Two Track Resolution in Longitudinal Direction in a TPC with Radial Drift Field

M. Konrad

Max-Planck-Institut für Physik, München, Germany

Abstract

A non homogeneous drift field in a TPC causes usually a spacial dependence of the drift velocity v. This $v(\vec{x})$ affects in general the distance of two charge clouds during their drift in the electric drift field as well as the distribution of the charge inside the cloud.
In the following the case of a radial drift field will be discussed, thus, the drift velocity becomes a one variable function $v(r)$. Formulae are derived to evaluate the seperation of two cloud centroids in $\hat{r}$-direction (= two track seperation) as well as the width of the cloud, both in dependence of r.
It is shown, that an increase of v with increasing r enlarges the distance between two clouds, however in $\hat{r}$-direction not the distance is measured but the difference of the arrival time at the readout chamber. This time difference is constant for every arbitrary fixed point on their driftpath. Therefore the width of the charge distribution is determined to judge the capability of a radial TPC to resolve two adjacent tracks in $\hat{r}$ - direction (= two track resolution). The results will be illustrated by an example.

1. INTRODUCTION

The main argument to choose a TPC with a radial drift field is the improved two track resolution in the azimuthal ($\hat{\Phi}$-) direction (= transverse direction). The transverse separation of two charge clouds increases linarly with increasing r, but the width of the charge distribution of the cloud only increases with $\sqrt{r}$; thus, an improvement of resolution can be gained, as discussed in [Dra92],[Bra93a] and [Bra93b].

An improved two track resolution also in radial ($\hat{r}$-) direction (= longitudinal direction) can be achieved if the drift velocity v decreases with the drift field E [Dra94]. The radial drift field $E(r)$ itself decreases with increasing r.

With $\frac{dv(r)}{dr} \neq 0$ the radial distance between the centroids of two charge clouds varies during their way through the drift field, however at every arbitrarily fixed position r they pass, the time difference measured between the passby of the centroid of the first and of the second cloud remains constant.

One has to look at the ratio of two quantities [Dra94]: First, the required time for each charge cloud to pass from the drift field into the readout chamber (= pass the cathode grid). Second, the time interval between the passage of the two charge centroids through the grid.

2. TWO TRACK SEPARATION

The $\hat{r}$-coordinate in a radial TPC is derived from the measurement of the arrival time of the charge cloud. To gain some information about the two track separation in $\hat{r}$-direction one needs to have an understanding of the relation between the measured time difference and the original spatial separation of the two particles, traversing the TPC.

The following notation will be used (see also fig. 1): r_i denotes the radius at which the

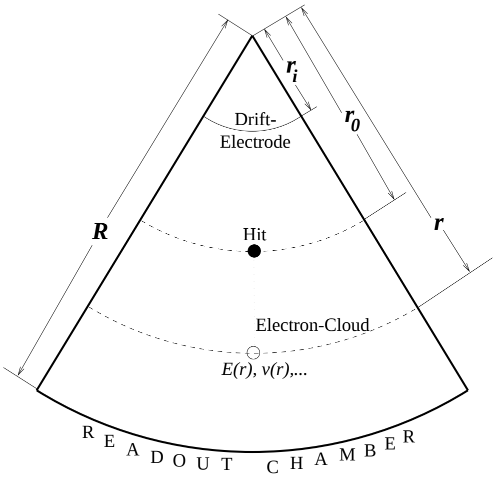

Figure 1. Schematic sketch of a sector of a radial TPC, illustrating the used denotations r_i, r_0, r and R.

drift electrode is positioned and R is the radius where the cathode grid is mounted. The radius at which a particle enters the chamber is named r_0; r is the variable, which denotes the actual position of the centroid of the drifting charge cloud, so this is the variable on which most quantities (e.g. electric fieldstrength $E(r)$, drift velocity $v(r)$) depend.

In the radial case the drift field $E(r)$ is a (non constant) function of r. Usually the drift velocity $v(E)$ is a (also non constant) function of E, which can be calculated or measured. Hence, there is in general a non trivial relation between v and r. To determine the time required for a charge cloud to move from r_0 to r, one has to evaluate

$$t(r) = \int_{r_0}^{r} \frac{d\tilde{r}}{v(\tilde{r})} \quad . \tag{1}$$

Because $v(r)>0$, $t(r)$ is monotonically increasing and therefore $r(t)$ always exists, but in general it will not be possible to give an analytical expression of $r(t)$.

