Rhic Spin Note # 79
Star Note # 405
12 July, 1999

NOTE ON RHIC POLARIMETRY

H. Spinka

High Energy Physics Division
Argonne National Laboratory
Argonne, Illinois 60439

1 Introduction

For physics measurements with polarized colliding beams, beam polarizations and relative luminosities must both be determined. As an example, the spin-spin correlation parameter with longitudinally polarized beams, A_{LL}, is given by:

$$A_{LL} \simeq \frac{1}{P_a P_b} \cdot \frac{(\text{Events/Lum})_{\text{parallel}} - (\text{Events/Lum})_{\text{antiparallel}}}{(\text{Events/Lum})_{\text{parallel}} + (\text{Events/Lum})_{\text{antiparallel}}},$$

where P_a and P_b are the average beam polarizations for the two colliding beams. Predictions for spin observables of many interesting physics processes at RHIC are quite small in magnitude. This requires high statistics measurements of relative luminosities and careful control of systematic errors.

Discussions about the polarized beams at RHIC often presume that the polarization and intensity of each bunch within a fill will be known quite well from measurements by the RHIC polarimeters. The purpose of this note is to give a description of the knowledge that can actually be obtained from these polarimeters. In particular, the following questions will be addressed:

- What assumptions are made about the beam and polarimeters, and how (well) can these be tested?
- With what accuracy can the polarization and intensity of each bunch be measured?
- What systematic errors might be present, and are there actions that can be taken to reduce these potential systematic errors?

The nominal assumptions about the beam and about the polarimeter construction and operation will be described in Sec. 2. A discussion of Poisson statistics for measurements of luminosities and polarimeter rates will also be included. Equations for polarimeter rates and quantities that can be deduced from these rates will be presented in Sec. 3. The information that can be found from a comparison of polarimeter and collider detector measurements will be described in Sec. 4, and possible problems associated with the pattern of bunches within each beam are discussed in Sec. 5. The summary and conclusions will be given in Sec. 6.

This note was mostly written before and during the March 1999 polarized beam run at the AGS. During this run, there were sizeable changes to the beam from spill to spill observed with the AGS internal polarimeter (E880), as well as with the extracted beam. As a consequence, large systematic errors were seen in the measured asymmetries from the E880 polarimeter. In addition, a significant spin-correlated change in rates for two beam veto counters was observed in E925. This change could have been caused by a difference in the horizontal position of the beam at the veto counters. These are examples of problems that may also arise at RHIC, since the AGS will be the RHIC injector.

2 Assumptions and Poisson Statistics

The RHIC beams will be assumed to have nearly identical phase space from bunch to bunch at the same relative time within the fill (to allow for variation of phase space as the luminosity decays or as the RHIC beams are tuned). That is, the transverse dimensions and distributions, the longitudinal size and average offset from the nominal position, the distributions of angles of beam protons from the nominal orbit, and the momentum spread will all be assumed to be the same at a particular point around RHIC. The beam polarization direction will be taken to be vertical at the polarimeter. Measurements with the polarimeter will be assumed to occur approximately once an hour, during a period of about 120 sec. A total of 60 bunches with spacing about 220 nsec are expected during early RHIC operation.

The polarimeter for each RHIC beam will be assumed to have approximately symmetric left and right arms and to be operated identically. Similarly, the polarimeter target will be assumed to have uniform thickness and to be centered between the arms and on the beam. The consequences of relaxing these conditions will be described briefly in Sec. 3.3. Thus the left and right arm analyzing powers, A_L and A_R, and the products of efficiency and solid angle, $d\Omega_L$ and $d\Omega_R$, are both assumed to be nearly equal. On the basis of past experience,

$$\begin{aligned} \left|\epsilon_A = \frac{A_L - A_R}{A_L + A_R}\right| &\leq 0.1 \\ \left|\epsilon_\Omega = \frac{d\Omega_L - d\Omega_R}{d\Omega_L + d\Omega_R}\right| &\leq 0.1, \end{aligned} \tag{1}$$

should be achievable. However, both quantities depend on details of the physics process involved and of the polarimeter construction. Thus it may be difficult to satisfy Eqs. (1) in practice. For example, there were times when polarimeters at the ANL-ZGS and LAMPF were operated so that $|\epsilon_\Omega|$ was significantly larger than 0.1.

It will also be assumed that the luminosity will be measured for each pair of colliding beam bunches for at least one of the detectors (BRAHMS, PHENIX, PHOBOS, PP2PP, STAR) using some physics process that is approximately spin independent.

Each proton bunch in the RHIC beams will be $\leq 1-2$ nsec long, and thus it will be impractical to scale multiple interactions or events from a single bunch or bunch crossing for either the polarimeter or the luminosity monitors. Multiple interactions may occur, but the detectors are not expected to be capable of distinguishing one from two or more interactions in a bunch. As a consequence, for each bunch the luminosity monitors or polarimeter arms will either fail to detect an event or will succeed in detecting at least one event; Poisson statistics governs this situation.

If the probability of detecting one or more interactions per bunch is p, then the probability to detect i events in n bunches is

$$\frac{n!}{i!(n-i)!}p^i(1-p)^{n-i}.$$

The mean number of events detected in n bunches will be np, and the variance will be $\sigma^2 = np(1-p)$. Therefore, the relative error is

$$\sigma/\text{Mean} = \sqrt{\frac{1-p}{np}}.$$

This gives relative errors of $\sim 10/\sqrt{n}$, $3/\sqrt{n}$, $1/\sqrt{n}$, $1/3\sqrt{n}$ for $p = 0.01$, 0.1, 0.5, and 0.9, respectively. As p decreases, the relative error approaches $1/\sqrt{np}$, as expected.

For the polarimeter, each bunch would be measured n times, where

$$n = \frac{120\,\text{sec}}{(60\,\text{bunches})(220\,\text{nsec/bunch})} \simeq 9 \times 10^6.$$

If the polarimeter operation will be compromised by multiple events in the same bunch, then probably p will need to be in the range $0.01 - 0.1$, or smaller. In fact, p will possibly vary considerably from fill to fill depending on the peak luminosity achieved and the accelerator operating conditions. For p in this range,

$$\sigma/\text{Mean} \simeq (1-3.3) \times 10^{-3}. \tag{2}$$

Similarly, for the luminosity monitors, the number of times each bunch will be measured and the relative error will be

$$\begin{aligned} n &= \frac{(58\,\text{min})(60\,\text{sec/min})}{(60\,\text{bunches})(220\,\text{nsec/bunch})} \simeq 2.6 \times 10^8 \\ \sigma/\text{Mean} &\simeq (1.8-6) \times 10^{-4}. \end{aligned} \tag{3}$$

The relative error should be about an order of magnitude smaller than for the polarimeter because of the much longer measurement time (factor of 29). The effects of the beam lifetime have not been explicitly taken into account in this error analysis, since the polarimeters and luminosity monitors are likely to rely on much different physics reactions and to have different solid angles. They will both be operated to give high count rates. In addition, the present polarimeter designs will not operate reliably when a large fraction of the events correspond to multiple interactions in the same bunch.

