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# EMC Nonlinearities and Their Effects on Physics Analyses

Thomas J. LeCompte
*Argonne National Laboratory*

**Abstract**

I discuss the effects of calorimeter nonlinearities on the STAR physics program. From this, I attempt to quantify the degree of nonlinearity that I can tolerate, with an eye towards justifying a non-linearity specification.

## 1 Introduction

Ideally, the electronic signal from a calorimeter is proportional to the energy deposited. This is not always the case, and the deviation from this strict proportionality is called *nonlinearity*. In this note, I attempt to quantify the effect of this nonlinearity on the capability of STAR to perform various physics measurements. This in turn lets us define a specification for how much nonlinearity is tolerable.

There are a number of potential sources of non-linearity. For example, the light or charge output from the detector might be quite linear, but the amplification may not be. Alternatively, the detector response itself may not be. In calorimeters that use scintillation or Cerenkov light, the light collection can often be a source of non-linearity. In a simple lead-glass calorimeter, for example, the attenuation length of the glass causes late developing showers to appear brighter than early developing showers. Since the depth of a shower grows logarithmically with energy, this causes the energy response to be nonlinear. (Actually, superlinear: the signal grows faster than it would for a truly linear device)

Nonlinearities can be benign. Consider the case where the detector output is perfectly linear, but the readout electronics are not - but each channel of readout electronics is identical. In this case, the nonlinearity can be easily calibrated away, perhaps with a constant intensity light source and a series of neutral density filters.

Nonlinearities can also severely compromise performance. Consider the case where there are large variations in light output from each fiber of a tile-fiber calorimeter. Nonlinearities caused by these deviations in response will differ from tower to tower. Instead of calibrating a single detector, all 4800 towers (in STAR's case) would have to be treated separately. Worse, since the problem here is in the light generation, it has to be calibrated with some sort of data: cosmic rays, electrons, perhaps radioactive sources. It's not as simple as injecting light.

In this note, I am concerned only with the second type on nonlinearity. I assume that any nonlinearities of the first type will be calibrated away and no longer pose any effect.[1]

## 2 Calibration

The first problem that a nonlinearity will cause is difficulty in *in situ* calibration. Experiments that work hard and are very careful can calibrate calorimeters to $\pm 2\%$ before they see beam. However, other experiments that have also worked hard and are very careful have found that the calorimeter in colliding beam data was off by as much as 6%. Because of this, it would be irresponsible to publish data on e.g. $d\sigma/dE_T$ without some sort of *in situ* calibration from the data.

The basic strategy for this sort of calibration is the following: The STAR EMC has 120 towers in $\phi$ and 40 in $\eta$. The physics is $\phi$ symmetric if the beam is centered at $(0, 0, z_0)$. One can use a high statistics process to set the response of all the $\phi$ towers in the same $\eta$ ring to the same value. My preference is to use towers without tracks pointing at them, but consistent with having some electromagnetic energy based on SMD (Shower Maximum Detector) data. This is a sample of photon-like objects. Other people have suggested using total tower energy, or energy in towers with single high $p_T$ tracks, to get a sample of minimum ionizing-like objects. Any of these samples can be used to normalize, and in principle the results should be the same.

Once the $\phi$ variations have been calibrated away, there are now 40 $\eta$ rings that need to be calibrated. [2] This can be done with electrons and $E/p$, assuming that

[1] This is not quite correct. Superlinear calorimeters tend to have reduced dynamic range, and sublinear calorimeters have poorer resolution. However, these effects are small for small deviations from linearity, so I ignore them here.

[2] One may ask, why 40? Why not take advantage of $\eta$ symmetry and only use 20? The difficulty with this is that it is not certain that the beam diamond will be centered on the detector, nor that it will be perfectly symmetric. It's a complicated function of cogging, and the RHIC $\beta$-functions in the $x$ and $y$ planes. We might be lucky and be able to use 20 $\eta$ rings, but we shouldn't plan on it.

the sample is large enough, or it can be done with $\pi^0 \rightarrow \gamma\gamma$ decays. Using $\pi^0$ with $p_T > 3$ GeV gives us a 2% calibration after 2-3 days of dedicated running or about a month of the standard mix of central and less central collisions.[1] Note that without the $\phi$ integration, this process will take one year of dedicated running or ten years of running with the standard trigger mix.

The difficulty with nonlinearities and calibrations is that a nonuniform depth response makes the $\phi$ integration unreliable.

