SVT Internal Alignment Package

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Abstract

We report on the development of a new software package developed in C++, and fully integrated in the STAR Analysis Framework (STAF) for the purpose of the geometry calibration (internal alignment) of the STAR Silicon Vertex Tracker (SVT). The software package, dubbed SAL (SVT Alignment), has been tested with simulated Au+AU collisions generated with the HIJING[3] model and processed with STAR GEANT (GSTAR). The alignment is based on straight tracks measured in a null magnet field. The alignment is achieved through the minimization of a functional which includes individual track residues as well as the size of the primary collision vertex reconstructed with all measured tracks. Simulations performed on the basis of ten Au+Au events indicate it is possible to achieve a position reconstruction of less than 15 microns.

1 Introduction

This document describes the Silicon Vertex Tracker (SVT) local alignment package (SAL). The SAL package is part of the STAR/SVT analysis software and is developed to be fully compatible with the STAF environment. It is written in C++ and should thus be easily portable to future versions of STAF. SAL is designed to perform a local alignment or internal geometry calibration of the SVT wafers. The alignment method is based on a calculation of residues and the minimization of primary vertex size (uncertainty of the vertex position) of tracks recorded with a null magnetic field. The alignment performed is local only. Further software components are required to perform global alignment with the TPC and other detectors of STAR. These are described in an upcoming note [1].

We present the design, algorithm, and performance of the SAL package. Sect. 2 presents a brief analysis of the alignment problem and discusses methods envisioned to solve the problem in the context of STAR. Sect. 3.5 presents the algorithm and method implemented in SAL to solve the local alignment problem. The performance of the package has been evaluated based on simulated events produced with HIJING and propagated through a computer model of the SVT with GSTAR. The simulation technique is presented in Sect. 4. Results illustrating the performance of the alignment package are presented in Sect. 5.

2 The Alignment Problem

2.1 Introduction

In its three layer configuration, the SVT will include 216 Silicon Drift (SDD) wafers. These will be mounted in three concentric barrels around the beam within the STAR Time Projecting Chamber (TPC). Each barrel will be subdivided in two sub-layers to permit the integration of the readout electronics. The mechanical assembly of the detector will be performed under stringent tolerance conditions using a high precision assembly tool. Nonetheless, one can foresee that the actual positions of all 216 wafers will differ from their nominal (ideal) positions and orientations. Moreover, it is possible that small displacements or shifts may occur when the SVT detector is moved from the assembly hall to its location in STAR, or when the magnetic field is switched "ON".

Considering the SDD wafers as rigid bodies, the positional shifts or misalignment of each wafer can be broken down into 4 categories and six parameters (3 translation and 3 rotations).

- planar translation (2 parameters)
- planar rotation (1 parameter)
- radial translation (1 parameter)
- out-of-plane rotation (2 parameters)

Once the SVT detector is positioned within the STAR TPC, it will not be possible to perform a detailed survey of each wafer position and orientation. A data-driven alignment, or geometry calibration, will thus be required to compensate for the positional shifts. A data-driven alignment can be accomplished using a number of different techniques. Basically, most techniques amount to a variation of the wafer alignment parameters (position and rotation) to minimize some functional representative of the alignment of the wafers. In the simplest cases, the functional corresponds to the track residues. In more sophisticated analysis, vertex size minimization and various other constraints are also included. The alignment procedures can be applied for data collected with or without magnetic field. The advantage of using a zero field alignment resides in the fact that no apriori knowledge of the field, particle ID, and momenta are necessary. Also, in the context of a very high track multiplicity environment such as STAR, it minimizes the risk of biases associated with ghost tracks reconstruction. Zero field alignment parameters are rarely found to be completely satisfactory however, and a second step alignment procedure at "full" field is also usually required.

In the context of STAR, we envision to perform the alignment of SVT in multiple steps as follows:

- Zero Field Local Alignment
- Zero Field Global Alignment

- Nominal Field Local Alignment
- Nominal Field Global Alignment

The local alignment consists of a variation of the wafer position and orientation parameters designed to determine the internal relative position and orientation of the SVT wafers. The global alignment further requires the parameters adjusted such that tracks be well matched with other detectors and that the detector position be correctly calibrated relative to the interaction vertex. In this note, we focus mainly on the internal alignment problem.

2.2 Local Alignment Problem

Consider for simplicity, that the SVT consists of three wafers mounted in a projective geometry at a nominal distance from the collision vertex. Given that STAR is a collider detector, it is not possible to assume that the tracks emerge from a fixed location in space. However, one may capitalize on the large track multiplicities produced in Au+Au collisions as a major fraction of the tracks, in each event, are produced at a common vertex, the primary vertex. With six degrees of freedom (6 parameters) per wafer, there are overall 18 alignment parameters per projective combination of wafer triplets. We first proceed to demonstrate that it is not possible to determine those 18 parameters uniquely using only data from these three wafers without any additional information from other parts of the detector.

To demonstrate this point, consider the simplified geometry illustrated in Fig.1 showing three wafers mounted perpendicular to a radial axis pointing away from the interaction point. Let Δx_i, Δy_i and Δz_i ($i = 1, 2, 3$) denote the i-th wafer shift (translation) along the corresponding coordinate axes as illustrated. Let α_i, β_i, and γ_i denote deviation angles of the i-th wafers in the xy, xz, and yz planes respectively. We consider a track produced near the origin and propagating with slopes (a, b) in the xz and yz planes respectively.

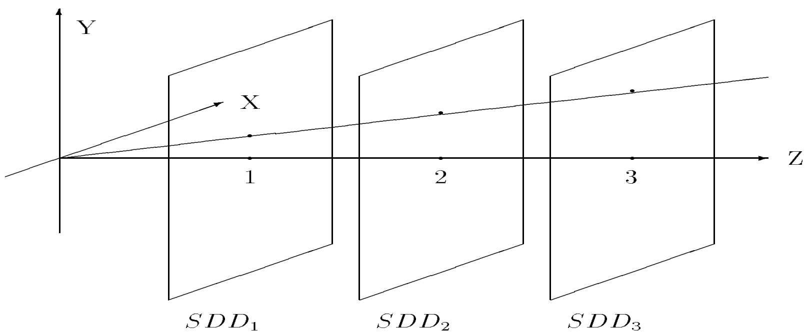

Figure 1: Simple model of SVT system

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For ideal wafer positions, which we assume arbitrarily, and without lost of generality, to be at z=1, 2, and z=3 and perpendicular to the z-axis, the coordinates of the intersection points would be $(a, b, 1)$, $(2a, 2b, 2)$ and $(3a, 3b, 3)$ (see Fig.1). Supposing the angles and shifts to be sufficiently small, the effect of arbitrary misalignments can be described with the following transformations:

$$
\begin{aligned}
(a, b, 1) &\rightarrow (a + \Delta x_1 + \alpha_1 b, b + \Delta y_1 - \alpha_1 a, 1 + \Delta z_1 + a\beta_1 + b\gamma_1) \\
(2a, 2b, 2) &\rightarrow (2a + \Delta x_2 + 2\alpha_2 b, 2b + \Delta y_2 - 2\alpha_2 a, 2 + \Delta z_2 + 2a\beta_2 + 2b\gamma_2) \\
(3a, 3b, 3) &\rightarrow (3a + \Delta x_3 + 3\alpha_3 b, 3b + \Delta y_3 - 3\alpha_3 a, 3 + \Delta z_3 + 3a\beta_3 + 3b\gamma_3)
\end{aligned}
$$