3 Polarimeter Equations

The number of good polarimeter events detected in the left arm for a + polarization bunch (j) will be denoted $L_+^{(j)}$, and similarly for events in the right arm (R) or for a – polarization bunch (k). These can be written (see Refs. [1, 2, 3] and also [4, 5]):

$$\begin{aligned} L_+^{(j)} &= NB_+^{(j)} d\Omega_L (1 + P_+^{(j)} A_L) \\ L_-^{(k)} &= NB_-^{(k)} d\Omega_L (1 - P_-^{(k)} A_L) \\ R_+^{(j)} &= NB_+^{(j)} d\Omega_R (1 - P_+^{(j)} A_R) \\ R_-^{(k)} &= NB_-^{(k)} d\Omega_R (1 + P_-^{(k)} A_R), \end{aligned} \tag{4}$$

where N is an overall normalization factor. These equations are written so that all beam polarizations ($P_+^{(j)}$, $P_-^{(k)}$) are positive or all are negative, depending on the sign of the analyzing power of the physics reaction used for the polarimeter. It will be assumed that $P_+^{(j)}, P_-^{(k)} \geq 0$. As noted before, slight differences in construction or alignment or operating conditions might yield $A_L \neq A_R$ and $d\Omega_L \neq d\Omega_R$. It is also expected that there will be bunch to bunch differences in beam intensity ($B_+^{(j)} \neq B_-^{(k)}$) and polarization.

In analogy with Eqs. (1), the following asymmetries can be defined:

$$\begin{aligned} \left| \epsilon_B^{(jk)} = \frac{B_+^{(j)} - B_-^{(k)}}{B_+^{(j)} + B_-^{(k)}} \right| &\leq 0.1 \\ \left| \epsilon_P^{(jk)} = \frac{P_+^{(j)} - P_-^{(k)}}{P_+^{(j)} + P_-^{(k)}} \right| &\leq 0.05, \end{aligned} \tag{5}$$

where the limits are guessed values only. It is expected that the polarized source operation and tuning in the Booster and AGS should give small changes to the beam polarization during the period to fill RHIC with beam. However, the uniformity of beam intensity from bunch to bunch could be much poorer than the limit shown in Eq. (5).

In the following analysis, the quantities

$$\epsilon_A, \;\; \epsilon_\Omega, \;\; \epsilon_B, \;\; \epsilon_P, \;\; PA$$

will all be assumed to be small and similar in magnitude. The average beam polarization, $P = (P_+ + P_-)/2$, will hopefully be in the range 0.5 – 0.7. Two types of polarimeters are being considered for early spin experiments. One involves pC or pp elastic scattering at very small angles in the Coulomb-nuclear interference (CNI) region. For this case, $A = (A_L + A_R)/2 \simeq 0.03$ and $PA \sim 0.02$. The second type of polarimeter would use inclusive production of charged pions off a C or H target, where $A \simeq 0.15$ and $PA \sim 0.1$. In both cases, there will be systematic uncertainties in the values of A that will affect the determination of the beam polarization from the polarimeter data. Similar to the definitions of the averages P and A, the mean values for the beam intensity and solid angle times efficiency will be denoted $B = (B_+ + B_-)/2$ and $d\Omega = (d\Omega_L + d\Omega_R)/2$.

Eqs. (4) are written assuming the beam is "stable," so that the phase space changes slowly with respect to the bunch spacing. If there are sizeable bunch to bunch changes, then these equations need to be modified. For example, $d\Omega_L$ would be replaced by $d\Omega_L^{(j)}$ or $d\Omega_L^{(k)}$, and A_R by $A_R^{(j)}$ or $A_R^{(k)}$, etc.. With these modifications the number of unknown quantities grows substantially and the equations to solve for either PA or beam intensity become more complicated than those shown later in this section. In effect, there is too little information to solve for the desired physical quantities if the phase space changes are too large.

3.1 Single Bunch Analysis

In this subsection, an asymmetry is defined using information from a single bunch. This asymmetry is shown to depend on both the beam polarization of the bunch, and also on the asymmetry in solid angle times efficiency, ϵ_Ω. An attempt is made to estimate ϵ_Ω, by averaging over all bunches in the fill, assuming it is approximately constant. The limits of accuracy of this method for the determination of the beam polarization of a single bunch are derived. Also, an estimate of the beam intensity in a single bunch is presented, and its dependence on other factors also described.

The natural asymmetry to compute for a single bunch j or k is [2]:

$$\begin{aligned}
\alpha_1^{(j)} &= \frac{L_+^{(j)} - R_+^{(j)}}{L_+^{(j)} + R_+^{(j)}} = \frac{\epsilon_\Omega + P_+^{(j)} A(1 + \epsilon_A \epsilon_\Omega)}{1 + P_+^{(j)} A(\epsilon_A + \epsilon_\Omega)} \\
&\simeq P_+^{(j)} A + \epsilon_\Omega + [-P_+^{(j)} A \epsilon_\Omega^2 - P_+^{(j)2} A^2 (\epsilon_A + \epsilon_\Omega) + \mathcal{O}(\epsilon^4)] \\
\alpha_1^{(k)} &= \frac{L_-^{(k)} - R_-^{(k)}}{L_-^{(k)} + R_-^{(k)}} = \frac{\epsilon_\Omega - P_-^{(k)} A(1 - \epsilon_A \epsilon_\Omega)}{1 - P_-^{(k)} A(\epsilon_A + \epsilon_\Omega)} \\
&\simeq -P_-^{(k)} A + \epsilon_\Omega + [P_-^{(k)} A \epsilon_\Omega^2 - P_-^{(k)2} A^2 (\epsilon_A + \epsilon_\Omega) + \mathcal{O}(\epsilon^4)].
\end{aligned} \tag{6}$$

The terms in brackets are of order ϵ^3 or higher, while the first two terms are $\mathcal{O}(\epsilon)$. Thus the terms in brackets can be ignored to a good approximation. The remaining two terms are comparable in magnitude, or perhaps ϵ_Ω might be considerably larger than PA. The statistical error on α_1 is given by

$$\begin{aligned}\delta\alpha_1 &= \frac{2LR}{(L+R)^2}[(\delta L/L)^2 + (\delta R/R)^2]^{\frac{1}{2}} \\ &\simeq \frac{1}{2}[(\delta L/L)^2 + (\delta R/R)^2]^{\frac{1}{2}}.\end{aligned}$$

