STAR-Note # NNNN
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Determining the Lifetime of Charged Kaons from Kinks in STAR

1 Introduction

The determination of the proper mean lifetime for charged Kaons in STAR is complicated by both limited lifetime acceptance and momentum-dependent cuts. A number of more elegant mathematical approaches were investigated to compensate for these effects, but in the end it was seen that the simplest and most reliable approach is a thorough analysis of simulated tracks embedded in a real event. This approach should be effective in any collider experiment.

2 Lifetime Acceptance in STAR

The proper lifetime of a particle is simply related to its lifetime, t_L, and velocity, v, in the laboratory by

$$t_0 = \frac{t_L}{\gamma}, (\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}, \beta \equiv \frac{v}{c}). \tag{1}$$

The time in the laboratory is related to the distance traversed by the particle by $t_L = s/v = s/(\beta c)$, so the proper lifetime can be expressed in terms of the distance traversed in the laboratory and the particle's mass, m, and momentum, $p = mv\gamma$, as:

$$ct_0 = \frac{s}{\beta\gamma} = s\frac{mc^2}{pc}. \tag{2}$$

The mean proper lifetime of the charged Kaon is $c\tau = 371.3$ cm. The fiducial volume in STAR that is searched for kink decays is 133 cm < r < 179 cm. Given these radial limits, it is useful to find a simple expression for the path length traversed between two points by a helix representing a particle with charge q, total momentum p, and transverse momentum $p_T \equiv |\overrightarrow{p} \times \hat{z}|$ given a magnetic field $\overrightarrow{B} = B\hat{z}$. The *dip angle* of the helix is $\lambda = cos^{-1}(\frac{p_T}{p})$.

As a practical matter, STAR tracking does not find tracks which cover more than half the period of the helix. This is because track finding is done from the outside of the TPC in, and so all tracks are monotonically increasing in the radial direction. This greatly simplifies the task of finding path length along a track; we need only know their separation in the transverse plane ($2A$ in Figure 1). It is easy to see that

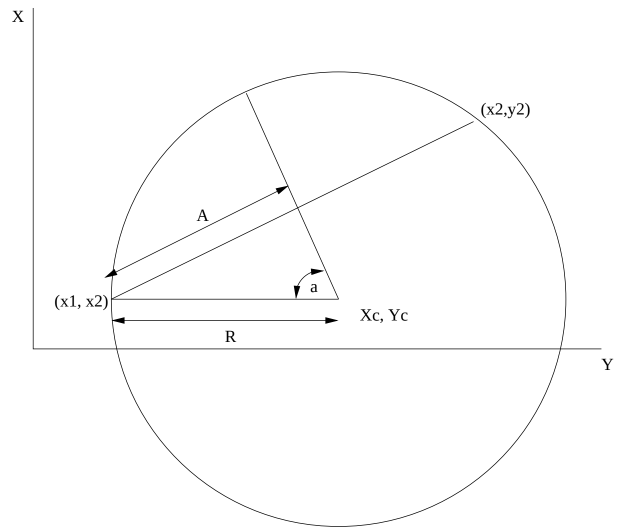

Figure 1: Transverse projection of helix

the azimuthal angle between the points $(x1, y1)$ and $(x2, y2)$ relative to the helix axis is $a = sin^{-1}(\frac{A}{R})$ and so the path length in the transverse plane is $s_t = 2Rsin^{-1}(\frac{A}{R})$. $R = p_T/qB$ here is the radius of curvature of the helix. The full path length then is just

$$s = \frac{s_t}{cos(\lambda)} = 2\frac{p_T}{qB}sin^{-1}(A\frac{qB}{p_T})\frac{p}{p_T} = \frac{2p}{qB}sin^{-1}(A\frac{qB}{p_T}) \tag{3}$$

This translates readily into lifetime:

$$ct = \frac{mc^2}{pc}\frac{2p}{qB}sin^{-1}(A\frac{qB}{p_T}) = \frac{2mc}{qB}sin^{-1}(A\frac{qB}{p_T}) \tag{4}$$

Note: This formula assumes nonzero p_T. The TPC doesn't see tracks with $p_T = 0$, so this is fine.