We now show that the displacements Δx_i, Δy_i, Δz_i and rotations α_i, β_i, and γ_i cannot all be resolved. Given linear trajectories, the coordinates of the track crossing on the 3rd wafer are extrapolated from the crossing coordinates on the first and second wafer with the following expressions:

$$
\begin{aligned}
x_1 + \frac{x_2 - x_1}{z_2 - z_1}(z_3 - z_1) &= x_3 \\
y_1 + \frac{y_2 - y_1}{z_2 - z_1}(z_3 - z_1) &= y_3
\end{aligned}
$$

where (x_i, y_i, z_i) are coordinates of the track intersection points with the i-th wafer.
Using the parameterization given above one obtains:

$$
\begin{aligned}
&\frac{(a + (\Delta x_2 - \Delta x_1) + b(2\alpha_2 - \alpha_1))}{(1 + (\Delta z_2 - \Delta z_1) + a(2\beta_2 - \beta_1) + b(2\gamma_2 - \gamma_1))} \cdot \\
&(2 + (\Delta z_3 - \Delta z_1) + a(3\beta_3 - \beta_1) + b(3\gamma_3 - \gamma_1)) + (a + \Delta x_1 + \alpha_1 b) = 3a + \Delta x_3 + 3\alpha_3 \\
&\frac{(b + (\Delta y_2 - \Delta y_1) + a(2\alpha_2 - \alpha_1))}{(1 + (\Delta z_2 - \Delta z_1) + a(2\beta_2 - \beta_1) + b(2\gamma_2 - \gamma_1))} \cdot \\
&(2 + (\Delta z_3 - \Delta z_1) + a(3\beta_3 - \beta_1) + b(3\gamma_3 - \gamma_1)) + (b + \Delta y_1 - \alpha_1 a) = 3b + \Delta y_3 - 3\alpha_3
\end{aligned}
$$

Using the fact that all the deviations are small and the approximate formula $1/(1+a) \approx 1-a$ (where a is small) one has:

$$
\begin{aligned}
(-\Delta x_1 + 2\Delta x_2 - \Delta x_3) + b(\alpha_1 - 4\alpha_2 - 3\alpha_3) + a(\Delta z_3 - 2\Delta z_2 + \Delta z_1) + \\
+a^2(3\beta_3 + 4\beta_2 - \beta_1) + ab(3\gamma_3 - 4\gamma_2 + \gamma_1) = 0 \\
(-\Delta y_1 + 2\Delta y_2 - \Delta y_3) + a(\alpha_1 - 4\alpha_2 - 3\alpha_3) + b(\Delta z_3 - 2\Delta z_2 + \Delta z_1) + \\
+ab(3\beta_3 - 4\beta_2 + \beta_1) + a^2(3\gamma_3 - 4\gamma_2 + \gamma_1) = 0
\end{aligned}
$$

The above equations should be correct for any track i.e. any slopes a and b. This leads to the six null conditions:

$$\begin{array}{ll} \Delta x_3 - 2\Delta x_2 + \Delta x_1 = 0 & 3\alpha_3 + 4\alpha_2 - \alpha_1 = 0 \\ \Delta y_3 - 2\Delta y_2 + \Delta y_1 = 0 & 3\beta_3 - 4\beta_2 + \beta_1 = 0 \\ \Delta z_3 - 2\Delta z_2 + \Delta z_1 = 0 & 3\gamma_3 - 4\gamma_2 + \gamma_1 = 0 \end{array} \tag{1}$$

There are 18 parameters and 6 constraints equations. 12 parameters are thus left unconstrained. The requirement of track linearity can be used only for the reconstruction of one wafer position with respect to the other two.

An additional set of 3 constraints is obtained by requiring that multiple tracks passing through the 3 wafers in a same event originate from a common vertex point. One can readily verify that this does not affect the shifts $\Delta x_i, \Delta y_i, \Delta z_i$, so for the sake of simplicity, we set $\Delta x_i = \Delta y_i = \Delta z_i = 0$ in the following. Using x_v, y_v, z_v to denote the vertex coordinates, the condition of a common vertex can be written as follows:

$$(a + \alpha_1 b) + \frac{(a + b(2\alpha_2 - \alpha_1))}{(1 + a(2\beta_2 - \beta_1) + b(2\gamma_2 - \gamma_1))}(z_v - 1 - a\beta_1 - b\gamma_1) = x_v,$$

$$(b + \alpha_1 a) + \frac{(b + a(2\alpha_2 - \alpha_1))}{(1 + a(2\beta_2 - \beta_1) + b(2\gamma_2 - \gamma_1))}(z_v - 1 - a\beta_1 - b\gamma_1) = y_v. \tag{2}$$

Again using the fact that angles have small values and supposing the vertex to be near the beam pipe center (i.e. x_v and y_v are small) we have :

$$2az_v + 2b(\alpha_1 - \alpha_2) + a^2(2\beta_2 - 2\beta_1) + ab(2\gamma_2 - 2\gamma_1) \approx 0 \tag{3}$$

$$2bz_v + 2a(\alpha_1 - \alpha_2) + b^2(2\gamma_2 - 2\gamma_1) + ab(2\beta_2 - 2\beta_1) \approx 0 \tag{4}$$

$$\begin{array}{rl} & \alpha_1 = \alpha_2 \\ \rightarrow & \beta_1 = \beta_2 \\ & \gamma_1 = \gamma_2 \end{array} \tag{5}$$

Taking into account the expressions obtained earlier this means:

$$\begin{array}{l} \alpha_3 = \alpha_1 \\ \beta_3 = \beta_1 \\ \gamma_3 = \gamma_1 \end{array} \tag{6}$$

So we can reconstruct the rotation of two wafers with respect to the third one. Thus, data from wafers allow one to reconstruct only 9 of 18 required parameters and we need some additional information to reconstruct the rest of the parameters. In practice, this will need to done using all SVT detectors as well as the TPC data points.

The SVT comprises 216 wafers mounted in a barrel geometry. Given all wafers are of equal size, they are many more wafers on the outer layers. To be specific, there are 32, 72, and 112 wafers on the inner, middle, and outer layers respectively. Also, the geometry is projective only in so far as many wafers are used on the 2nd and 3rd layers to cover cone projections associated with the 32 wafers of the first layer. This implies that in practice less than nine (9) parameters per wafer triplet are actually independent.

3 Internal Alignment Method

The purpose of the alignment or geometry calibration is to determine the positions and angles of all wafers relative to a minimal set of wafers chosen as reference. The alignment procedure is data driven; i.e. one will use data collected with the detector at known magnetic field settings to provide constraints for the determination of the wafer alignment parameters. The alignment procedure is based on the implicit assumption that reconstructed tracks, consisting of triplet of points with one point per SVT layer, can be described with a definite model. In the simplest case, one plans to use straight tracks reconstructed with a null magnetic field. In the ideal case of perfect alignment, perfect position resolution, and in the absence of multiple scattering, the point triplets should form perfect straight lines. If the wafer positions are known incorrectly, reconstructed triplets should form two line segments that are not perfectly co-linear. The local internal alignment procedure thus consists in a variation of selected wafer parameters (positions and angles) in order to restore the co-linearity of the two line segments. As discussed in the previous paragraph, such a procedure does not however provide sufficient constraints to resolve the parameters of all three wafers of a triplet. In Au+Au collisions, the number of produced particles originating from the primary collision point is quite large. Such a knowledge provide additional constraints on the wafer positions and orientations readily exploitable in the alignment procedure by including the primary vertex explicitly in the track reconstruction process.