Using Eq. (2),

$$\delta\alpha_1 \simeq (0.7 - 2.3) \times 10^{-3}.$$

The problem with using Eqs. (6) to determine $P_+^{(j)}$ or $P_-^{(k)}$ is the presence of the term ϵ_Ω. This term arises because of slight differences in the construction or operation of the two polarimeter arms that could give unequal L and R counts even for unpolarized beam. In order to estimate ϵ_Ω, an obvious solution would be to average over all m positive polarization and q negative polarization bunches,

$$\begin{aligned}\langle\alpha_1\rangle &= \frac{1}{m+q}\left(\sum_{j=1}^{m}\alpha_1^{(j)} + \sum_{k=1}^{q}\alpha_1^{(k)}\right) \\ &\simeq \frac{A}{m+q}\left(\sum_{j=1}^{m}P_+^{(j)} - \sum_{k=1}^{q}P_-^{(k)}\right) + \epsilon_\Omega.\end{aligned}$$

On the average, $P_+ = P(1+\overline{\epsilon_P})$ and $P_- = P(1-\overline{\epsilon_P})$, and assuming a random variation of bunch polarizations, then $|\overline{\epsilon_P}| \simeq |\epsilon_P^{(jk)}|/\sqrt{(m+q)/2}$. If $m = q$,

$$\langle\alpha_1\rangle \simeq PA\overline{\epsilon_P} + \epsilon_\Omega. \tag{7}$$

The first term in Eq. (7) would have a magnitude less than 0.0002 or 0.001 for the CNI or π-inclusive polarimeter, respectively, while ϵ_Ω could be as large in magnitude as 0.1 for the assumption in Eq. (1). The statistical error on $\langle\alpha_1\rangle$ would be approximately

$$\begin{aligned}\delta\langle\alpha_1\rangle &\simeq (0.7 - 2.3) \times 10^{-3}/\sqrt{m+q} \\ &\simeq (1 - 3.3) \times 10^{-4},\end{aligned}$$

where $m + q$ was chosen to be somewhat less than 60 bunches, as explained in Sec. 5.

Forming the differences,

$$\begin{aligned}\alpha_1^{(j)} - \langle\alpha_1\rangle &\simeq P_+^{(j)}A - PA\overline{\epsilon_P} + \mathcal{O}(\epsilon^3) \\ \alpha_1^{(k)} - \langle\alpha_1\rangle &\simeq -P_-^{(k)}A - PA\overline{\epsilon_P} + \mathcal{O}(\epsilon^3),\end{aligned} \tag{8}$$

then ignoring the term $PA\overline{\epsilon_P}$ would lead to systematic errors on the beam polarization for each bunch with somewhat smaller magnitude than the statistical errors. In particular, the statistical and systematic uncertainties on $P_+^{(j)}$ or $P_-^{(k)}$ for the CNI polarimeter would be 0.05 and 0.007, respectively. For the π-inclusive polarimeter, they would be 0.01 and 0.0014. Note that the averaging for $\langle\alpha_1\rangle$ must be done with equal numbers of positive and negative polarization bunches ($m = q$), or the systematic error would be considerably larger.

Thus the combined statistical and systematic uncertainties will limit the knowledge of beam polarization of individual bunches to $\pm 0.02 - 0.05$. It will also be important to monitor the AGS beam polarization during the filling of RHIC, in order to minimize $|\epsilon_P|$. Injecting alternate pulses with opposite beam polarization into successive bunches in RHIC would tend to cancel slow drifts in the polarized ion source or AGS operating conditions, and thus also minimize the polarization asymmetry.

In order to monitor the beam intensity, useful quantities are

$$\begin{aligned}
\sqrt{L_+^{(j)}R_+^{(j)}} &= \sqrt{Nd\Omega_L B_+^{(j)}(1+P_+^{(j)}A_L)}\cdot\sqrt{Nd\Omega_R B_+^{(j)}(1-P_+^{(j)}A_R)} \\
&\simeq B_+^{(j)}\cdot[N\sqrt{d\Omega_L d\Omega_R}]\cdot\{1+P_+^{(j)}A\epsilon_A - P_+^{(j)2}A^2/2+\mathcal{O}(\epsilon^4)\} \qquad (9) \\
\sqrt{L_-^{(k)}R_-^{(k)}} &\simeq B_-^{(k)}\cdot[N\sqrt{d\Omega_L d\Omega_R}]\cdot\{1-P_-^{(k)}A\epsilon_A - P_-^{(k)2}A^2/2+\mathcal{O}(\epsilon^4)\}.
\end{aligned}$$

The magnitude of $P_\pm^2 A^2/2$ is 0.0002 or 0.005, and of $P_\pm A\epsilon_A$ is ≤ 0.002 or 0.01, for the CNI or π-inclusive polarimeter, respectively. It would be difficult to obtain ϵ_A from these equations, since they also contain the beam polarization, which has its own systematic errors as noted above. Also, $P_\pm A\epsilon_A$ is a second order correction to $B_\pm$, and the variation in the beam intensity from bunch to bunch is likely to be sizeable. The relative statistical error on $\mathcal{I}_+ = \sqrt{L_+R_+}$ is the same as $\delta\alpha_1$, or $(0.7-2.3)\times 10^{-3}$. This is comparable to or smaller than the systematic error due to the term $PA\epsilon_A$ in Eqs. (9), so systematic effects may dominate the beam intensity measurement.

An alternate estimate of the relative beam intensity could be:

$$\begin{aligned}
\mathcal{I}'_+ &= L_+^{(j)} + R_+^{(j)} \\
&= B_+^{(j)}\cdot[2Nd\Omega]\cdot\{1+P_+^{(j)}A(\epsilon_A+\epsilon_\Omega)\},
\end{aligned}$$

and similarly for $\mathcal{I}'_- = L_-^{(k)} + R_-^{(k)}$. These expressions have comparable systematic effects to those of $\mathcal{I}_+$ and $\mathcal{I}_-$ in Eqs. (9), and thus could also be used for monitoring the beam intensity.

In conclusion, the individual bunch intensities may be affected by systematic errors of roughly the same magnitude as the statistical uncertainties. It will be shown in a following subsection that the same may be true for individual bunch polarizations.

