Event asymmetry effect on event mixing distributions

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The effect of event azimuthal asymmetry in event mixing invariant mass distributions is investigated for proton-proton and gold-gold collisions reconstructed by the Time Projecting Chamber of the STAR experiment. Two-particle invariant mass distributions obtained by the classical event mixing method are disturbed by the asymmetry of reconstructed events. The formalism of Fourier expansion of azimuthal particle distributions is proposed in order to describe the effect and to calculate the correction. The result of the correction applied to experimental data is finally given.

1 Introduction

The STAR experiment is dedicated to the observation of ultra-relativistic ion collisions provided by the RHIC collider. The detector [1] allows the tracking, momentum analysis and identification of particles emitted at mid-rapidity over a pseudorapidity range of $|\eta| < 1.7$ (main TPC acceptance) and with full azimuthal coverage. Short lived particles emitted from the collision are observed via their decay into daughter particles. The event mixing technique is commonly used to extract the signal. The invariant mass distribution of the short lived particle is obtained by associating all selected tracks of the daughter species. The large combinatorial background is reproduced by combination of selected tracks from different events and is subtracted from the first distribution. Thus the method allows to extract a signal while the original signal is not distinguishable from the background.

The event mixing technique is notably required for the analysis of resonances production where the topological approach applied to strange hyperon analysis for example is not possible. Nevertheless the combinatorial background in general is not perfectly reproduced by event mixing. The final distribution – after the subtraction of the mixed background – is not flat and can show a non trivial shape described in section 2.1. This remaining background is probably a sum of the contributions of many physical effects. However we will show in section 2 and 3 that the effect can be understand in terms of the global event azimuthal asymmetry and that it can be corrected. Section 4 contains experimental results and the comparison between the calculated effect and the observed one.

2 Event azimuthal asymmetry effect

2.1 Waves in invariant mass distributions

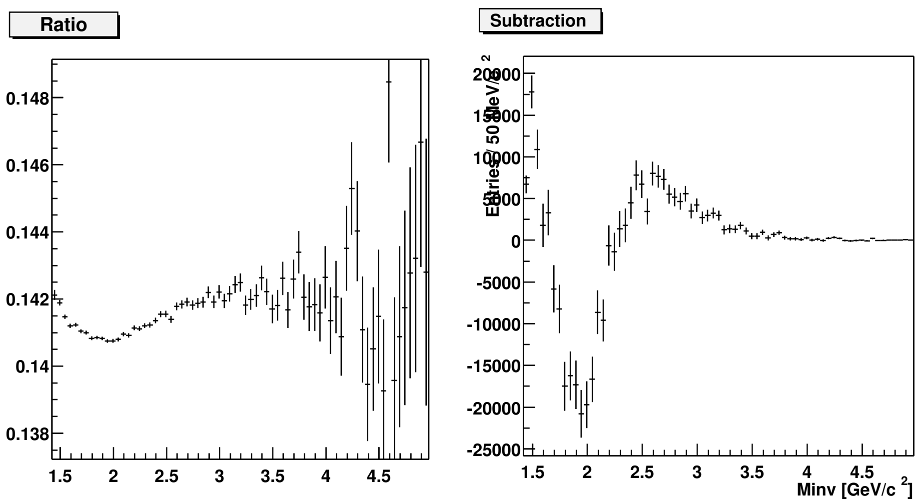

Figure 1: Ratio (left) and subtraction (right) of real and mixed invariant mass distributions.

This work has been originally done in order to extract the $\Lambda^*(1520)$ signal. Therefore experimental proton kaon invariant mass distributions are taken as examples. Protons and kaons are identified via their energy loss in the TPC. Only tracks coming from the primary vertex, i.e. their distance of closest approach to the primary vertex is lower than 3 cm, are selected. We will call real distribution the original invariant mass distribution and mixed distribution the invariant mass distribution which is constructed by event mixing. The signal is extracted by dividing or subtracting these two distributions. Before the subtraction, the mixed distribution is normalized by a scaling factor in order to reproduce the amplitude of the combinatorial background. The investigated effect is visible in figure 1 for the ratio on the left and for the subtraction on the right. The ratio distribution exhibits an undulating shape from the beginning to the end of the spectrum. The subtraction distribution shows the same wave shape modulated by the number of combinatorial pairs in each bin. Thus, in the subtraction distribution, the effect is large at small invariant mass and decreases with the increasing invariant mass.

