

# Critical fluctuations at STAR BES & FXT

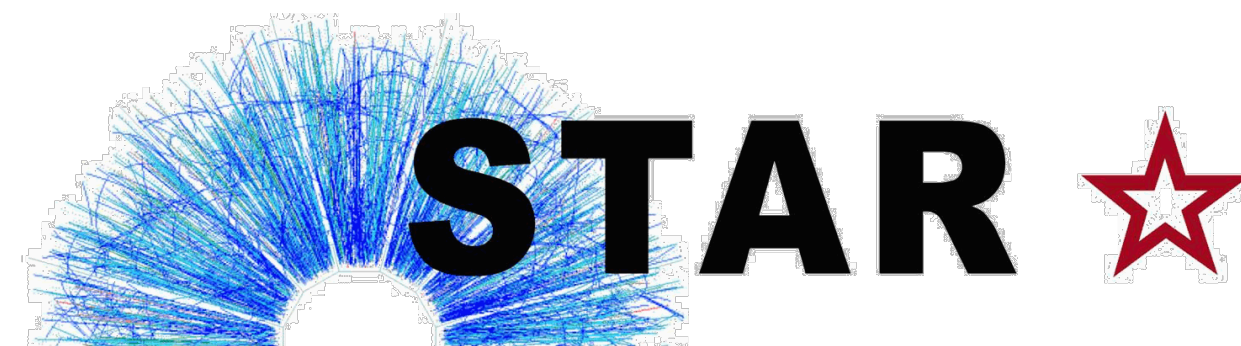
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***Toshihiro Nonaka*** for the STAR collaboration

*University of Tsukuba*

3rd workshop on Physics performance studies at FAIR and NICA

*Supported in part by*



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# Outline

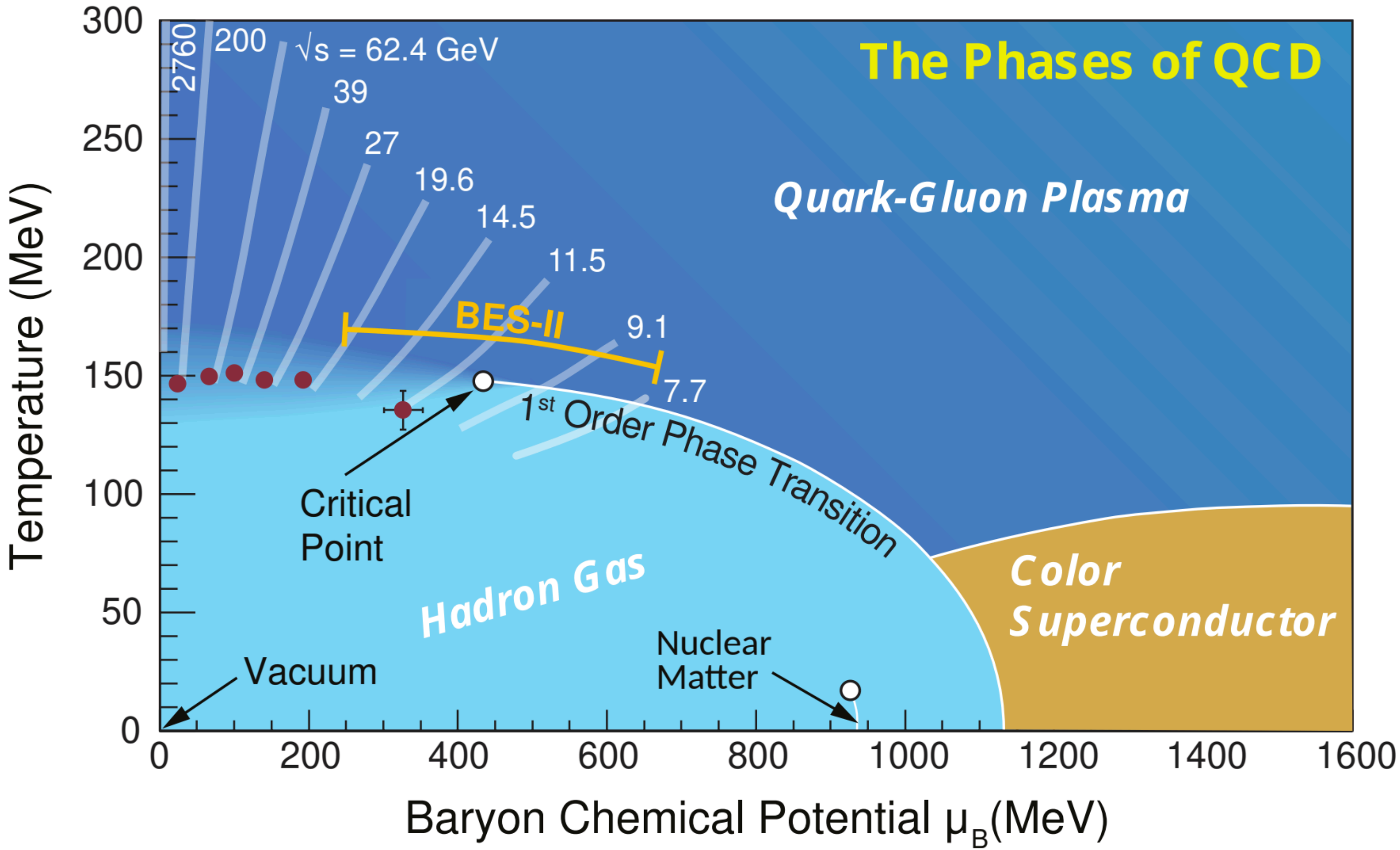
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- ***Introduction***
- ***Analysis techniques***
- ***$C_4/C_2$  for critical point search***
- ***$C_5/C_1$  and  $C_6/C_2$  for crossover search***
- ***Future perspective***
- ***Summary***



# QCD phase diagram

✓ QCD phase structure in wide ( $\mu_B, T$ ) region.



- **Crossover at  $\mu_B = 0$  MeV**  
*Y. Aoki et al, Nature 443, 675(2006)*
- **1st-order phase transition at large  $\mu_B$ ?**
- **Critical point?**



# Beam Energy Scan

✓ Need to investigate the QCD phase structure in wide ( $\mu_B, T$ ) region.

$\sqrt{s_{NN}}$ (GeV)	No. of events (million)	$T_{ch}$ (MeV)	$\mu_B$ (MeV)
200	238	164.3	28
62.4	47	160.3	70
54.4	550	160.0	83
39	86	156.4	160
27	30	155.0	144
19.6	15	153.9	188
14.5	20	151.6	264
11.5	6.6	149.4	287
7.7	3	144.3	398

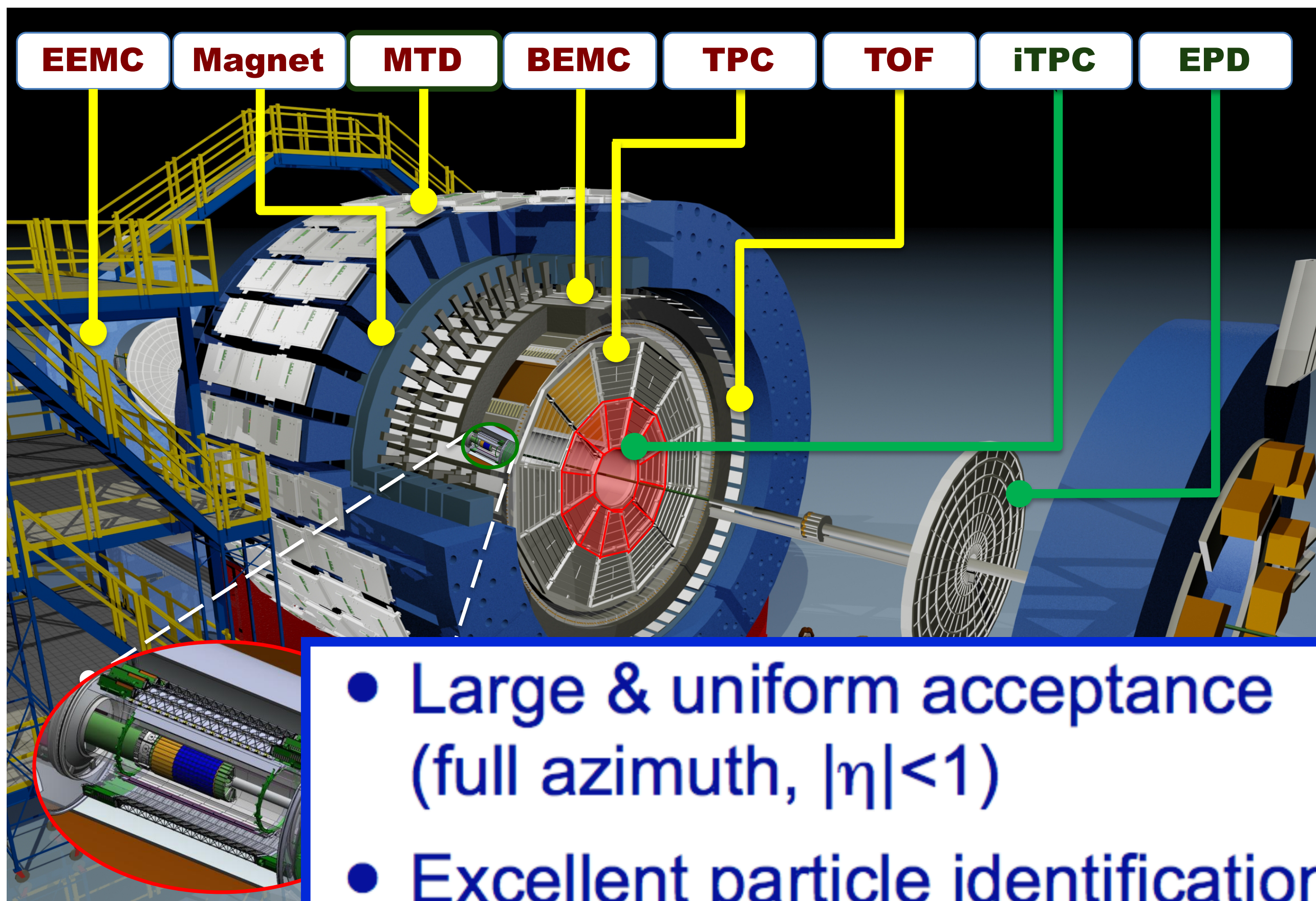
2010-  
2017

- **Crossover at  $\mu_B = 0$  MeV**  
*Y. Aoki et al, Nature 443, 675(2006)*
- **1st-order phase transition at large  $\mu_B$ ?**
- **Critical point?**

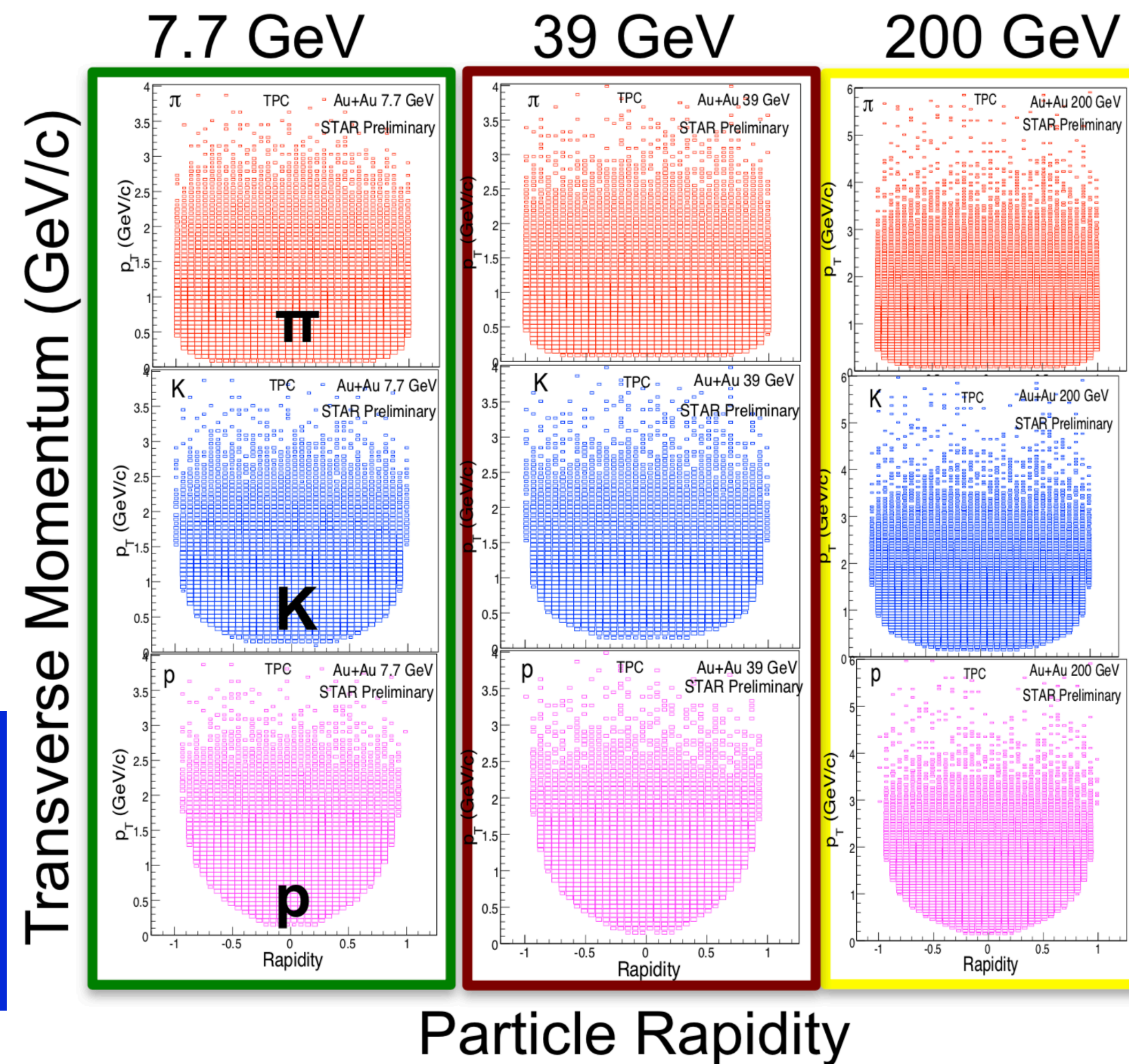


# STAR detector

✓ TPC and TOF are used for fluctuation analysis.



- Large & uniform acceptance (full azimuth,  $|\eta| < 1$ )
- Excellent particle identification





# Particle identification

✓  $dE/dx$  measured with TPC is used for particle identification at low  $p_T$  region.

✓  $< 2\sigma$  on  $dE/dx$  distribution

✓ Track quality is assured based on # of hits in TPC

✓ The combined PID with  $m^2$  from TOF is used at high  $p_T$  region.

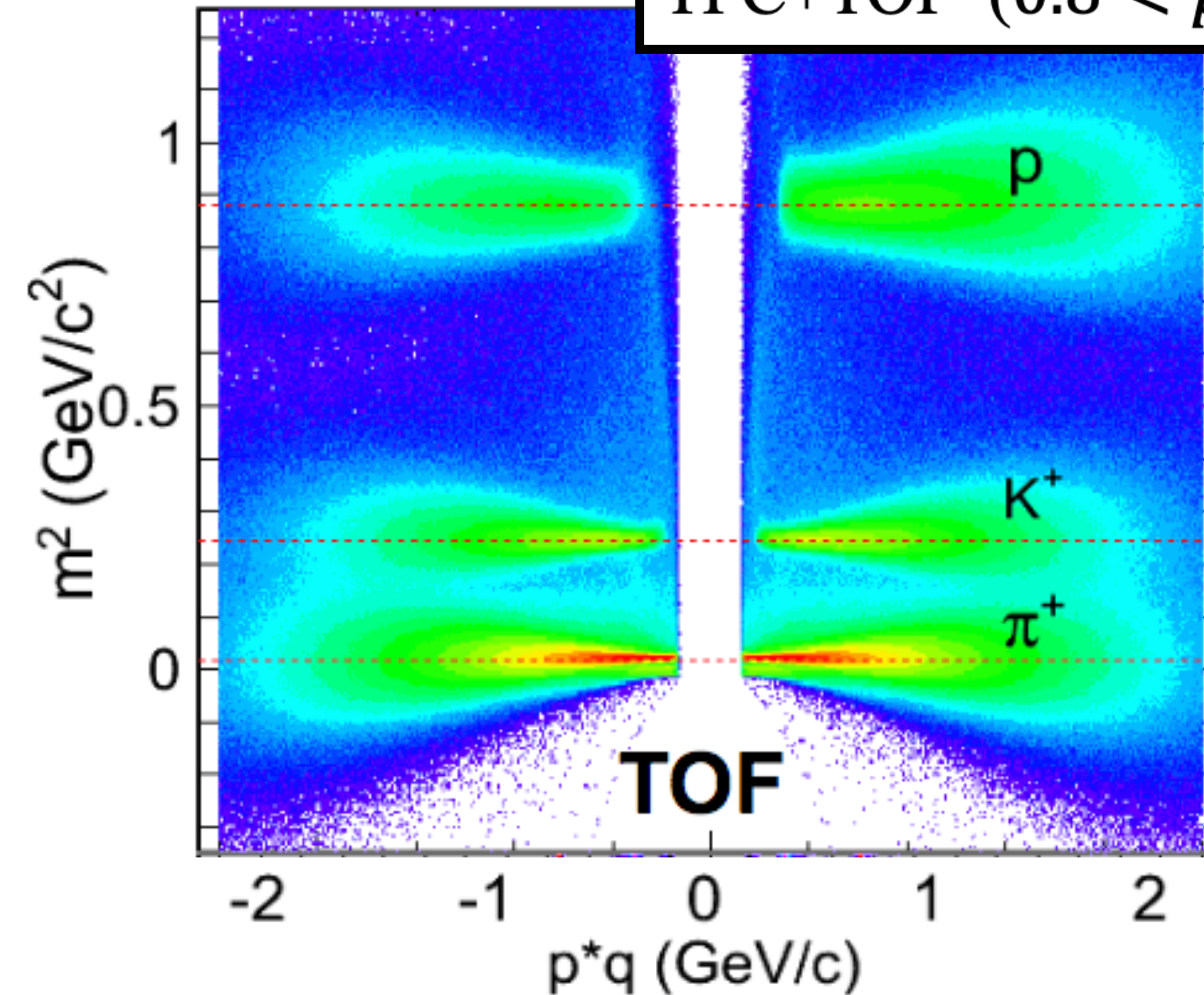
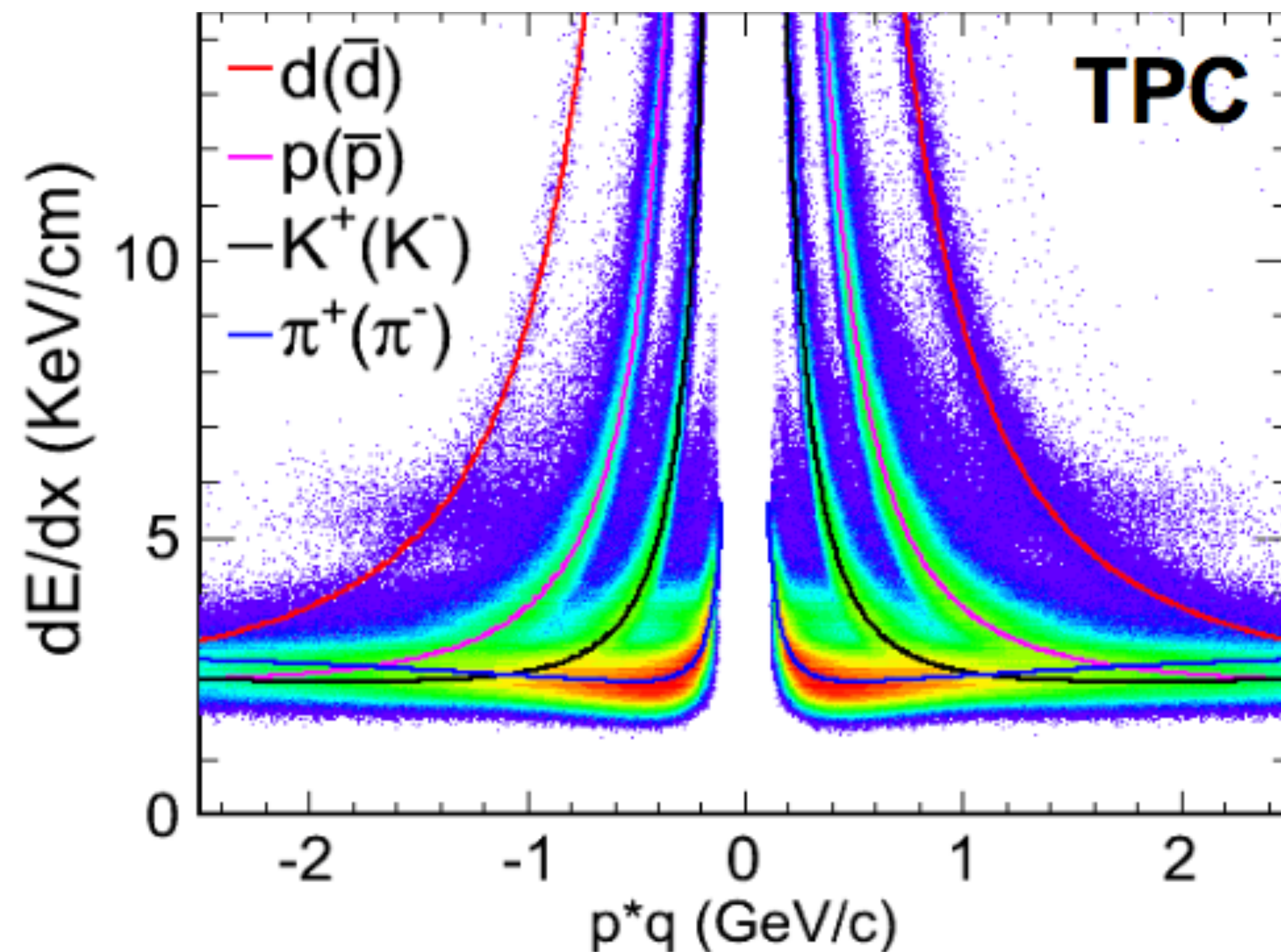
**Analysis window:**