In practice, the SDD detectors provide measurements of the track crossing points with finite accuracy and multiple scattering effects are also finite. This implies that the actual tracks cannot be perfectly measured and that in fact they are not perfect straight lines ($B = 0$) or helices ($B > 0$). In the absence of particular biases, it should however be possible by accumulating a large ensemble of tracks to reconstruct average trajectories that correspond to straight lines (or helices) and use these to provide constraints for the determination of the alignment parameters. The alignment procedure becomes a statistical process based on ideally a very large number of tracks and independent events (collisions).

The straight track model and the common track origin or vertex are used conjointly to make explicit use of the large track multiplicities produced in one event. We first outline the alignment procedure. Details are presented next.

Given measured points (x_d,x_a), where x_d, and x_a are respectively measured positions in the drift and anode directions taken to be in the SDD wafer planes, one first calculates the 3-dimensional coordinates (x,y,z) of those points (P) in the STAR reference frame using the measured (survey) wafer positions and orientations. A track finding procedure is then invoked to reconstruct the straight tracks and form triplets of points (P_1,P_2,P_3). The (approximately straight) tracks thus reconstructed are used to determine the main vertex position (V_x,V_y,V_z). The tracks and the vertex position are fitted together to evaluate the track residues,(i.e. deviations between the fitted straight tracks and measured point positions) to provide an estimator of the alignment quality. The residues and track χ^2 are used as a measure of the alignment quality. For a perfect alignment, the residues should be null on the average and the track χ^2 should be consistent with effects due to finite position resolution and multiple scattering. The wafer parameters are varied with a steepest descent (gradient) algorithm to find the parameter values which minimize the track residues. The common vertex constraint is included by adding the vertex to each track. The steepest descent procedure is iterated while the track χ^2 keeps decreasing . It is interrupted when the variation of the alignment parameters leads to no significant improvement of the tracks. At that point, the wafer parameters (positions, and orientations) are calculated, written out and saved. The alignment procedure is completed.

The specificities of the alignment procedure are described next.

3.1 Reference Frames

Two reference frames are used within the SAL package to express the coordinates of tracks crossing the detector wafer planes.

The first reference frame is the "local" reference frame associated to a wafer. In that reference frame, crossing positions or hits can be specified by a couple $\vec{X}^l = (x_d, x_a)$ where x_d, and x_a express the hit position along the drift and anode direction respectively. The origin of the reference frame is arbitrarily chosen to be the geometrical center of the wafer.

The other reference frame used in this package is a global reference frame used to express the detector positions (and orientations) as well as the the hit positions. In this frame, we distinguish three different instances of the hit positions. The first is referred to as "measured" frame. This frame corresponds to the reference frame in which the wafer positions and orientations are measured (surveyed). For the time being, we assume the origin of that reference frame to be the geometrical center of the SVT. The location (position) of each wafer are expressed as $\vec{W}^M = (W_x, W_y, W_z)$. The "z" axis is taken to be the beam direction. The "x" and "y" axes are respectively taken as vertical and horizontal directions. The orientations of all wafers can be described by three angles (α,β,γ) relative to that frame.

We express the measured hit positions $\vec{X}^M$ in the "measured" global frame as follows:

$$\vec{X}^M = \vec{W}^M + \Omega^M(\alpha,\beta,\gamma)\vec{X}^l \tag{7}$$

where is Ω^M is a rotation matrix determined by the three measured angles α^M,β^M, and γ^M.

The positions $\vec{W}^M$ and rotations Ω^M are known with finite accuracy. It is the purpose of the alignment procedure to determine the actual (real) positions $\vec{W}^R$ and orientations Ω^R of the wafers. The alignment method is iterative, so it is convenient, in practice, to define the notation $\vec{W}^i$ and Ω^i to refer to the "i"th version of the position and rotation matrix. Ideally when the procedure is completed, after the Nth iteration, one should have $|\vec{W}^N - \vec{W}^R| < \epsilon$ as well as $|\alpha^N - \alpha^R| < \epsilon$, $|\beta^N - \beta^R| < \epsilon$, and $|\gamma^N - \gamma^R| < \epsilon$. with ϵ "arbitrarily" small. In view of multiple scattering effects and finite position resolution, one anticipates that the wafer position and orientation reconstruction should be feasible with a resolution of the order of $\epsilon \approx 10$ microns and 10 milli-radians respectively.

3.2 Track Reconstruction

The tracks are reconstructed using a standard search technique. Points on the first (P_1) and third (P_3) layers are used to initiate a search for tracks. Line segments are formed to join P_1 and P_3. If the intercept of these segments with the second layer are found within a given radial distance from points P_2 of the second layer, then the triplet (P_1,P_2,P_3) is considered to form a straight line candidate. The reconstructed tracks (P_1,P_2,P_3) are fitted using the following parameterization:

$$\vec{P_i} = \vec{a} + \lambda_i \vec{b} \tag{8}$$

where $\vec{P_i}$ is the i-th point on the track,$\vec{b}$ - vector, determining track direction in space, $\vec{a}$ is a vector defining the minimal distance between the beam axis and the track in a plane orthogonal to the beam axis (by definition, one has $(\vec{b} \cdot \vec{a})|_{xy} = 0$), and λ is a parameter that determines the position of the points along the track (see Fig.2). The above parameterization allows a quick, non-iterative, determination of the track parameters.

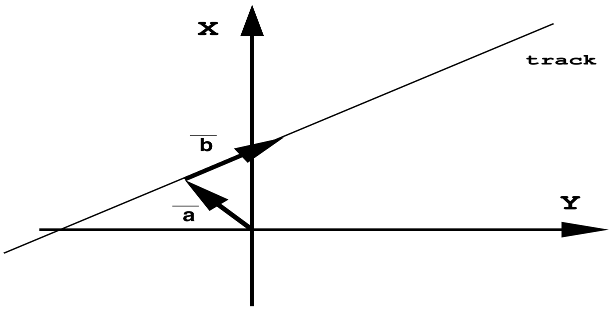

Figure 2: Track parameterization

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One proceeds as follow: first, a preliminary vector $\vec{b_o} = \vec{P_1} - \vec{P_2}$ is formed in the XY plane (transverse to the beam axis). Given that by construction, one has $\vec{a} \cdot \vec{b_o} = 0$, one readily determines the values of the λ coefficients.

$$\lambda_i = \frac{\vec{p_i} \cdot \vec{b_o}}{\vec{b_o} \cdot \vec{b_o}} \tag{9}$$

One wishes to find the vectors $\vec{a}$ and $\vec{b}$ such the sum of the square of the track residues, L, is minimized.

$$L = \sum_{i=1}^{3} \left(\vec{a} + \lambda_i \vec{b} - \vec{P_i} \right)^2 \tag{10}$$