The observed effect introduce two major problems in event mixing analysis. The inaccuracy of the calculated scaling factor is the first one. In principle the scaling factor can be calculated by the total number of entries in the real distribution divided by the number of entries in the mixed distribution. By definition this ratio normalizes the mixed distribution. However this value is influenced by the signal and other correlations at low invariant mass. A better scaling factor is thus performed by dividing the real distribution by the mixed distribution in an invariant mass region where no correlation are present, in general at high invariant mass. However the ratio distribution of figure 1 is affected in the whole range by the event asymmetry effect, the ratio being a function of the mass. Therefore the scaling factor can not be accurately determined. The other major point is simply the fact that signal extraction is highly perturbed by the non flat background. The final invariant mass distribution on the right of figure 1 exhibits a high slope in the signal region, i.e 1.52 GeV/c^2 for the $\Lambda^*(1520)$. The peak is thus hardly separated from the background.

2.2 Origin of the event asymmetry effect

Real particle distributions contain the azimuthal asymmetry of the event. On the contrary event mixing distributions are essentially symmetric. The example of elliptic azimuthal distributions is shown on figure 2. On the left, the two particle species have a preferential emission direction in the transverse plane which is the same since they come from the same event. The azimuthal distribution of the first particle species is represented by a dash line and the second by a solid line. On the right, event mixing technique combines one of these particle species with the other from events with different preferential emission directions. The angular distribution in the transverse plane of particles of the second particle species tends then to be perfectly flat in the case of event mixing. Furthermore the calculation of the invariant mass involves the scalar product of particle momenta and take into account the angle between the two associated particles. It is therefore logical that the difference of azimuthal distributions affects the final invariant mass distributions.

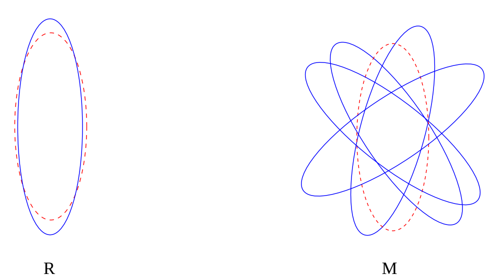

Figure 2: combination of two particle distributions for real event (left) and mixed event (right).

2.3 Description in terms of azimuthal Fourier expansion.

The contribution of the event asymmetry to the shape seen on invariant mass distributions can be calculated. The difference between real distribution and mixed distribution will be described in this section within the formalism of the azimuthal Fourier expansion of particle distributions. Although this formalism is similar to the one used in elliptic flow analysis [2, 3, 4], the developed description is a technical view of the azimuthal asymmetry effect and doesn't make hypothesis on the physical meaning of the asymmetry. The contribution to the invariant mass distribution is calculated bin per bin. The number of entries in a bin of the final mass distribution is the difference of two terms :

$$N = R - \alpha M \tag{1}$$

R is the number of entries in the bin of the real distribution, M is the number of entries in the mixed distribution bin and α is the scaling factor. The number of entries in R and M are equals to the number of proton-kaon associations which have the corresponding invariant mass. The R term can be expressed as the function of the two triple differential distributions

$$R = \int d^3p_1 d^3p_2 \left| \left(\frac{d^3N}{d^3p_1}\right)\left(\frac{d^3N}{d^3p_2}\right)\right|_{real}, \tag{2}$$

The integration is done over the phase space corresponding to the invariant mass range of the bin. The azimuthal dependence is then introduced for the two particle species by a Fourier expansion [3]