Full magnetic field strength in STAR is $B = 0.5$ T $= 5$ kG $= 5\text{x}10^{-14}\ \frac{\text{GeV s}}{\text{cm}^2} = 0.0015\ \frac{\text{GeV/c}}{\text{cm}}$. Thus a mid-rapidity charged Kaon in STAR that originates at the beam line and has mean p_T ($< p_T > \approx$ 600MeV) will enter and leave the fiducial volume after traveling 133.6 cm and 180.5 cm, respectively.

Since the charged Kaon mass is $m = 493.7\ MeV/c^2$, these correspond to proper lifetimes, ct_0, of 109.9 cm and 148.5 cm, or 0.3 and 0.4 of the mean proper $c\tau$ for charged Kaons. One can see in Figure 2 how limited this range of 0.1 $c\tau$ is. We shall see later on how this renders ineffective some techniques for finding the lifetime.

3 Fitting with Maximum Likelihood

This is a summary and application of the Maximum Likelihood fitting method described in Reference [1]. Note that the example in the book deals with V_0 particles

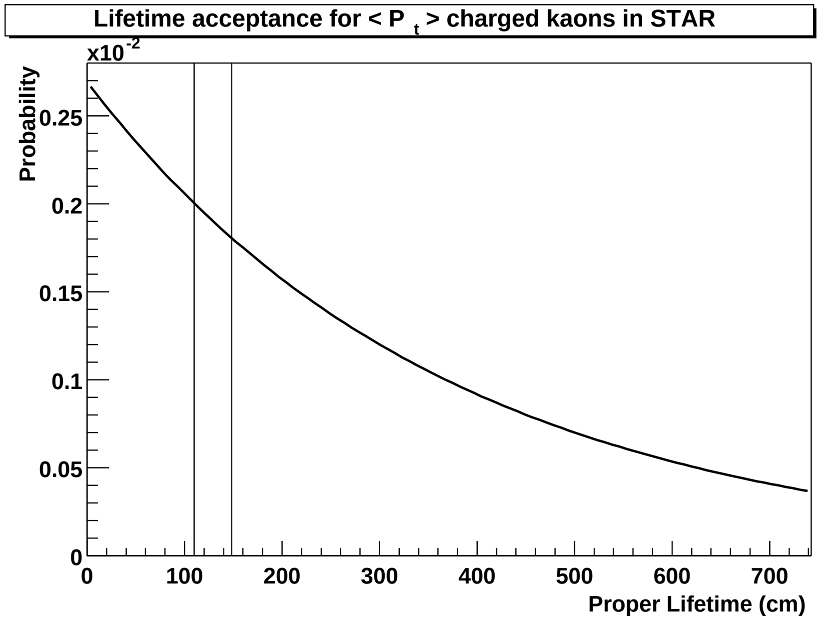

Figure 2: STAR lifetime acceptance for $< p_T >$ charged Kaons

in a fixed target experiment, where one typically has shorter lifetimes and larger acceptances. First we look at the probability of one *event* , an observed particle in the detector, has a lifetime t_i. (*Proper* lifetime will be implied for the rest of the discussion.) We factor the probability in terms of observation probability and lifetime probability. The probability of observing a particle with mean lifetime τ, momentum $\overrightarrow{p_i}$, and position $\overrightarrow{r_i}$ multiplied by the conditional probability that the observed particle has a given lifetime is equal to the probability that the particle has a given lifetime (whether observed or not):

$$P(observation)P(observation \mid lifetime\ t_i) = P(lifetime\ t_i) \tag{5}$$

$$\Rightarrow P(observation \mid lifetime\ t_i) = \frac{1}{P(observation)} P(lifetime\ t_i) \tag{6}$$

$$\Rightarrow P_i = A_i \frac{e^{-\frac{t_i}{\tau}}}{\tau}, \tag{7}$$

where A_i is a factor representing the detector efficiency. In this case A_i^{-1} is the probability that a particle having mean lifetime τ, momentum $\overrightarrow{p_i}$, and position $\overrightarrow{r_i}$ will be observed to decay in the detector. $e^{-\frac{t_i}{\tau}}/\tau$ is the probability that a particle having mean lifetime τ will live time t_i.