## 2.1 Quantifying the Effect

The calorimeter response can be parameterized as:

$$E_{meas} = c + bE_{real} + aE_{real}^2$$

Typically, the offset $c$ is absorbed into the pedestal. For a linear calorimeter, then, $a = 0$ and $b = 1$. We can rewrite this as:

$$E_{meas}/E_{real} = b + aE_{real}$$

To determine these coefficients in terms of calibration, we look at GEANT samples of 1,2, and 5 GeV positrons.[2] We look at the amount of energy deposited in the front part (first five layers) and the back part (last sixteen layers).

| Electron Energy (GeV) | Energy Fraction In Front Section |
|---|---:|
| 1 | 59.6% |
| 2 | 50.5% |
| 5 | 39.6% |

From this, we introduce a difference in response, replacing:

$$E = E_{\text{front}} + E_{\text{back}}$$

with

$$E = \left(1 - \frac{P}{2}\right) E_{\text{front}} + \left(1 + \frac{P}{2}\right) E_{\text{back}}$$

We can now express $a$ and $b$ in terms of $P$:

$$a = .04686P$$

$$b = 1 - .1239P$$

Recall that $b$ is tuned to get the average right in a given sample.

We can then work out the measured energy for a 1 GeV (for example) photon:

$$E = \left(1 - \frac{P}{2}\right) E_{\text{front}} + \left(1 + \frac{P}{2}\right) E_{\text{back}}$$

$$E_{\text{meas}} = (1 - 10.67a)E_{\text{front}} + (1 + 10.67a)E_{\text{back}}$$

$$E_{\text{meas}} = (1 - 10.67a)0.596 + (1 + 10.67a)0.404(\text{GeV})$$

$$E_{\text{meas}} = 1 - 2.085a(\text{GeV})$$

Because $a$ is something that can vary tower to tower, to keep the tower-to-tower variation under 2%, we need the magnitude of $a$ to be less than about 0.01.

We can now turn our attention to the response to hadrons. We look at a GEANT sample of $\pi^+$'s with the following cuts:

- $E(EMC) = 0.2 - 0.4$ GeV
- $E(SMD) < 5$ mips
- Non-zero energy in both depths

The energies deposited by these pions (according to GEANT) are shown in the following table.

| Pion Energy (GeV) | Fraction In Front Section | EMC Energy (MeV) |
|---|---|---|
| 1 | 31.3% | 263 |
| 2 | 29.7% | 263 |
| 5 | 26.3% | 264 |

From this, I define a "generic pion" that deposits 82 MeV in the front compartment and 181 MeV in the rear. The measured energy will be

$$E = \left(1 - \frac{P}{2}\right) E_{\mathrm{front}} + \left(1 + \frac{P}{2}\right) E_{\mathrm{back}}$$

$$= \left(1 - \frac{P}{2}\right) 82 + \left(1 + \frac{P}{2}\right) 181\mathrm{MeV}$$

$$= (1 - 10.67a)82 + (1 + 10.67a)181(\mathrm{MeV})$$

$$= 263 + 1056a\mathrm{MeV}$$

So, to get response uniform to 2% requires that $a < 0.005$ or so.

This specifies the requirement for setting the calibration at a single point. What it does not specify is the requirement necessary for taking this calibration and extrapolating to higher energy.

# 3 Transverse Energy Spectra

One of the first measurements we plan to make is the measurement of the transverse energy spectrum. The particle spectrum falls pretty steeply. If it goes as $p_T^{-5}$, the uncertainty in the cross section is five times larger than the calibration uncertainty. A 10% cross section measurement requires a 2% calibration. This is true for the calorimeter cell spectrum, and it's also true for the spectrum of individual particles: $\pi^0$'s, $\eta$'s and direct photons.

To convince ourselves that we really do understand the detector response, I would propose that we need to use at least two methods to balance the detector response in $\phi$, and show they agree. The requirements on $a$ are therefore set by the most restrictive measurement, in this case the minimum bias response. For a calibration of 2%, we require $|a| < 0.005$. More generally, for a calibration good to Q%, $|a| < 0.0025Q$.

Hopefully, we can check the calibration scale as we go to higher energy by:

- Checking that the number of photons of moderate to high $E_T$ is constant tower to tower. (Because of the steeply falling $p_T$ spectrum, counting is more sensitive than mean energy)

- Checking the mass of reconstructed $\pi^0$'s in the energy range of interest to see that it is correct in all bins.

- Checking the width of reconstructed $\pi^0$'s in the energy range to see that it is well understood. Poor tower to tower calibration is likely to produce non-Gaussian tails and increased peak widths.

This gives us some handles on extrapolating the calibration, or rather, to confirm that the calibration at these energies, which relies on the low energy tower-to-tower calibration, is still valid. Because we are at ten times the energy of the 1 GeV or so tower-to-tower calibration, the specification on the linearity is ten times more restrictive: $|a| < 0.0005$ gives us a 2% systematic measurement on 10 GeV neutral mesons.

# 4 Direct Photons

Triggering on direct photons is a strong function of resolution. To be efficient for photons of one energy, one has to set the energy threshold down 2 or more $\sigma$. Since everything is a steeply falling function of $E_T$, the rate increases dramatically. As a rule of thumb, each 1% the threshold moves makes a 5% change on the trigger rate.

The trigger resolution is affected by three things:

- The intrinsic energy resolution
- The uncertainty on $E_T$ due to vertex smearing
- The calorimeter nonlinearity.