$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi p_t}\frac{d^2N}{dp_t dy}\left\{1 + 2\sum_{n=1}^{\infty} V_n \cos n(\phi - \Psi_R)\right\}, \tag{3}$$

where Ψ_R is the azimuthal reaction plane angle of the event and ϕ the azimuthal angle of particles. For flow measurement the reaction plane is defined by the beam direction and the impact parameter vector b. Vn coefficients characterize the amplitude of event asymmetry for all harmonic n. Using equation 3 for particle species 1 and 2, R can be rewritten as

$$\begin{aligned} R = & \int dp_{t_1} d\phi_1 dy_1 dp_{t_2} d\phi_2 dy_2 d^2\Psi_R \left| \left(\frac{d^2N}{dp_{t_1}dy_1}\right)\left(\frac{d^2N}{dp_{t_2}dy_2}\right)\right|_{real} \\ & \times \left\{1 + 2\sum_{n1=1}^{\infty} V_{n1}^{(1)} \cos n_1(\phi - \Psi_R)\right\}\left\{1 + 2\sum_{n2=1}^{\infty} V_{n2}^{(2)} \cos n_2(\phi - \Psi_R)\right\}. \end{aligned} \tag{4}$$

$V_n^{(1)}$ and $V_n^{(2)}$ are the Fourier coefficients for the two particle species. The sum over events is made by the integration over Ψ_R. This integration is performed by writing

$$\begin{aligned} & \int_0^{2\pi} d^2\Psi_R \left\{1 + 2\sum_{n1=1}^{\infty} V_{n1}^{(1)} \cos n_1(\phi - \Psi_R)\right\}\left\{1 + 2\sum_{n2=1}^{\infty} V_{n2}^{(2)} \cos n_2(\phi - \Psi_R)\right\} \\ & = 4\pi^2\left\{1 + 2\sum_{n=1}^{\infty} V_n^{(1)} V_n^{(2)} \cos n(\phi_1 - \phi_2)\right\}. \end{aligned} \tag{5}$$

There is no dependence on the reaction plane angle after integration from 0 to 2π. The final expression for the number of entries in the real invariant mass distribution is then

$$\begin{aligned} R &= \int dp_{t_1} d\phi_1 dy_1 dp_{t_2} d\phi_2 dy_2 \left| \left(\frac{d^2 N}{dp_{t_1} dy_1} \right) \left(\frac{d^2 N}{dp_{t_2} dy_2} \right) \right|_{real} \\ &\times \left\{ 1 + 2 \sum_{n=1}^{\infty} V_n^{(1)} V_n^{(2)} \cos n(\phi_1 - \phi_2) \right\}. \end{aligned} \tag{6}$$

This is the expression of the number of entries in a bin of the real invariant mass distribution. This number is a function of the event asymmetry via the $V_n^{(1)} V_n^{(2)}$ term.

The mixed distribution term M can also be expressed with the same formalism. This time we combine two particle species from different events. Azimuthal distributions are therefore calculated according to two different random reaction plane angles, namely Ψ_R^1 and Ψ_R^2.

$$\begin{aligned} M &= \int d^3 p_1 d^3 p_2 \left| \left(\frac{d^3 N}{d^3 p_1} \right) \left(\frac{d^3 N}{d^3 p_2} \right) \right|_{mix} \\ M &= \int dp_{t_1} d\phi_1 dy_1 dp_{t_2} d\phi_2 dy_2 d\Psi_R^1 d\Psi_R^2 \left| \left(\frac{d^2 N}{dp_{t_1} dy_1} \right) \left(\frac{d^2 N}{dp_{t_2} dy_2} \right) \right|_{mix} \\ &\times \left\{ 1 + 2 \sum_{n1=1}^{\infty} V_{n1}^{(1)} \cos n_1(\phi - \Psi_R^1) \right\} \left\{ 1 + 2 \sum_{n2=1}^{\infty} V_{n2}^{(2)} \cos n_2(\phi - \Psi_R^2) \right\} \end{aligned} \tag{7}$$