Given an estimate $\vec{b_o}$, one finds a corresponding estimate for $\vec{a}$ with:

$$\vec{a_o} = \vec{P_1} - \lambda_1 \vec{b_o} \tag{11}$$

Better estimates are sought by rewriting the functional L as a function of adjusted vectors $\vec{a}$ and $\vec{b}$ defined as follows:

$$\vec{a} = \alpha \vec{a}_o \tag{12}$$
$$\vec{b} = \vec{b}_o + \beta \vec{a} \tag{13}$$

Derivatives of L with respect to α and β must be zero.

$$0 \;=\; \frac{\partial L}{\partial \alpha} = 2 \sum_{i=1}^{3} \left(\alpha \vec{a}_o + \lambda_i (\vec{b}_o + \beta \vec{a}_o) - \vec{P}_i \right) \cdot \vec{a}_o \tag{14}$$
$$0 \;=\; \frac{\partial L}{\partial \beta} = 2 \sum_{i=1}^{3} \left(\alpha \vec{a}_o + \lambda_i (\vec{b}_o + \beta \vec{a}_o) - \vec{P}_i \right) \cdot \lambda_i \vec{a}_o \tag{15}$$

These yield the solution :

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{1}{a_o^2 (3 \sum \lambda_i^2 - \sum \lambda_i)} \begin{pmatrix} \sum \lambda_i^2 & -\sum \lambda_i \\ -\sum \lambda_i & 3 \end{pmatrix} \begin{pmatrix} \sum \vec{P}_i \cdot \vec{a}_o \\ \sum \lambda_i \vec{P}_i \cdot \vec{a}_o \end{pmatrix} \tag{16}$$

Having minimized L in the XY plane, one then considers the z-projection component of the L functional so far neglected.

$$L_z = \sum (a_z + \lambda_i b_z - \vec{i} \cdot \vec{P}_i)^2 \tag{17}$$

One seeks once again to minimize this function taking derivatives with respect to a_z and b_z. One gets

$$\begin{pmatrix} a_z \\ b_z \end{pmatrix} = \frac{1}{(3 \sum \lambda_i^2 - (\sum \lambda_i)^2)} \begin{pmatrix} \sum \lambda_i^2 & -\sum \lambda_i \\ -\sum \lambda_i & 3 \end{pmatrix} \begin{pmatrix} \sum \vec{P}_i \cdot \vec{i} \\ \sum \lambda_i \vec{P}_i \cdot \vec{i} \end{pmatrix} \tag{18}$$

The solution $(\vec{a}, \vec{b})$ can then be expressed as

$$\vec{a}_n \;=\; \alpha \vec{a}_o + a_z \vec{i} \tag{19}$$
$$\vec{b} \;=\; \vec{b}_o + \beta \vec{a}_o + b_z \vec{i} \tag{20}$$

In general, $\vec{a}_n$ is not perpendicular to $\vec{b}$. So for convenience, one redefines $\vec{a}$ such that any component along $\vec{b}$ is removed.

$$\vec{a} = \vec{a}_n - \frac{\vec{b} \cdot \vec{a}_n}{\vec{b} \cdot \vec{b}} \tag{21}$$

3.3 Vertex reconstruction

With all track parameters in hand, one proceeds to determine the position of the primary vertex. An initial estimate is obtained by a simple counting method. The space surrounding the beam axis is discretized in small cells. To find the approximate position of the main vertex, one simply determines how many times each of those cells are crossed by a track. The cell with the largest number of crossing contains the vertex. Its center is used as a seed for the fitting procedure that follows.

A fitting procedure is used to determine the vertex position with more accuracy. One considers the optimal estimate of the vertex position as the position which minimizes the sum of the distance of closest approach to all tracks.

Given the (unknown) vertex position $\vec{V}$, and a track, j, whose direction is given by $\vec{b}_j$, the point of closest approach to the vertex, $\vec{P}_c$ is obtain by solving:

$$(\vec{P}^c - \vec{V}) \cdot \vec{b} = 0 \tag{22}$$

Using the same track model as in Sect. 3.2 , one finds

$$(\vec{a} + \lambda^c \vec{b}_j - \vec{V}) \cdot \vec{b}_j = 0 \tag{23}$$

Clearly, this yields

$$\lambda_j^c = \frac{(\vec{V} - \vec{a}_j) \cdot \vec{b}_j}{b_j^2} \tag{24}$$

The optimal vertex position is obtained by minimization of the functional K define as the sum of the square of the closest approach distance of all tracks.

$$K = \sum_{j=1}^{N_t} \omega_j \left(\vec{a}_j + \lambda_j^c \vec{b}_j - \vec{V} \right)^2 \tag{25}$$

A weight, ω_j, is used for each track to to make **robust** (see, for example, [4]) the whole procedure, i.e. to reduce the effect of ghost tracks or tracks properly reconstructed but not pointing to the primary vertex. We use the Tukey's bi-squared weights [2], which are calculated as follows:

$$\omega_j = \{ \begin{array}{l} \left(1 - \frac{g_j}{(3\sigma^{n-1})^2}\right)^2 \text{ for } g < [3\sigma^{n-1}]2 \\ 0 \text{ otherwise} \end{array} \tag{26}$$

where "n" is the iteration number, and

$$g_j = (\vec{V} - \vec{a}_j) \cdot (\vec{V} - \vec{a}_j) - \frac{((\vec{V} - \vec{a}_j) \cdot \vec{b}_j)^2}{(\vec{b}_j \cdot \vec{b}_j)} \tag{27}$$

$$\sigma^n = \frac{\sum_{j=1}^{N_t} \omega_j^{n-1} g_j^{n-1}}{\sum_{j=1}^{N_t} \omega_j^{n-1}} \tag{28}$$

N_t is the number of tracks in the event. The notation X^{n-1} refers to quantities calculated during the previous iteration.

The vertex position, $\vec{V}$, is determined by minimizing the functional K relative to the vertex position.

$$\frac{\partial K}{\partial \vec{V}} = \sum_{j=1}^{N_t} \omega_j \left(\vec{a}_j + \frac{(\vec{V} - \vec{a}_j) \cdot \vec{b}_j}{b_j^2} \vec{b}_j - \vec{V} \right) \cdot \left(\frac{\vec{b}_j \vec{b}_j}{b_j^2} - 1 \right) = 0 \tag{29}$$