Now assume a particle traversing the TPC at radius r_0 and a second one passing through the TPC at the same time and at the same azimuth at radius r_0+s_0; $s(r)$ denotes the separation in $\hat{r}$-direction of the two tracks when the charge cloud of the first particle has reached the radial position r, thus $s_0 = s(r_0)$. The values of the drift velocity shall be v_1 at r_0 and v_2 at R.

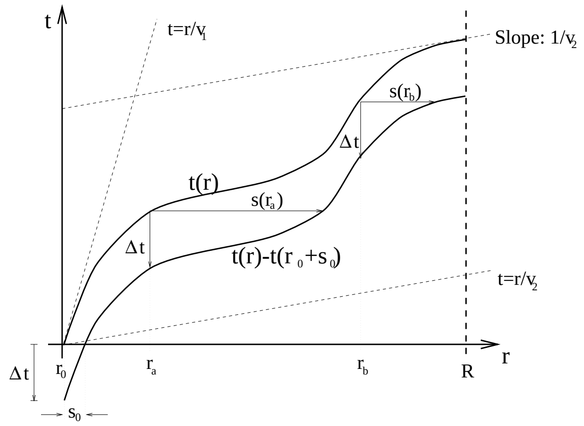

Figure 2. Curves of the t - r - dependence for two particles passing through the chamber at radii r_0 and $r_0 + s_0$.

In Figure 2 the t - r - dependences of the two charge clouds produced by the particles are sketched (the curves are arbitrarily chosen, but they fulfill the conditions mentioned above). The curve labeled with $t(r)$ gives the t - r - relation for the cloud starting to drift at position r_0 and ending up at R. The similar curve for the cloud starting at r_0+s_0 can be obtained from the former one by shifting it in negative t - direction, until the point of intersection of curve and $r-$ axis is at s_0. So there is a time offset $\Delta t = t(r_0 + s_0)$ between the two curves, which remains constant†.

The t - r - behaviour for constant drift velocity v_1 or v_2 is illustrated by the straight lines starting from the origin. Taking into account the relation

$$\frac{dt}{dr} = \frac{1}{v(r)} \tag{2}$$

it is obvious that the slope of $t(r)$ is just the inverse of the drift velocity $v(r)$.

With the knowledge of the slope and the constant Δt one can easily determine the (spatial) separation of the cloud centroids as a function of r as depicted in figure 3.

Extending equation 2 to

$$\frac{dt}{dr} = \frac{1}{v(r)} = \frac{\Delta t}{s(r)} \tag{3}$$

† That this is the right way to determine the second curve can additionally be proven by a consideration of the initial drift velocities at the starting radii of the two clouds.

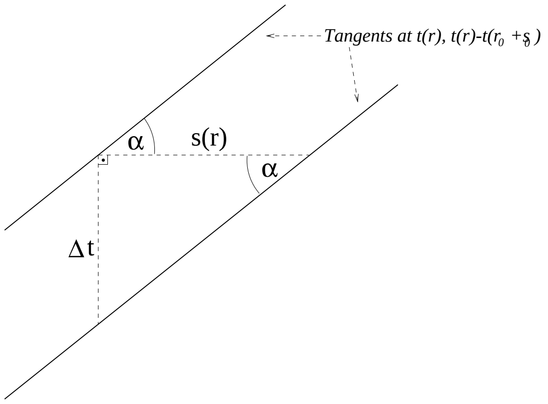

Figure 3. The (spatial) separation $s(r)$ of two charge clouds is determined by the drift velocity $v(r)$ and the quantity Δt; Δt is determined by s_0 and $v(r_0 + s_0)$.

one finds that

$$s(r) = \Delta t \cdot cot\big(\alpha(r)\big) \tag{4}$$

with

$$\alpha(r) = arctan\Big(\frac{1}{v(r)}\Big)$$

The change of seperation $M(r_a, r_b)$ in $\hat{r}-$ direction of two charge clouds from r_a to r_b (see fig. 2) can be defined to be $M(r_a, r_b) := \frac{s(r_b)}{s(r_a)}$ and using equation 3 one finds that $M(r_a, r_b) = \frac{v(r_b)}{v(r_a)}$.