Here azimuthal dependences vanish, due to the integration over distinct Ψ_R^1 and Ψ_R^2. The expression of M can then be formulated by :

$$M = \int dp_{t_1} d\phi_1 dy_1 dp_{t_2} d\phi_2 dy_2 \left| \left(\frac{d^2 N}{dp_{t_1} dy_1} \right) \left(\frac{d^2 N}{dp_{t_2} dy_2} \right) \right|_{mix}.$$

The number of entries in each bin of the final invariant mass distribution can finally be written as the sum of three terms a, b and $c.d$:

$$\begin{aligned} N &= R - \alpha M \qquad (8) \\ N &= \int dp_{t_1} d\phi_1 dy_1 dp_{t_2} d\phi_2 dy_2 \left| \left(\frac{d^2 N}{dp_{t_1} dy_1} \right) \left(\frac{d^2 N}{dp_{t_2} dy_2} \right) \right|_{real} \qquad (a) \\ &- \alpha \int dp_{t_1} d\phi_1 dy_1 dp_{t_2} d\phi_2 dy_2 \left| \left(\frac{d^2 N}{dp_{t_1} dy_1} \right) \left(\frac{d^2 N}{dp_{t_2} dy_2} \right) \right|_{mix} \qquad (b) \\ &+ \int dp_{t_1} d\phi_1 dy_1 dp_{t_2} d\phi_2 dy_2 \left| \left(\frac{d^2 N}{dp_{t_1} dy_1} \right) \left(\frac{d^2 N}{dp_{t_2} dy_2} \right) \right|_{real} \\ &\times 2 \sum_{n=1}^{\infty} V_n^{(1)} V_n^{(2)} \cos n(\phi_1 - \phi_2) \qquad (c) \end{aligned}$$

The event asymmetry effect can directly be seen by examining equation 8. Without any azimuthal asymmetry, the number of entries in the bin is equal to the difference between the first term (a), coming from the real distribution, and the second term (b) coming from the mixed distribution. The result of this subtraction is zero if the bin is outside the signal peak, otherwise it is equal to the signal value of the bin. Nevertheless, in the case of an azimuthal asymmetry, the third term (c) is added to the bin value. Indeed, due to the $V_n^{(1)}V_n^{(2)}$ term, the event asymmetry modifies the number of entries in the bin of the real distribution. The mixed distribution does not provide a correction to this behavior. Thus the modification of the number of entries by the event asymmetry in the real distribution is still there after subtraction of the mixed distribution.

3 Correction

3.1 Correction of the azimuthal asymmetry effect

The correction of the azimuthal asymmetry effect is doable in two ways.

The first one is to mix only events with similar particle azimuthal distributions. Thus event mixing will combine the same particle azimuthal distributions as the real event combination do. From a mathematical point of view, this method tends to make equal the Ψ_R^1 and Ψ_R^2 angles in equation 7. The equation of the number of entries in the bins of the mixed distribution becomes then similar to the equation of the real distribution (Eq. 5). However the method is applied by making reaction plane angle bins. Therefore it is only possible when the reaction plane angle is precisely measured. In any case the method suffers from the resolution of the reaction plane angle and from the bin width.

The second method consists in the explicit calculation of the mixed equivalent to the third term (c) in equation 8 of section 2.3. Indeed the bin per bin calculation of

$$\int dp_{t_1} d\phi_1 dy_1 dp_{t_2} d\phi_2 dy_2 \left| \left(\frac{d^2N}{dp_{t_1}dy_1}\right)\left(\frac{d^2N}{dp_{t_2}dy_2}\right)\right|_{mix} 2\sum_{n=1}^{\infty} V_n^{(1)}V_n^{(2)} \cos n(\phi_1 - \phi_2) \tag{9}$$