This expression yields the solution

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = M^{-1} \begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix} \tag{30}$$

with the matrix "M" defined as

$$M = \begin{pmatrix} \sum \omega \frac{1-(\vec{b}\cdot\vec{i})(\vec{b}\cdot\vec{i})}{b^2}) & \sum -\omega \frac{(\vec{b}\cdot\vec{i})(\vec{b}\cdot\vec{j})}{b^2} & \sum -\omega \frac{(\vec{b}\cdot\vec{i})(\vec{b}\cdot\vec{k})}{b^2} \\ \sum -\omega \frac{(\vec{b}\cdot\vec{j})(\vec{b}\cdot\vec{i})}{b^2} & \sum \omega \frac{1-(\vec{b}\cdot\vec{j})(\vec{b}\cdot\vec{j})}{b^2}) & \sum -\omega \frac{(\vec{b}\cdot\vec{j})(\vec{b}\cdot\vec{k})}{b^2} \\ \sum -\omega \frac{(\vec{b}\cdot\vec{k})(\vec{b}\cdot\vec{i})}{b^2} & \sum -\omega \frac{(\vec{b}\cdot\vec{k})(\vec{b}\cdot\vec{j})}{b^2} & \sum \omega \frac{1-(\vec{b}\cdot\vec{k})(\vec{b}\cdot\vec{k})}{b^2}) \end{pmatrix} \tag{31}$$

and the matrix "I" defined as

$$I = \begin{pmatrix} \sum \omega((\vec{a} \cdot \vec{i}) - (\vec{a} \cdot \vec{b}) \frac{(\vec{b}\cdot\vec{i})}{b^2}) \\ \sum \omega((\vec{a} \cdot \vec{j}) - (\vec{a} \cdot \vec{b}) \frac{(\vec{b}\cdot\vec{j})}{b^2}) \\ \sum \omega((\vec{a} \cdot \vec{k}) - (\vec{a} \cdot \vec{b}) \frac{(\vec{b}\cdot\vec{k})}{b^2}) \end{pmatrix} \tag{32}$$

where the sums are on all "N_t" tracks reconstructed in an event.

3.4 Alignment Functional

The alignment procedure is based on the found tracks and vertices of an (ideally) large number of events. The tracks are characterized by a functional $L_{j,ev}$. "j" refers to the jth track of event "ev". For each event, a vertex is found, $\vec{V}_{ev}$ which is characterized by the functional K_{ev}. One expects that a better alignment should yield lower values for all $L_{j,ev}$ and K_{ev}. The alignment thus proceeds to minimize the functional "G" calculated as:

$$G = \sum_{ev=1}^{N_{events}} \left(\sum_{j=1}^{N_{t,ev}} L_{j,ev} + K_{ev} \right) \tag{33}$$

The first term of this functional takes into account each track residue, defined as follow:

$$\begin{aligned} L_{j,ev} = (\sum_{i=1}^{3}((a.x_j + \frac{(\bar{P}_i, \bar{b})|_{xy}}{(\bar{b}, \bar{b})|_{xy}} b.x_j - P_i.x)^2 + \\ ((a.y_j + \frac{(\bar{P}_i, \bar{b})|_{xy}}{(\bar{b}, \bar{b})|_{xy}} b.y_j - P_i.y)^2 + \\ ((a.z_j + \frac{(\bar{P}_i, \bar{b})|_{xy}}{(\bar{b}, \bar{b})|_{xy}} b.z_j - P_i.z)^2))/N_{t,ev} \end{aligned}$$

The second term takes into account the vertex width

$$K_{ev} = \frac{\sum_{j=1}^{N_{t,ev}}((\bar{a}'_j, \bar{a}'_j) - \frac{(\bar{a}'_j, \bar{a}'_j)^2}{(\bar{b}, \bar{b})})}{N_{t,ev}}$$

where $\bar{a}'_j$ is defined as:

$$\bar{a}'_j = \bar{V} - \bar{a}_j$$

3.5 G Functional Minimization Procedure

The minimization of the functional G is performed using a steepest descent method. The method proceeds iteratively. During each iteration one first calculates the track and vertex parameters in order to evaluate the functional G. Given alignment parameters μ_i obtained from the previous iteration (from surveyed position in the first iteration), one proceeds to calculate numerical derivatives of the functional relative to the alignment parameters μ_i as follows:

$$\frac{\partial G}{\partial \mu_i} = \frac{G(\mu_i + \delta\mu_i) - G(\mu_i - \delta\mu_i)}{2\delta\mu_i}, \tag{34}$$

where μ_i is one of the wafer alignment parameters (position and angles). $\delta\mu_i$ is the parameter step size used during the i-th iteration. The step size are calculated and varied separately for each parameters category (x, y, z, α, β, γ).

Once all derivatives have been calculated, new parameter values are recalculated in accordance with the corresponding derivatives. In order to increase the speed of the descent, and to avoid local minima, the parameter changes are normalized according to the maximum of all derivatives. For each iteration j, one calculates :

$$\mu_i^j = \mu_i^{j-1} - \frac{\partial G}{\partial \mu_i} \frac{\delta \mu_i}{MAX(\frac{\partial G}{\partial \mu_i})} \Delta_j,$$

where Δ_j is a dimensionless step size common to all parameters.

Next, the functional value is calculated using the new geometry obtained during this recalculation. If the new functional value has decreased, the alignment procedure is continued. If the functional value has increased, the iteration step Δ_j is reduced and the alignment parameters are recalculated.

The steepest descent procedure is iterated while the track χ^2 keeps decreasing. The procedure is interrupted when the variation of alignment parameters leads to no significant improvement of the tracks, i.e.when χ^2 reaches a stable minimum. At that point, the wafer parameters (positions, and orientations) are calculated, written out and saved. The alignment procedure is completed.

The alignment procedure for the local internal alignment task is divided into two stages. Both stages are based on the gradient descent method. An investigation of the effects produced on the functional value by different types of misalignment parameters showed that the influence of rotations in the wafers plane (alpha) exceeds that of all other misalignment parameters types. So, in the first stage we make alignment for all alpha-angles only. Without this preliminary procedure, the alpha angles influence dominates the influence of all other parameters. This can lead to one of the numerous (in case of minimizing very complicated dependence) local minima. Organizing the preliminary alpha-alignment in a separate stage is also motivated by the unique character of its influence on the functional value and by the absence of correlation with other parameters types. After the above described preliminary alignment procedure, the same algorithm was realized for all 1296 parameters (including alpha) simultaneously. The results are discussed in Sect. 5.

4 Performance Evaluation Method

The performance of the Alignment reconstruction algorithm is evaluated by means of Monte Carlo simulations. The purpose of the performance evaluation is to determine (1) the accuracy one can practically reach in the alignment reconstruction, and (2) to provide a test bed for the alignment reconstruction code.

The simulations of the internal alignment reconstruction involves the following steps:

- Generation of particle final states using models such as HIJING, or FRITIOF to simulate central Au+Au collisions at $\sqrt{s} = 200$ GeV.
- Propagation of particles within a realistic model (but with ideal geometry) of STAR and in particular the SVT detector to determine track crossing position in the detector, i.e. hit positions.
- Generation of Detector misalignment

- Modification (Shift) of the hit positions obtained with the ideal geometry to take into account the generated detector misalignment.
- Track Reconstruction, Vertex reconstruction and Alignment reconstruction.

4.1 Event Generation

The generation of particle final states has been performed so far with HIJING for purely practical reasons. We do not foresee any need to test the alignment method with events produced by another model given that it is quite unlikely that the specifics of the particle final state should have much of an impact on the alignment.

The propagation of particles through an idealized model of STAR is performed with GSTAR. It is worth noting that it is necessary to run GSTAR with high digitization accuracy to enable an accurate evaluation of the alignment. Early simulations with position digitization accuracy limited to 8 bits were found to be wrongly biased by the limited accuracy. The SVT hits are currently stored with 16 bits to alleviate that problem. Likewise, the particle momenta must also be stored with increased accuracy.