To point out the basic statement of these considerations once again: With $\frac{dv(r)}{dr} \neq 0$ the spatial separation between two charge cloud centroids varies, but not the time difference with which they pass by a given point. As the *arrival time* measures the $\hat{r}-$ coordinate this effect alone does not improve the two track resolution, compared to a homogeneous drift field.

3. SIGNAL WIDTH

For a statement concerning the improvement of two track resolution in a radial TPC in comparison to the case of a homogeneous drift field (with constant drift velocity) it is necessary to determine an expression for the width $\sigma(r)$ in $\hat{r}$ - direction of the charge distribution inside the cloud. In [Bra93a] the expression

$$\sigma(r) = \Big[\sigma_0^2 + \int\limits_{r_0}^{r} C_l^2(\tilde{r})\, d\tilde{r}\Big]^{\frac{1}{2}}$$

is suggested; σ_0 denotes the width at r_0 and $C_l(r)$ is the longitudinal diffusion coefficient. However, in this formula only a possible r - dependence of the diffusion coefficient is taken into consideration, the effect of a non vanishing gradient of the drift velocity on the width is neglected. To consider this, a different approach has been chosen for the following calculations. The basic assumptions are:

- C_l = constant.
- Change of the width due to $\frac{dv(r)}{dr} \neq 0$.

The second point is illustrated in figure 4.

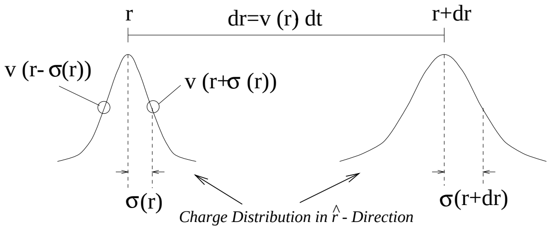

Figure 4. The drift velocity varies over the dimension of the charge cloud. A larger v at larger r causes an increase of the width just due to the velocity gradient.

A distribution with a width of $\sigma(r)$ at radius r will have a larger $\sigma(r + dr)$ at the radius $r + dr$ if the value $v\big(r + \sigma(r)\big)$ is larger than the one of $v\big(r - \sigma(r)\big)$. To bring this into a formula, additionally regarding the diffusion, one has to write:

$$\sigma^2(r + dr) = C_l^2 dr +$$
$$\Big\{\tfrac{1}{2}\Big[v\big(r + \sigma(r)\big) - v\big(r - \sigma(r)\big)\Big]dt + \sigma(r)\Big\}^2 \tag{5}$$

To simplify equation 5 the expression in brackets is squared and only terms of maximum first order in dr are considered. The expressions $v\big(r + \sigma(r)\big)$ and $v\big(r - \sigma(r)\big)$ are replaced by their Taylor expansion at point r up to first order. Finally (5) can be written as

$$\frac{d\big(\sigma^2(r)\big)}{dr} = C_l^2 + 2\sigma^2(r)\frac{v'(r)}{v(r)} \quad . \tag{6}$$

The solution of this differential equation is

$$\sigma^2(r) = \sigma_0^2 \frac{v^2(r)}{v^2(r_0)} + C_l^2 v^2(r) \int\limits_{r_0}^{r} \frac{1}{v^2(\tilde{r})}\, d\tilde{r} \quad . \tag{7}$$

4. IMPROVEMENT OF THE RESOLUTION

Up to this point all the basic tools have been developed, to really calculate the enhancement of the resolution in $\hat{r}$ - direction.

Assume the case where one cloud starts at r_0 with velocity v_1 and the other is located at $r_0 + s_0$ (s_0 small compared to r_0). At radius R the drift velocity is v_2. Define the following:

- δt = time interval required for the charge cloud to enter the readout chamber.
- Δt = arrival time difference of the two centroids of the clouds.
- Resolution factor $RF := \frac{\delta t}{\Delta t}$

For the radial field the result is:

$$\delta t = \frac{\sigma(R)}{v_2} \; ; \; \Delta t = \frac{s_0}{v(r_0 + s_0)} \approx \frac{s_0}{v_1}$$
$$\Rightarrow \quad RF_{rad} = \frac{\delta t}{\Delta t} = \frac{\sigma(R)}{s_0} \cdot \frac{v_1}{v_2} \tag{8}$$