$0.4 < p_T < 2.0$  GeV/c,  $|y| < 0.5$

**Proton identification:**

TPC ( $0.4 < p_T < 0.8$  GeV/c)

TPC+TOF ( $0.8 < p_T < 2.0$  GeV/c)

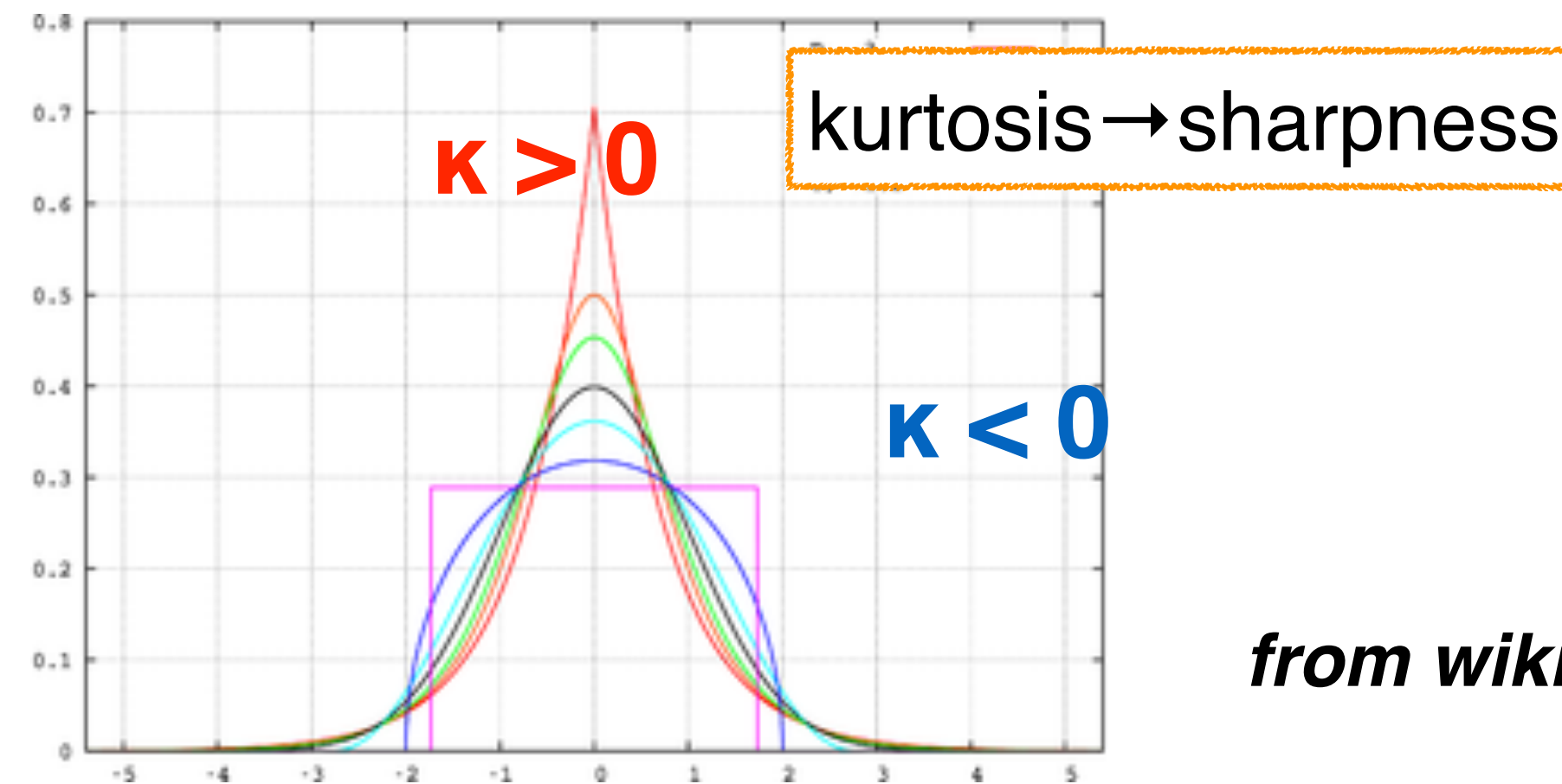
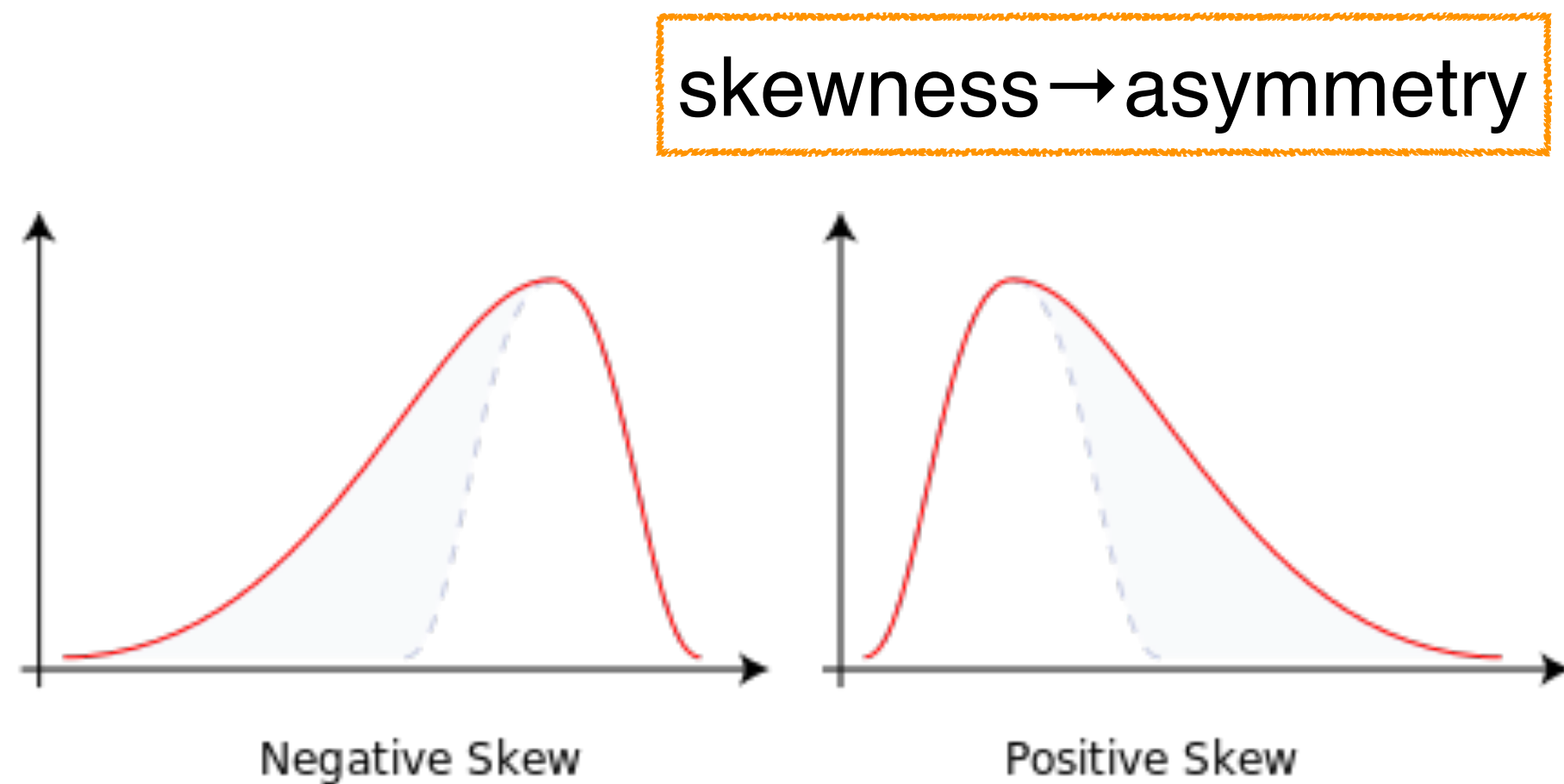




# Higher-order fluctuations

◆ Moments and cumulants are mathematical measures of “shape” of a distribution which probe the fluctuation of observables.

- ✓ Moments: mean ( $M$ ), standard deviation ( $\sigma$ ), skewness ( $S$ ) and kurtosis ( $\kappa$ ).
- ✓  $S$  and  $\kappa$  are sensitive to non-gaussian fluctuations.



from wikipedia

✓ Cumulant  $\Leftrightarrow$  Central Moment

$$\langle \delta N \rangle = N - \langle N \rangle$$

$$C_1 = M = \langle N \rangle$$

$$C_2 = \sigma^2 = \langle (\delta N)^2 \rangle$$

$$C_3 = S\sigma^3 = \langle (\delta N)^3 \rangle$$

$$C_4 = \kappa\sigma^4 = \langle (\delta N)^4 \rangle - 3 \langle (\delta N)^2 \rangle^2$$

✓ Cumulant : additivity

$$C_n(X + Y) = C_n(X) + C_n(Y)$$

→ proportional to volume

◆ **Net baryon, net charge and net strangeness**

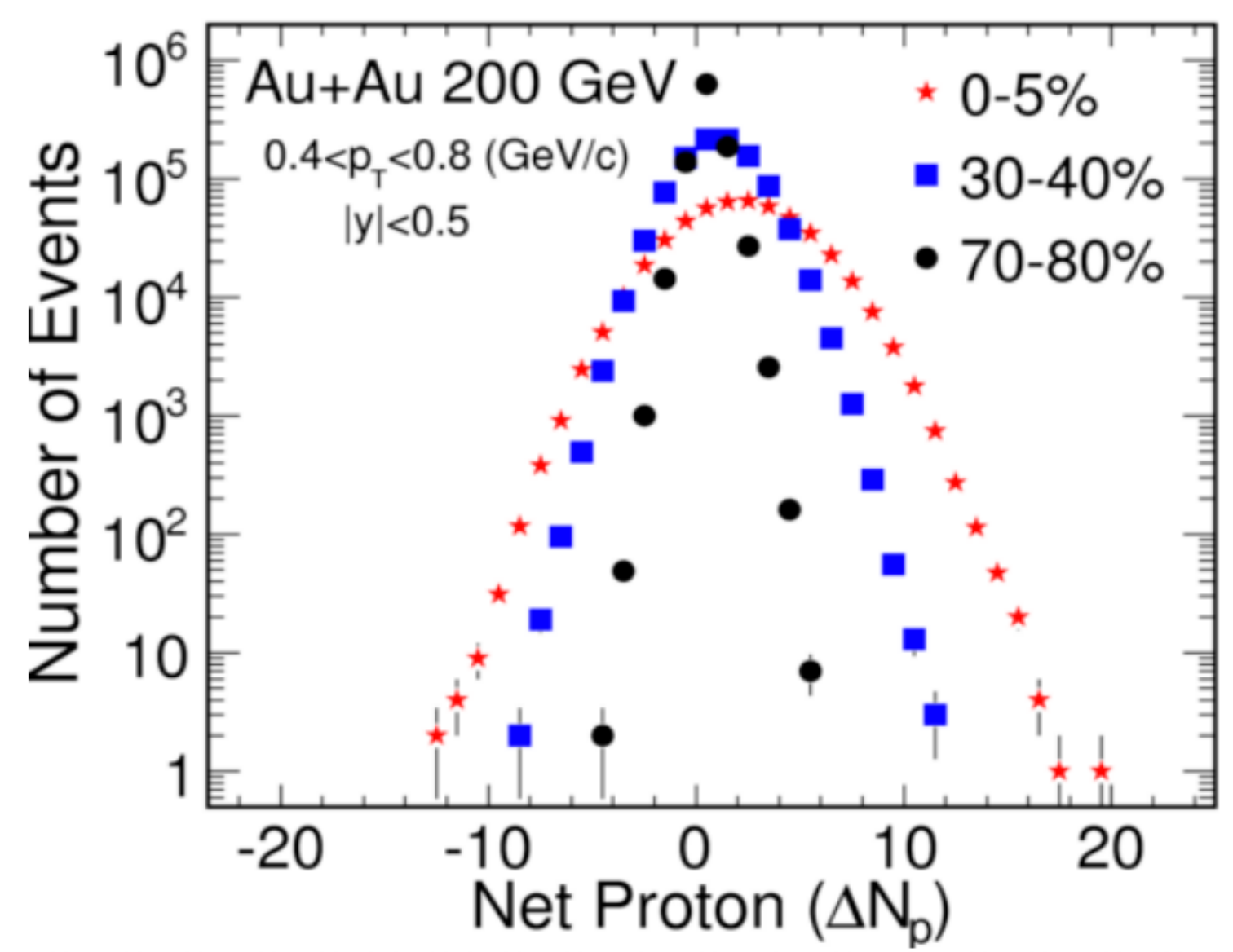
“Net” : positive - negative

$$\Delta N_q = N_q - N_{\bar{q}}, \quad q = B, Q, S$$

Fill in histograms  
over many collisions

No. of **positively charged**  
particles in one collision

No. of **negatively charged**  
particles in one collision



→ neutrons cannot be measured

**(1) Sensitive to correlation length**

$$C_2 = \langle (\delta N)^2 \rangle_c \approx \xi^2 \quad C_5 = \langle (\delta N)^5 \rangle_c \approx \xi^{9.5}$$

$$C_3 = \langle (\delta N)^3 \rangle_c \approx \xi^{4.5} \quad C_6 = \langle (\delta N)^6 \rangle_c \approx \xi^{12}$$

$$C_4 = \langle (\delta N)^4 \rangle_c \approx \xi^7$$

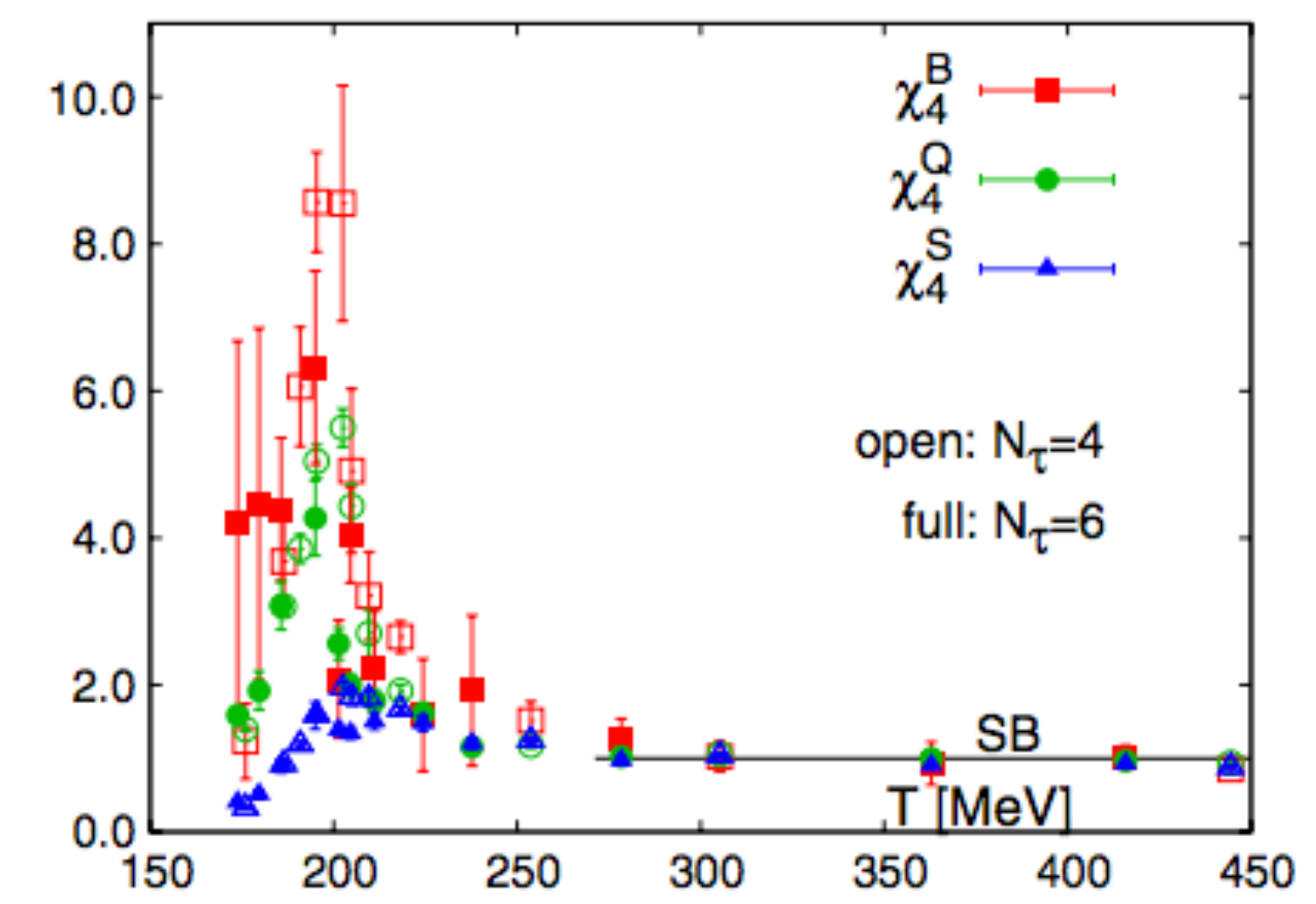
M. A. Stephanov, *Phys. Rev. Lett.* 102, 032301 (2009)  
 M. A. Stephanov, *Phys. Rev. Lett.* 107, 052301 (2011)  
 M. Asakawa, S. Ejiri and M. Kitazawa, *Phys. Rev. Lett.* 103, 262301 (2009)

**(2) Direct comparison with susceptibilities.**

M. Cheng et al, *PRD* 79, 074505 (2009)

$$S\sigma = \frac{C_3}{C_2} = \frac{\chi_3}{\chi_2} \quad \kappa\sigma^2 = \frac{C_4}{C_2} = \frac{\chi_4}{\chi_2}$$

$$\chi_n^q = \frac{1}{VT^3} \times C_n^q = \frac{\partial^n p/T^4}{\partial \mu_q^n}, \quad q = B, Q, S$$