The GSTAR particle (track) propagation through the apparatus and hit generation is performed with an ideal SVT geometry. The hits generated are modified subsequently to take into account misalignment effects. Such an approach is deemed much simpler than completely realistic simulations where the misalignments would be directly plugged into GEANT. This would require painful and detailed modifications of the SVT geometry on a per wafer basis and would be likely to reduce the simulation speed and increase the complexity of the task considerably. In contrast, the modification of the hit positions after the fact, i.e. after GEANT, enables multiple usage of the same GSTAR event files and thus simplifies greatly the process of testing the alignment code under different misalignment conditions.

4.2 Misalignment Generation

The misalignment of the wafers is generated by an independent procedure which uses the nominal (ideal) wafer positions and angles as input. The positions and angles of each wafer are randomly shifted using flat (uniform) random number generators. The magnitude of the shifts and orientations are input parameters. It is thus possible to generate misalignments of arbitrary magnitude.

The hit positions generated by GSTAR must be modified to take into account the wafer position shifts and rotations. This is accomplished in a separate module by using the GSTAR generated points, the track Monte Carlo information, the ideal, and the shifted wafer geometries as inputs. The hit position are recalculated on the basis of the construction illustrated in Fig.3: the calculation of the shifted hit positions requires not only the wafer original and shifted positions but also the direction of the tracks at the crossing point.

The ideal position of a point is given by

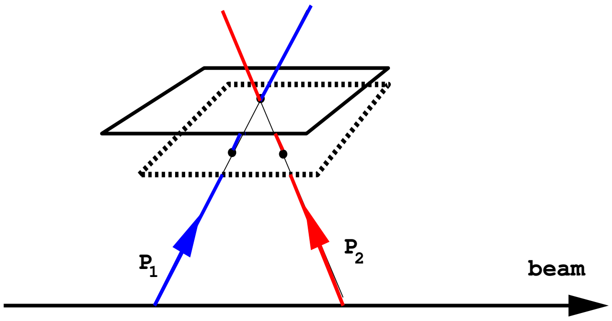

Figure 3: Hit position recalculation for different tracks

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$$\vec{X} = \vec{W} + x_d\vec{d} + x_t\vec{t} \tag{35}$$

where $\vec{W}$ points to the wafer nominal (ideal) position, $\vec{d}$ a unit vector in the wafer plane pointing in the drift direction, $\vec{t}$, a unit vector in the wafer plane pointing in the anode or transverse direction. x_d and x_t are distances relative to the wafer center in the drift and anode directions respectively. The shifted positions are expressed in the same way using primed quantities.

$$\vec{X}' = \vec{W}' + x'_d\vec{d}' + x'_t\vec{t}' \tag{36}$$

The shifted wafer parameters $\vec{W}'$, $\vec{d}$, and $\vec{t}$ are known (by construction). One needs to calculate the values x_d and x_t. First we express the position of the unshifted point as $\vec{X} = \vec{a} + \lambda\vec{p}$ to emphasize that it lies along a straight line whose direction is given by the (local) momentum vector $\vec{p}$. Then we define $\vec{a} \cdot \vec{p} = 0$ by construction and calculate the value of λ.

$$\lambda = \frac{\vec{x} \cdot \vec{p}}{\vec{p} \cdot \vec{p}} \tag{37}$$

The momentum vector $\vec{p}$, stored by GSTAR for every hits in the SVT, corresponds to the particle direction while crossing the wafer, giving $\vec{a} = \vec{X} - \lambda\vec{p}$. One now looks for the point $\vec{X}'$ corresponding to a straight line crossing the shifted wafer. By construction, every point in the (shifted) wafer plane satisfies the equation

$$\vec{n}' \cdot \vec{X}' = \vec{n}' \cdot \vec{W}' \tag{38}$$

The shifted point $\vec{X}'$ can thus be characterized by

$$\lambda' = \frac{\vec{n}' \cdot \vec{W}' - \vec{n} \cdot \vec{a}}{\vec{n}' \cdot \vec{p}} \tag{39}$$

and the shifted coordinates are given by

$$\vec{X}' = \vec{a} + \lambda'\vec{p} \tag{40}$$

4.3 Performance Evaluation Criteria

We use three methods to evaluate the alignment reconstruction.

4.3.1 Direct Comparison of the reconstructed positions and angles

Within the context of a Monte Carlo simulation, one has apriori knowledge of the actual wafer positions and orientations. We can thus evaluate the quality of the parameter reconstruction by comparing the reconstructed parameters directly with the nominal (input) parameters. This is done in the following way: one calculates the deviation of the parameters $X_r - X_a$ (reconstructed - actual) and plot histograms of those deviations for all 216 wafers. It is necessary to notice that the reconstruction method leaves unconstrained at least 6 degrees of freedom, i.e. three global translations, and three global rotations, it is possible that the reconstructed parameters might display an overall offset with the nominal parameters.

4.3.2 Comparison of the vertex width

Yet another way of evaluating the reconstruction performance consists in comparing the distribution of track impact parameters, i.e. the modulus of the impact parameter vector $\vec{a}$, obtained with the reconstructed geometry with that obtained with the ideal geometry. Results are shown in the next section.

4.3.3 Comparison of the track χ^2 distributions

The last method used to evaluate the quality of the reconstruction is based on track residues and chi-squares. The track chi-square is defined as

$$\chi^2 = \sum_{i=0}^{3} \frac{(\vec{X}_i - \vec{a} - \lambda_i \vec{b})^2}{\sigma_i^2} \tag{41}$$

$\vec{X}_i, i = 0, \ldots, 3$ are the reconstructed vertex (i=0), and the three measured points (i=1,...,3). $\vec{a}$ and $\vec{b}$ are best fit to the track. σ_i is an estimate of the error on the measurement of the position $\vec{X}_i$. The error includes two components. The first component is directly connected to the actual position measurement resolution σ_m, the second, σ_{MCS}, arises because of multiple Coulomb scattering (MCS) in the detector.

Given that the internal alignment procedure is based on straight tracks reconstructed in a zero magnetic field, one does not have information (in the actual reconstruction) on the particle momenta. It is therefore impossible to evaluate MCS effects quantitatively on a per track basis. We thus resorted to an average procedure. Using HIJING events, we propagated the tracks with GSTAR and determined the average track deviation, $\sigma_{avg,i}$ due to MCS on each SVT layer. Tracks had momenta ranging from < 100 MeV/c to a few GeV/c. The average deviation obtained with GSTAR has a value of $81.3\mu m$ and is used in the calculation of Eq.41.

5 Results

Ten HIJING events have been generated and processed through GSTAR and G2T to convert the GSTAR output into tables readable with STAF. These events were further processed to simulate a misalignment of the wafers in order to provide an input to the alignment reconstruction package. The events were central Au+Au collisions. They contained roughly 1600 tracks (per event) going through the SVT volume. The events were generated at random locations along the beam axis to simulate the finite size of the interaction region. Interactions were generated within $\pm 3cm$ of the origin.

5.1 Wafer Parameters Reconstruction Accuracy

On input, the wafer positions were shifted randomly within a range of $\pm 50\mu m$ along three axes. The angles of the wafers were also changed by adding random values in a range of ± 5 milli-radian to the nominal wafer angles. Shaded areas histograms a, b, and c, shown in Fig.4 display the generated wafer misalignments along the x, y, and z axes respectively. Histograms d, e, and f show the generated angular deviations along the same three axes.