allows the construction of the new number of entries in the mixed distribution

$$\begin{aligned} M' &= \int dp_{t_1} d\phi_1 dy_1 dp_{t_2} d\phi_2 dy_2 \left| \left(\frac{d^2N}{dp_{t_1}dy_1}\right)\left(\frac{d^2N}{dp_{t_2}dy_2}\right)\right|_{mix} \\ &\times \left\{1 + 2\sum_{n=1}^{\infty} V_n^{(1)}V_n^{(2)} \cos n(\phi_1 - \phi_2)\right\} \end{aligned} \tag{10}$$

which reproduces now the combinatorial background of equation (6). Indeed the azimuthal asymmetry dependence in real distribution is reproduced in the mixed term M'. The bin per bin corrected invariant mass distribution is then obtained by

$$N' = R - \alpha'.M' \tag{11}$$

The scaling factor α'normalizes the new mixed distribution containing the numbers of entries M'.

3.2 V_n measurement

In order to calculate the new term (9) one has to know the coefficients V_n. These coefficients have been measured via two-particle azimuthal correlation function which is defined as [5]

$$C(\Delta\phi) = \frac{N_{cor}(\Delta\phi)}{N_{uncor}(\Delta\phi)}$$

where $N_{corr}(\Delta\phi)$ is the observed $\Delta\phi$ distribution for track pairs selected from the same event, and $N_{uncorr}(\Delta\phi)$ is the $\Delta\phi$ distribution for particle pairs selected from mixed events. The applied method is currently used for elliptic flow measurements [6]. However what we want here is not a flow measurement but the global characterization of event azimuthal asymmetry, including all contributing phenomena. An example of $\Delta\phi$ distributions is shown on figure 3. The resulting correlation function $C(\Delta\phi)$ is then fitted assuming

$$C(\Delta\phi) \propto 1 + 2V_1^2\cos\Delta\phi + 2V_2^2\cos 2\Delta\phi + 2V_3^2\cos 3\Delta\phi + 2V_4^2\cos 4\Delta\phi.$$

The coefficients V_1 to V_4 for both particle species are extracted from the measured correlation functions. The measurement of Vn coefficients has been done as a function of the transverse momentum of particles. Furthermore, one of the major advantage is that there is no need to determine the reaction plane angle. In particular this method is required for systems where the reaction plane angle is not defined or poorly measured. It can be the case in systems with a few number of particles, i.e. proton-proton collisions or peripheral gold-gold collisions, or with a high level of symmetry like central gold-gold events.

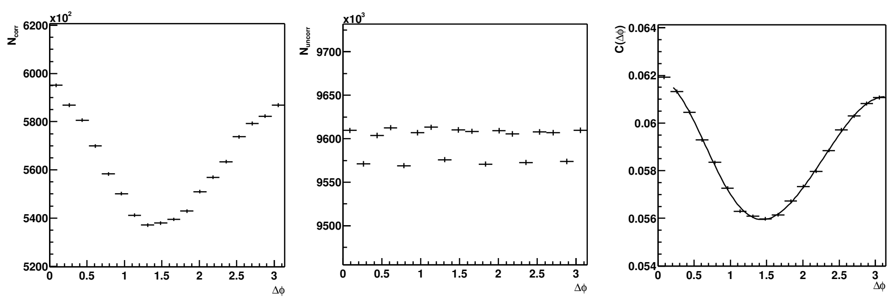

Figure 3: $\Delta\phi$ distributions for particles from the same event (left) and from mixed events (middle). The resulting correlation function is shown on the right.

4 Experimental results

The study has been performed over five sets of experimental data. The first set contains data from proton-proton collisions. The other sets are data from gold-gold collisions in four centrality classes from peripheral to most central events: 80% to 60%, 60% to 40%, 40% to 10%, and 10% or less of the cross section. The first step of the analysis consists in the measurement of the coefficients V_n for the particle species. V_1, V_2, V_3, and V_4 have been measured. V_1 and V_2 coefficients are shown in figure 4 for both particle species and for the five data sets.