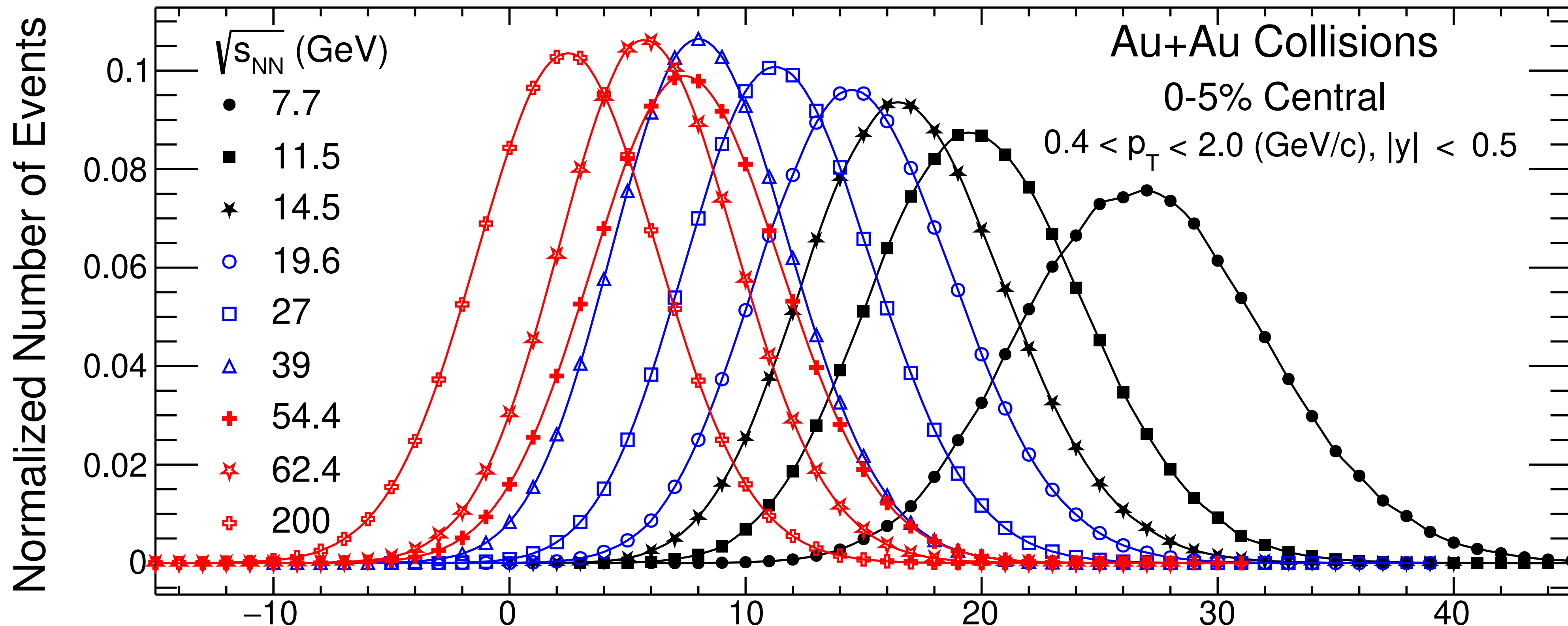
M. Cheng et al, *PRD* 79, 074505 (2009)





# Raw net-proton distribution

- ✓ Avoid auto-correlation effects : New centrality definition
- ✓ Suppress initial volume fluctuation : Centrality bin width correction
- ✓ Detector efficiency correction : Binomial model



X.Luo, J. Xu, B. Mohanty and N. Xu. *J. Phys. G*40,105104(2013)  
 M. Kitazawa : *PRC*.86.024904(2012)  
 A. Bzdak and V. Koch : *PRC*.86.044904(2012), X. Luo : *PRC*.91.034907(2016)  
 T. Nonaka, M. Kitazawa, S. Esumi : *PRC*.95.064912(2017), *NIMA*906 10-17 (2018),  
*NIMA*984(2020)164632  
 X. Luo, T. Nonaka : *PRC*.99.044917(2019)

Net-proton ( $\Delta N_p = N_p - N_{\bar{p}}$ )

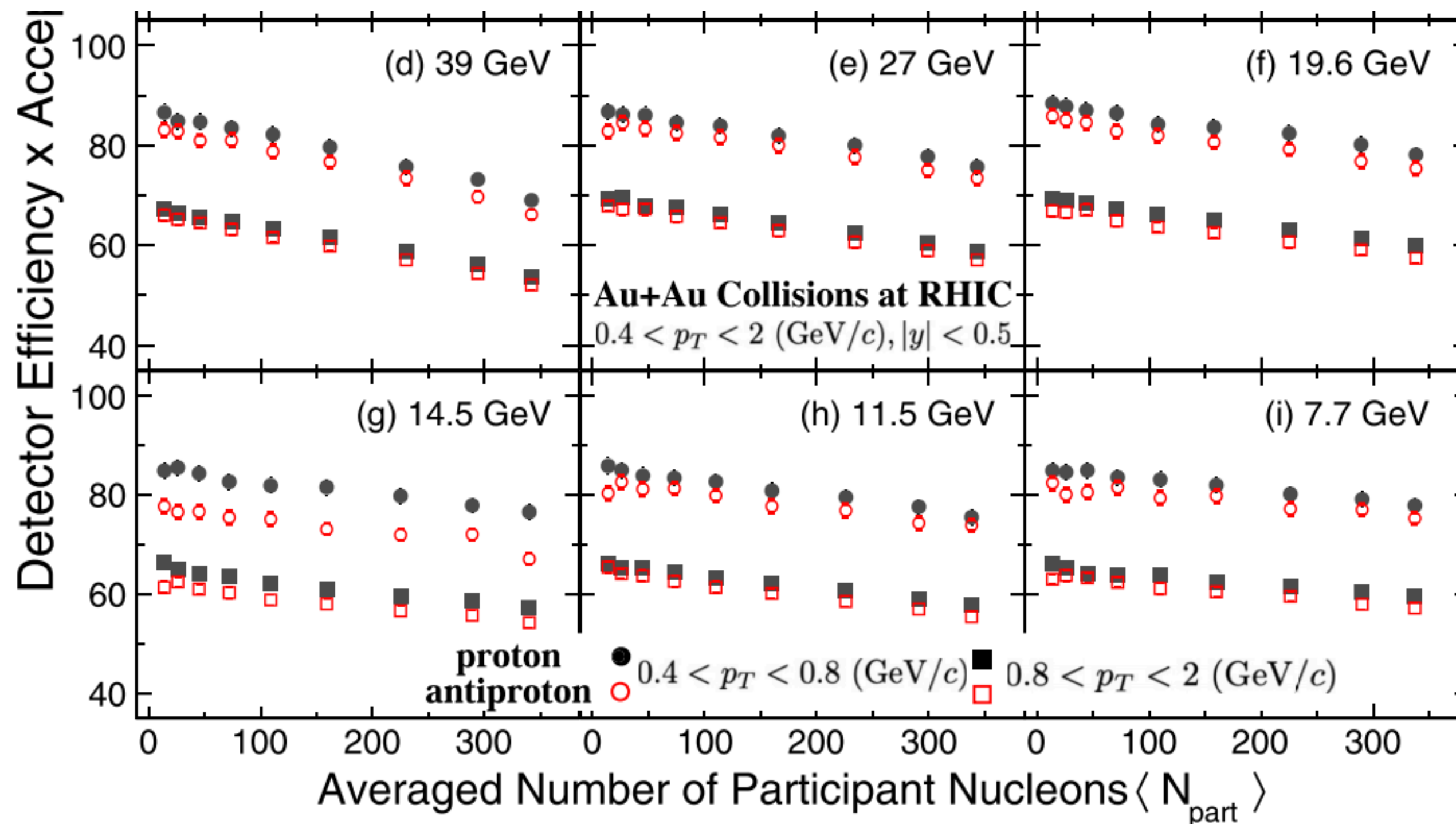
**STAR Collaboration,**  
***PRL*.126.092301(2021)**  
***PRC*.104.024902(2021)**



# Efficiency correction

## Proton identification

- $0.4 < p_T < 0.8$  (GeV/c) TPC only
- $0.4 < p_T < 0.8$  (GeV/c) TPC only
- $0.8 < p_T < 2$  (GeV/c) TPC+TOF
- $0.8 < p_T < 2$  (GeV/c) TPC+TOF



- ✓ Efficiencies are assumed to follow binomial distributions.

$$B_{p,N}(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}$$

- ✓ Multiplicity and  $p_T$  dependence are taken into account.

*M. Kitazawa : PRC.86.024904(2012)*  
*A. Bzdak and V. Koch : PRC.86.044904(2012)*  
*X. Luo : PRC.91.034907(2016)*  
*T. Nonaka, M. Kitazawa, S. Esumi : PRC.95.064912(2017)*  
*X. Luo, T. Nonaka : PRC.99.044917(2019)*

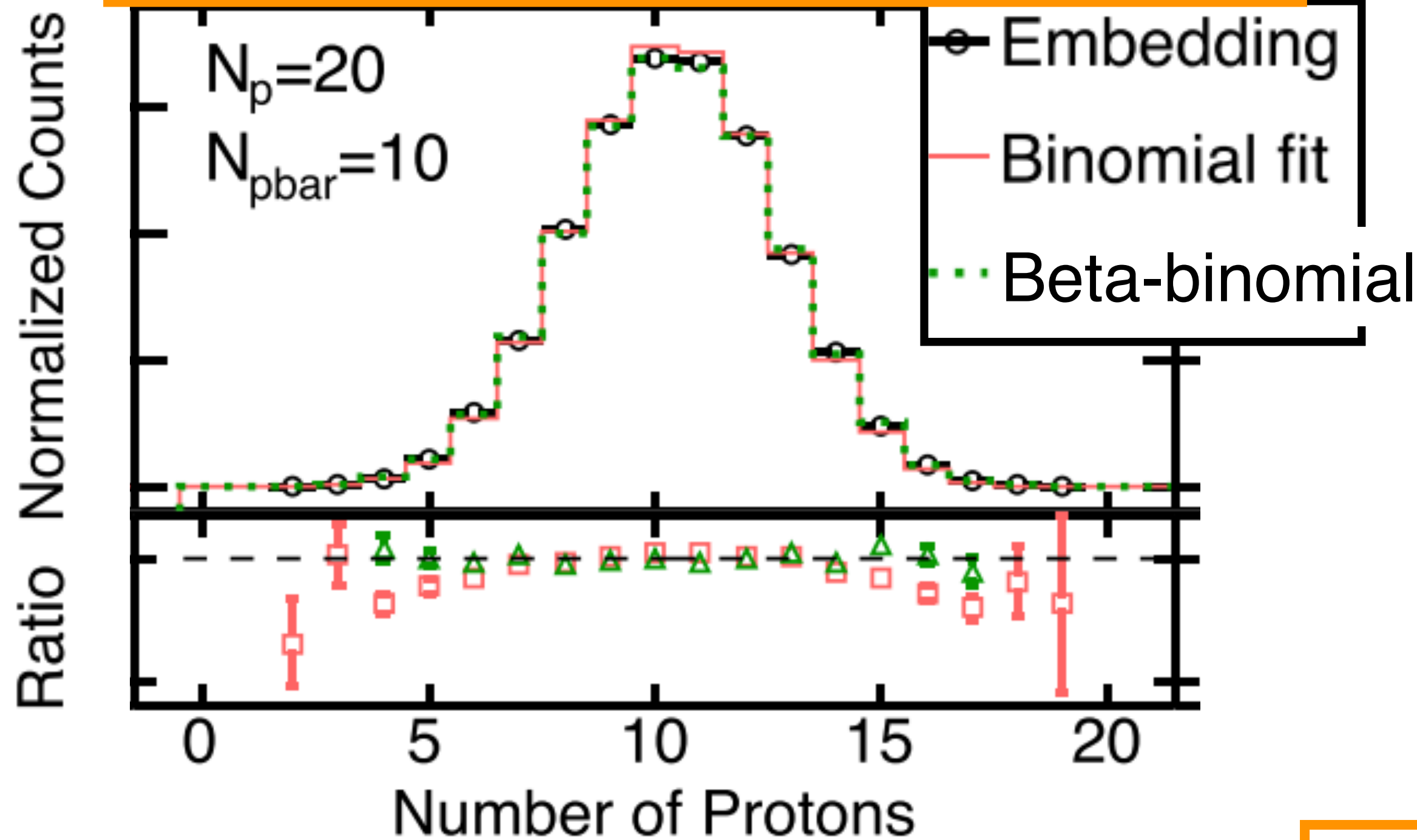


# Non-binomial efficiency correction

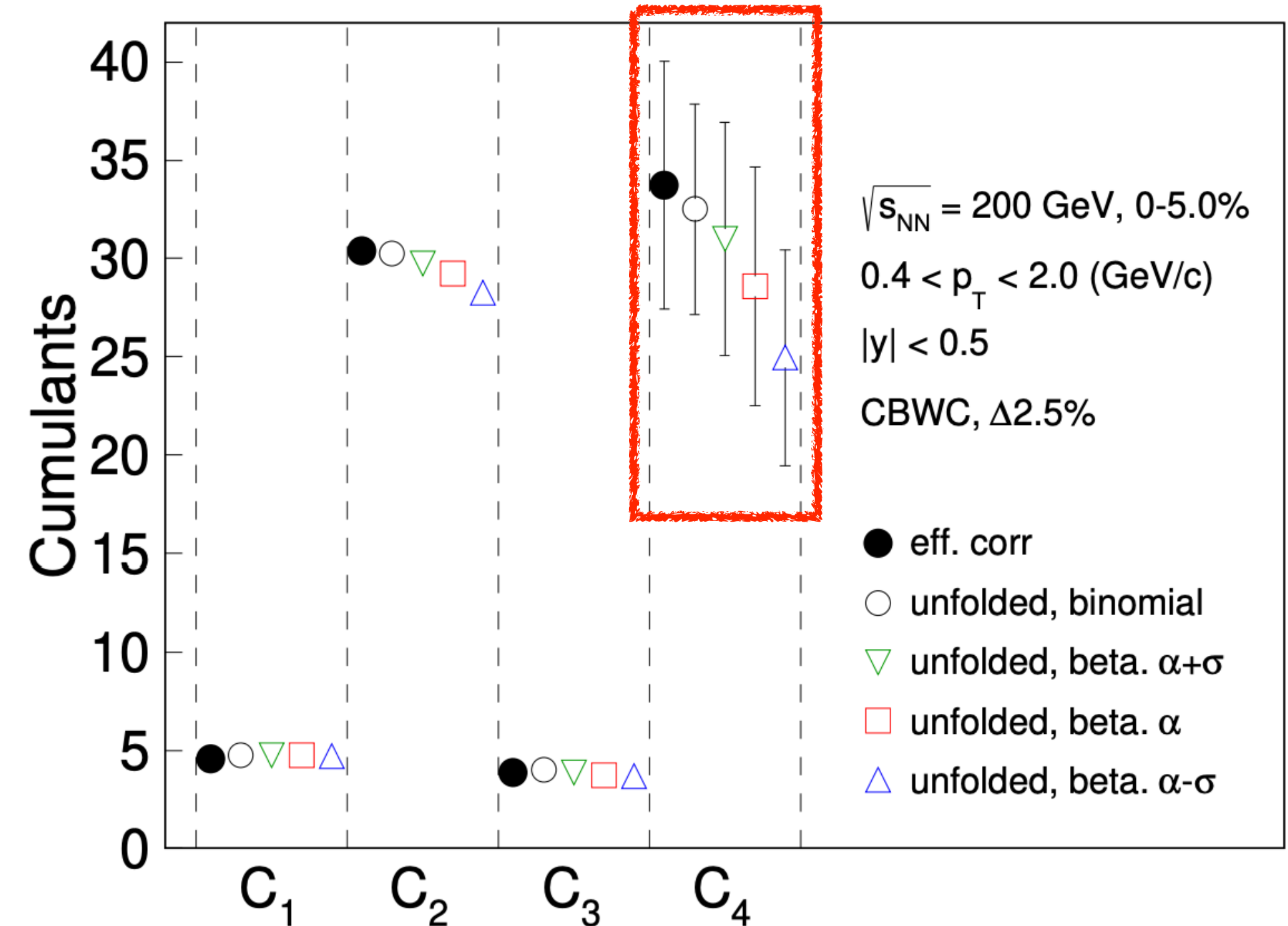
✓ Efficiency distributions need to be carefully studied through simulations.

✓ Higher-order cumulants from non-binomial efficiency corrections are consistent with binomial corrections.

*Slightly deviates from binomial distribution*



STAR Collaboration, PRC.104.024902(2021)  
STAR Collaboration, PRL.126.092301(2021)

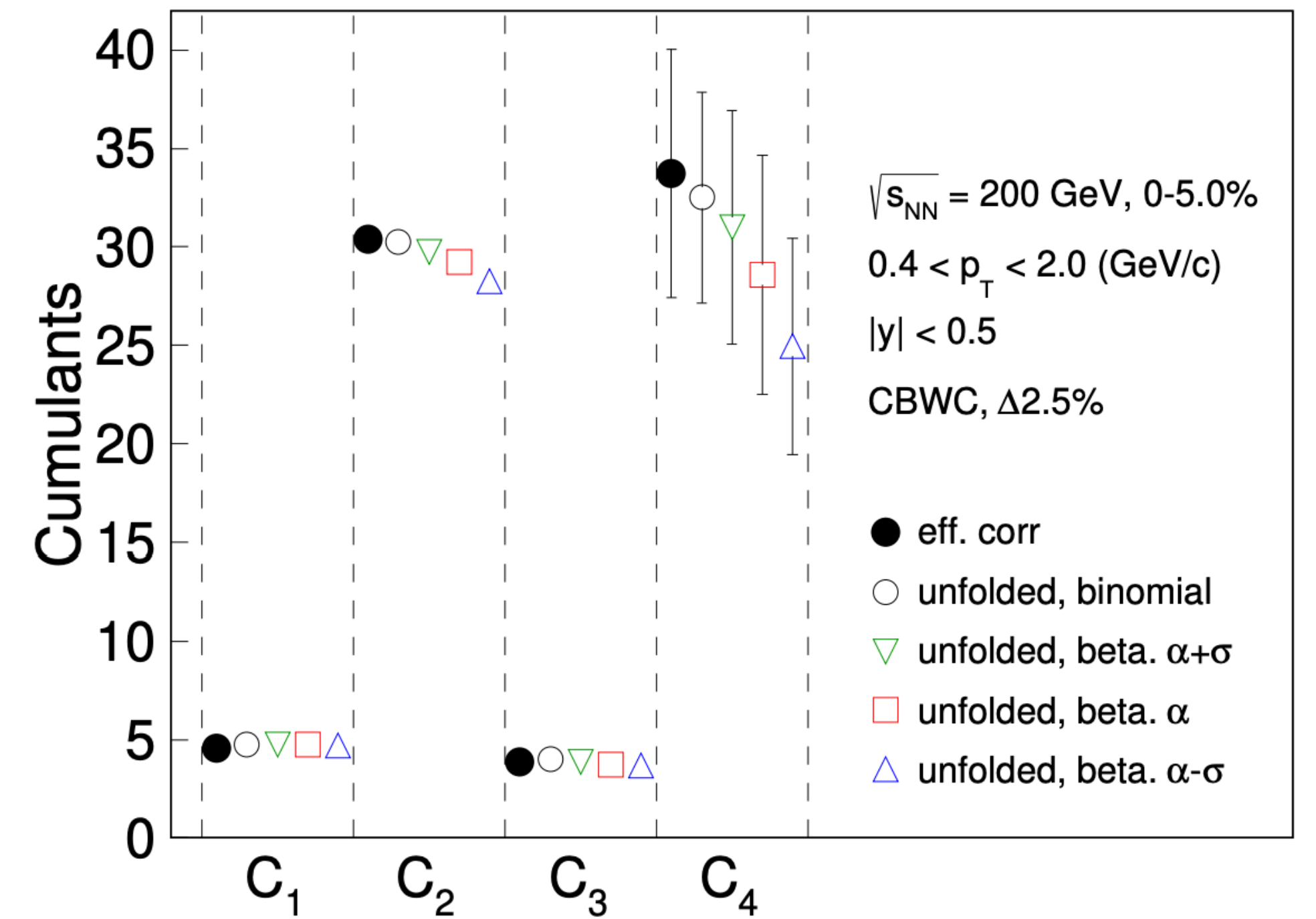
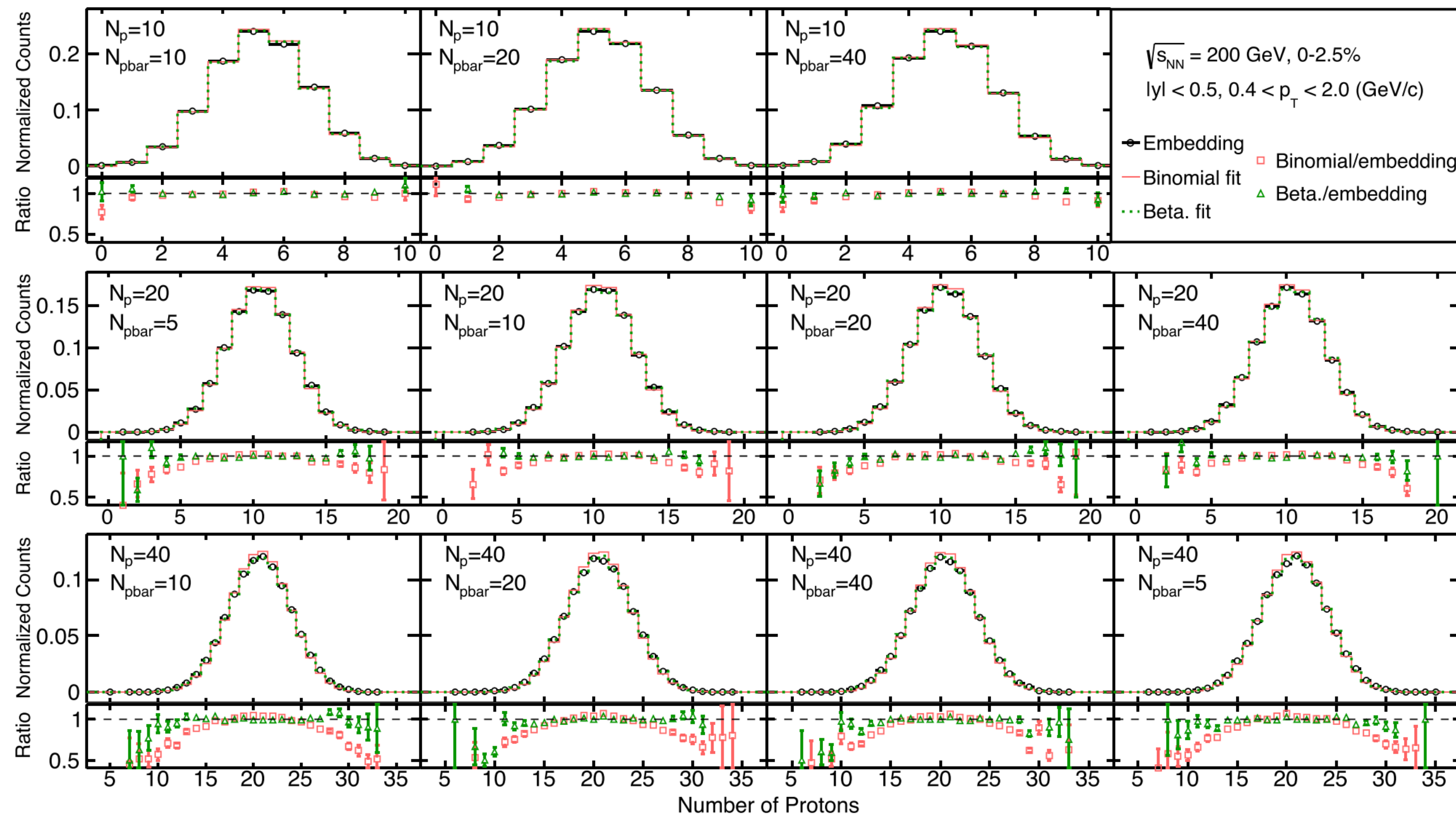


**Non-binomial efficiency correction**  
Unfolding: Esumi, Nakagawa, Nonaka, NIMA.987.164802(2021)  
Moment expansion: Nonaka, Kitazawa, Esumi, NIMA906 10-17 (2018)



# Non-binomial efficiency correction

- ✓ Efficiency distributions are studied via simulations to check the binomial assumption.
- ✓ Results of non-binomial efficiency corrections are consistent with those from conventional binomial corrections.



