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Method for the Absolute PMT Calibration

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Abstract.

Just to remind about the statistical approach for the photomultiplier (PMT) absolute calibration ... The accuracy of the method is estimated.

Introduction

In some application, it is important to know, with a reasonably good accuracy, the absolute value of the PMT amplification coefficient

$$K(U) = \overline{Q}/e\overline{N}_{phe} \tag{1}$$

where e is the electron charge, $\overline{N}_{phe}$ is the average number of photoelectrons emitted from the photocathode, and $\overline{Q}$ is the average charge-per-pulse collected at the PMT anode. The amplification K strongly depends on the PMT bias U that is reflected in the formula above. However, it is sufficient to measure the absolute value of K just at any one particular U setting. The complete $K(U)$ dependence can be derived then, using the known $\overline{Q}(U)$ dependence at some fixed light exposure. The measurement of $\overline{Q}(U)$ is out of attention in this note because it represents a so called "relative" type of measurements which are quite simple compared to the "absolute" ones.

One of the usual ways to measure K is based on the one- or two-photoelectron signal measurements assuming that the one- or even two-photoelectron peaks are well separated from the PMT noise and also one from the other (see, for example, [1, 2]). However, for the ordinary PMTs that are not specially designed for the single-photoelectron detection, this might not be the case. As a result, for this kind of PMTs, the absolute PMT calibrations based on the approach above have either low accuracy or are impossible at all.

The statistical method described below can be used with a reasonable accuracy for the absolute calibration of any PMT irrespectively of the ability to see clearly its single-photoelectron signal.

1 Statistical Method

In the method to measure the calibration constant K I'm reminding[1] here, the light source can be chosen bright enough to produce, in average, tens or even hundreds of photoelectrons per pulse for getting rid of the problem of the PMT signal–noise separation, the pedestal jitter of the Q measuring device (for example, ADC), and so forth. The method is based on the relation between the signal distribution width and the average number of photoelectrons emitted from the photocathode per pulse.

The more detailed, than Eq. (1), relation between the average number of photons, $\overline{N}_{ph}$, striking the photocathode, and the average output charge, $\overline{Q}$, is the following:

$$\overline{Q} = e \cdot \overline{N}_{ph} \cdot \epsilon \cdot \kappa_1 \cdot \kappa_2 \cdot \ldots \cdot \kappa_M \tag{2}$$

where ϵ is the photocathode quantum yield times electron collecting efficiency at the first dynode, and constants $\kappa_1, \ldots, \kappa_M$ represent the secondary emission factors of each of the M PMT's dynodes along with the electron collection efficiency at the subsequent dynodes (anode, after the last, M-th dynode).

The fluctuations of the output signal are due to the fluctuations of the number of photons from the light source and also the fluctuations of the number of electrons produced at each stage of the PMT's amplification chain[2].

Let's denote $n_{i,j}$ the number of electrons, emerging from the hit number "j" by a single electron (photon, for $i = 0$) of the i-th dynode (photochathode, for $i = 0$), which then reach $(i+1)$-th dynode. The number of these electrons at each event (pulse) is:

[1] See acknowledgments.

[2] The estimates of the signal fluctuation can be found in many textbooks, articles, and/or reports. Here I follow Ref. [3].

$$N_i^e = \sum_{j=1}^{N_{i-1}^e} n_{i,j} \tag{3}$$

where N_{i-1}^e is the number of "primary" electrons which arrived to the i-th dynode from the $(i-1)$-th one (or, for $i = 0$, photons which struck the photocathode). The variance, $(\sigma_i^e)^2$, of N_i^e distribution, can be calculated as following:

$$(\sigma_i^e)^2 = \overline{(\frac{\partial N_i^e}{\partial N_{i-1}^e})^2} \cdot (\sigma_{i-1}^e)^2 + \overline{\sum_{j=1}^{N_{i-1}^e} (\frac{\partial N_i^e}{\partial n_{i,j}})^2 \cdot \sigma_{i,n}^2} = (\overline{n}_i \cdot \sigma_{i-1}^e)^2 + \overline{N}_{i-1}^e \cdot \sigma_{i,n}^2 \tag{4}$$