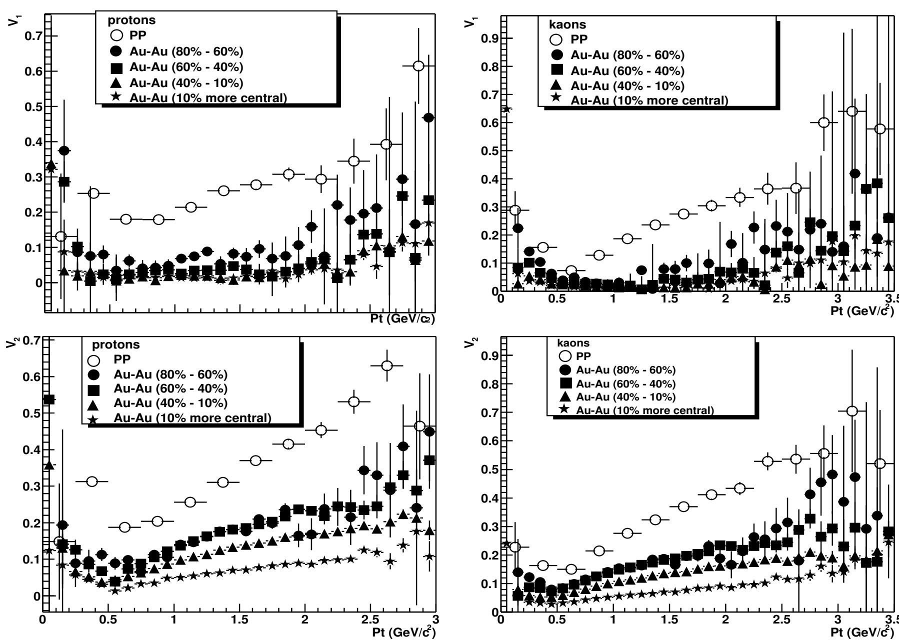

Figure 4: Measured V_1 and V_2 for protons (left) and kaons (right) for proton-proton collisions and gold-gold collisions at different centralities.

Then real and mixed invariant mass distributions are calculated. Final invariant mass distributions are obtained by the subtraction of these two distributions. The corrected mixed invariant mass would be calculated with equation (11) including the new term of equation (9). However, for the purpose of the method illustration, we show separately on figure 5 the classical event mixing spectra, and the calculated event asymmetry represented by solid lines. They are formed by the bin per bin values of equation (9) which are then normalized by the scaling factor. The first distribution is the invariant mass distribution in proton-proton collisions, followed by distributions from gold-gold collisions. The asymmetry effect is visible in all systems. It's shape is similar in all spectra. However the amplitude is dependent of the system, probably because of different physical origins of the event azimuthal asymmetry. In general, the calculated event asymmetry effect represented by solid lines reproduces the shape and the amplitude of distributions. It has been noticed that the large multiplicity fluctuation of some systems introduce a supplementary shape in invariant mass distributions. Although event mixing is done per multiplicity bin, the difference is still visible but largely removed in proton-proton and peripheral gold-gold distributions.

Conclusion

A wave shape structure is observed in invariant mass distributions from event mixing technique. The effect of event azimuthal asymmetry on these event mixing distributions has been therefore investigated in order to explain this behavior. The effect is calculated within the framework of the azimuthal Fourier expansion of particle distributions. A possible correction has been calculated via the developed formalism. It is based on the measurement of the event azimuthal asymmetry via a two particle azimuthal correlation function. Comparing with experimental data, the calculated behavior reproduces the shape and the amplitude of the observed effect. It is therefore possible to calculate a correction term which allows to remove a large part of the wave shape structure. Analysis based on event mixing grandly benefit from this correction especially near threshold where the signal is hardly visible. Indeed the remaining background is made flat in the experimental spectra, decreasing the possible bias due to the background behavior.