STAR Collaboration, PRC.104.024902(2021)

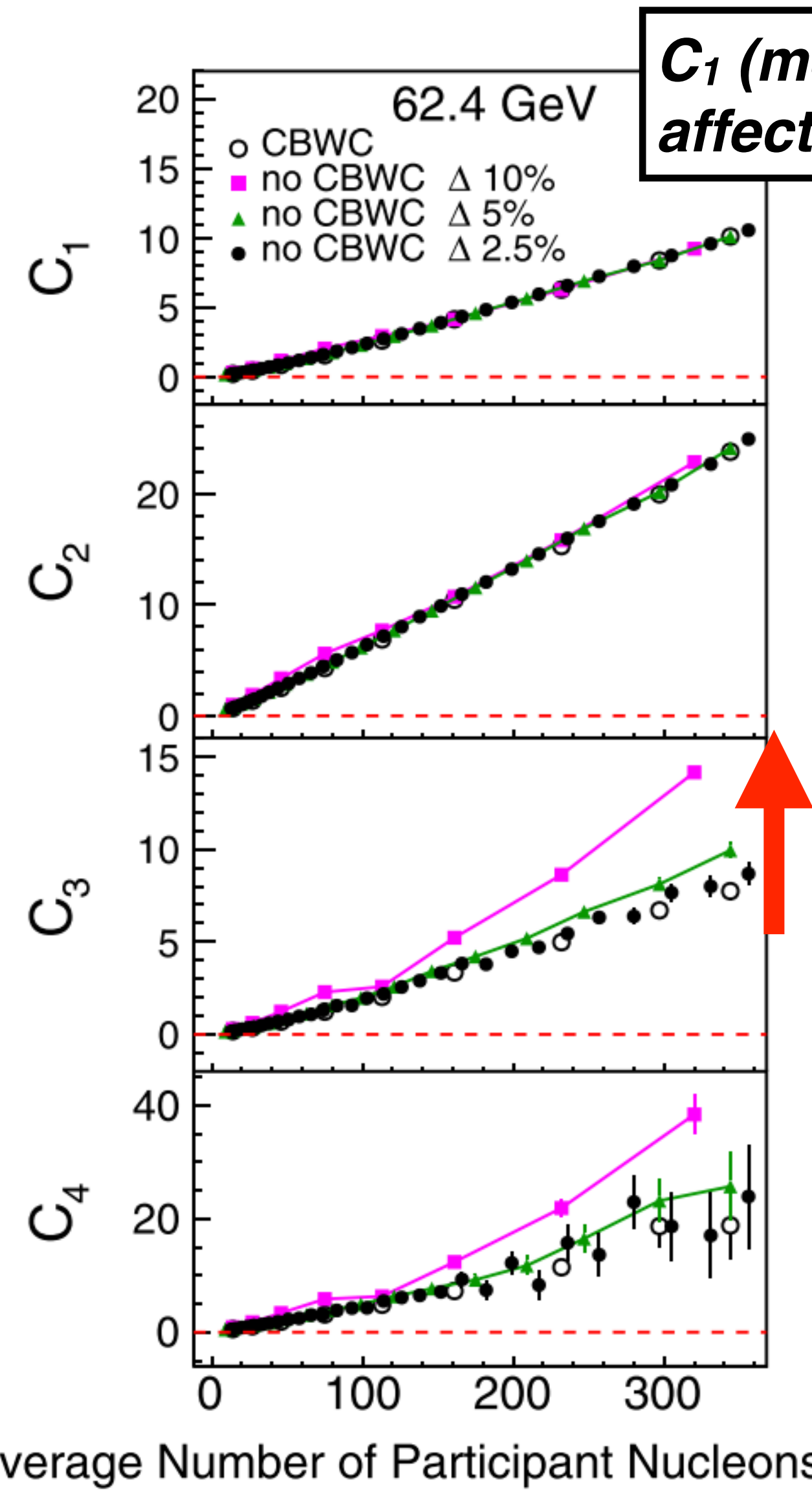
STAR Collaboration, PRL.126.092301(2021)

**Unfolding:** Esumi, Nakagawa, Nonaka, NIMA.987.164802(2021)

**Moment expansion:** Nonaka, Kitazawa, Esumi, NIMA906 10-17 (2018) Nonaka, 3rd workshop on Physics performance studies at FAIR and NICA 12



# Centrality bin width correction



$C_1$  (mean) is not affected by definition

- ✓ Final state multiplicity and initial geometry are not one-to-one corresponding  $\rightarrow$  volume fluctuation
- ✓ Higher-order cumulants are artificially enhanced by volume fluctuations.
- ✓ Data driven approach (CBWC) is applied.

X.Luo et al, J. Phys. G40,105104(2013)

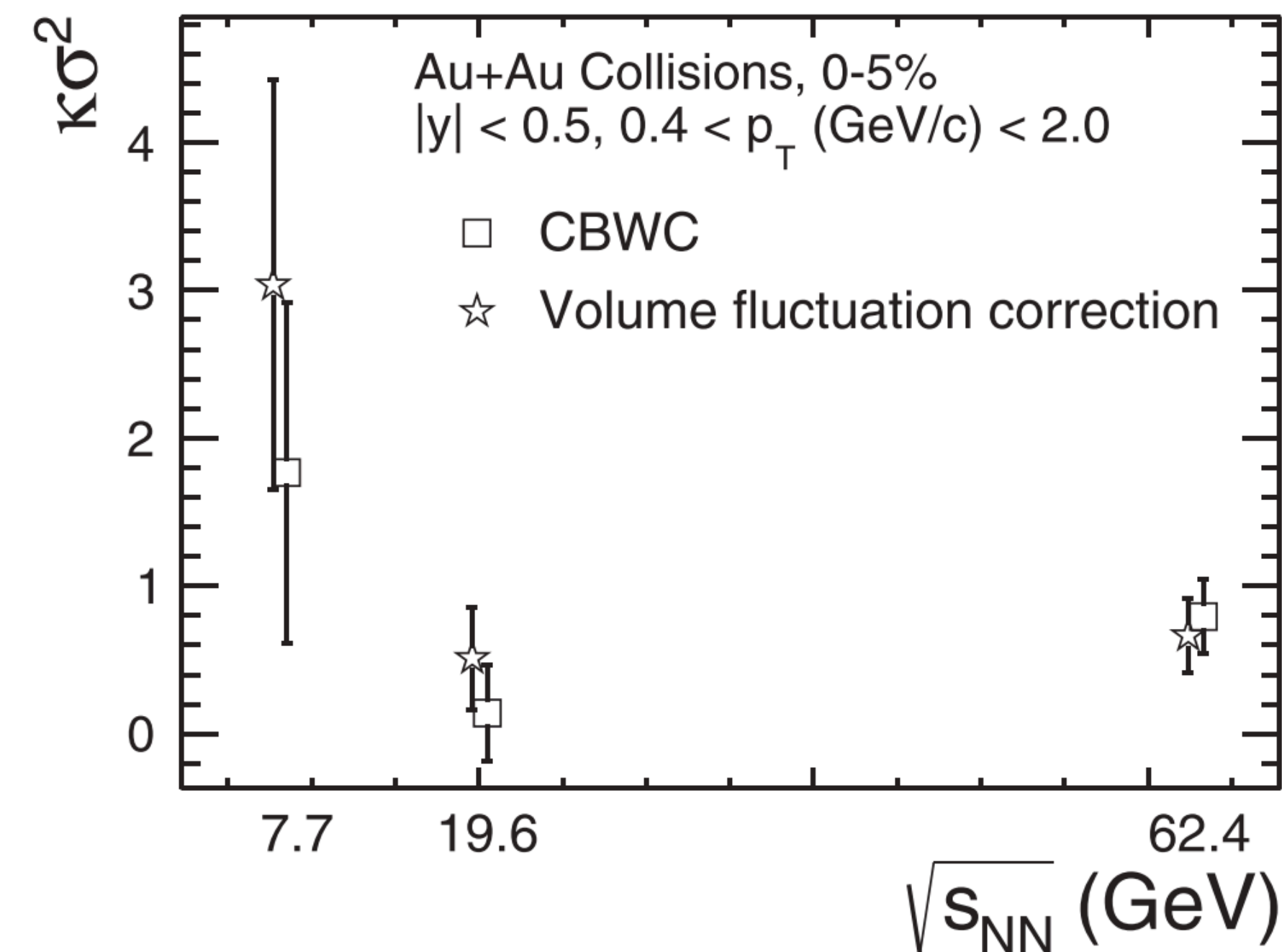
■ no CBWC  $\Delta$  10%  
 ▲ no CBWC  $\Delta$  5%  
 ● no CBWC  $\Delta$  2.5%

↑ Wider centrality bin: larger value of higher-order cumulants



# Volume fluctuation correction

- ✓ Data driven approach (CBWC) and model dependent method (VFC) are consistent with each other in BES-I data.



Measured cumulant	True cumulant	
$\kappa_1(\Delta N) = \langle N_W \rangle \kappa_1(\Delta n)$	$\langle N_W \rangle \kappa_1(\Delta n)$	
$\kappa_2(\Delta N) = \langle N_W \rangle \kappa_2(\Delta n) + \langle \Delta n \rangle^2 \kappa_2(N_W)$	$\langle N_W \rangle \kappa_2(\Delta n) + \langle \Delta n \rangle^2 \kappa_2(N_W)$	Additional terms appear from the event by event participant fluctuation
$\kappa_3(\Delta N) = \langle N_W \rangle \kappa_3(\Delta n) + 3 \langle \Delta n \rangle \kappa_2(\Delta n) \kappa_2(N_W) + \langle \Delta n \rangle^3 \kappa_3(N_W)$	$\langle N_W \rangle \kappa_3(\Delta n) + 3 \langle \Delta n \rangle \kappa_2(\Delta n) \kappa_2(N_W) + \langle \Delta n \rangle^3 \kappa_3(N_W)$	
$\kappa_4(\Delta N) = \langle N_W \rangle \kappa_4(\Delta n) + 4 \langle \Delta n \rangle \kappa_3(\Delta n) \kappa_2(N_W) + 3 \kappa_2^2(\Delta n) \kappa_2(N_W) + 6 \langle \Delta n \rangle^2 \kappa_2(\Delta n) \kappa_3(N_W) + \langle \Delta n \rangle^4 \kappa_4(N_W)$	$\langle N_W \rangle \kappa_4(\Delta n) + 4 \langle \Delta n \rangle \kappa_3(\Delta n) \kappa_2(N_W) + 3 \kappa_2^2(\Delta n) \kappa_2(N_W) + 6 \langle \Delta n \rangle^2 \kappa_2(\Delta n) \kappa_3(N_W) + \langle \Delta n \rangle^4 \kappa_4(N_W)$	

P. Braun-Munzinger, A. Rustamov, J. Stachel: *NPA.2017.01.011*

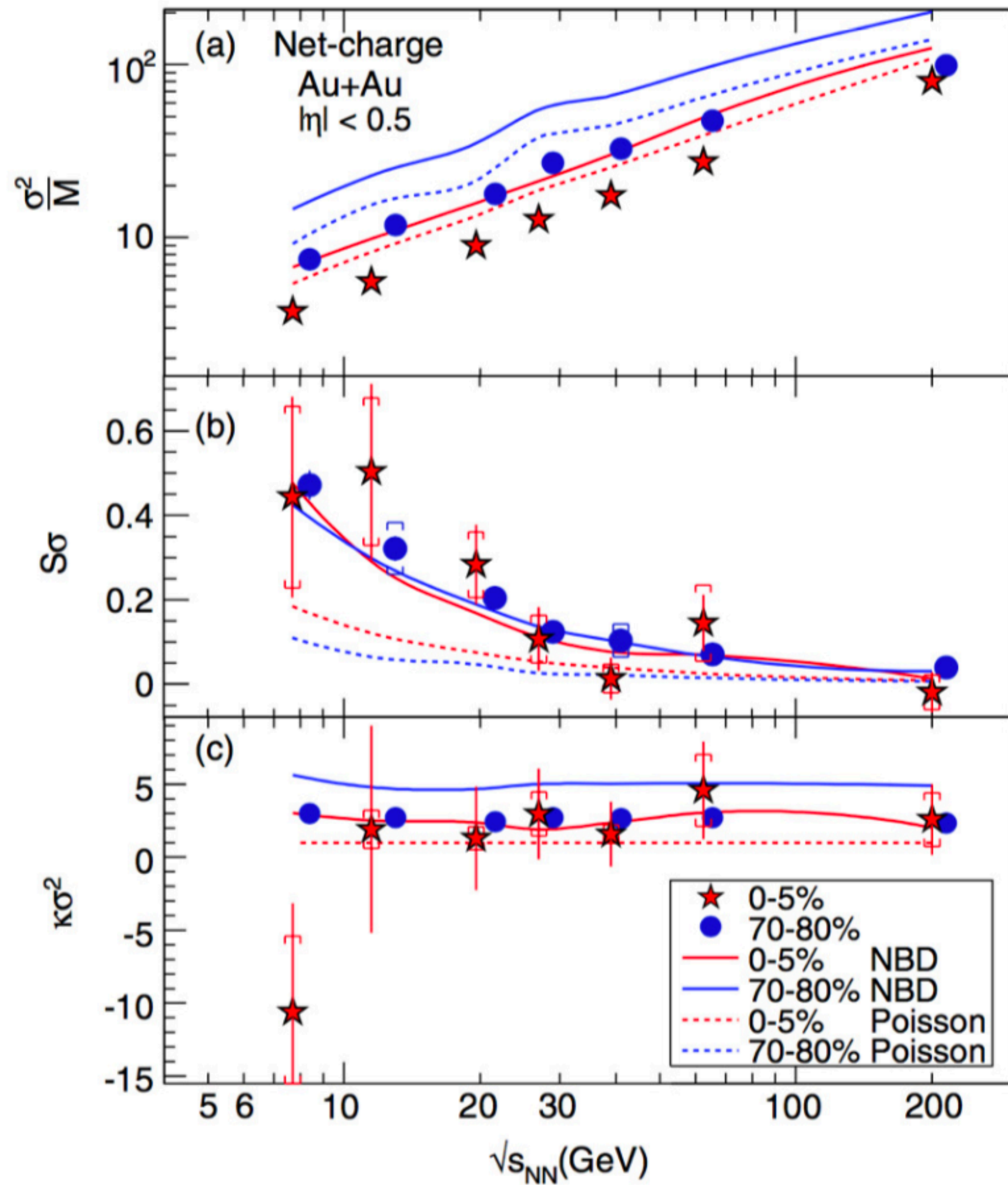
$\Delta n$  : net-proton per  $N_W$   
 $\Delta N$  : net-proton

**Taylor expansion around the mean volume employed in HADES:**  
*Phys.Rev.C 102 (2020) 2, 024914*

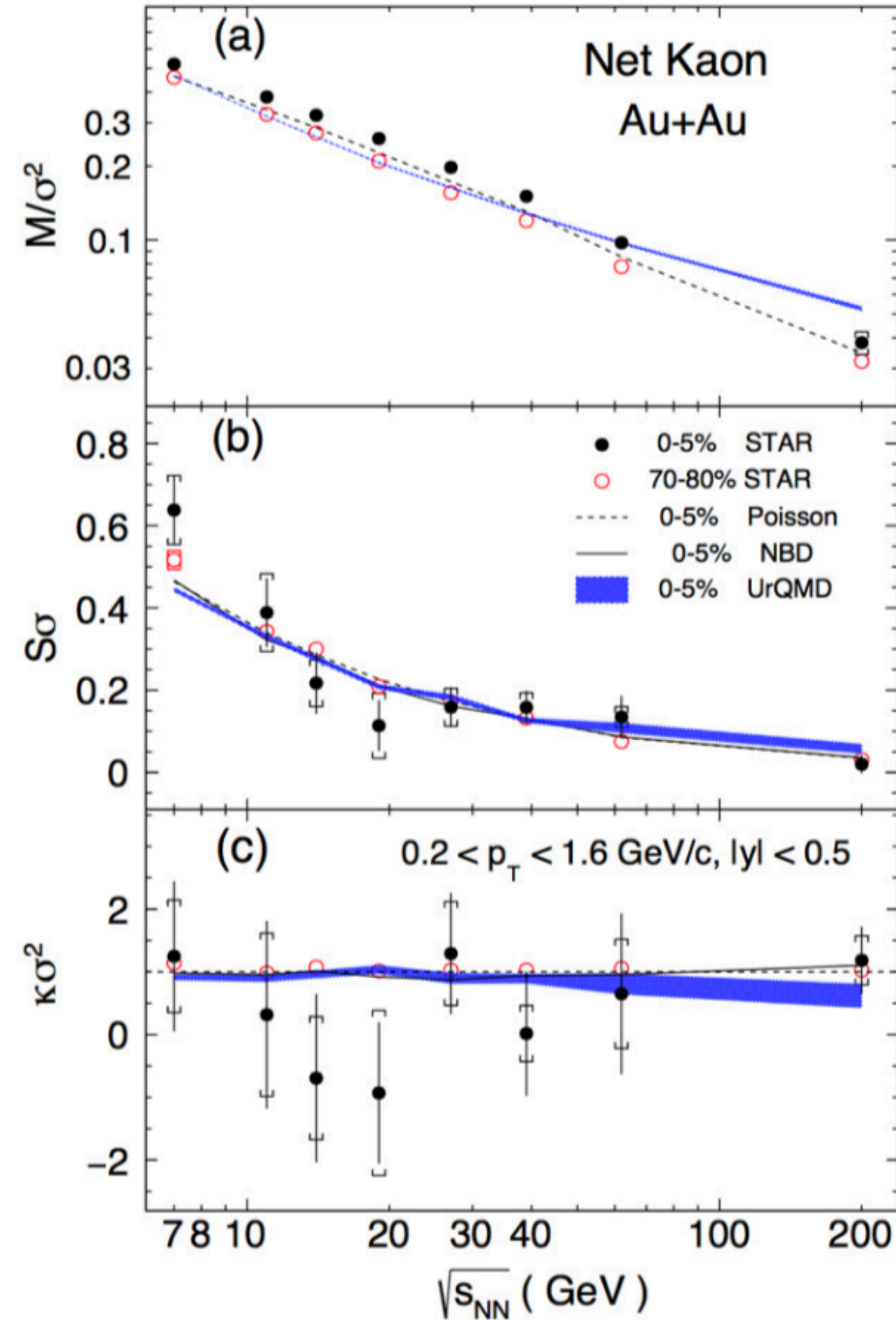


# Net-charge and net-kaon

STAR: PRL 113, 092301(2014)