The ten HIJING events were processed simultaneously with the SAL package. The final wafer positions are compared to the actual positions in Fig.4. The RMS positional and angular deviations are listed in Table 1 (Single pass). The position deviations, obtained with such a single pass, are of the order of 20 microns and the angular deviations of the order of 1-2 milli-radians; an alignment that should in principle be quite acceptable. We found however that with one pass on the data, the reconstructed parameters may vary considerably depending on the initial wafer parameters used as a seed to the multi-parameter search. This stems in part from the sizeable MCS effects and in part from the very large number of parameters simultaneously minimized. We thus proceeded to implement a multi-stage search where the steepest descent algorithm is repeated "N" times using, each time, a different "seed". The parameters reconstructed by the five passes are averaged individually and compared to the actual parameter values. Table 1 lists the final RMS deviations obtained by averaging the parameters reconstructed in the five passes. The RMS position deviations are significantly improved. It is thus clear that the multiple pass analysis of a large number of events (>10) should lead to very robust and reliable alignment results.

5.2 Vertex Width Reconstruction

Figure 5 shows histograms of the track impact parameters, i.e. distance of closest approach to the main vertex, obtained with the ideal geometry, the misaligned geometry, and the reconstructed geometry. The average track impact parameter, obtained with the misaligned geometry (Fig.5a) amounts to 0.09 mm, and the distribution has a standard deviation of 0.048 mm. It is to be compared to the impact parameter distribution, obtained after alignment, shown in Fig.5*b*). In this case, the average impact parameter is 0.056 mm, and the distribution has a standard deviation of 0.032 mm. Both distributions are compared to the ideal impact

Table 1: Precision of the wafers position determination. σ_x, σ_y, and σ_z are RMS position deviations in microns. σ_α, σ_β, and σ_γ are RMS angular deviations in milli-radians. RMS error are shown for single pass and multiple passes reconstructions. See text for details.

	σ_x	σ_y	σ_z	σ_α	σ_β	σ_γ
Single Pass	22.5	17.4	25.1	1.1	1.7	1.8
Multiple Passes	11.8	8.0	12.9	0.5	1.0	1.2

parameter distribution obtained with the nominal geometry and a perfect alignment shown in the same figures with dashed lines. In this case, the average impact parameter is 0.052 mm, and the distribution has a standard deviation of 0.029 mm.

It is worth noting that the misalignment can have a dramatic impact on the track impact parameter resolution, and in particular, for the identification of short lived particles decaying near the primary vertex as exemplified by the very large mean and rms of the misaligned track impact parameter distribution. The alignment procedure reported in this work fortunately provides for an excellent wafer parameter determination. As shown in Fig.5b, the ideal and reconstructed impact parameter distributions are nearly indistinguishable. The impact parameter determination is thus essentially limited by MCS effects only.

5.3 Track Chi-squares

Figure 6, shows histograms of the track χ^2 distributions for tracks reconstructed with the misaligned geometry (a), the ideal geometry (perfect alignment) (b), and the reconstructed geometry (c).

The wafer misalignment implies that straight tracks cannot in general be reconstructed as such; the χ^2 thus becomes considerably larger on the average.

The mean value and the shape of the distribution obtained after alignment (c) are very close to the distribution obtained with an ideal geometry. The alignment procedure is thus shown to provide for an excellent reconstruction of the positional and angular parameters. In addition, the χ^2 distribution obtained after alignment is in excellent agreement with the theoretical χ^2 distribution for one degree of freedom (shown in all three panels of Fig.6). As such, this provides a method by which one can judge of the goodness of the reconstructed alignment in a realistic case when the actual misalignments are not known.

6 Conclusion

We have developed a fast and accurate geometry calibration (i.e. alignment) procedure for the STAR SVT. The alignment procedure is based on a steepest descent minimization of simple functionals calculated for straight tracks in a zero magnetic field. The performance of the method was evaluated by comparing the reconstructed wafer positional and angular parameters, as well as the vertex size and straight track distributions. We found the positions and angles are on the average reconstructed with negligible bias. In addition, we determined that it is possible, with only ten central Au+Au events, to obtain a very satisfactory reconstruction such that the RMS wafer position deviation is found to be smaller than $15\mu m$ while the angular RMS deviation is smaller than $1.5 mrad$. The reported algorithm, as implemented in the SAL STAF package, thus provides a solid foundation for the STAR SVT alignment.

References

[1] O. Yu. Barannikova *et al.* , in preparation.

[2] F. Mosteller, W. Tukey, Data analysis and regression: a second course in statistics, Addison - Wesley (1987).

[3] Xin-Nian Wang and Miklos Gyulassy, Phys. Rev. **D44** (1991) 3501.

[4] H.Agakishiev, O.Barannikova, F.Ceretto, U.Faschingbauer, P.Glassel, E.Kolganova, G.Ososkov, Yu.Panebratsev, J.Rak, N.Saveljic, I.Tserruya, T.Ullrich, J.P.Wurm *New Robust Fitting Algoritm For Vertex Reconstruction in the* **CERES** *Experiment*, Nucl. Instr. and Methods **A394** (1997) 225-231.