Since an event *did* occur (we observed a particle with properties τ, $\overrightarrow{p_i}$, $\overrightarrow{r_i}$), we can say that the integral of P_i over all observable lifetimes for a given τ, $\overrightarrow{p_i}$, and $\overrightarrow{r_i}$ must be unity. This allows us to determine A_i:

$$1 = \int_{t_i^{min}}^{t_i^{max}} P_i dt \tag{8}$$

$$\Rightarrow A_i^{-1} = (e^{-\frac{t_i^{min}}{\tau}} - e^{-\frac{t_i^{max}}{\tau}}). \tag{9}$$

The upper and lower limits on the integral are functions of $\overrightarrow{p_i}$, $\overrightarrow{r_i}$, and the detector *fiducial volume* , the region where particles may be detected. Let us examine the specific example of charged Kaon decays in STAR. Here, the fiducial volume, as mentioned above, is $133\,cm < r < 179\,cm$. That means that we can express the integral limits using Equation 4 as

$$ct_i^{min} = \frac{2mc}{qB} sin^{-1}(\frac{r^{min}}{2}\frac{qB}{p_T^i}) \tag{10}$$

$$ct_i^{max} = \frac{2mc}{qB} sin^{-1}(\frac{r^{max}}{2}\frac{qB}{p_T^i}) \tag{11}$$

Now that the individual decay probabilities have been determined, we can look at the *joint probability* associated with a set of events. This is just the probability of N events occurring concurrently and is called the the *likelihood function*:

$$L \equiv \prod_{i=1}^{N} P_i = \tau^{-N} e^{\frac{-1}{\tau}\sum_{i=1}^{N} t_i} \prod_{i=1}^{N} A_i \tag{12}$$

As a practical matter, this number will be very small for large N, and on a computer it is more useful to consider the logarithm of the likelihood function:

$$M \equiv ln(L) = -N ln(\tau) - \frac{1}{\tau}\sum_{i=1}^{N} t_i + \sum_{i=1}^{N} A_i \tag{13}$$

The likelihood function can be used to determine any parameter in the probability distributions. For our purposes, we wish to find which value of the mean lifetime τ maximizes L (or M). We know that the observed events *did* occur and so the τ' which maximizes M is the most probable value. Furthermore, for a large number of events L is Gaussian around the most probable value of a parameter,which allows one to extract the uncertainty in the parameter as well:

$$L(\tau) \propto e^{-\frac{(\tau-\tau')^2}{2\sigma^2}} \tag{14}$$

$$M(\tau) = -\frac{(\tau-\tau')^2}{2\sigma^2} + constant \tag{15}$$

The general procedure when looking at experimental data is as follows:

1. Select a set $\{\tau_j\}$ of trial mean lifetimes in the vicinity of the hypothesized value of τ.
2. For each τ_j, calculate $M(\tau_j)$ as shown in Equation 15.
3. Take the exponential of M to get L and fit L to a Gaussian near its peak. This will yield the most likely value, τ', for the mean lifetime and the uncertainty in that value.