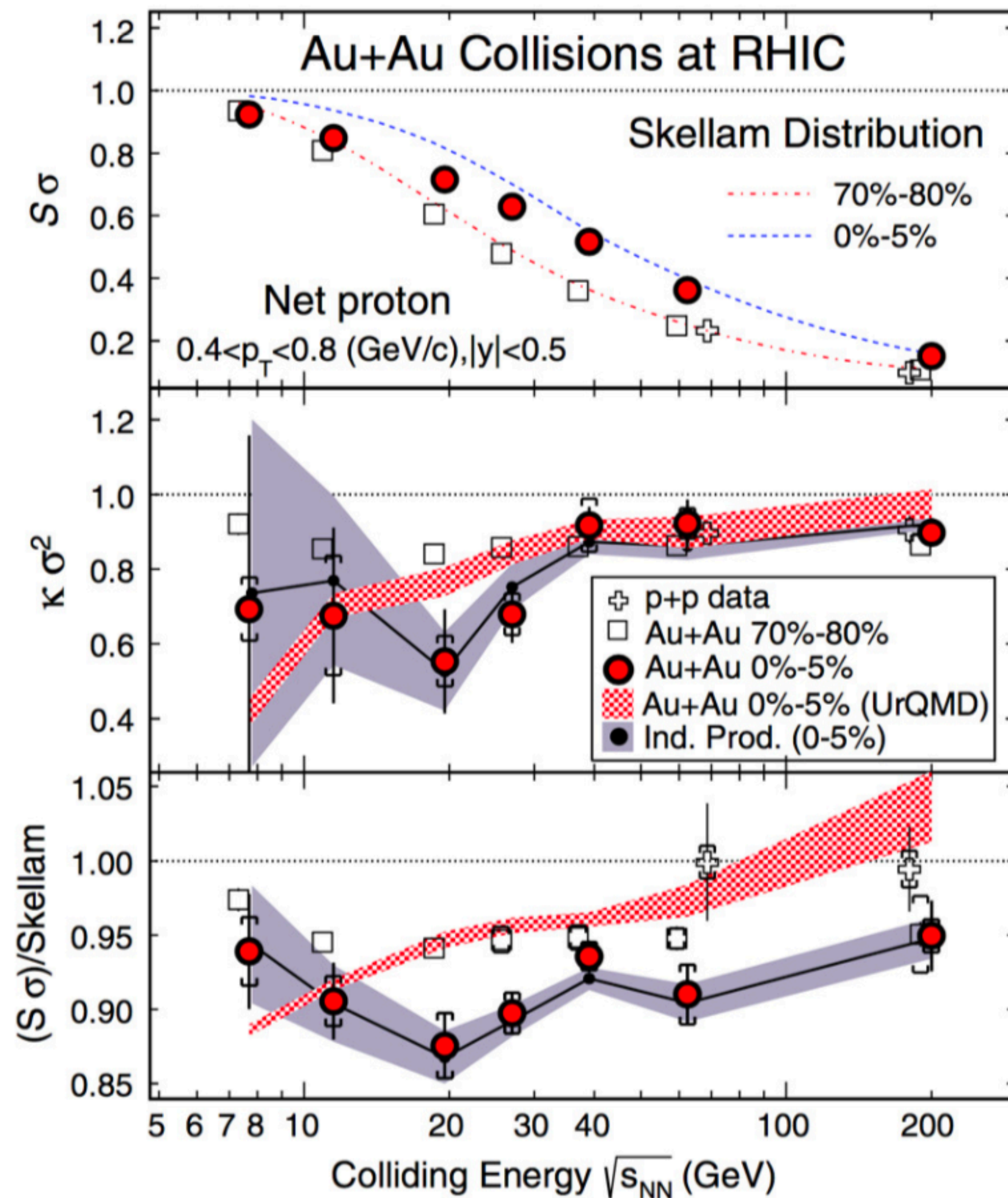
STAR: PLB, 785, 551(2018)



✓ Statistical errors depend on width of the distribution and detector efficiencies.

$$error(\kappa\sigma^2) \propto \frac{\sigma^2}{\epsilon^2} \frac{1}{\sqrt{N_{evts}}}$$

✓ Large statistical uncertainties, need more data.



- ✓ Deviation below Poisson baseline (unity).
- ✓ Both 3<sup>rd</sup>- and 4<sup>th</sup>-order fluctuations have their minima at  $\sqrt{s_{NN}} = 19.6 \text{ GeV}$ .

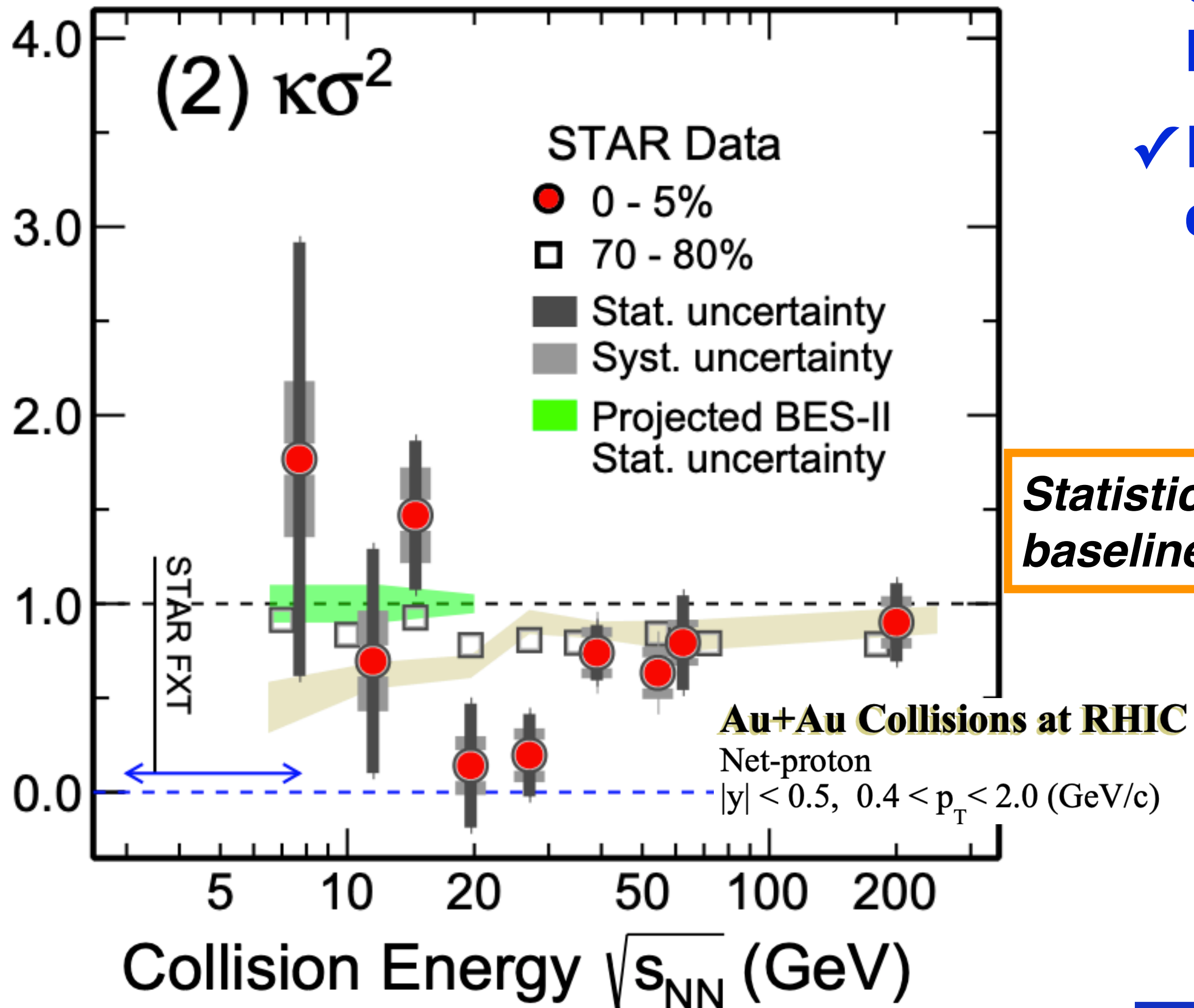
***PRL 112, 032302(2014): STAR Collaboration***



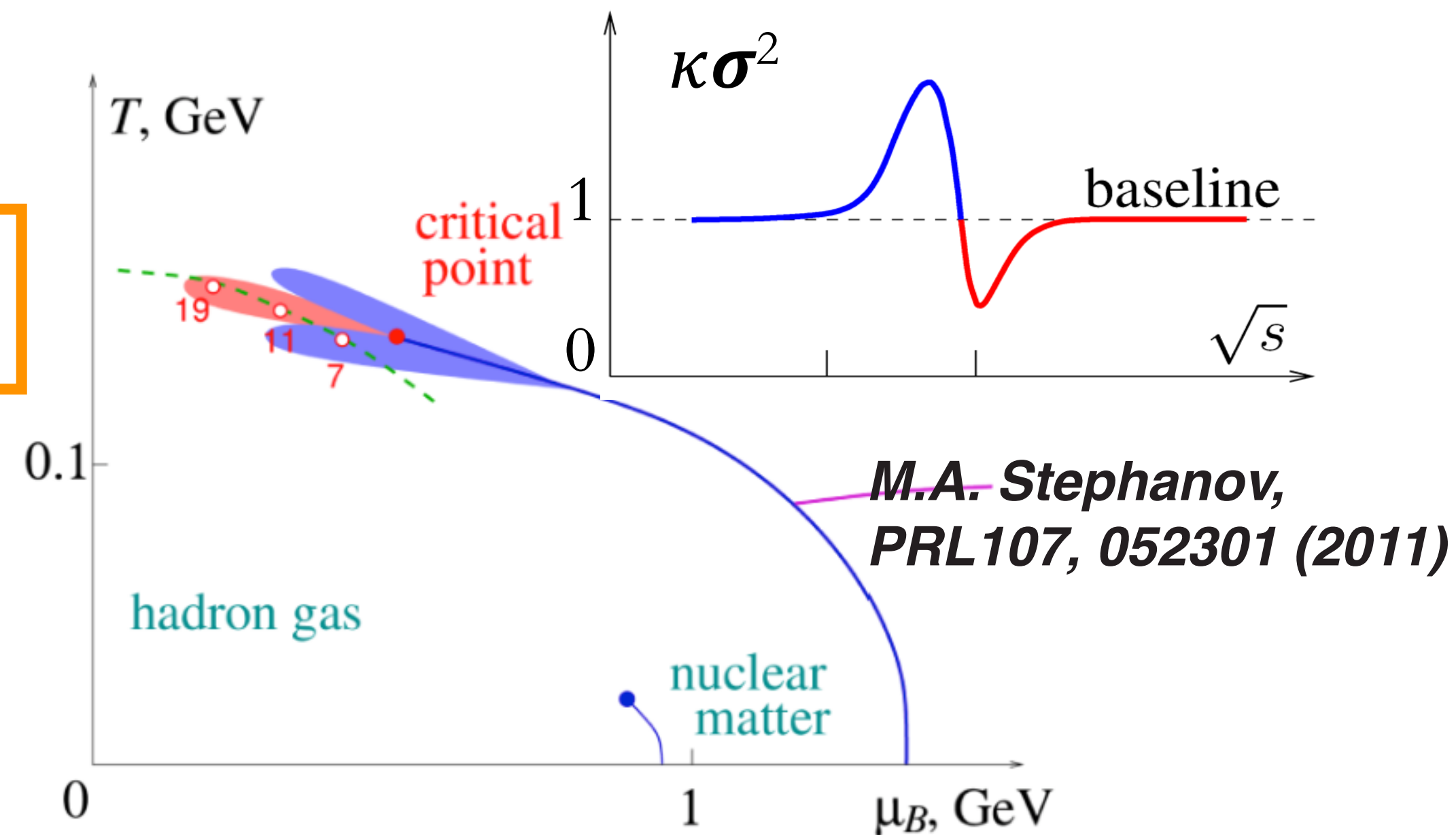


# Net-proton $C_4/C_2$ with extended $p_T$ coverage

STAR Collaboration, PRL.126.092301(2021)



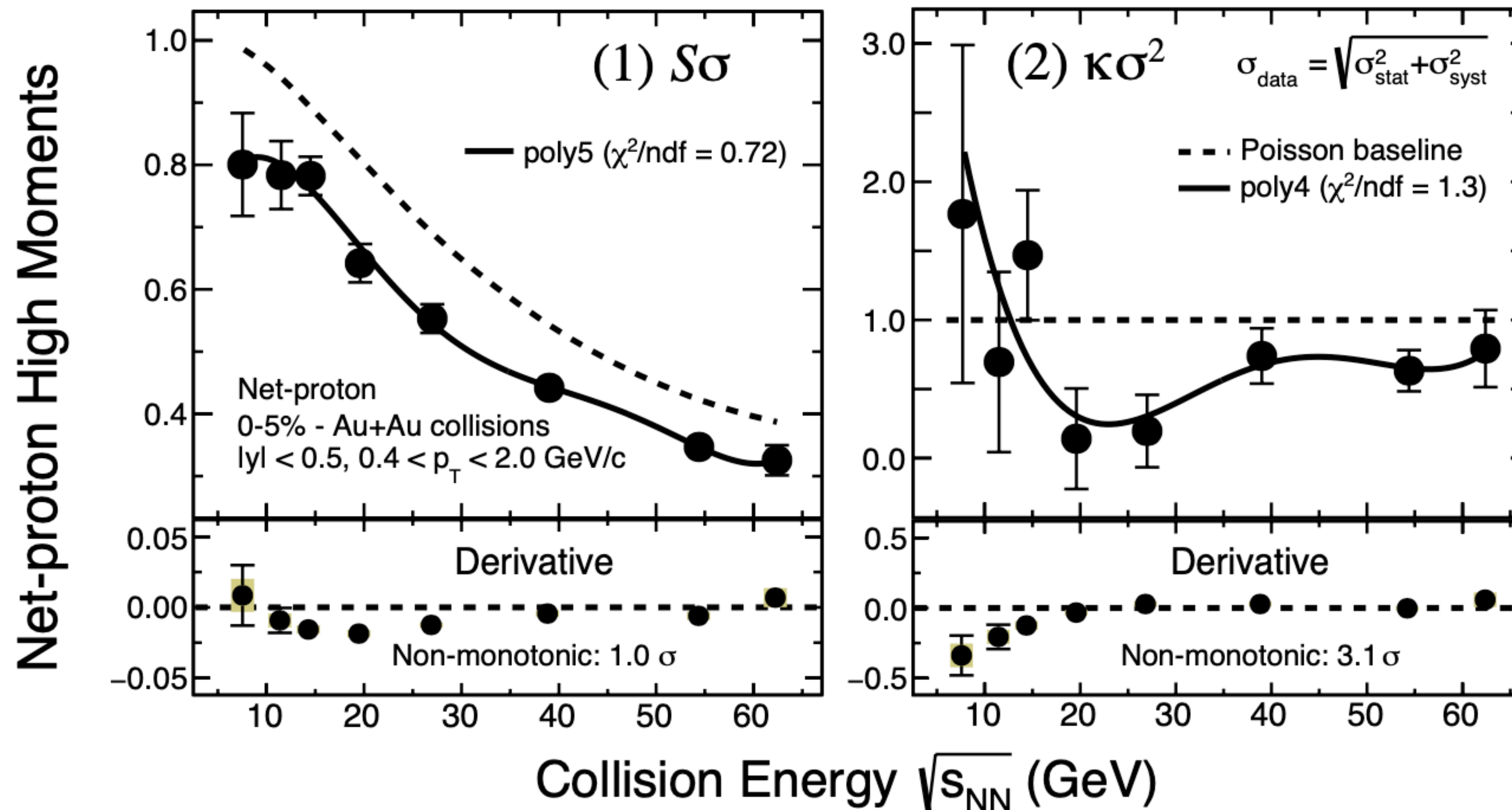
- ✓ Net-proton  $\kappa\sigma^2$  ( $C_4/C_2$ ) shows a non-monotonic behaviour. The trend is consistent with the expectation from theoretical calculations having a critical point.
- ✓ Enhancement at low beam energies cannot be explained by baryon number conservation.





# Non-monotonicity

- ✓ Polynomial fits are done varying the data point within uncertainties.
- ✓ Check the probability that at least one point of derivatives at 8 energies has different sign from others → **3.1 $\sigma$  significance of non-monotonicity for  $\kappa\sigma^2$**