3.2 Bunch Pair Analysis

As opposed to the methods in Sec. 3.1, here measurements from a + polarization and a – polarization bunch are combined using the so-called square root technique. Any pair of + and – bunches in a fill could be used. Alternately, scalers from many or all + bunches could be added together, and from all – bunches could be added together, and the sums could be used. In this subsection, three quantities will be defined [2, 3, 5] and evaluated in the framework of Eqs. (4). Then several possible methods to treat the data will be considered. Also, a connection to the single bunch analysis of Sec. 3.1 will be shown.

In the following expressions, the superscripts denoting the bunch numbers will be suppressed temporarily for clarity. Then the three useful asymmetries are:

$$\begin{aligned}
\alpha_9 &= \frac{\sqrt{L_+R_-} - \sqrt{L_-R_+}}{\sqrt{L_+R_-} + \sqrt{L_-R_+}} \simeq PA[1 + \mathcal{O}(\epsilon^3)] \\
\alpha_{10} &= \frac{\sqrt{L_+L_-} - \sqrt{R_+R_-}}{\sqrt{L_+L_-} + \sqrt{R_+R_-}} \simeq \epsilon_\Omega + PA\epsilon_P + \mathcal{O}(\epsilon^3) \qquad (10) \\
\alpha_{11} &= \frac{\sqrt{L_+R_+} - \sqrt{L_-R_-}}{\sqrt{L_+R_+} + \sqrt{L_-R_-}} \simeq \epsilon_B + PA\epsilon_A + \mathcal{O}(\epsilon^3).
\end{aligned}$$

For example, the average beam polarization times analyzing power, PA, is determined with small systematic error of order ϵ^4, unlike the case for individual bunch polarizations from Eqs. (6) or even (8). For example, the systematic error in Eqs. (8) is $\mathcal{O}(\epsilon^2)$, and an averaging over bunches is required to estimate ϵ_Ω. Recall that it is assumed that the beam phase space is constant from bunch to bunch. The statistical error on α_9 for two bunches is

$$\begin{aligned}
\delta\alpha_9 &= \frac{\sqrt{L_+L_-R_+R_-}}{(\sqrt{L_+R_-} + \sqrt{L_-R_+})^2} \left[\left(\frac{\delta L_+}{L_+}\right)^2 + \left(\frac{\delta L_-}{L_-}\right)^2 + \left(\frac{\delta R_+}{R_+}\right)^2 + \left(\frac{\delta R_-}{R_-}\right)^2 \right]^{\frac{1}{2}} \\
&\simeq (0.5 - 1.6) \times 10^{-3}.
\end{aligned}$$

This is just $\sqrt{2}$ smaller than $\delta\alpha_1$, as expected since counts from two bunches are included instead of only one bunch.

The distribution of $\alpha_9^{(jk)}$ for all positive polarization bunches j and negative bunches k in a fill can be computed. The width, $\sigma(\alpha_9^{(jk)})$, of this distribution will be the statistical uncertainty $\delta\alpha_9^{(jk)}$ in quadrature with the intrinsic width of the $PA = (P_+^{(j)} + P_-^{(k)})A/2$ distribution. Assuming A varies significantly less from bunch to bunch than $P_\pm$, and that the $P_+^{(j)}$ and $P_-^{(k)}$ distributions are the same and are independent, then the width $\sigma(PA)$ is just $A\,\sigma(P_\pm)/\sqrt{2}$. It is likely that $\sigma(P_\pm)$, and especially P, will vary from fill to fill due to changes in the source and accelerator operating conditions.

The expression for α_{10} has the same form as $\langle\alpha_1\rangle$ in Eq. (7). The distribution of $\alpha_{10}^{(jk)}$ for all positive and negative bunches in the same fill can also be computed.

The mean value, $\langle\alpha_{10}\rangle$, should be approximately ϵ_Ω if the $P_+^{(j)}$ and $P_-^{(k)}$ distributions are the same and if A is essentially constant. The asymmetry ϵ_Ω may also change from fill to fill. The width of the $\alpha_{10}^{(jk)}$ distribution, $\sigma(\alpha_{10}^{(jk)})$, will be the statistical uncertainty $\delta\alpha_{10}^{(jk)}$, the intrinsic width of ϵ_Ω or $\sigma(\epsilon_\Omega)$, and the intrinsic width of $PA\epsilon_P = (P_+^{(j)} - P_-^{(k)})A/2$, all in quadrature. The statistical uncertainty on α_{10} is

$$\delta\alpha_{10} = \frac{\sqrt{L_+L_-R_+R_-}}{(\sqrt{L_+L_-}+\sqrt{R_+R_-})^2}\left[\left(\frac{\delta L_+}{L_+}\right)^2+\left(\frac{\delta L_-}{L_-}\right)^2+\left(\frac{\delta R_+}{R_+}\right)^2+\left(\frac{\delta R_-}{R_-}\right)^2\right]^{\frac{1}{2}},$$

or nearly identical to $\delta\alpha_9$. Furthermore, if $P_+^{(j)}$ and $P_-^{(k)}$ are independent, then the widths of the $PA\epsilon_P$ and PA distributions should be the same! Under these conditions,

$$\sigma^2(\alpha_{10}^{(jk)}) - \sigma^2(\alpha_9^{(jk)}) \simeq \sigma^2(\epsilon_\Omega) \geq 0. \tag{11}$$

A study of this difference for many fills should clearly demonstrate either a variation of solid angle times efficiency from bunch to bunch if this variation is sizeable, or put a limit on such a variation. Strong evidence for a nonzero $\sigma(\epsilon_\Omega)$ would suggest differences in phase space of the bunches within a fill.

The quantity α_{11} gives a measure of the asymmetry in beam intensity in the two (sets of) bunches. A similar procedure could be attempted for the determination of $\alpha_{11}^{(jk)}$ for all negative and positive bunches. From Eq. (10), $\langle\alpha_{11}\rangle \simeq PA\epsilon_A$ assuming the bunch intensities $B_+^{(j)}$ and $B_-^{(k)}$ are independent and have the same distribution. However, for a single fill the uncertainty on ϵ_A will be sizeable, since the distribution of ϵ_B will likely be broad and since PA will be small. Thus, a good estimate of ϵ_A can only be obtained from an average over a number of fills assuming small changes in A_L, A_R, and beam phase space with time.