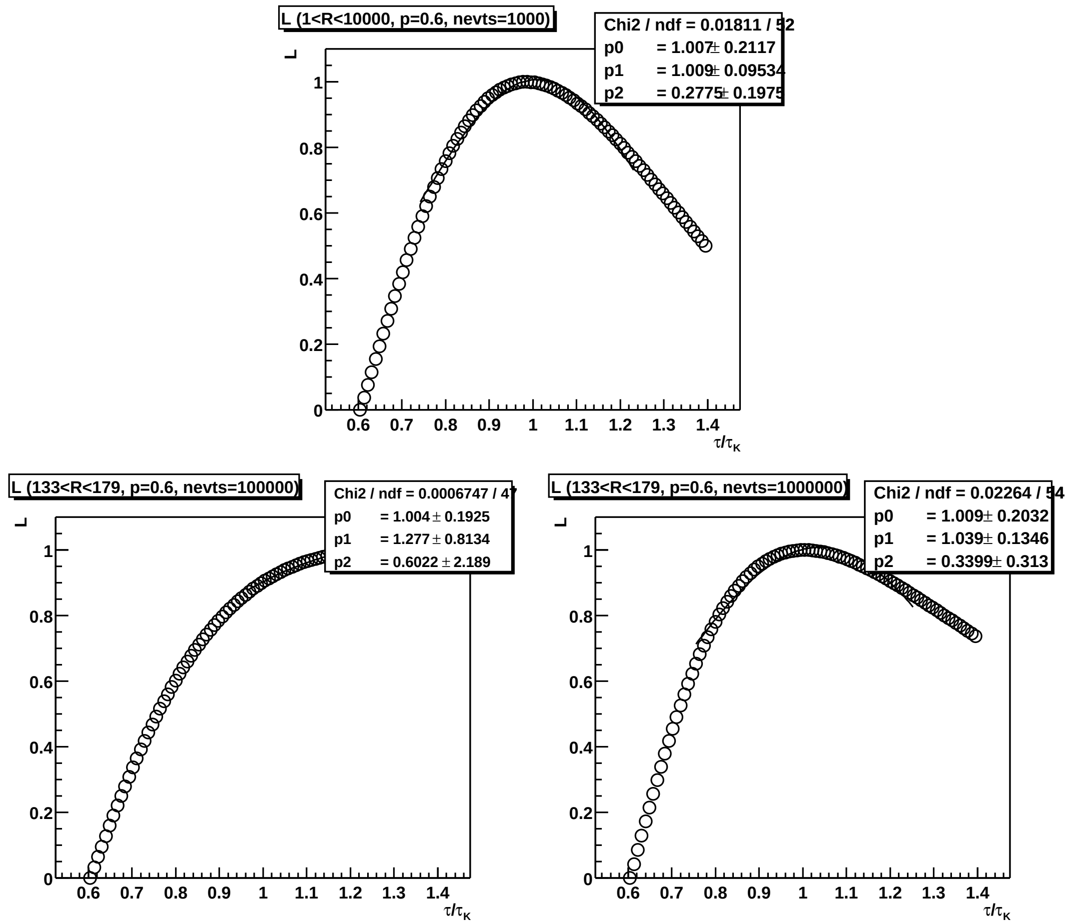

Figure 3: Gaussian fits to likelihood function (p1=center, p2=width)

In Figure 3, we show the results of the method applied to simple one dimensional Monte Carlo data which was thrown with flat momentum and a mean lifetime of τ_K. In many situations, this give excellent results. If the range of experimentally observed lifetimes is large relative to the mean lifetime, the correct mean lifetime can be extracted from a small number (1000) of events. This can be seen in the topmost figure. For very narrow lifetime acceptance, however, as many as a million events are needed for the method to converge. One can from the bottom figures see the that the correct lifetime is not reproduced for even 100 thousand events.

There is even more to the story, however, as the method described above assumes perfect efficiency within the detector fiducial volume. This is not the case in STAR because 1) not all decays in the fiducial are reconstructed and 2) momentum dependent cuts are applied in order to separate Kaon signal from correlated and combinatorial background. The second factor in particular biases the observed lifetime distribution a great deal because momentum dependence translates directly into lifetime dependence for a fixed acceptance.

4 Fitting with Virtual Particles

This method attempts to restore the signal which is lost outside of the fiducial volume. It proceeds as follows:

1. For each event (detected Kaon), calculate the lifetime acceptance based on its momentum and the coordinate-space acceptance. In STAR, for example, this involves finding the upper t_{max} and lower t_{min} lifetime limits based on the outer and inner radii of the fiducial volume using Equation 4.

2. Using simple Monte Carlo, generate two virtual particles using the ideal lifetime distribution (Equation 7) and the accepted value for the charged Kaon mean lifetime. The first particle should have lifetime $0 < t_{under} < t_{min}$ and the second should have lifetime $t_{max} < t_{over} < \infty$. These particles will represent particles which have fallen outside the detector acceptance

3. Weigh the three particles by the integral of the probability distribution in each region:

$$w = \int_{t_{min}}^{t_{max}} \rho(t)dt \tag{16}$$

$$w_{under} = \int_{0}^{t_{min}} \rho(t)dt \tag{17}$$

$$w_{over} = \int_{t_{max}}^{\infty} \rho(t)dt \tag{18}$$

4. Fill the lifetime histogram (dN/dt) with the lifetimes for the real event and the two virtual particles, weighted as above. The weights above ensure that the virtual particles will fall into the ideal lifetime distribution relative to the weight of the ideal particles. Fitting this histogram will extract the mean lifetime.

This method reproduces the ideal lifetime distribution for ideal Monte Carlo events, but it faces certain complications for real data. Firstly, it shares the problem of efficiency with the Maximum Likelihood method above because it assumes 100% reconstruction of particles in the fiducial volume. Secondly, it is seriously affected by the limited lifetime acceptance of STAR. The virtual particles which are generated for each real particle have weights much greater than the weight of the real particle. That means that the real signal is overpowered by the virtual particles, and the hypothesized mean lifetime is always reproduced, regardless of the lifetime of the real particles.