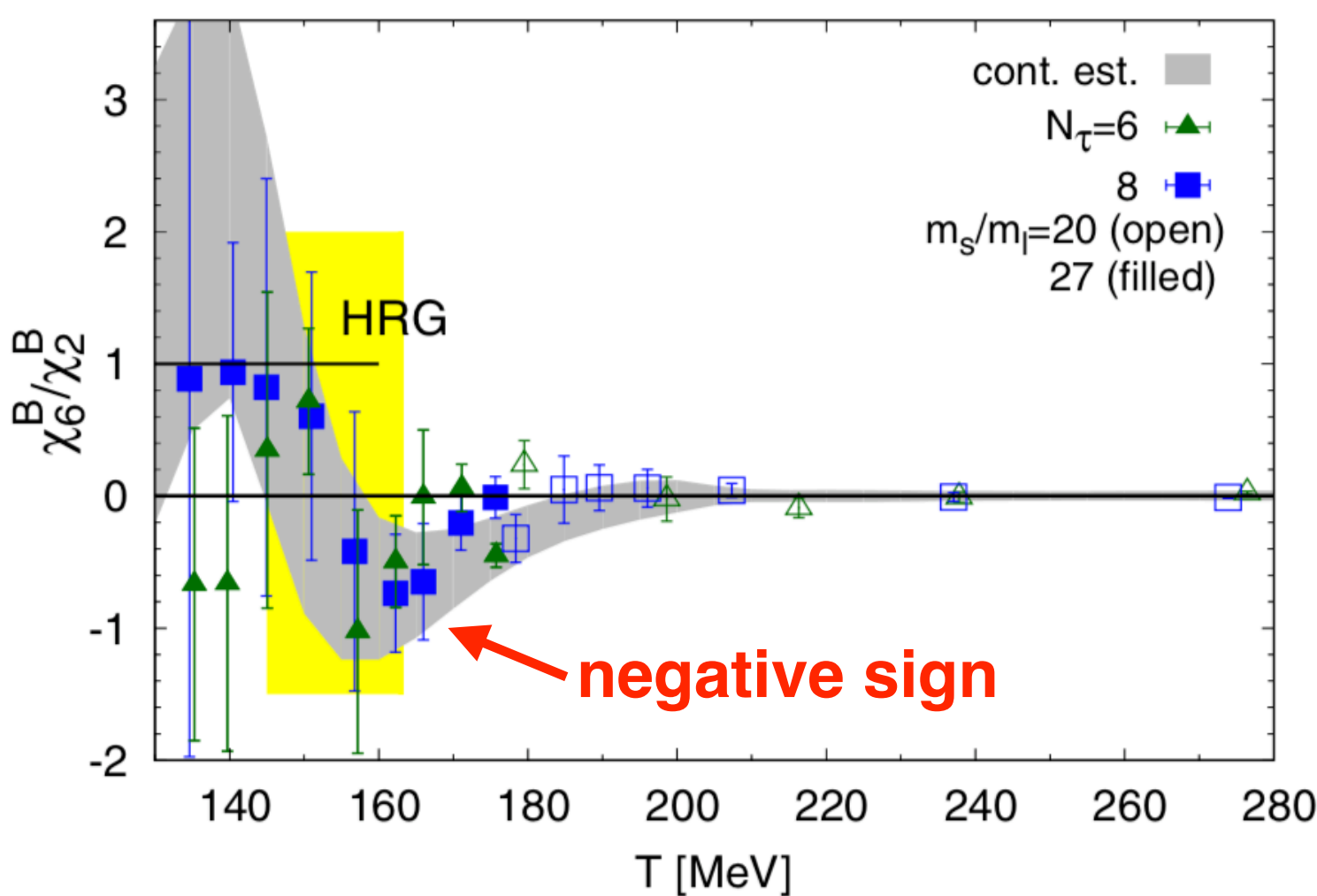




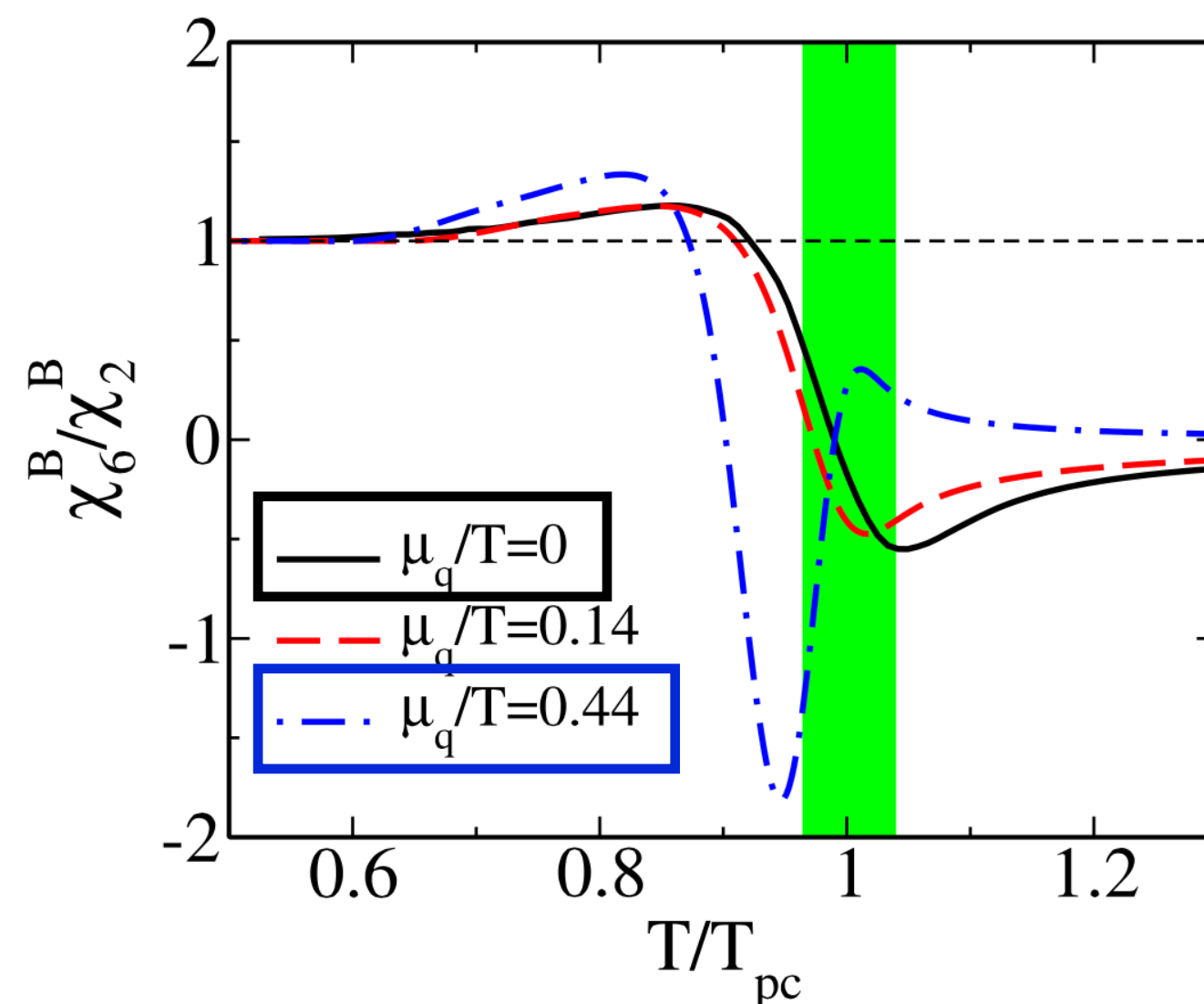
# $C_6/C_2$ for crossover search

- ✓ There isn't yet any direct experimental evidence for the smooth crossover at  $\mu_B \sim 0$ .
- ✓  $C_6/C_2 < 0$  is predicted as a signature of crossover transition.
- ✓ High-statistics data sets at  $\sqrt{s_{NN}} = 27, 54.4, \text{ and } 200 \text{ GeV}$  are analyzed to look for the **experimental signature of crossover transition**.

A. Bazavov et al,  
PhysRevD.95.054504 : LQCD



Friman et al, Eur. Phys. J. C (2011)  
71:1694 : PQM model



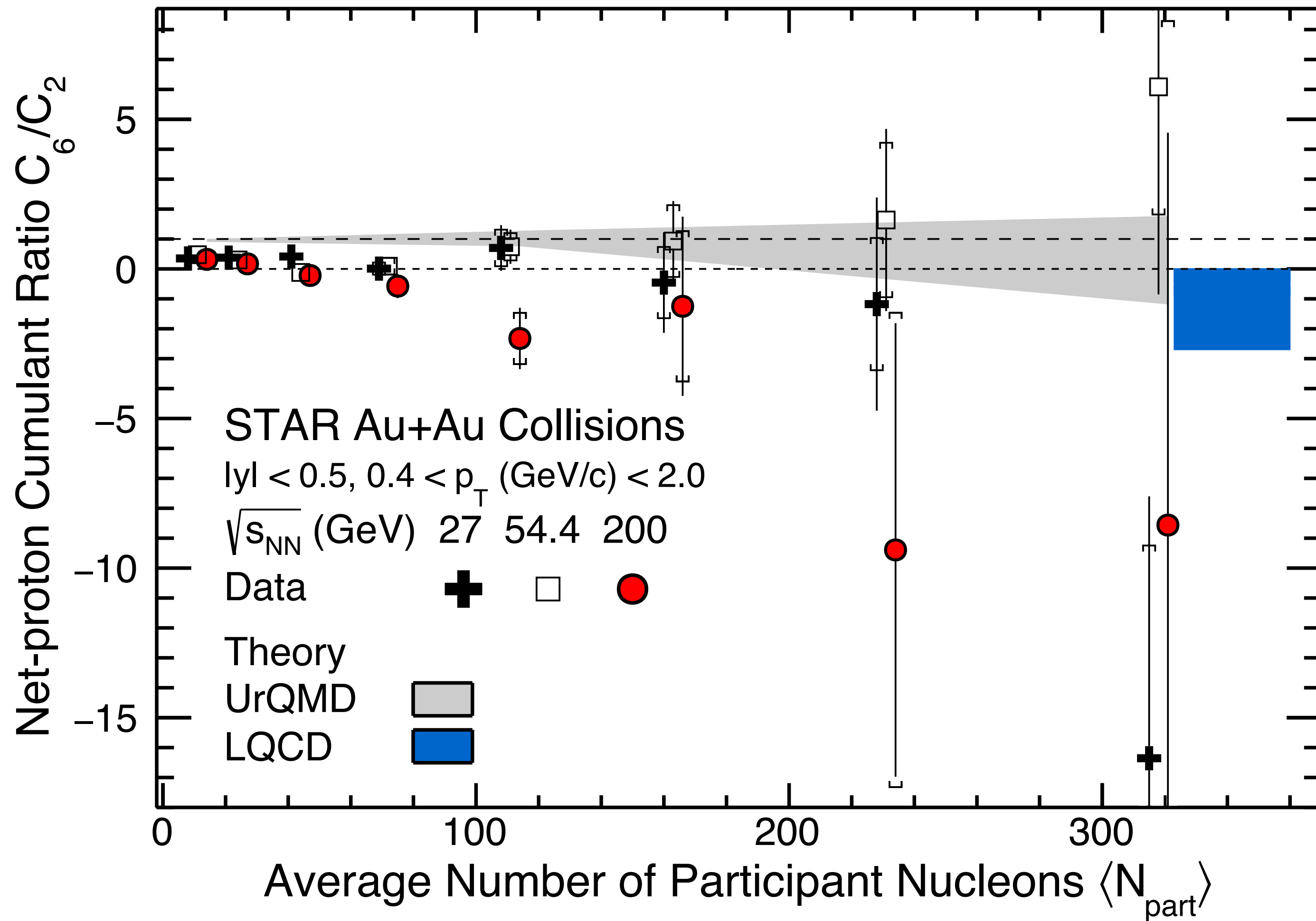
C.Schmidt, Prog.Theor.Phys.Suppl.186,563–566(2010)  
Cheng et al, Phys. Rev. D 79, 074505 (2009)  
Friman et al, Eur. Phys. J. C (2011) 71:1694

Freeze-out conditions	$\chi_4^B/\chi_2^B$	$\chi_6^B/\chi_2^B$	$\chi_4^Q/\chi_2^Q$	$\chi_6^Q/\chi_2^Q$
HRG	1	1	$\sim 2$	$\sim 10$
QCD: $T^{\text{freeze}}/T_{pc} \lesssim 0.9$	$\gtrsim 1$	$\gtrsim 1$	$\sim 2$	$\sim 10$
QCD: $T^{\text{freeze}}/T_{pc} \simeq 1$	$\sim 0.5$	$< 0$	$\sim 1$	$< 0$

**Predicted scenario for this measurement**



# Centrality dependence



- ✓  $C_6/C_2$  values are progressively negative from peripheral to central collisions at 200 GeV, which is consistent with LQCD calculations.
- ✓ Could suggest a smooth crossover transition at top RHIC energy.

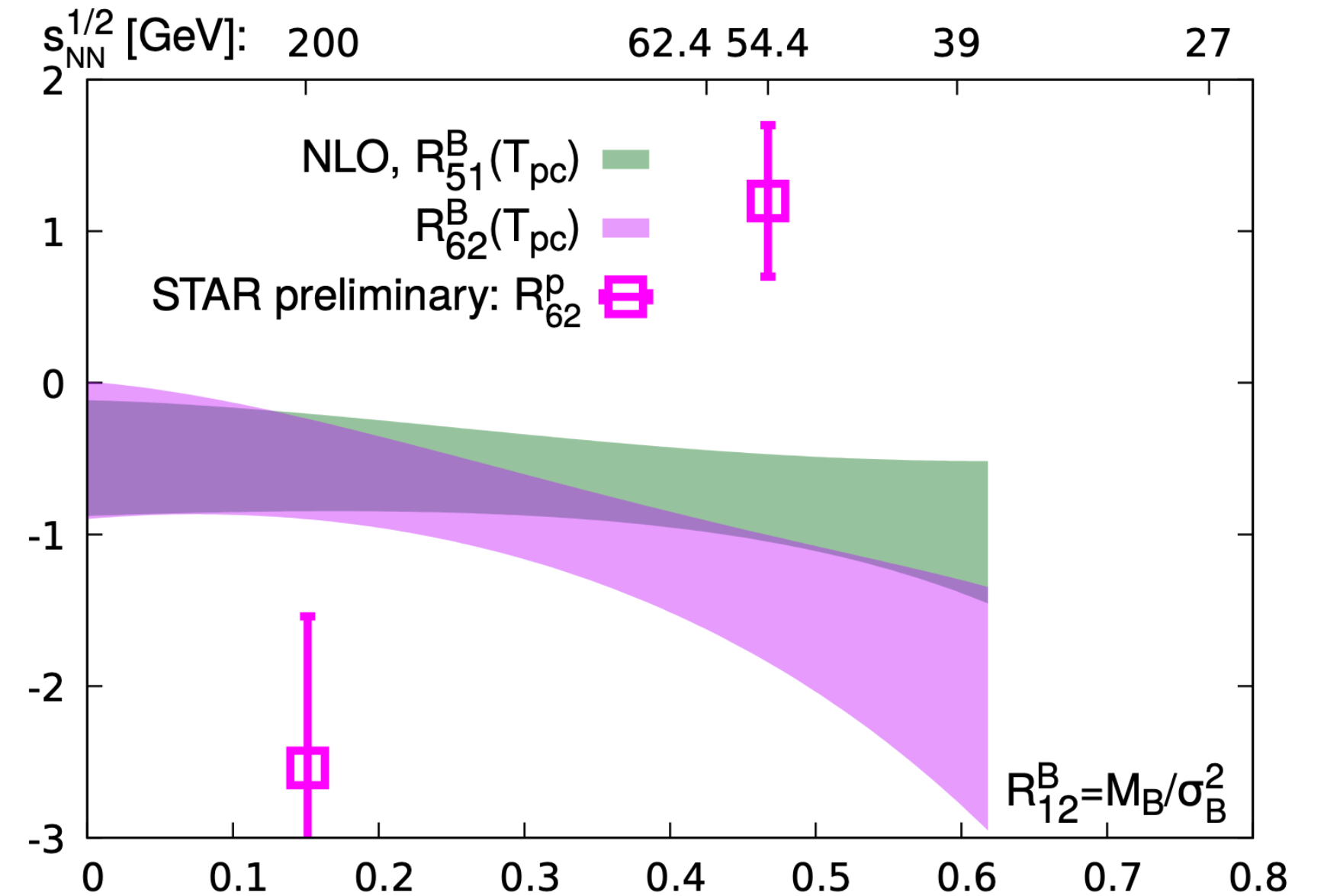
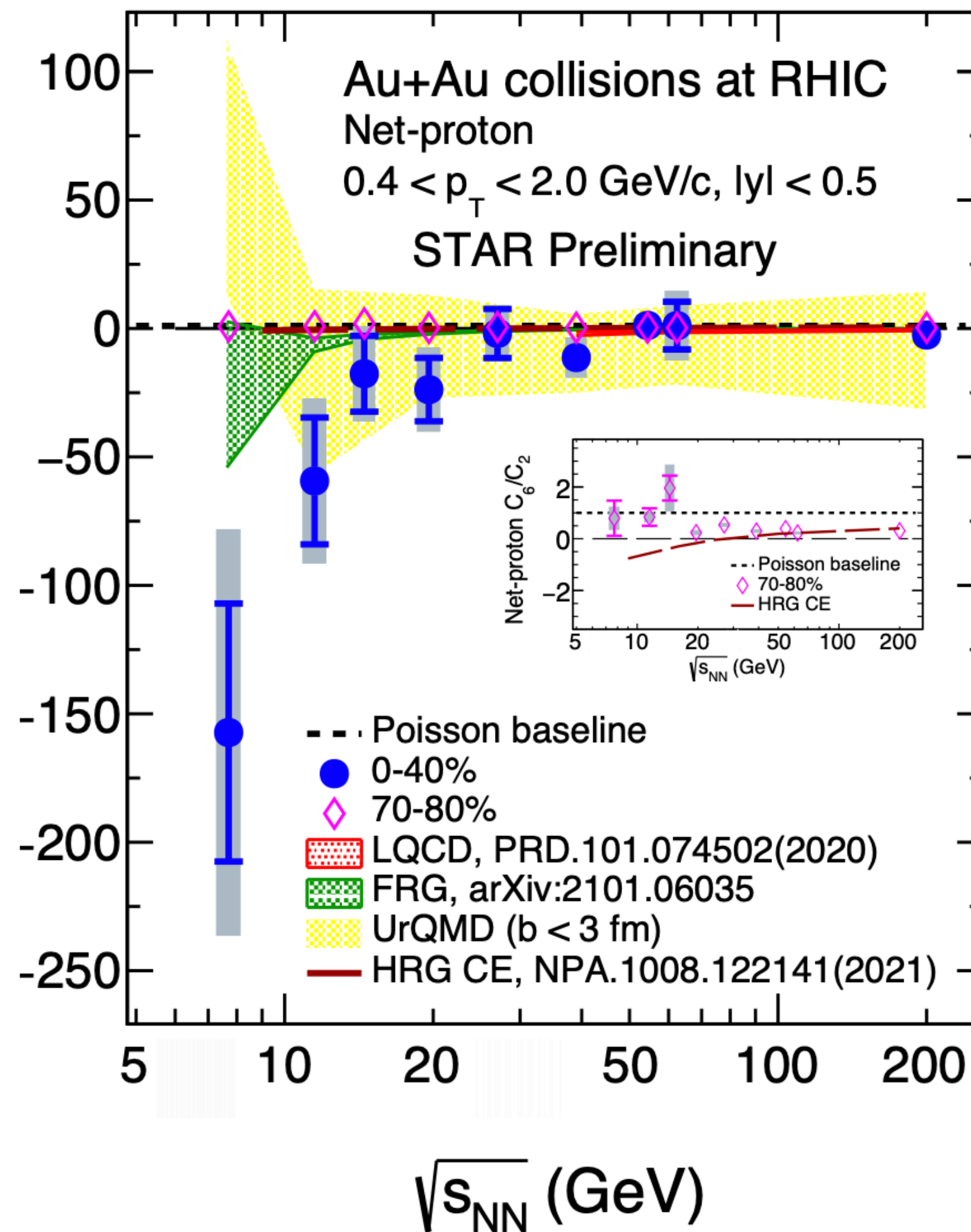
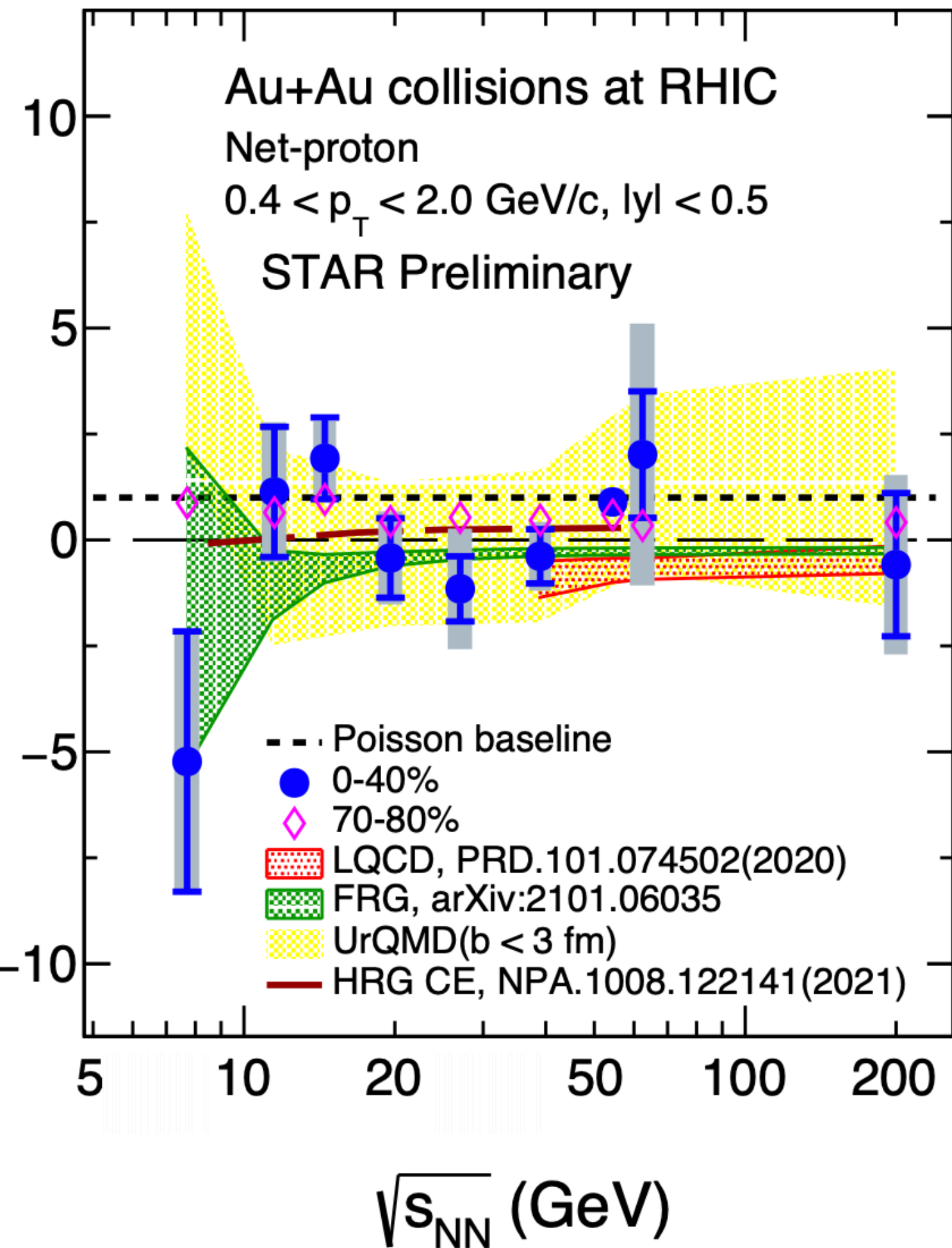


# Energy dependence of $C_5/C_1$ and $C_6/C_2$

$C_5/C_1$

$C_6/C_2$

- ✓ Weak collision energy dependence observed for 0-40% centrality.
- ✓ Deviations from zero at a level of  $< 2\sigma$  observed for 0-40% centrality.