For RHIC spin measurements with both beams polarized, it will be desired to determine the average beam polarization for all bunches in one beam that have parallel spin directions to those of the other beam, and independently to those that have antiparallel directions. This would involve an average,

$$\begin{aligned}\langle\mathcal{P}\rangle A = \langle\mathcal{A}\rangle &= \frac{1}{m_1+q_1}\left(\sum_{j=1}^{m_1}\alpha_1^{(j)} - \sum_{k=1}^{q_1}\alpha_1^{(k)}\right) \\ &\simeq \frac{A}{m_1+q_1}\left(\sum_{j=1}^{m_1}P_+^{(j)} + \sum_{k=1}^{q_1}P_-^{(k)}\right) + \frac{m_1-q_1}{m_1+q_1}\epsilon_\Omega.\end{aligned} \tag{12}$$

The first term is the desired result. The equality of the number of bunches, $m_1 = q_1$ must hold or there would be a sizeable systematic error. For example, if $m_1 = 13$, $q_1 = 12$, and $\epsilon_\Omega = 0.1$, then the last term in Eq. (12) would produce a systematic error of 0.13 or 0.03 for the average polarization from the CNI or π-inclusive polarimeter, respectively. The equality of the number of bunches does not apply to the results in Eqs. (10). When $m_1 = q_1 = 12$ or 13, the statistical uncertainty on $\langle \mathcal{P} \rangle = \langle \mathcal{A} \rangle / A$ in Eq. (12) would be approximately $\pm(0.005 - 0.016)$ and $\pm(0.001 - 0.003)$ for the CNI and π-inclusive polarimeters.

3.3 Effects of Beam Motion

The derivation of Eqs. (7, 8, 10 - 12) assumed ϵ_Ω was constant within a fill. Bunch to bunch changes in phase space could introduce differences in ϵ_Ω that would increase the systematic error; this will be discussed further in Secs. 4,5. One type of phase space change would be to have the + bunches systematically to the left and the – bunches systematically to the right of the nominal beam position. As a consequence, the solid angle and number of events detected will be higher than expected for the left arm and lower than expected for the right arm during + bunches, and vice versa for – bunches, producing a false asymmetry.

For example, with the changes to Eqs. (4) described above, then α_9 in Eq. (10) becomes

$$\alpha_9 \simeq PA + \frac{1}{2}(\epsilon_{\Omega L} - \epsilon_{\Omega R}) + \mathcal{O}(\epsilon^3), \tag{13}$$

where

$$\begin{aligned}
d\Omega_L^{(j)} &= d\Omega_L(1 + \epsilon_{\Omega L}) \\
d\Omega_L^{(k)} &= d\Omega_L(1 - \epsilon_{\Omega L}) \\
A_L^{(j)} &= A_L(1 + \epsilon_{AL}) \\
A_L^{(k)} &= A_L(1 - \epsilon_{AL}) \\
d\Omega_R^{(j)} &= d\Omega_R(1 + \epsilon_{\Omega R}) \\
&etc.
\end{aligned}$$

and $A = (A_L + A_R)/2$ is now an average over bunches as well as the left and right arms. The corrections to α_9 involving $\epsilon_{\Omega L}$ and $\epsilon_{\Omega R}$ are the same order as PA! Similarly, $\alpha_1^{(j)}$ in Eq. (6) becomes

$$\begin{aligned}
\alpha_1^{(j)} \;\; \simeq \;\; & P_+^{(j)} \, A \; + \; \epsilon_\Omega \; + \; \frac{1}{2}(\epsilon_{\Omega L} - \epsilon_{\Omega R}) \; + \; \left[\frac{1}{2} \, P_+^{(j)} \, A \; (\epsilon_{AL} + \epsilon_{AR}) \right. \\
& \left. - \; \frac{1}{4}(\epsilon_{\Omega L}^2 - \epsilon_{\Omega R}^2) \; + \; \mathcal{O}(\epsilon^3) \right].
\end{aligned}$$

Averaging over a number of bunches will not necessarily remove the third term involving $\epsilon_{\Omega L}$ and $\epsilon_{\Omega R}$. Hence, this correction term must be kept small compared to the statistical precision on α_1 or α_9 to avoid sizeable systematic errors.

As a numerical example, if the + polarization bunches are systematically δx to the left and the – polarization bunches are systematically δx to the right of the nominal beam position, and if the distance to the detector that defines the solid angle in the polarimeter is r, then

$$\epsilon_{\Omega L} \simeq -\epsilon_{\Omega R} \simeq 2\delta x \sin\theta_L / r,$$

where θ_L is the laboratory angle of the polarimeter arm detectors. In order to keep systematic errors small, $2\delta x \sin\theta_L/r \ll \delta\alpha_1 \sim 1.5 \times 10^{-3}$. For the CNI polarimeter, $\sin\theta_L \simeq 1$. If $r = 15$ cm, then $\delta x \ll 0.1$ mm would be required for the variation of beam position at the polarimeter. Note these systematic errors occur even for a 2 arm polarimeter with matched analyzing powers and solid angles times efficiencies, and for equations for either single bunches or for the square root asymmetry, α_9.

A narrow polarimeter target may limit the effects of such changes to bunch positions, provided its location is stable. If this target vibrates, then it may produce a similar systematic error unless the period is long compared to (60 bunches) $\times$ (220 nsec) and/or the amplitude of the vibration is $\ll 0.1$ mm for the CNI polarimeter. A $\pi-$inclusive polarimeter would not be very sensitive to this particular systematic error because $\sin\theta_L$ would be small and r would be very much larger than 15 cm.

Finally, a polarized gas jet target running with the CNI polarimeter for an absolute beam polarization calibration would be sensitive to such beam motion correlated with polarization sign. If a fit is needed to determine both PA and $(\epsilon_{\Omega L} - \epsilon_{\Omega R})$ in Eq. (13) for such a calibration, this would probably increase the uncertainty on PA over the case without these systematic effects.

4 Luminosities and Relative Beam Intensities

For measurements of spin observables with the RHIC detectors, it will be important to monitor the luminosities. The number of good events detected must be normalized by the luminosity for each pair of colliding bunches in order to calculate the spin observables. Alternately, the events can be summed for each of the four combinations of beam spins $(+\,+, +\,-, -\,+, -\,-)$, and these sums normalized by the integrated luminosity in that combination. For the case of longitudinally polarized beams, one spin observable of interest is A_{LL},

$$A_{LL} \simeq \frac{1}{P_a P_b} \cdot \frac{n_{++} + n_{--} - n_{+-} - n_{-+}}{n_{++} + n_{--} + n_{+-} + n_{-+}},$$

where the two average beam polarizations are P_a and P_b, and the normalized numbers of good events are n_{ij}.