This situation can be seen in Figure 4, which shows the results of applying this method to simple Monte Carlo data in the STAR fiducial volume. Particles with monochromatic momentum were thrown according to an input mean lifetime. Acceptance cuts were then applied to coincide with the fiducial. This data is labeled “uncorrected” in the histograms. The data labeled “corrected” includes both the

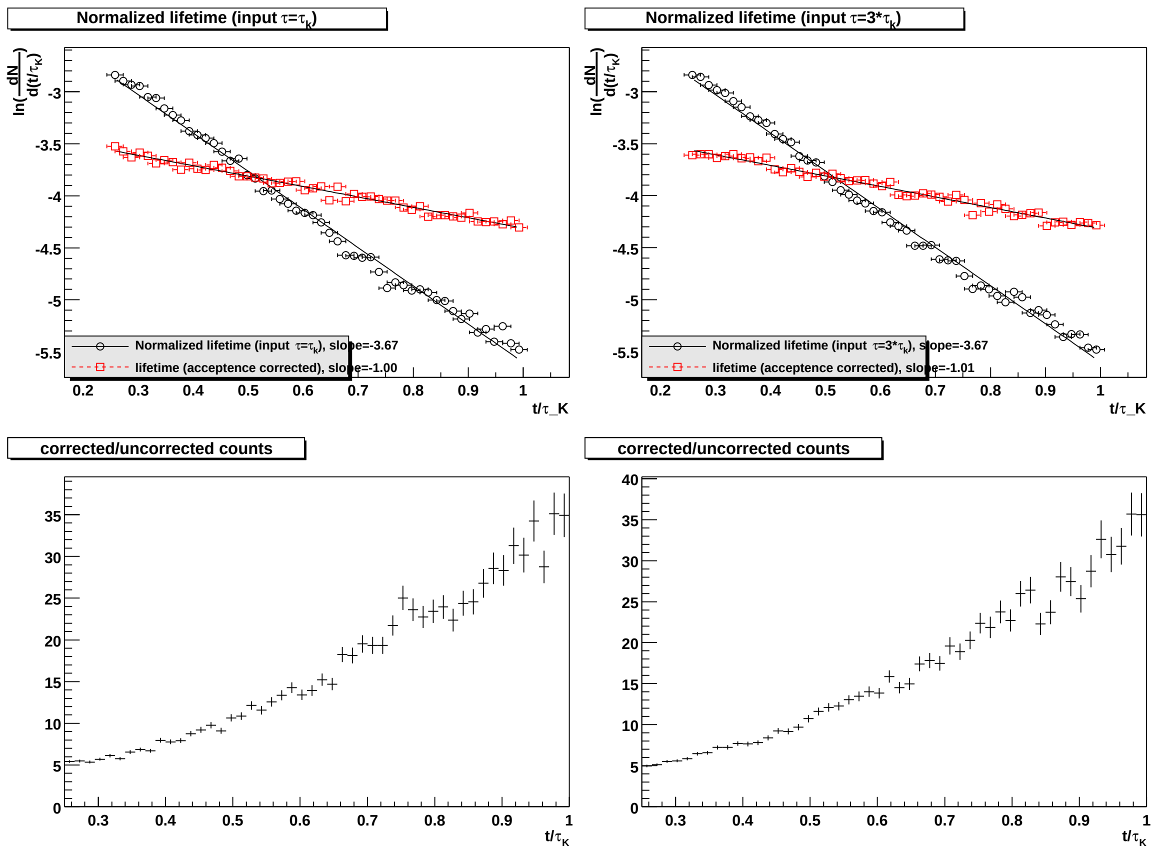

Figure 4: Results of Fitting with Virtual Particles

input particles and virtual particles thrown according to the hypothesized mean lifetime. The histograms on the right show the result of an input mean lifetime τ_K equal to the hypothesized mean lifetime. Those on the left show the same exercise with an input mean lifetime of 3 times that of the hypothesized lifetime. In both cases, the lower histograms show that the sum of the virtual and input particles is much greater than the input particles alone. This is seen clearly in the upper figures, which show that in both cases, the "corrected" data follows the hypothesized lifetime distribution. The mean lifetime of the input particles is not recovered.

