chemical equilibrium. The pion gas can be thermally equilibrated due to elastic collisions, but it will be out of chemical equilibrium, if the rate of inelastic reactions is low. We have considered the pion properties during the thermalisation process. For this purpose we have calculated the momentum-dependent collective potential (both real and imaginary parts) for π-mesons propagating in pionic matter. We obtained the finite pion collision time $\tau \sim 10fm$ for pion momentum $p \rightarrow 0$ and mean damping factor $\bar{\gamma} \sim T^3$ instead of $\bar{\gamma} \sim T^5$ in the leading order chiral pertubation theory. This difference is connected with the pion chemical potential $\mu \simeq 100MeV$.

We found the degree of thermalisation of pion gas near freezout at the level $\sim 70\%$ for S+S collisions and $\sim 90\%$ for O+Au collisions at CERN. For initial $n_0 = 0.5fm^{-3}, \epsilon_0 = 0.24\frac{GeV}{fm^3}, \alpha = 4.3$ the calculated curve is close to experimental spectra of pions, measured in the central region of O+Au collisions. We have shown, that the effect of chemical potential on the pion spectra is small for $p_\perp \geq 200MeV$, but for $p_\perp \leq 200MeV$ the effect of the chemical potential is more pronounced than the temperature effect. We hope that this behaviour can be observed in future experiments at RHIC.

V.E. is gratefull to the members of the Department of Physics, of the University of Ioannina for their warm hospitality and to the Greek Ministry of Finance, for a NATO Fellowship.

References

[1] V. Cerny, P. Lichard and J. Pisut, Phys. Rev. D**16** (1977) 2822; Phys. Rev. D**18** (1978) 2409.

[2] K. Kajantie and T. Matsui, Phys. Lett.B**164** (1985) 573

[3] G. Baym, Phys. Lett. B**138** (1984) 18.

[4] J. Bjorken, Phys. Rev. D**27** (1983) 140.

[5] R. Hwa and K.Kajantie, Phys. Rev. Lett.**56** (1986) 696

[6] K. Geiger, Phys. Rev. D**48** (1992) 4965; Phys. Rev. D**47** (1993) 133.

[7] M. Asakawa and T. Matsui, Phys. Rev. D**43** (1991) 2871.

[8] B. Kampfer and O. Pavlenko, Phys. Lett. B**289** (1992) 127.

[9] K. Geiger and J. Kapusta, Phys. Rev. Lett. **70** (1993) 1920.

[10] J. W. Harris (NA35 Collab.), Nucl. Phys.**A498** (1989) 133c.

[11] S. Gavin, Nucl. Phys.**B351** (1991) 561.

[12] S. Gavin and P. Ruuskanen, Phys. Lett. B**262** (1991) 326.

[13] B. Martin, D. Morgan and G. Show, Pion - Pion Interactions in particle physics, Academic Press, London, 1976.

[14] E. Shuryak, Phys. Lett. B**207** (1988) 345.

[15] J. Goity and H. Leutwyler, Phys. Lett. B**207** (1988) 517.

[16] B. Kampfer, P. Koch and O. Pavlenko, Phys. Rev. C**49** (1994) 1132.

[17] E.Shuryak, Nucl. Phys.**A533** (1991) 761.

[18] A.Schenk, Nucl. Phys.**B363** (1991) 97.

[19] G.Chanfray and D. Davesne, Z. Phys.**A349** (1994) 65.

[20] G. Welke, R Venugopalan and M. Prakash, Phys. Lett. B**245** (1991) 137.