where the mean value,

$$\overline{n}_i = \lim_{L\to\infty} (\frac{1}{L} \sum_{j=1}^{L} n_{i,j}) \ ,$$

is the quantum efficiency, κ_i, of the i-th dynode (for the photocathode, $\overline{n}_0 = \epsilon$); and $\sigma_{i,n}^2$ is the variance of the $n_{i,j}$ distribution,

$$\sigma_{i,n}^2 = \lim_{L\to\infty} (\frac{1}{L} \sum_{j=1}^{L} (n_{i,j} - \overline{n}_i)^2) \ .$$

It is usually assumed [3] that, for the photocathode, the quantum yield from a single photon is binomially distributed between zero and one with $\sigma_{0,n} = \sqrt{\epsilon(1-\epsilon)}$, while for the dynodes, the better choices are Poissonian distributions with $\sigma_{i,n} = \sqrt{\kappa_i}$, $i = 1, \ldots, M$.

Now, using Eq. 4, the following recurrent formula for the relative widths, $\sigma_i^e / \overline{N}_i^e$, can be written:

$$\frac{\sigma_i^e}{\overline{N}_i^e} = \sqrt{(\frac{\overline{n}_i \cdot \sigma_{i-1}^e}{\overline{N}_i^e})^2 + \frac{\overline{N}_{i-1}^e \cdot \sigma_{i,n}^2}{(\overline{N}_i^e)^2}} = \sqrt{(\frac{\sigma_{i-1}^e}{\overline{N}_{i-1}^e})^2 + \frac{\sigma_{i,n}^2}{\overline{n}_i \cdot \overline{N}_i^e}} \ . \tag{5}$$

For the fluctuations of the number of photoelectrons, approaching the first dynode, formula (5) gives the following[3]:

$$\frac{\sigma_{phe}}{N_{phe}} \equiv \frac{\sigma_0^e}{N_0^e} = \sqrt{(\frac{\sigma_{ph}}{N_{ph}})^2 + \frac{\epsilon \cdot (1-\epsilon)}{\epsilon \cdot N_{phe}}} = \sqrt{\frac{1}{N_{ph}} + \frac{1-\epsilon}{N_{phe}}} = \sqrt{\frac{\epsilon}{N_{phe}} + \frac{1-\epsilon}{N_{phe}}} = \sqrt{\frac{1}{N_{phe}}} \ . \tag{6}$$

In Eq. (6), the light source has been assumed to be stable enough[4] for the number of photons, striking the photocathode, to be Poissonian distributed with the width $\sigma_{ph} = \sqrt{N_{ph}}$.

Applying Eq. (5) sequentially to all dynodes, we will obtain:

$$\frac{\sigma_1^e}{N_1^e} = \sqrt{\frac{1}{N_{phe}} + \frac{\kappa_1}{\kappa_1 \cdot N_1^e}} = \sqrt{\frac{1}{N_{phe}} + \frac{1}{\kappa_1 \cdot N_{phe}}} = \sqrt{\frac{1 + \frac{1}{\kappa_1}}{N_{phe}}} \ ;$$

[3] In these equation as well as elsewhere below the "mean value" signs (over-lines) are omitted for brevity.

[4] ...i.e the relative variation of its light output due to instability is much less than $1/\sqrt{N_{ph}}$. This assumes also that, in the calibration procedure, the photons hit just a small area of the photocathode to avoid the fluctuations of the signal due to variations of the photocathode quantum yield and the electron collecting efficiency at the dynodes.