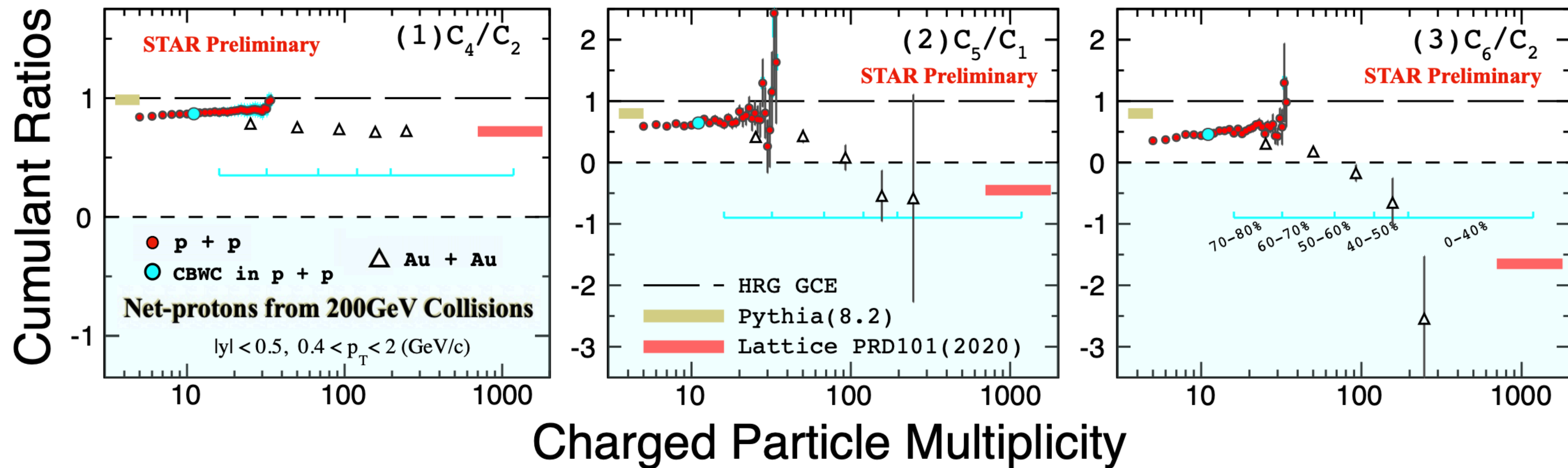


Bazavov et al., Phys.Rev.D101,074502 (2020)



# System size dependence

- ✓  $C_5/C_1$  and  $C_6/C_2$  are positive for p+p collisions, while negative for central Au+Au collisions.
- ✓ Lattice calculations imply chiral phase transition in the thermalized QCD matter, which is not the case in 200 GeV p+p collisions.



- Only statistical errors are shown for Au+Au results
- Efficiency is not corrected for x-axis

STAR Collaboration,  
 PRC.104.024902(2021)

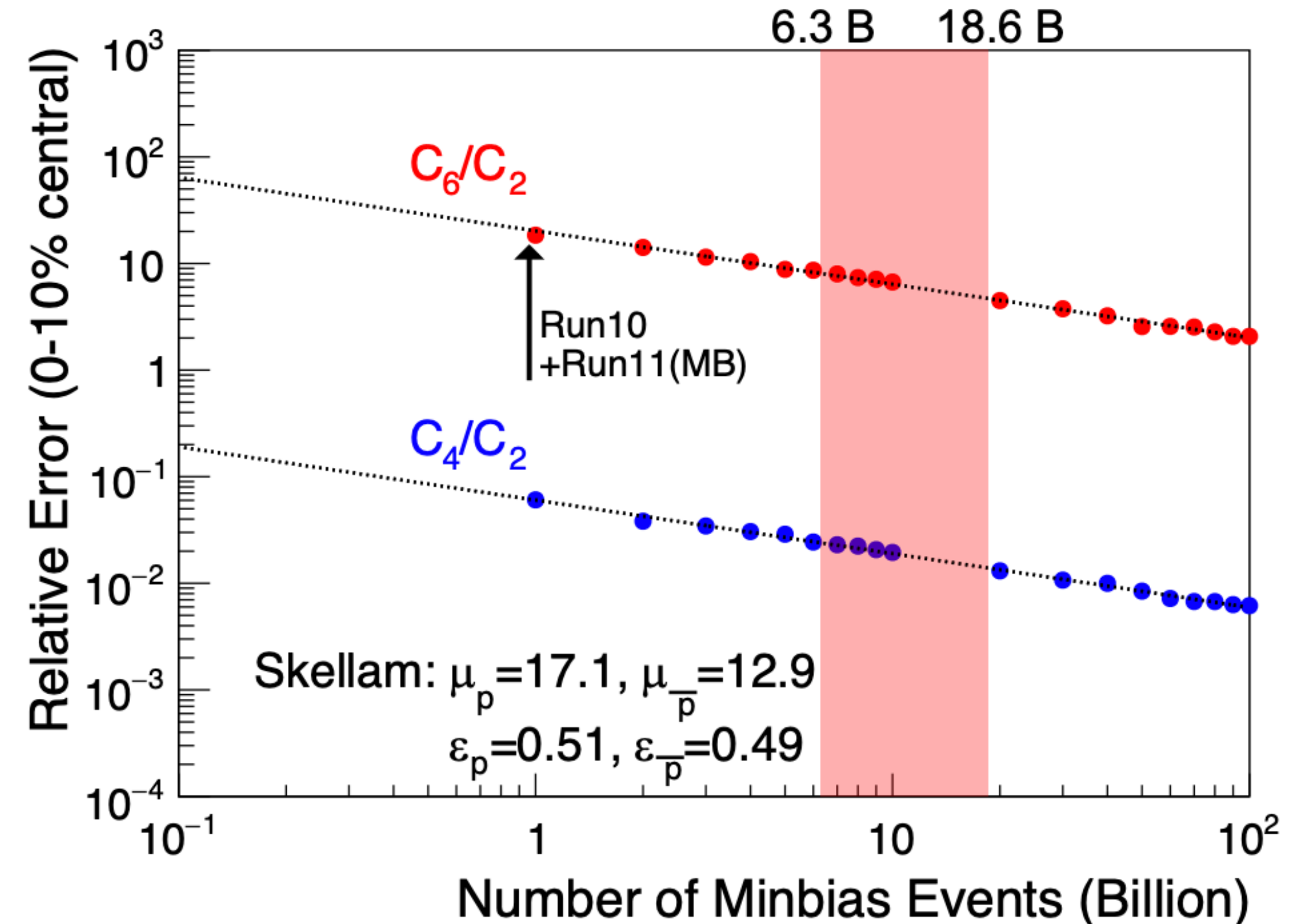
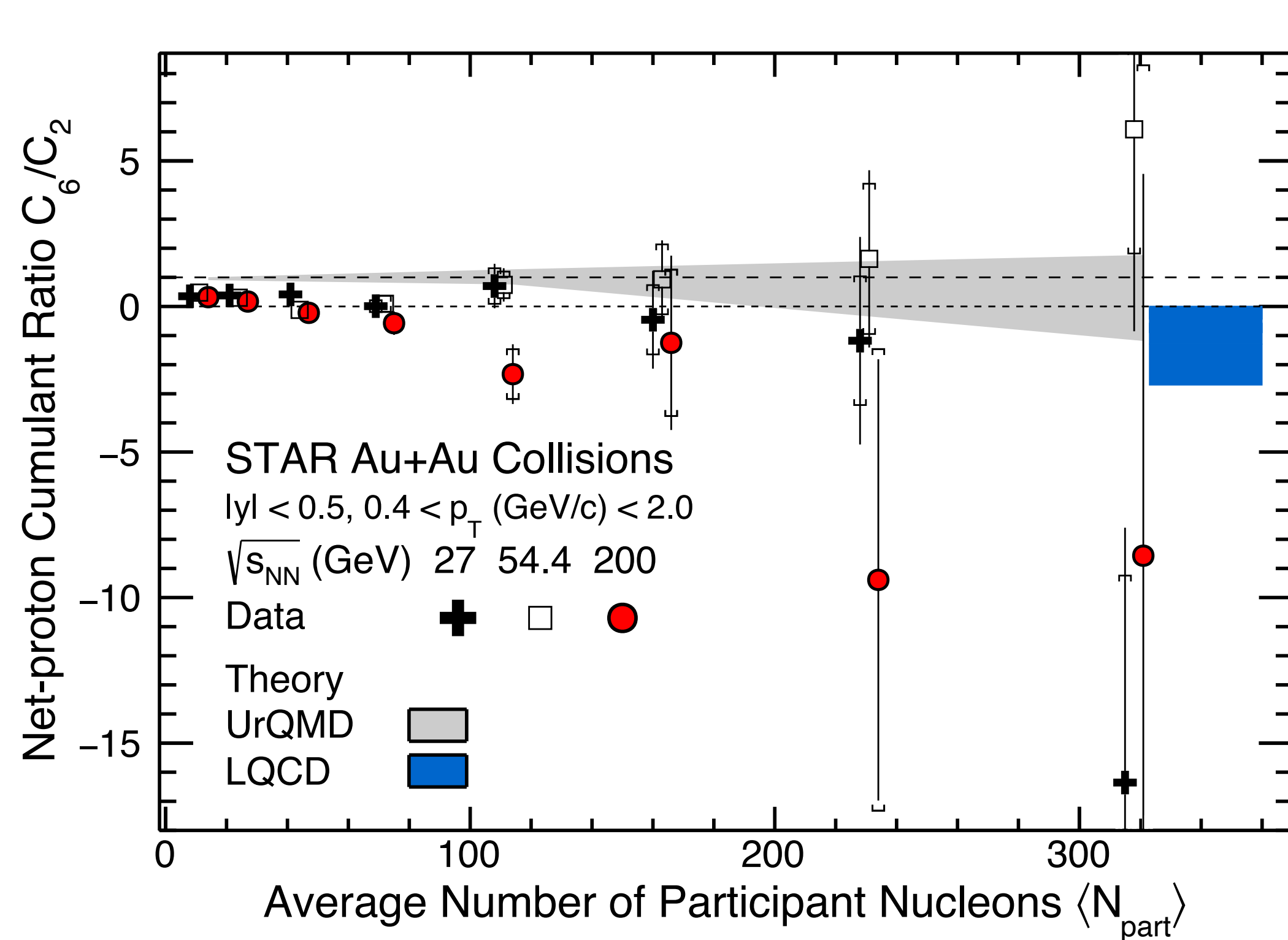
LQCD : Phys. Rev. D 101, 074502 (2020)

STAR Collaboration,  
 Nuclear Physics A, 1005,  
 121882 (2021)



# Future perspective @small $\mu_B$

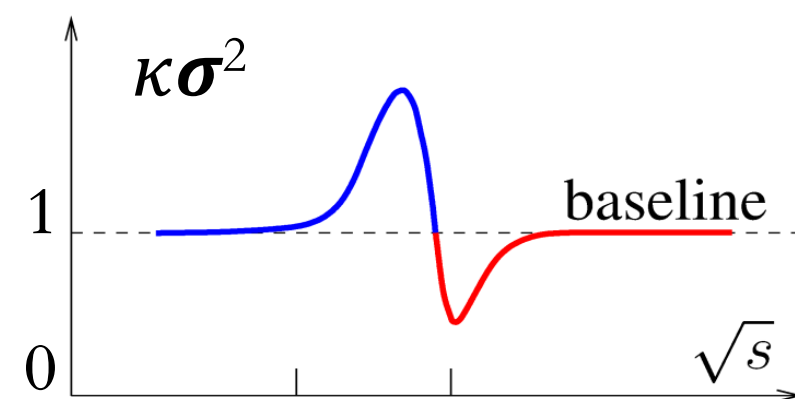
- ✓ ~18 times event statistics in Au+Au collisions at 200 GeV for 2023-2025.
- ✓ More definitive signature of smooth crossover at  $\mu_B \sim 20$  MeV.



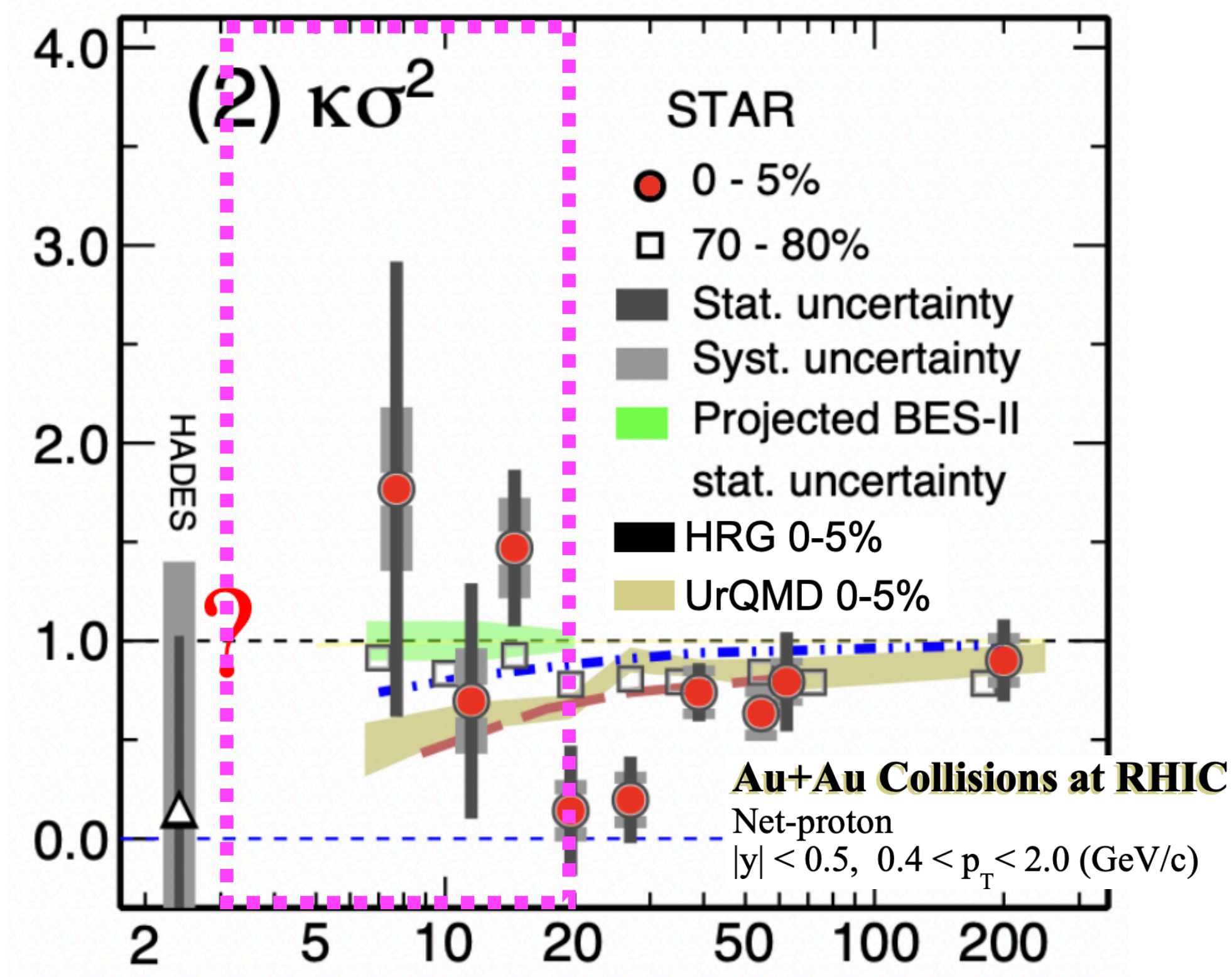
**SN0773, "The STAR Beam Use Request for Run-22 and data taking in 2023-2025"**



# Future perspective



M.A. Stephanov,  
PRL107, 052301 (2011)



Collision Energy  $\sqrt{s_{NN}}$  (GeV)

STAR Collaboration, PRL.126.092301(2021)

HADES Collaboration, PRC.102.024914(2020)

**BESII  $7.7 < \sqrt{s_{NN}}$  GeV < 19.6:**

10-20 times larger statistics than BES-I have been successfully collected.

**HADES  $\sqrt{s_{NN}} = 2.4$  GeV:**

Consistent with zero, drop from STAR 7.7 GeV. HADES, Phys.Rev.C 102 (2020) 2, 024914

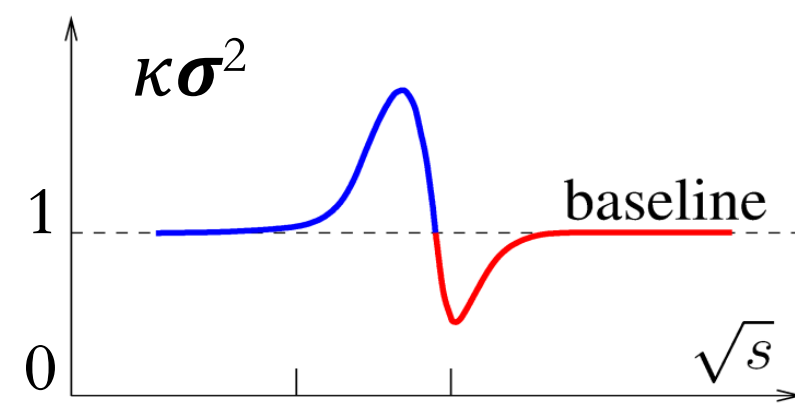
**FXT  $3.0 < \sqrt{s_{NN}}$  GeV < 13.7:**

Bridging the gap between BES-II and HADES

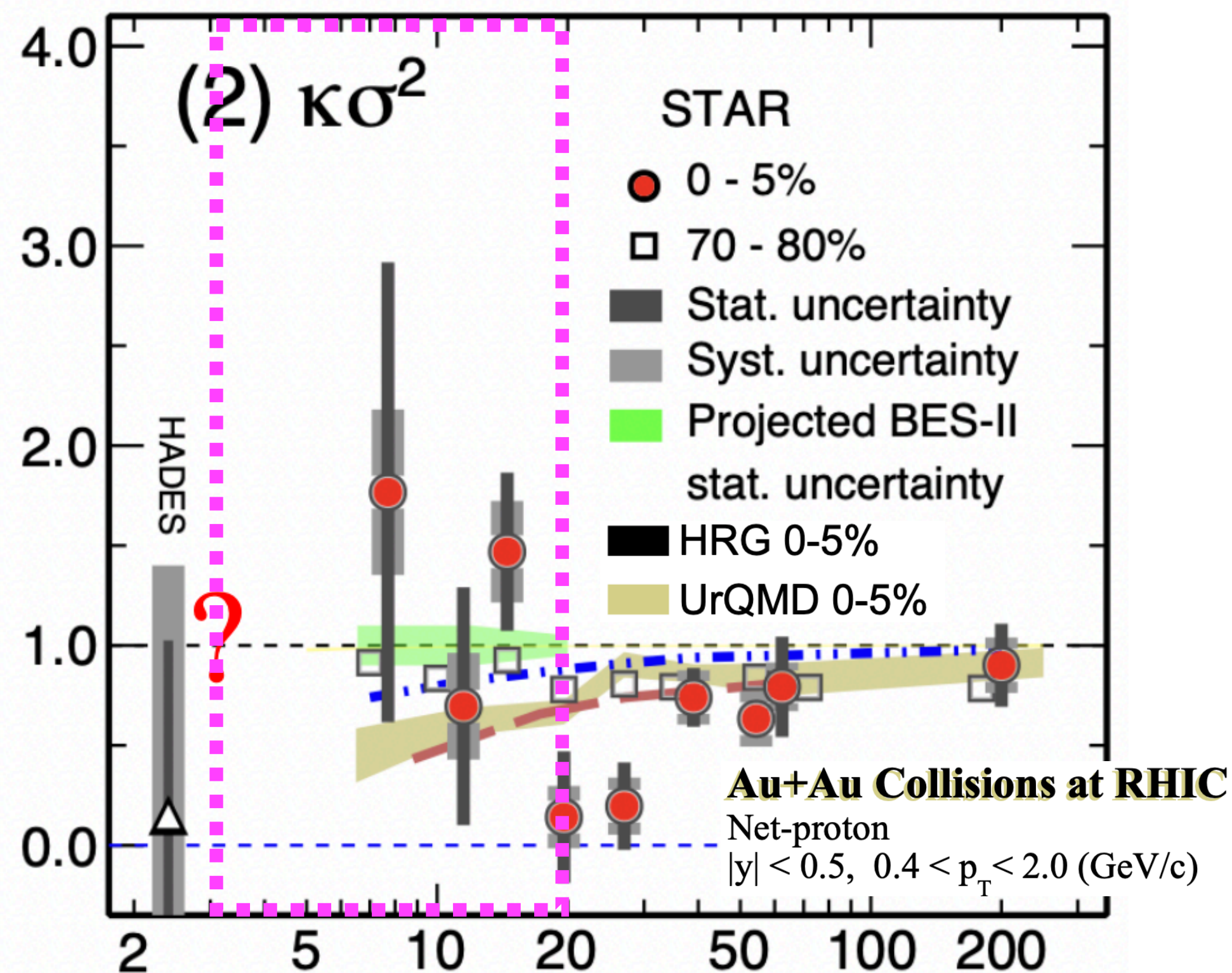




# Future perspective



M.A. Stephanov,  
PRL107, 052301 (2011)