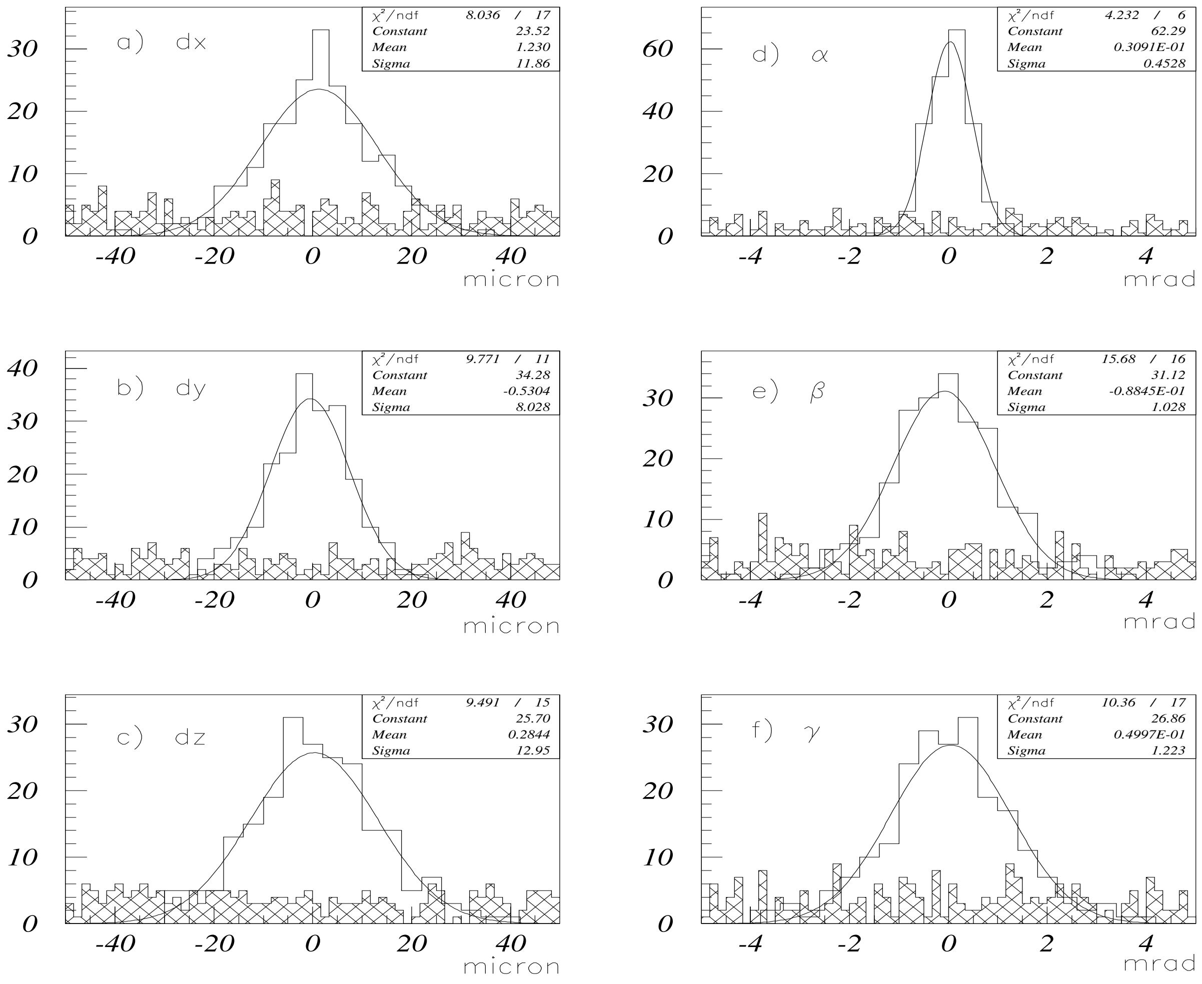

Figure 4: Parameters definition errors

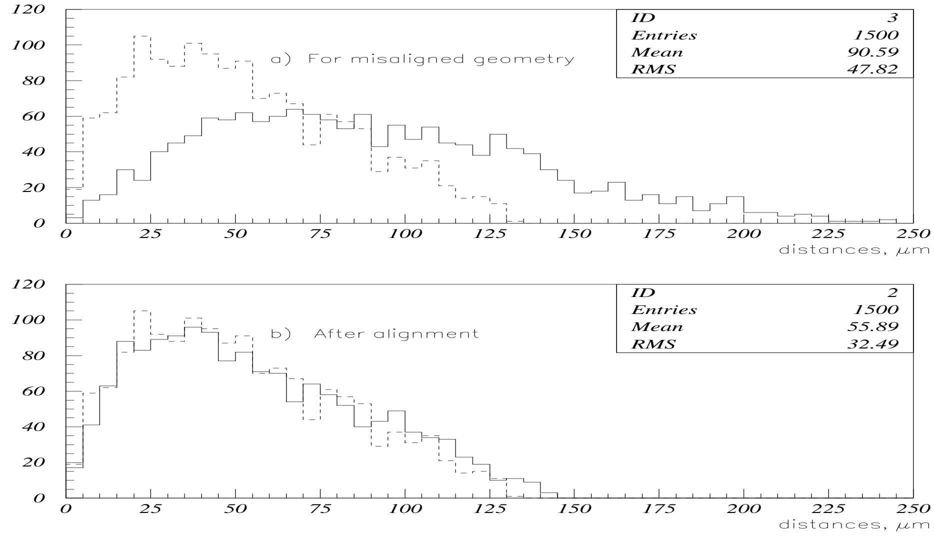

Figure 5: Vertex width

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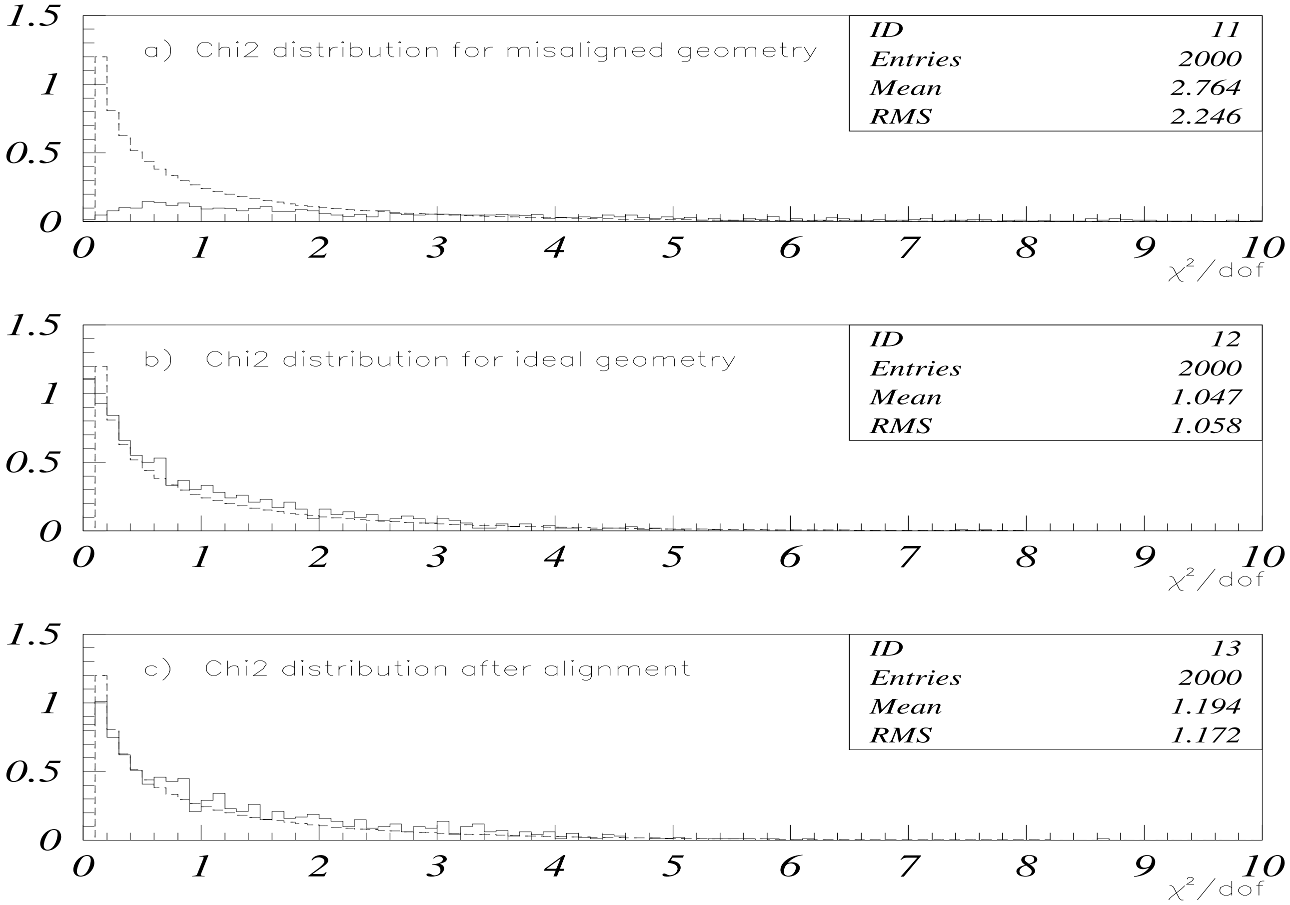

Figure 6: Track Chi-squares

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