If the phase space of all bunches is the same in the clockwise circulating beam "a", and they remain equal during the fill, changing together with time, and if the same is true for the anti-clockwise beam "b", then the luminosities will be proportional to the product of relative beam intensities,

$$\mathcal{L}_{[l,m]} \propto \mathcal{I}_{a,l}\ \mathcal{I}_{b,m}.$$

The bunches will be labeled l and m, respectively. The constant of proportionality will depend on the sizes and profiles of the bunches and other factors. Each RHIC detector "d" will use some monitor of the luminosity at its intersection region, and the polarimeters will give the relative intensities, $\mathcal{I}$, from Eqs. (9). Then

$$\mathcal{L}^{(d)}_{[l,m]} = C_d\ \mathcal{I}_{a,l}\ \mathcal{I}_{b,m}. \tag{14}$$

One test for the equality of the phase space for each bunch in a beam will be the variation in C_d for each pair $[l, m]$. If C_d is found to vary outside statistics, then one possible explanation would be phase space differences from bunch to bunch. Such a test could be performed for each detector, for each fill, or for each time a polarization measurement occurs; such tests would generally yield different values of C_d.

Additional equations apply when two or more RHIC detectors are operated simultaneously. If the two detectors are 180° apart on the RHIC ring, then they will have the same bunch pairs $[l, m]$ colliding, and

$$\mathcal{L}^{(d)}_{[l,m]}/\mathcal{L}^{(d')}_{[l,m]} = \mathcal{R}_{dd'} = C_d/C_{d'} \tag{15}$$

should apply. Note $\mathcal{R}_{dd'}$ may change if there are changes in RHIC operating conditions. Variations in $\mathcal{R}_{dd'}$ outside statistics among the 60 bunch pairs that collide at detectors d and d' would indicate systematic errors in at least one of the luminosity measurements, or else phase space changes from bunch to bunch.

If the two detectors are 60° or 120° apart on the RHIC ring, different pairs of bunches will collide. Assigning numbers to the bunches from $1 - 60$ consecutively as shown in Fig. 1, starting at the polarimeters, then the bunch pairs for the polarimeters at 12 o'clock and the detector (STAR) at 6 o'clock will have the form $[i, i]$. Similarly, for the 2 (BRAHMS, PP2PP) and 8 (PHENIX) o'clock interaction regions they will be $[i, i + 20]$ or $[i, i - 40]$, and for the 4 and 10 (PHOBOS) o'clock locations they will be $[i, i + 40]$ or $[i, i - 20]$.

Consider the same assumptions as those for Eq. (14) at the two large RHIC detectors. STAR at 6 o'clock will measure luminosities

$$\mathcal{L}^{(st)}_{[i,i]} = C_{st}\ \mathcal{I}_{a,i}\ \mathcal{I}_{b,i} \quad (i = 1 - 60),$$

and PHENIX at 8 o'clock will measure

$$\mathcal{L}^{(ph)}_{[i,i+20]} = C_{ph}\ \mathcal{I}_{a,i}\ \mathcal{I}_{b,i+20} \quad (i = 1 - 40)$$
$$\mathcal{L}^{(ph)}_{[i,i-40]} = C_{ph}\ \mathcal{I}_{a,i}\ \mathcal{I}_{b,i-40} \quad (i = 41 - 60).$$

As a consequence of these relations, the following equations apply:

$$\begin{aligned} \frac{C_{st}}{C_{ph}} = \mathcal{R}_{st,ph} &= \frac{\mathcal{L}^{(st)}_{[i,i]}}{\mathcal{L}^{(ph)}_{[i,i+20]}} \frac{\mathcal{I}_{b,i+20}}{\mathcal{I}_{b,i}}, \\ &= \frac{\mathcal{L}^{(st)}_{[i+20,i+20]}}{\mathcal{L}^{(ph)}_{[i,i+20]}} \frac{\mathcal{I}_{a,i}}{\mathcal{I}_{a,i+20}}, \end{aligned} \tag{16}$$

etc.. Eqs. (16) are mixtures of results from the polarimeters and two detectors separated by 60° or 120° around RHIC. The statistical errors will probably be dominated by the relative intensities from the polarimeters, because of the shorter measurement periods. Also, the polarimeter bunch intensities may have significant systematic errors; see Sec. 3.1.

Another set of relations can be derived that do not involve the polarimeter data,

$$\mathcal{L}^{(st)}_{[i,i]}\ \mathcal{L}^{(st)}_{[i+20,i+20]}\ \mathcal{L}^{(st)}_{[i+40,i+40]} = \mathcal{K}\ \mathcal{L}^{(ph)}_{[i,i+20]}\ \mathcal{L}^{(ph)}_{[i+20,i+40]}\ \mathcal{L}^{(ph)}_{[i+40,i]} \tag{17}$$

for $i = 1 - 20$. The constant $\mathcal{K}$ is just

$$\mathcal{K} = (C_{st}/C_{ph})^3 = \mathcal{R}^3_{st,ph},$$

which should be determined accurately from a comparison of the average luminosities at the two detectors. Eq. (17) should provide a stringent test on the bunch to bunch phase space variations, assuming the luminosity monitors for the two detectors have small systematic errors. Of course, the Eqs. (15 - 17) should apply to any pair of RHIC detectors, and to unpolarized beam as well as polarized proton bunches. They should also apply to multiple measurements at different times within a fill, though the constants $\mathcal{R}_{d,d'}$ may differ with time.

5 Some Complications

It has been noted numerous times in this paper that deviations in phase space from bunch to bunch could cause systematic errors in the measurement of the polarization or intensity for each bunch. These deviations could be the result of variations in the beam properties upon injection into RHIC, for example due to differences in the ion source or AGS operation. This section describes another possible cause for such deviations.

In order to minimize systematic errors due to drifts in the collider detector efficiencies, it will be desirable to have frequent changes of beam polarization directions for the two beams. For example, the following pattern would be satisfactory:

$$\begin{array}{lllll} a & \rightarrow & +-+- & +-+- & +-+- & \ldots \\ b & \rightarrow & ++-- & ++-- & ++-- & \ldots . \end{array}$$

This would give 15 bunch crossings each for the four beam spin combinations (+ +, + −, − +, − −) for the a and b beams, respectively.

Backgrounds at the collider detectors from sources other than interactions in the bunch crossings are often studied by inserting empty bunches in each beam. For instance, if the bunches in each beam were arranged in three successive groups of

20, each with the following patterns,

$$\begin{array}{lllllll} a & \rightarrow & +-+- & +-+0 & 0-+- & +-+- & +-+- \\ b & \rightarrow & ++-- & ++-- & ++-- & ++0- & +0-- \end{array} ,$$

then there would be 12 bunch crossings each for the spin combinations (+ +, + −, − +, − −), and three each for (0 +, 0 −, + 0, − 0). Having the bunches in three identical groups of 20 guarantees the same pattern of bunch pair spin combinations at each RHIC detector.