The first and third components are self-evident. The second arises because what is measured is $E$, the shower energy, and $z_{cal}$, the position of the tower where the shower develops. The transverse energy, $E_T$ is therefore

$$E_T = \frac{RE}{\sqrt{R^2 + (z_{cal} - z_0)^2}}$$

where $R$ is the calorimeter radius and $z_0$ is the position along the $z$ axis of the interaction vertex. Because $z_0$ is not available at the trigger level, this causes an uncertainty in $E_T$. It can be quantified:

$$\frac{\partial E_T}{\partial z_0} = -2E_T \frac{z_{cal} - z_0}{R^2 + (z_{cal} - z_0)^2}$$

$$\frac{\partial E_T}{\partial z_0} = \frac{E_z}{E} \frac{-2E_T}{\sqrt{R^2 + (z_{cal} - z_0)^2}}$$

Near $\eta = 1$, $E_z \approx E/\sqrt{2}$, and $\sqrt{R^2 + (z_{cal} - z_0)^2} \approx \sqrt{2}R$. Near $\eta = 0$, $E_z \approx 0$. Therefore, this effect is largest at the $\eta = 1$ side, so we approximate it there:

$$\frac{\partial E_T}{\partial z_0} = \frac{-E_T}{R}$$

$$\frac{\Delta E_T}{E_T} = -\frac{\Delta z_0}{R}$$

For a $z$ vertex smearing of $\pm 40$ cm and a EMCal radius $R$ of 220 cm, this induces a smearing of 18%. This is substantially greater than the calorimeter resolution. However, $\eta = 1$ is not the best spot to be looking for direct photons: it is difficult to make an isolation cut near the calorimeter edge. The photon "sweet spot" is between $\eta = -0.5$ and $\eta = 0.5$, so the *average* smearing due to the vertex effects is closer to 5%. This is comparable to the resolution for photons of 10 GeV.

It is reasonable to require that the nonlinearity be no worse than the vertex smearing. It has been shown[3] that the spot at which we might have to prescale direct photons (above this, trigger thresholds don't matter: we get all the events anyway) is around 20 GeV (corresponding to 30 Hz into a software based trigger that can presumably separate photons from jets). Requiring no more than a 5% deviation at 20 GeV implies that the magnitude of $a$ be less than 0.0025.

# 5 Offline Energy Resolution

Another way to go about the problem is to just consider the energy resolution. Our stochastic specification is $20\%/\sqrt{E}$. So far as I know, we haven't specified a constant term, but the Conceptual Design Report mentions 1 to 1.5%, consistent with similar calorimeters. If we take our resolution specification as

$$\frac{20\%}{\sqrt{E}} \oplus 1.5\%$$

we can ask what resolution we need at the maximum energy. The maximum energy seems governed by the highest energy direct photon we will reconstruct in the 500 GeV spin program (driven largely by the 2.5 times greater energy per nucleon than other STAR run conditions), and that is in the 35-40 GeV ballpark. [3] In that range, we require a resolution of 3.5-3.7%.

Typical test beam stochastic terms have been around $16\%/\sqrt{E}$. That would allow us a constant term of as large as 2.4-2.5% and have the same overall resolution. If one assumes that this constant term is solely due to the nonlinearity, the largest the $a$ coefficient can be is $2.4\%/E$, or 0.0006. This agrees with the requirement based on measuring the spectra of neutral mesons, and is our most restrictive requirement.

# 6 Other Parameterizations

One can consider parameterizing the nonlinearity in some other way. A possibly reasonable suggestion would be

$$E_{meas} = C + BE_{real} + A(E_{real} \log E_{real})$$

Again, the constant $C$ gets absorbed in the pedestal. It also absorbs the negative logarithm of a reference energy (so that the argument of logarithm is dimensionless), in this case 1 GeV. The idea here is that since the shower depth grows logarithmically with energy, perhaps the logarithm of the energy is a better variable to use to characterize the nonlinearity than the energy itself. To some level, the parameterization isn't entirely relevant. What we have are requirements at various energies: the coefficients might change, but the requirements themselves do not.

[3] The $W$ and $Z$ program has comparably energetic electrons, but the physics depends more on electron identification than energy measurement.

# 7 Conclusions

The most restrictive requirement on linearity comes extrapolating the tower-to-tower calibration to higher $p_T$ to measure the spectra of neutral mesons. This corresponds to and agrees with the resolution specification of

$$\frac{20\%}{\sqrt{E}} \oplus 1.5\%.$$

That implies a maximum nonlinearity of 0.0005-0.0006 GeV/GeV$^2$.

# References

[1] STAR Computing Requirements Document, Draft 2.0, p.14

[2] T. LeCompte, "Dielectron Backgrounds and Their Impact on $J/\psi$ Physics", STAR Note 287.

[3] B. Hubner, G.D. Westfall, A.M. Vander Molen, "Trigger Level 0 Studies for Jets and Direct Photons in pp and pAu Interactions"