For a homogeneous field with arbitrary drift velocity v_{hom} the result is:

$$\delta t = \frac{\sigma_{hom}(R)}{v_{hom}} \; ; \; \Delta t = \frac{s_0}{v_{hom}}$$
$$\Rightarrow \quad RF_{hom} = \frac{\delta t}{\Delta t} = \frac{\sigma_{hom}(R)}{s_0} \tag{9}$$

The ratio κ of RF_{rad} and RF_{hom} is thus a measure for the two track resolution enhancement:

$$\kappa := \frac{RF_{rad}}{RF_{hom}} = \frac{\sigma(R)}{\sigma_{hom}(R)} \cdot \frac{v_1}{v_2} \tag{10}$$

So what is the meaning of this κ? Assume that $\sigma(r) = \sigma_{hom}(r)$ for every r, so κ is just $\frac{v_1}{v_2}$. The velocity v_1 determines the time difference between the two centroids, whereas v_2 determines the time, the charge cloud needs to pass from the drift volume at R into the readout chamber.

- Comparison with the homogeneous case of drift velocity v_1:
  The time difference between the two centroids is the same in both cases, but at R the necessary time for $\sigma(r)$ to pass is $\frac{\sigma(R)}{v_2}$ in the radial field but $\frac{\sigma(R)}{v_1}$ in the homogeneous case (remember: $v_2 > v_1$, $\sigma(r) = \sigma_{hom}(r)$). The improvement in the radial case is due to the smaller temporal size of the charge cloud at R.
- Comparison with the homogeneous case of drift velocity v_2:
  In the radial case the time difference of the centroids is (for small track separation) $\frac{s_0}{v_1}$; but in the homogeneous field it is $\frac{s_0}{v_2}$. At R the time for $\sigma(R)$ to pass is $\frac{\sigma(R)}{v_2}$ in both cases, so the improvement in the radial case is here due to a larger time difference between the centroids at r_0.

5. EXAMPLES

5.1. *Homogeneous Field*

All the formulae were not been derived to study the homogeneous case, but it is useful to check, whether the well known formulae are recovered by constant v. With $v(r) = v_0$ one obtains (1):

$$t(r) = \int_{r_0}^{r} \frac{d\tilde{r}}{v(\tilde{r})} = \frac{1}{v_0}(r - r_0) \quad \Rightarrow \quad r = v_0 t + r_0$$

The spatial distance of the cluster centroids remains constant along the drift, thus $M(r_1, r_2) \equiv 1$. By using (7) $\sigma^2(r)$ can be expressed as

$$\sigma^2(r) = \sigma_0^2 \frac{v_0^2}{v_0^2} + C_l^2 \frac{v_0^2}{v_0^2}(r - r_0) = \sigma_0^2 + C_l^2(r - r_0) \;\; .$$

As can be seen, the case of a homogeneous field is also included in these formulae.

5.2. *Radial Field and Linear Decrease of $v(E)$*

As it is commonly known, an electric field $E(r)$ between two, concentric equipotential tubes is

$$E(r) = \frac{V_0}{\ln\left(\frac{r_i}{R}\right)} \cdot \frac{1}{r} \tag{11}$$

with V_0 being the drift voltage and r_i and R being the radii as defined in chapter 2 (see fig. 1). Thus, the electric field can be parametrized as $E(r) = \frac{A}{r}$. If a behaviour of the drift velocity like $v(E) = -\alpha E + b$ with $\alpha, b > 0$ is assumed, the r - dependence of v can be written as

$$v(r) = \frac{-\alpha A}{r} + b = \frac{-\alpha A + br}{r} \tag{12}$$

By using (1) the result for $t(r)$ is

$$t(r) = \int_{r_0}^{r} \frac{\tilde{r}}{b\tilde{r} - \alpha A}\, d\tilde{r} =$$
$$\frac{1}{b^2}\Big[br + \alpha A \cdot \ln(br - \alpha A) - \rho_0\Big] \; ; \tag{13}$$
$$\text{with } \rho_0 = br_0 + \alpha A \cdot \ln(br_0 - \alpha A) \;\; .$$

Inserting expression (12) into equation (7), one obtains:

$$\sigma^2(r) = \frac{\sigma_0^2}{v^2(r_0)} \cdot \left(\frac{-\alpha A}{r} + b\right)^2 +$$
$$C_l^2 \left(\frac{-\alpha A}{r} + b\right)^2 \cdot \int_{r_0}^{r} \left(\frac{\tilde{r}}{b\tilde{r} - \alpha A}\right)^2 d\tilde{r} \tag{14}$$