If it is necessary to have three or four empty bunches in a row for injection of beams into RHIC, then one of the above groups of 20 in each beam could be replaced by

$$\begin{array}{lllllll} a & \rightarrow & +-+- & +-+0 & 000- & +-+- & +-+- \\ b & \rightarrow & ++-- & ++-- & ++-- & ++00 & 00-- \end{array} .$$

In this case there would be 11 bunch crossings each for the spin combinations (+ +, + −, − +, − −), and four each for (0 +, 0 −, + 0, − 0). For further discussion of possible options for empty bunches, and examples where a pair of empty bunches "collide," see Ref. [6].

The potential problem with the presence of empty bunches is that the bunches that "collide" with them at one of the intersection regions would suffer less interactions than bunches that only collide with + or − polarization bunches. Hence, it would not be surprising if the phase space of these two classes of bunches could evolve to be different, even if they began the same. Assuming some variation of the three groups of 20 is used at RHIC, the only bunches that could have problems for polarization measurements would be those that collided with the special ones added so as to have three empty bunches in a row. Any bunches that always collided with empty bunches could be ignored. Additional comments on these possible systematic errors can be found in Ref. [7]. The extension of the three groups of 20 bunches to six groups would cover the situation where RHIC was run with a total of 120 bunches per beam. Some empty bunches might be omitted to increase the fraction of collisions between non-empty bunches in this case.

6 Summary and Conclusions

This paper discusses the expected statistical errors from RHIC polarimeters for monitoring beam intensity and polarization of bunches in the two colliding RHIC beams. A detailed analysis in Secs. 2,3 indicates an uncertainty of $\delta P \sim \pm 0.05$ on the beam polarization and $\delta B/B \sim \pm 0.0015$ on the beam intensity for a single bunch can be achieved for the CNI polarimeter. The same statistical errors for the π-inclusive polarimeter are $\delta P \sim \pm 0.01$ and $\delta B/B \sim \pm 0.0015$. These estimates are very approximate because detailed polarimeter designs are not yet finalized, so the actual uncertainties may easily differ by a factor of two. Also, during early running of the RHIC polarized beams these uncertainties will probably be much larger due to lower beam intensities.

In addition to the statistical errors, there may be systematic effects that could bias these results. Most of this paper deals with estimates of these systematic effects and with methods to search for them or possibly to correct the measurements. One example is the expected differences in bunch intensity and polarization due to changes in the ion source and accelerator (linac, booster, AGS, transfer lines) operating conditions during the time to fill RHIC. These variations may lead to differences in the phase space from bunch to bunch, and thus to differences in effective solid angle in the polarimeter arms; changes in the transverse beam size or position would be most important. Slow drifts in polarimeter detector efficiencies may affect the solid angles or analyzing powers, especially if the efficiency changes with scattering angle. Some of these effects may be negligible for bunch to bunch variations within a fill, since the polarimeter measurement time is short, but may cause changes from fill to fill.

Conclusions from the studies of systematic errors include:

- The systematic error on the polarization of a bunch was estimated to be $\simeq$ 0.007 and $\simeq$ 0.0014 for the CNI and π-inclusive polarimeter, respectively. These are considerably smaller than statistical uncertainties, and arise from imperfect knowledge of the asymmetry in left and right arm solid angle times efficiency, ϵ_Ω. These estimates assume the polarizations are derived from Eqs. (6, 8) and that the average analyzing power is known. In practice, the average polarization of + and − bunches can be obtained with smaller systematic effects via Eq. (10). A larger systematic error could occur if the effective solid angle changes from bunch to bunch by more than roughly $\pm 0.1\%$.
- The systematic error on the relative intensity of a bunch derived from Eqs. (9) are estimated to be $\delta B/B \leq 0.002$ or 0.01 for the CNI or π-inclusive polarimeter data, respectively. These would be caused by possible differences in analyzing powers for the two polarimeter arms, and could be larger than statistical uncertainties.
- An estimate of the variation of beam polarization from bunch to bunch, $\sigma(P_\pm)$, and hence of the asymmetry of polarizations for a pair of bunches, ϵ_P, could be found from the width of the distribution of $\alpha_9^{(jk)}$ (see Eq. (10)) for all pairs of + and − bunches in a fill. This is described further in Sec. 3.2, and assumes the distributions of $P_+^{(j)}$ and $P_-^{(k)}$ are the same.
- An estimate of the variation of ϵ_Ω from bunch to bunch, $\sigma(\epsilon_\Omega)$, could be obtained from the widths of the $\alpha_9^{(jk)}$ and $\alpha_{10}^{(jk)}$ distributions; see Eqs. (10, 11). A nonzero value for $\sigma(\epsilon_\Omega)$ would probably indicate differences in the phase space from bunch to bunch.
- It will be difficult to obtain the estimate of the asymmetry in left and right arm analyzing powers, ϵ_A. This quantity appears in small corrections to large asymmetries. For example, $\alpha_{11} \simeq \epsilon_B + PA\epsilon_A$ from Eq. (10), and large changes

in beam intensity from bunch to bunch are expected, leading to large ϵ_B. In addition, PA is small for either the CNI or π-inclusive polarimeter.

- A number of tests for differences in phase space of bunches in a fill are possible using relative intensities measured by the polarimeters (Eqs. (9)) and the luminosities measured in one or more collider detectors. These tests include comparisons of results from the polarimeters and one detector in Eq. (14) or two detectors in Eqs. (15,16). A comparison of luminosities from two different detectors separated by 60° or 120° at RHIC (Eq. (17)) may provide the most stringent tests.

Many of these systematic errors arise from variations in $d\Omega_L$ and $d\Omega_R$ from bunch to bunch. Thus, to avoid these errors, even for reasonably matched left and right polarimeter arms, the efficiency of the polarimeter detectors and electronics must be kept stable to better than about 0.1% over time periods that are long compared to 60×220 nsec. Similarly, the bunch to bunch differences in phase space and target position changes must be minimized. This presumably places constraints on operation of the AGS, booster, and polarized ion source as well as RHIC. Some tests with unpolarized beam may be helpful to search for such systematic errors before polarized beam running begins.

Finally, it must be remembered that in typical fixed target polarization experiments at the AGS (and elsewhere), there were 60 beam spills with alternating polarization sign in $\sim 3-4$ min. At RHIC, there will be the same 60 bunches used repeatedly for ***hours***. The "averaging" over systematics effects such as beam position changes, etc., that occurred at the AGS will thus be much slower at RHIC, leading to possibly larger fluctuations in measured beam polarizations than expected on the basis of fixed target experience.