5 Fitting with Momentum Integral Weighting

This is one method which is not susceptible to the limited lifetime acceptance in STAR, but it does make certain assumptions. For each input particle, we calculate its momentum acceptance from its lifetime and the coordinate space acceptance. This requires knowledge of the input momentum distribution:

$$P(p_{min} < p < p_{max}) = \int_{p_{min}(r_{min},t)}^{p_{max}(r_{max},t)} \rho(p)dp \tag{19}$$

Then we simply weigh the lifetime of each particle with the inverse of this prob-

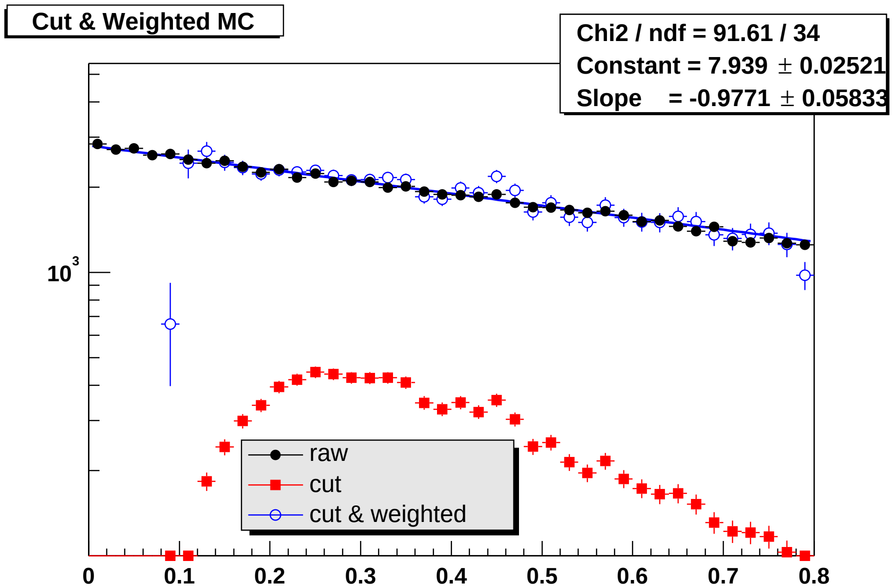

Figure 5: Lifetime distribution ($c\ \tau$) after momentum integral weighting to recover input slope ($c\ t_0 = 1$)

ability when filling the lifetime histogram, effectively correcting for the coordinate space acceptance in momentum space. If the input momentum distribution is know, this approach works, as see in Figure 5. The closed circles are the input Monte Carlo distribution. The closed squares are that distribution after fiducial cuts. The open circles are the cut distribution after weighting by the momentum integral. The slope is fit from this weighted distribution, and comes out very close to 1 (the input slope).

The weakness of this approach is that in practice the input momentum distribution is not known, in particular when momentum Dependant cuts are applied. It fails when applied to STAR data.

6 Correcting Data Using Embedding

Besides the problem of limited lifetime acceptance, a common flaw in the approaches presented above is that they assume perfect efficiency in the fiducial volume. Since we do not have perfect detection efficiency, we must use simulated data to calculate it. One can use embedded data (Monte Carlo tracks propagated through the detector and placed inside real events) to determine the efficiency by looking at how many of the Monte Carlo tracks in the fiducial volume are reconstructed. One could, for example, determine the coordinate space efficiency and then use the maximum likelihood method. Or the momentum efficiency could be calculated and used with the momentum weighting method above. But one could just as easily calculate the lifetime efficiency directly ($dN_{reconstructed}/dN^{Decayed}_{Embedded}$) and use that to correct the raw lifetime distribution without any more effort. In the end, this was our approach. Figure 6 shows our results using the embedding lifetime efficiency correction applied to year 2000 data. The ratio of the mean lifetime from the fit to the PDG lifetime is very close to 1. Note that Kaons with shorter proper lifetime only reach the STAR fiducial volume if they have high p_T. For example, Equation 4 shows that a Kaon with $ct = 20$ cm must have $p_T = 3.3$ GeV/c to reach the fiducial volume ($R = 133$ cm). The Kaon signal at larger momentum has higher contamination and is eliminated from the lifetime fit here.