References

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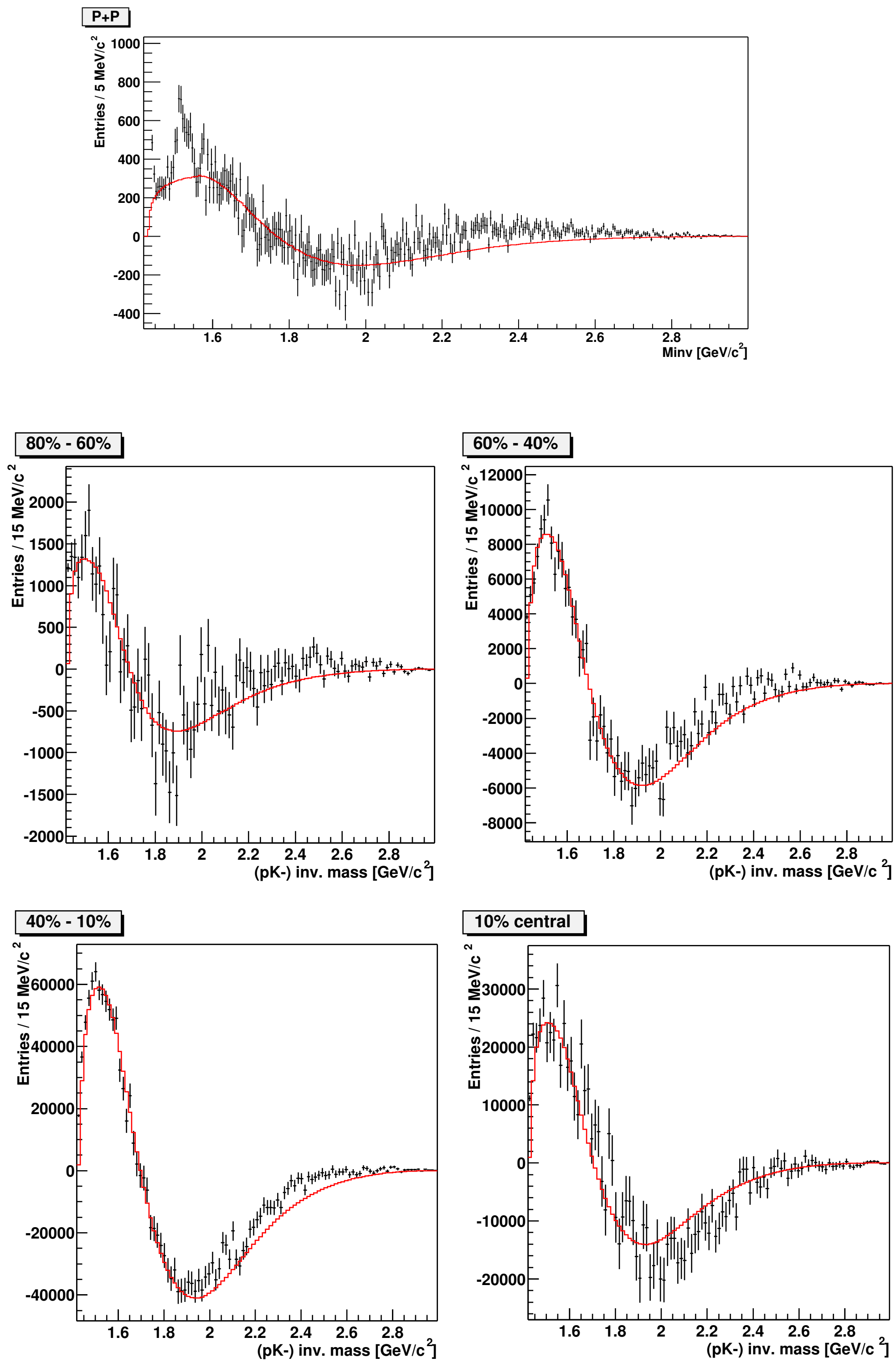

Figure 5: Classical event mixing distributions and the calculated corrections for proton-proton and gold-gold collisions.