References

[1] R.C. Hanna, in Proceedings of the Second International Symposium on Polarization Phenomena of Nucleons, Karslruhe, 1965, eds. P. Huber and H. Schopper, p280.

[2] H. Spinka, Argonne National Laboratory Report ANL-HEP-PR-80-02 (1980).

[3] H. Spinka, E. Colton, W.R. Ditzler, H. Halpern, K. Imai, R. Stanek, N. Tamura, G. Theodosiou, K. Toshioka, D. Underwood, R. Wagner, Y. Watanabe, A. Yokosawa, G.R. Burleson, W.B. Cottingame, S.J. Greene, S. Stuart, and J.J. Jarmer, Nucl. Instrum. Meth. **211**, 239 (1983).

[4] W. Haeberli, R. Henneck, Ch. Jacquemart, J. Lang, R. Müller, M. Simonius, W. Reichart, and Ch. Weddigen, Nucl. Instrum. Meth. **163**, 403 (1979).

[5] C.E. Allgower, J. Ball, L.S. Barabash, P.-Y. Beauvais, M.E. Beddo, Y. Bedfer, N. Borisov, A. Boutefnouchet, J. Bystrický, P.-A. Chamouard, M. Combet, Ph. Demierre, J.-M. Fontaine, V. Ghazikhanian, D.P. Grosnick, R. Hess,

Z. Janout, Z.F. Janout, V.A. Kalinnikov, T.E. Kasprzyk, Yu.M. Kazarinov, B.A. Khachaturov, R. Kunne, J.M. Lagniel, F. Lehar, J.L. Lemaire, A. de Lesquen, D. Lopiano, M. de Mali, V.N. Matafonov, G. Milleret, I.L. Pisarev, A.A. Popov, A.N. Prokofiev, D. Rapin, J.-L. Sans, H.M. Spinka, Yu.A. Usov, V.V. Vikhrov, B. Vuaridel, C.A. Whitten, and A.A. Zhdanov, accepted for publication in Phys. Rev. **C**, and Argonne National Laboratory report ANL-HEP-PR-99-66.

[6] D. Underwood, "Note on Polarized RHIC Bunch Arrangement," RHIC Spin Note No. 35, (1996) and Argonne technical report ANL-HEP-TR-96-73.

[7] D. Underwood, "Luminosity Monitor Topics for RHIC Spin and AA and pA Interactions," RHIC Spin Note No. 71, (1997) and STAR Note No. 320.

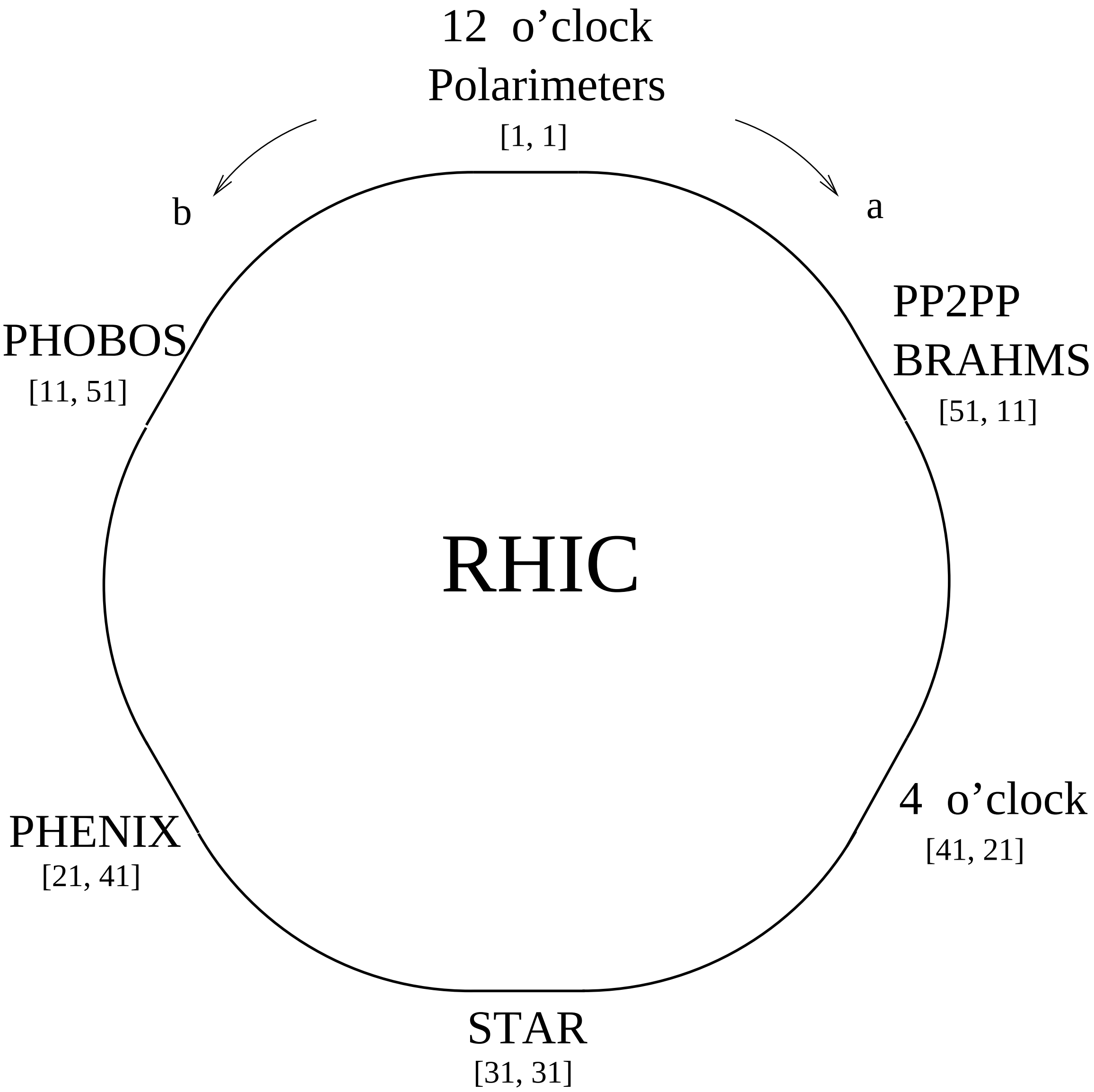

Figure 1: Schematic layout of the RHIC accelerator showing the location of the major detectors and the polarimeters. The labels for the colliding bunches at each intersection region are given by $[l, m]$, where bunch l occurs in beam a (clockwise), and bunch m in beam b (counterclockwise).