The integral in (14) has the solution

$$\int_{r_0}^{r} \left(\frac{\tilde{r}}{b\tilde{r}-\alpha A}\right)^2 d\tilde{r} =$$

$$\frac{1}{b^3}\left[(br-\alpha A) - \frac{\alpha^2 A^2}{br-\alpha A} + 2\alpha A \cdot \ln(br-\alpha A)\right]_{r_0}^{r} \quad (15)$$

As these formulae don't give an intuitive impression of the behaviour of the considered quantities, in the following a numerical example will be used to illustrate the qualitative behaviour. The following parameter values are used, still assuming a linear decrease of v with E:

- $r_0 = r_i = 10$ cm
- $R = 120$ cm
- $C_l = 0.25 \frac{mm}{\sqrt{cm}}$
- $\sigma_0 = 0$
- $v_1 = v(E(r_0)) = 2.5 \frac{cm}{\mu sec}$
- $v_2 = v(E(R)) = 5.2 \frac{cm}{\mu sec}$

In figure 5 the cloud width $\sigma(r)$ for the radial case is plotted and compared to the curve

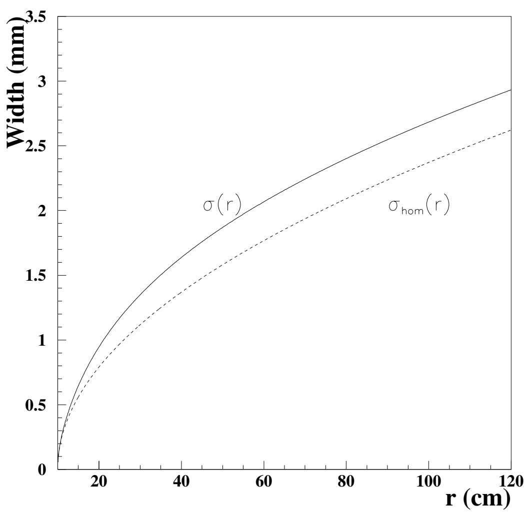

Figure 5. The value of $\sigma(r)$ for a charge cloud starting at $r_0 = 10$ cm. The width of the charge cloud in the radial case ($\sigma(r)$) gets its main increase in comparison to the homogeneous case ($\sigma_{hom}(r)$) at small radii.

$\sigma_{hom}(r) = C_l\sqrt{r-r_0}$ for the homogeneous case; as one can clearly see, $\sigma(r)$ is larger than $\sigma_{hom}(r)$ for every r and $\sigma(r)$ gets its main increase at small r; this is where the gradient of $v(r)$ is largest.

Next the case will be discussed, when two particles traverse the TPC at radii r_0 and $r_0 + s_0$ with $s_0 = 1$ cm. The relation between t and r for the centroids of the two charge clouds are depicted in figure 6 (see also fig. 2). It can be

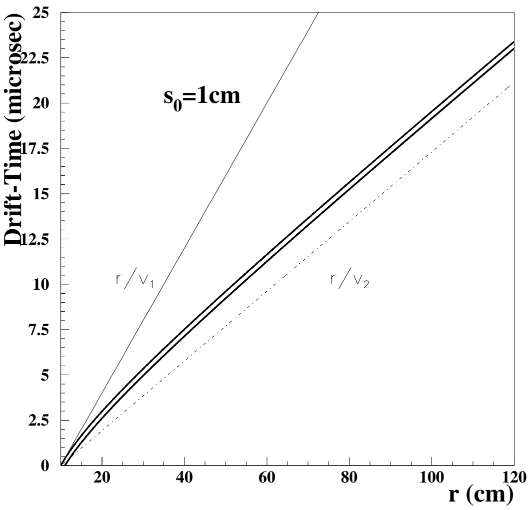

Figure 6. t - r - dependence in a radial drift field for two charge clouds generated at equal time at position r_0 and $r_0 + s_0$.

seen, that the upper one of both curves starts with a slope of $\frac{1}{v_1}$ and end up with a slope of $\frac{1}{v_2}$.