7 Summary

Several analytical methods were investigated to recover the mean proper lifetime for charged Kaons including Maximum Likelihood, Virtual Particles, and Momentum Integral Weighting. Due to limited acceptance and complicated momentum dependencies of the analysis cuts, none of these methods produces satisfactory results. The weighting function must be produced from detailed simulation of Kaon decays; this method reproduces very well the accepted value of the proper mean lifetime and should be applicable for any collider experiment.

References

[1] Philip R Bevington & D. Keith Robinson, Data Reduction and Error Analysis for the Physical Sciences, Second Edition.

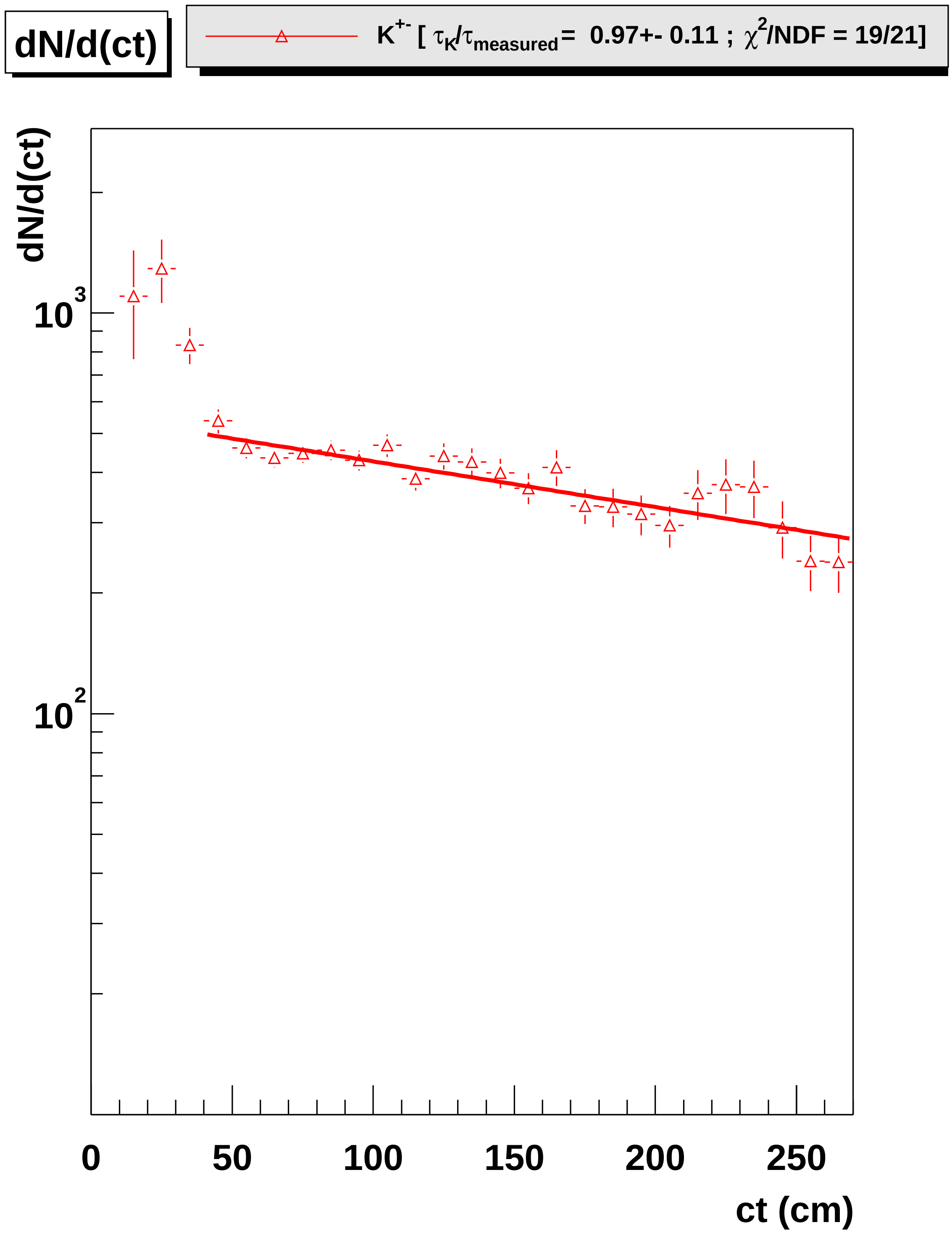

Figure 6: Results of Efficiency/Acceptance Correction from Embedding