$$\begin{aligned}
\frac{\sigma_2^e}{N_2^e} &= \sqrt{\frac{1+\frac{1}{\kappa_1}}{N_{phe}} + \frac{1}{N_2^e}} = \sqrt{\frac{1+\frac{1}{\kappa_1}}{N_{phe}} + \frac{1}{\kappa_1\kappa_2 N_{phe}}} = \sqrt{\frac{1+\frac{1}{\kappa_1}+\frac{1}{\kappa_1\kappa_2}}{N_{phe}}}\,; \\
\ldots &= \ldots\,; \\
\frac{\sigma_Q}{Q} \equiv \frac{\sigma_M^e}{N_M^e} &= \sqrt{\frac{1+\ldots+\frac{1}{\kappa_1\cdot\ldots\cdot\kappa_{M-1}}}{N_{phe}} + \frac{1}{\kappa_1\cdot\ldots\cdot\kappa_M\cdot N_{phe}}} = \sqrt{\frac{1+\frac{1}{\kappa_1}+\ldots+\frac{1}{\kappa_1\cdot\ldots\cdot\kappa_M}}{N_{phe}}}\,. \quad (7)
\end{aligned}$$

Formula (7) gives the idea how to estimate N_{phe} and the amplification K by measuring σ_Q/Q:

$$N_{phe} = G(\frac{Q}{\sigma_Q})^2; \qquad K = \frac{\sigma_Q^2}{eGQ} \tag{8}$$

where

$$G = 1 + \frac{1}{\kappa_1} + \frac{1}{\kappa_1\kappa_2} + \ldots + \frac{1}{\kappa_1\ldots\kappa_{M-1}} + \frac{1}{\kappa_1\ldots\kappa_M}\,. \tag{9}$$

2 Errors

The experimental error of measuring the PMT amplification, using this method, is a quadratic sum of errors $\delta\sigma_Q$, δQ, and δG:

$$\frac{\delta K}{K} = \sqrt{(\frac{2\cdot\delta\sigma_Q}{\sigma_Q})^2 + (\frac{\delta Q}{Q})^2 + (\frac{\delta G}{G})^2}\,. \tag{10}$$

The statistical parts of $\delta\sigma_Q$ and δQ can easily be made as small as it is necessary for them to be well below the systematics, originating from the light source and PMT gain instabilities, noise, pedestal jitter of the Q measuring device, its nonlinearity, etc. However, the main contribution to the systematic errors should be expected from the uncertainties, coming from a rather poor knowledge of the secondary emission factors, $\kappa_1, \ldots, \kappa_M$.

Nevertheless, even some very approximate knowledges about the design and characteristics of the used PMT can significantly reduce $\delta G/G$. Just one example.

Let's assume that we use the 12-stage PMT with the secondary emission factors, roughly, all equal, i.e. $\kappa_1 \simeq \kappa_2 \simeq \ldots \simeq \kappa_{12} \equiv \kappa$ and $G \simeq \kappa/(\kappa-1)$. Let's assume we also know that the PMT amplification K at the chosen bias is somewhere in between 10^5 and 10^6. This gives us the estimate for $\kappa \simeq K^{1/12}$ from 2.61 to 3.16, and the corresponding values for G are equal to 1.62 and 1.46. Surprisingly, starting from the ten fold uncertainty for K, we, apparently, were able to estimate G and, potentially, to measure K itself with the pretty descent accuracy of $\sim\pm 5\%$. The better prior knowledges of K and some other information, for example, the distribution of the supply voltage between dynodes, etc can also be used to reduce $\delta G/G$ even more.

3 Acknowledgments

It's pleasure to express my thanks to I. Danil'chenko who introduced me the basics of this method many years ago. I have no doubts that it was published somewhere long ago, and I apologize to the Author(s) that I have not applied enough efforts to find out the paper(s). The goal of this particular note is to remind to my colleagues in the STAR EMC team about this approach.

References

[1] **V. P. Zrelov**. *Vavilov–Čerenkov Radiation and Its Application in High Energy Physics*, V. II. Atomizdat, Moscow 1968 (in Russian).

[2] **S. Aota, et al**. *Quality Control of Mass Production of Tile/Fiber Units for the Plug EM Calorimeter Upgrade.* Fermilab CDF/PLUG_UPGR/DOC/CDFR/3141, August 26, 1995, Version 1.0.

[3] **Tatsuro Hayashi (Director of Research)**. *Use of Photomultiplier Tubes in Scintillation Applications.* The report: "APPLICATION RES-0790" of HAMAMATSU TV CO., LTD.