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Project

Solenoidal Tracker at RHIC (STAR)

Subproject

Time Projection Chamber - Assembly and Test

Title

Installation Beam Stress and Elastic Stability, rev. A

Revision A - Solid beam replaces welded rectangular tube section (supersedes SN0271 of 10/18/96)

1.0 Introduction

The Time Projection Chamber (TPC) will roll into the STAR Magnet on two beams, one at the 3 and one at the 9 o'clock position. The beams are bolted to support columns affixed to the Magnet's end rings. The span between supports is 295.5 inches (7.51 meters). The weight of the TPC at "roll-in" is about 18,300 lb.(8,200 N) and the design weight for calculational purposes is 19,000 lb. Calculation of the beam's stress, deflection and elastic stability follow. The initial installation beam design utilized a rectangular tube formed by welding two 12" heavy channels together. Substitution of a solid beam, machined from plate, was dictated by the relative difficulty of producing a straight member from a welded unit. The increase weight has been considered in the design and selection of the beam insertion hardware (covered under separate STAR Note). Refer to the Installation Beam Drawing TPC800-E-1 and TPC800-E-2 (LBL 24A43466B and 24A43476B).

Roll in weight of TPC: $W_{TPC} := 19000\ \mathrm{lbf}$

The approximate area moment of inertia in the stiff and compliant directions is:

$b := 2.75\ \mathrm{in}$ $\qquad E := 30\cdot 10^6 \cdot \frac{\mathrm{lbf}}{\mathrm{in}^2}$

$h := 12\ \mathrm{in}$

$I_{xx} := \frac{b\cdot h^3}{12}$ $\qquad I_{yy} := \frac{h\cdot b^3}{12}$

$I_{xx} = 396\ \mathrm{in}^4$ $\qquad I_{yy} = 20.797\ \mathrm{in}^4$

Length of beam $\quad L_b := 352\ \mathrm{in}$

Length between supports: $\quad L := 295.5\ \mathrm{in}$

Length between TPC Support rollers: $\quad a := 188.8\ \mathrm{in}$

Worst case deflection.

The TPC's roll in wt. is supported by four rollers.

$$P := \frac{W_{TPC}}{4}$$

Weight per inch of the installation beam.

$$w := \frac{116.8\ \text{lbf}}{12\ \text{in}}$$

The beam can be treated as a simply supported overhanging member for calculation of the deflection of the beam due to its own weight. After installation the beam is bolted rigidly to fixed support points at either end of the magnet. Therefore the worst case (maximum) deflection would result from assuming the half of the TPC's weight is supported by a single roller at the midpoint between fixed end beam supports. The moments, shear and deflections resulting from the point load and the distributed weight of the beam can be superimposed.

Deflection of Beam Under a Point Load and Fixed Ends

Assuming the TPC is centered in the magnet and the installation beam is simply supported, the maximum worst case displacement at the center can be calulated using ASIC 7th Ed. Section 2 Case 9 for two equal concentrated loads symmetrically placed :

where, $b := \frac{L - a}{2}$

$$\Delta y := -\frac{P \cdot b}{24 \cdot E \cdot I_{xx}} \cdot (3 \cdot L^2 - 4 \cdot b^2) \qquad \Delta y = -0.223\ \text{in}$$

Deflection due to distributed weight of the 352" beam section. The 352" beam section overhangs the support points at the columns significantly.

The deflection of a simply supported beam overhanging one support is:

Length of overhang: $L_a := L_b - L$

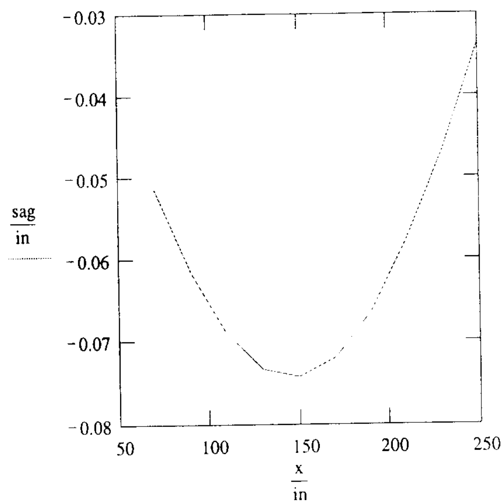
$i := 0 .. 9$

$x_i := i \cdot 20\ \text{in} + 70\ \text{in}$

$$sag_i := \frac{-w \cdot x_i}{24 \cdot E \cdot I_{xx} \cdot L} \cdot \left[\left[L^4 - 2 \cdot L^2 \cdot (x_i)^2 \right] + L \cdot (x_i)^3 - 2 \cdot L_a^{\,2} \cdot L^2 + 2 \cdot L_a^{\,2} \cdot (x_i)^2 \right]$$