However, when studying the resolution enhancement, one is in general not interested in the value of $\kappa(r)$ for a fixed r_0 but in $\kappa(R)$ as a function of r_0 ($r_0 > r_i$). In figure 7 the predicted width at the readout chamber at R is plotted as a function of the radius r_0 at which a particle traverses the TPC. It can be seen, that the main

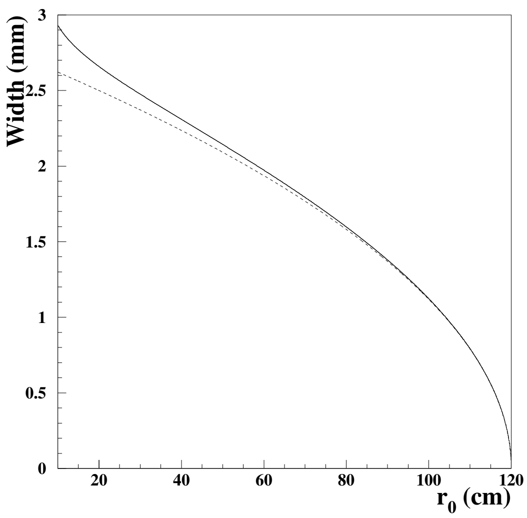

Figure 7. Calculated σ (upper curve) and σ_{hom} (lower curve) at R for a varying startpoint r_0.

width increase occurs for clouds starting at small radii.

Using the expressions for the width of the charge cloud and the drift velocity $v(r)$ one can calculate $\kappa(R)$ as a function of r_0. This dependence can be seen in figure 8. As

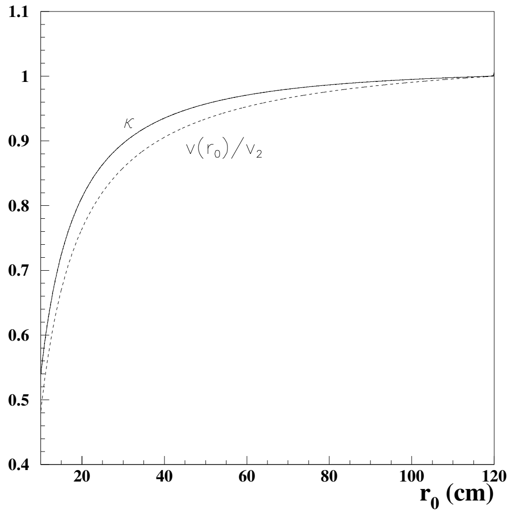

Figure 8. The upper curve is $\kappa(r_0)$; the lower one gives the ratio of the drift velocities $\frac{v(r_0)}{v(R)}$. The distance between them indicates the increase of κ by the increase of the width due to the velocity gradient.

expected, the improvement in comparison to the homogeneous case ist largest for clouds starting at small radii. However, due to the fact that the main drop of the electric field takes place in a short range of the radius, the improvement is not big in the upper two thirds of the range of r.

Comparing the curves $\kappa(r_0)$ and $\frac{v(r_0)}{v_2}$ in figure 8 it becomes clear that the deterioration of the two track resolution due to the increased signal width σ is small compared to the improvement, which can be achived by a suitable drift velocity gradient.

Varying in this example the radii (e.g. $r_i = 5$ cm, $R = 40$ cm) produces no major effects, because the dominating parameter is still the difference of the velocity at r_i and R; only the absolute values of $\sigma(r_0, r)$ become significantly smaller due to the shorter driftlength.

6. CONCLUSIONS

In the previous chapters expressions for the width of the charge distribution of a charge cloud in longitudinal direction in a radial drift field have been derived as well as for the enhacement of the two track resolution of a radial TPC in this direction in comparison to a homogeneous drift field.

Going from the one coordinate r to the general spatial coordinate $\vec{x}$, this formalism can also be used to derive equivalent formulae for arbitrary, non constant drift fields.

With the given formalism it is possible to calculate quickly the achieveable two track resolution for given geometry- and gas-parameters.

As a rough estimate an upper limit of improvement is given by the ratio of the maximum and minimum drift velocity for electrons in the drift field.

For a detailed discussion it is necessary to know the $v(E)$ dependence. A linear approximation of the $v(E)$ dependence might be sufficient for a semi-quantitative discussion. Assuming a linear decrease of $v(E)$, the major results are:

- Slight increase (compared to the homogeneous case) of the signal width due to the velocity gradient.
- Resolution enhancement is dominated by the ratio $\frac{v(r_0)}{v(R)}$; minor influence of $\frac{\sigma(R)}{\sigma_{hom}(R)}$.
- Best resolution enhancement at small r.

References

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