Sag of beam due to it's own wt.

x = (in)	sag = (in)
70	-0.051
90	-0.062
110	-0.069
130	-0.073
150	-0.074
170	-0.072
190	-0.066
210	-0.058
230	-0.047
250	-0.034



Therefore the total deflection of the beam under worst case loading conditions is:

$Y_{max} := sag_5 + \Delta y$ $\qquad Y_{max} = -0.295 \cdot \text{in}$

<u>Bending Stress</u>

Treating the span between the magnet mounted support columns as simply supported (worst case stress).

Maximum bending moment for a simply supported beam with with two equal concentrated moving loads from ASIC 7th Ed. Section 2-212 case 41:

For a > than .586L:

For present case: $a = 188.8 \cdot \text{in}$ $\qquad .586 \cdot L = 173.163 \cdot \text{in}$

The maximum moment occurs when one load is at center span and the other is off the span.

At center (x=L/2)

$$M_c := \frac{P \cdot L}{4}$$

Maximum bending moment for a beam overhanging a simple support occurs at x.

where $x := \frac{L}{2} \cdot \left(1 - \frac{L_a^{\,2}}{L^2}\right)$ $\quad x = 142.349 \cdot in$

$$M_{max} := \frac{w}{8 \cdot L^2} \cdot (L + L_a)^2 \cdot (L - L_a)^2$$

The maximum bending stress can be found by adding two maximum moments since the location of the maximum moment is close to coincident.

$$M := M_{max} + M_c$$

Stress : $\sigma_b := \frac{M \cdot \frac{h}{2}}{I_{xx}}$ $\quad \sigma_b = 6.811 \cdot 10^3 \cdot \frac{lbf}{in^2}$

Safety factor on ultimate: $SF := \frac{58000 \frac{lbf}{in^2}}{\sigma_b}$ $\quad SF = 8.516$

Stability of Edge Loaded Beam

Elastic stability per Roark and Young Art.14.2 Table 34 Case 13 for a straight uniform beam of narrow rectangular cross section under a center load applied at a point a above the centroid of the section: ends of beam simply supported and constrained against twisting.

Depth of beam: $d := 12\ in$

Thickness of beam: $b := 2.75\ in$

Distance from centroid of section to load: $a := 6.5\ in$

Poisson's Ratio for steel: $v := .28$

Shear modulus $G := \frac{E}{2 \cdot (1 + v)}$

Critical Load

$$P_{cr} := 2.82 \cdot b^3 \cdot d \cdot \frac{\sqrt{\left(1 - 0.63 \cdot \frac{b}{d}\right) \cdot E \cdot G}}{L^2} \cdot \left[1 - \frac{1.74 \cdot a}{L} \cdot \sqrt{\frac{E}{G \cdot \left(1 - 0.63 \cdot \frac{b}{d}\right)}}\right]$$

$$P_{cr} = 1.305 \cdot 10^5 \cdot lbf$$

Factor of safety on buckling:

At worst 1/2 the weight of the TPC can be concentrated on a single roller. Therefore the worst case load is P, as defined above.

$$SF_{buc} := \frac{P_{cr}}{P}$$

$$SF_{buc} = 27.48$$

<u>Conclusions</u>

The worst case deflection of the Installation Beams during the TPC's insertion in the STAR Magnet is of the order of 0.3 inches. The radial gap between the TPC and STAR Magnet is about 2 ft. for the initial installation since no EMC modules are installed. Once the EMC modules are installed a 4 inch gap will be created for installation or removal of the TPC by removing CTB modules as needed. Therefore, the amount of deflection is insignificant with respect to the amount of space available. The stress in the installation beam is well within the allowable limits. The calculated safety factor of 8 compares well with the typical safety factor of 4 on ultimate strength for lifting equipment. The elastic stability similarly has a relatively large safety factor of 13 on the theoretical critical load.