Collision Energy  $\sqrt{s_{NN}}$  (GeV)

STAR Collaboration, PRL.126.092301(2021)

HADES Collaboration, PRC.102.024914(2020)

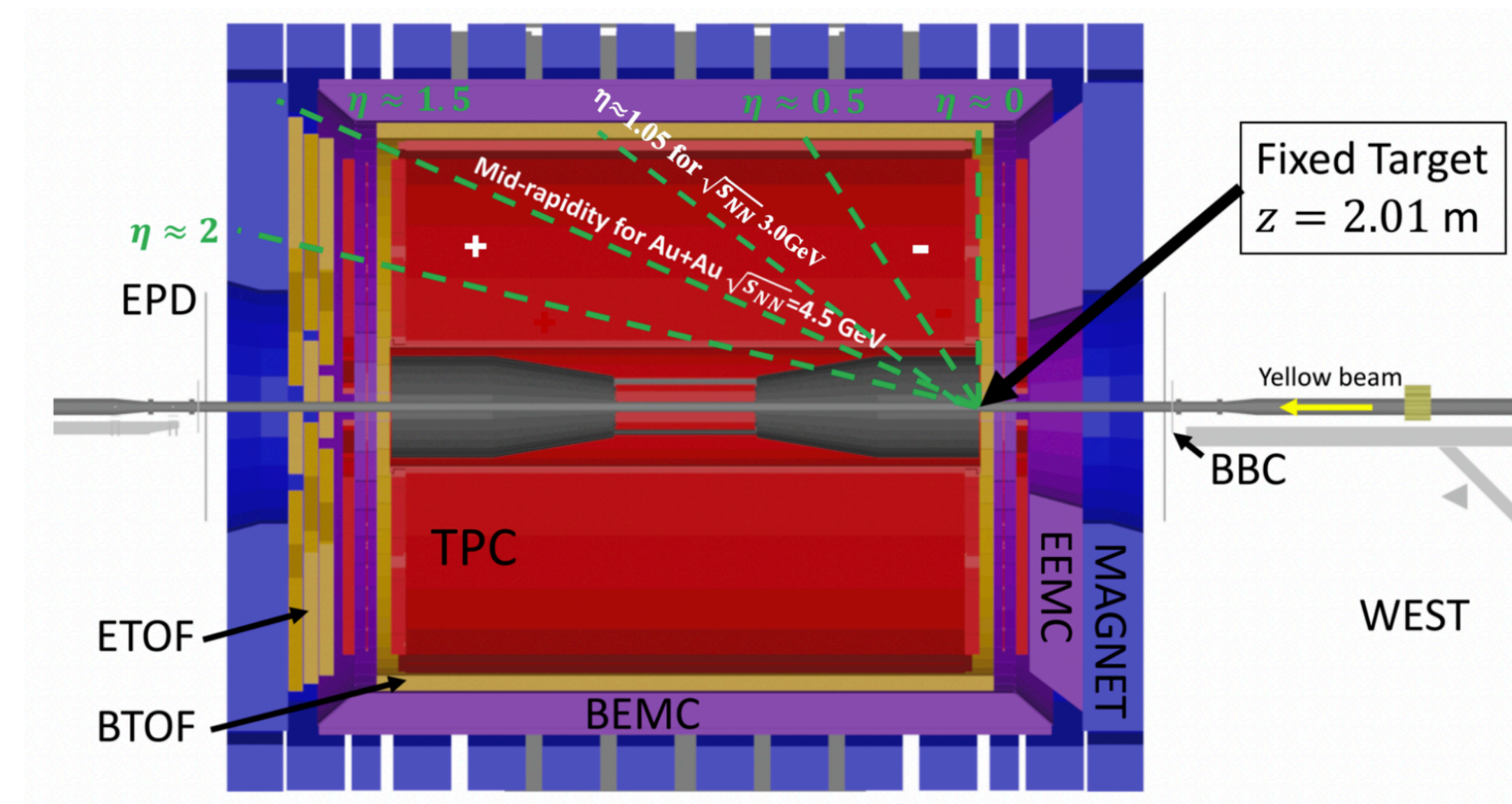
✓ 10-20 times larger statistics than BES-I have been successfully collected.

$\sqrt{s_{NN}}$ (GeV)	Beam Energy (GeV/nucleon)	Collider or Fixed Target	$y_{center\ of\ mass}$	$\mu_B$ (MeV)	Run Time (days)	No. Events Collected (Request)	Date Collected
200	100	C	0	25	2.0	138 M (140 M)	Run-19
27	13.5	C	0	156	24	555 M (700 M)	Run-18
19.6	9.8	C	0	206	36	582 M (400 M)	Run-19
17.3	8.65	C	0	230	14	256 M (250 M)	Run-21
14.6	7.3	C	0	262	60	324 M (310 M)	Run-19
13.7	100	FXT	2.69	276	0.5	52 M (50 M)	Run-21
11.5	5.75	C	0	316	54	235 M (230 M)	Run-20
11.5	70	FXT	2.51	316	0.5	50 M (50 M)	Run-21
9.2	4.59	C	0	372	102	162 M (160 M)	Run-20+20b
9.2	44.5	FXT	2.28	372	0.5	50 M (50 M)	Run-21
7.7	3.85	C	0	420	90	100 M (100 M)	Run-21
7.7	31.2	FXT	2.10	420	0.5+1.0+ scattered	50 M + 112 M + 100 M (100 M)	Run-19+20+21
7.2	26.5	FXT	2.02	443	2+Parasitic with CEC	155 M + 317 M	Run-18+20
6.2	19.5	FXT	1.87	487	1.4	118 M (100 M)	Run-20
5.2	13.5	FXT	1.68	541	1.0	103 M (100 M)	Run-20
4.5	9.8	FXT	1.52	589	0.9	108 M (100 M)	Run-20
3.9	7.3	FXT	1.37	633	1.1	117 M (100 M)	Run-20
3.5	5.75	FXT	1.25	666	0.9	116 M (100 M)	Run-20
3.2	4.59	FXT	1.13	699	2.0	200 M (200 M)	Run-19
3.0	3.85	FXT	1.05	721	4.6	259 M -> 2B(100 M -> 2B)	Run-18+21



# Fixed-target setup

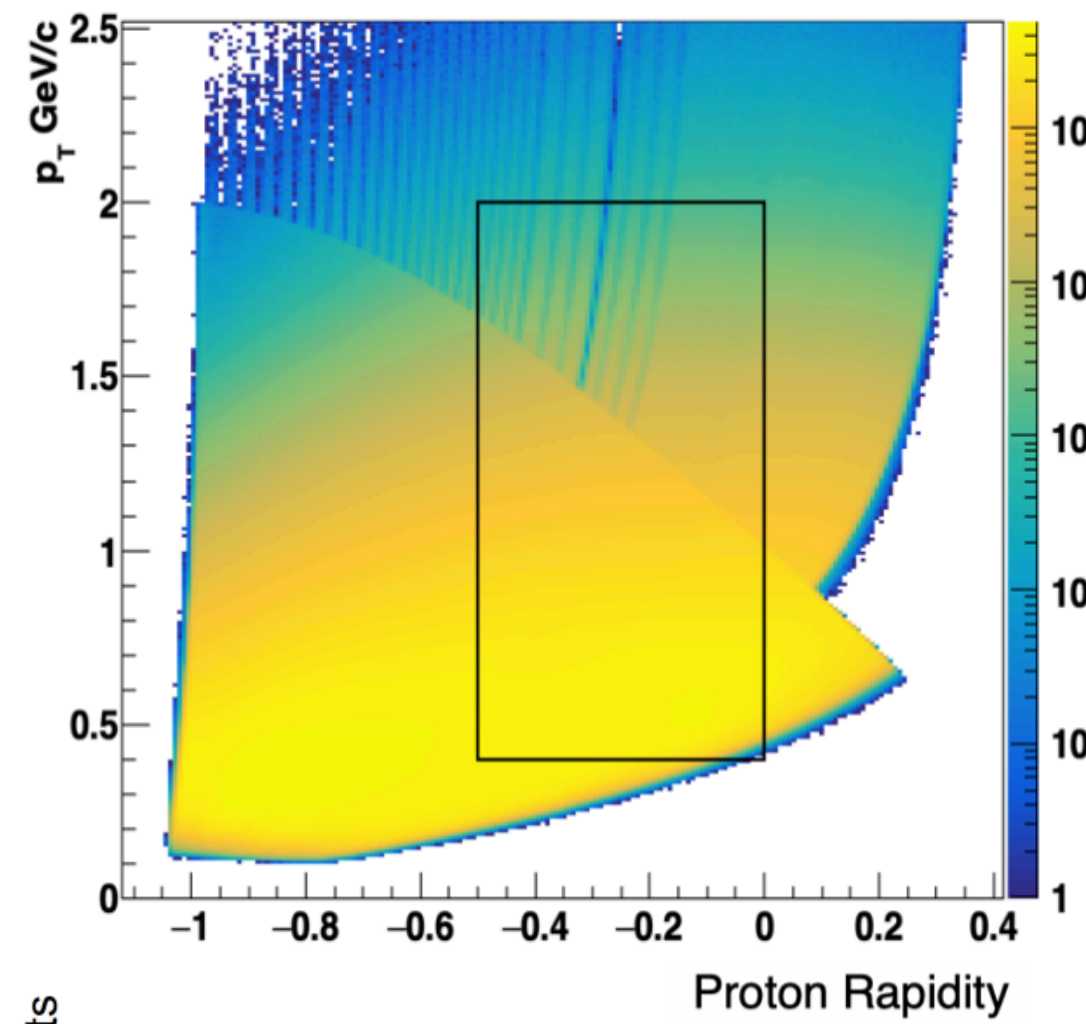
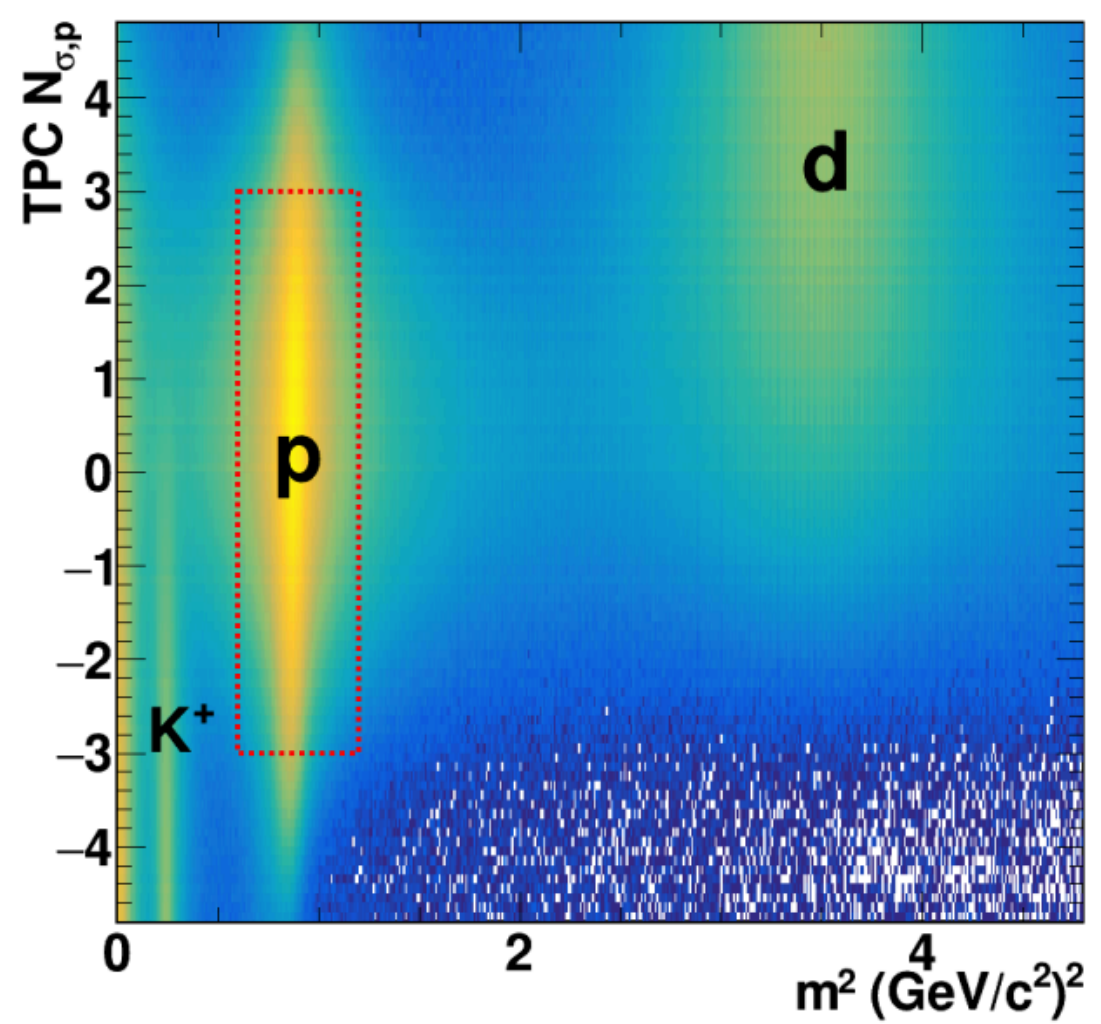
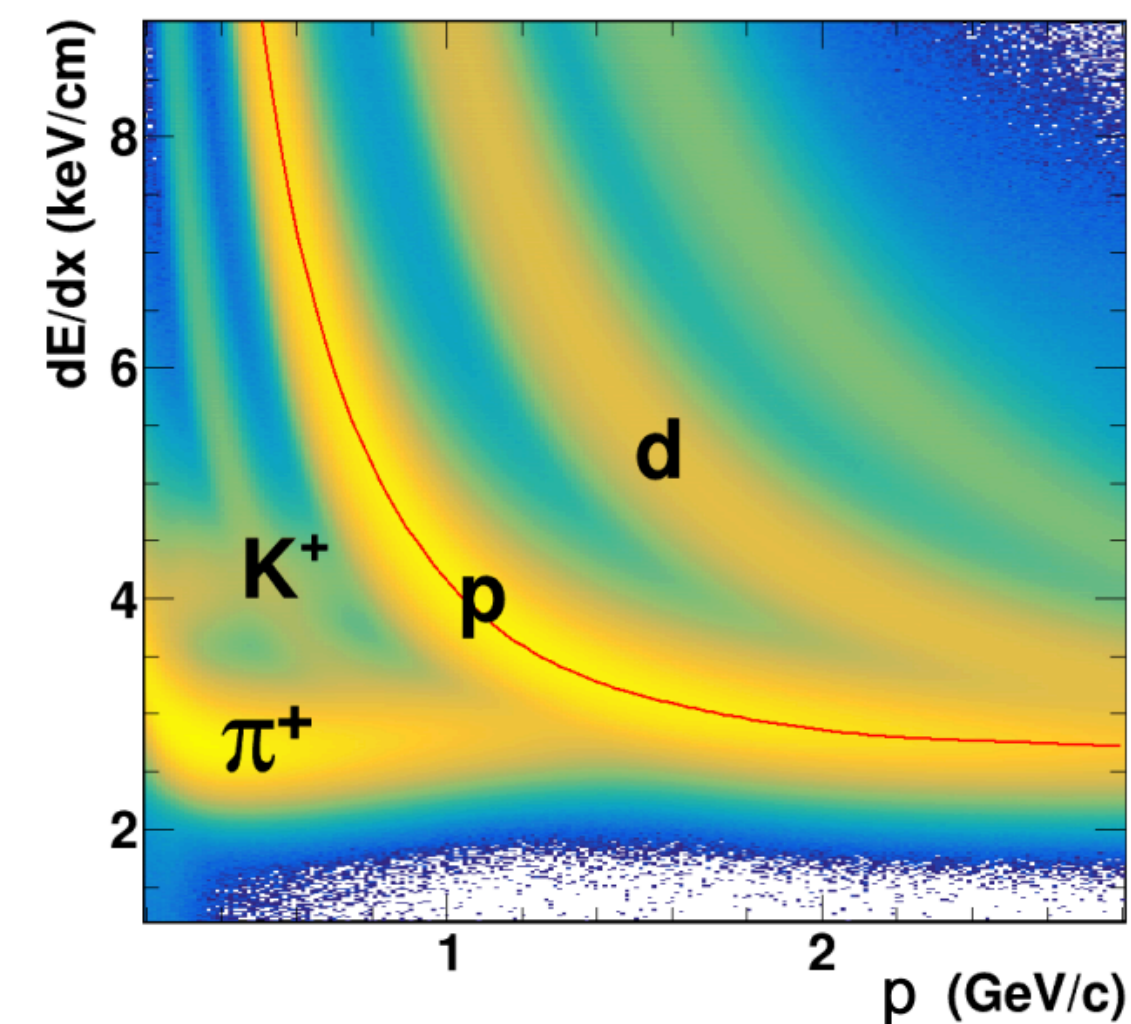
✓ Baryon chemical potential has been extended up to 720 MeV.



TPC PID

TOF PID

Proton Phase Space

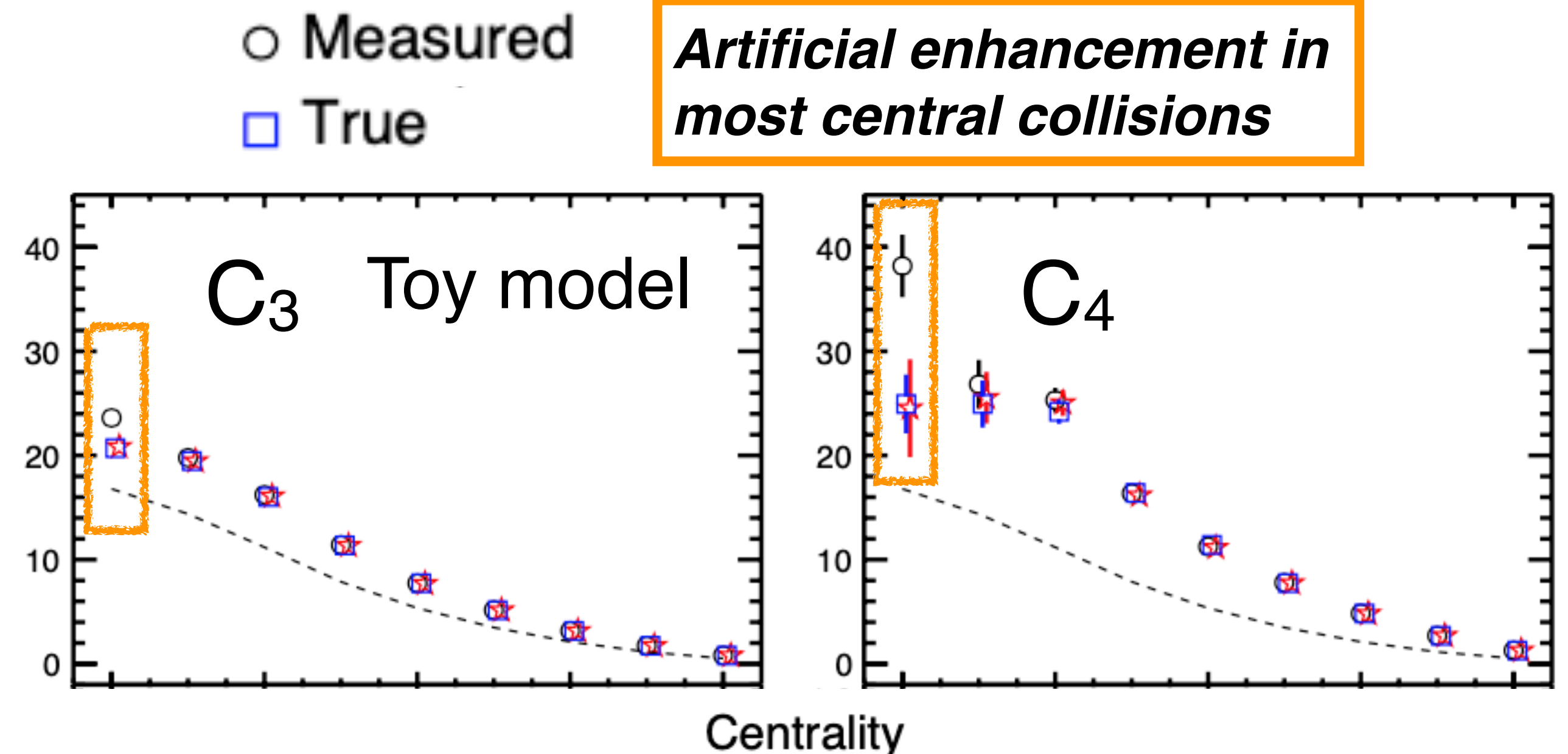
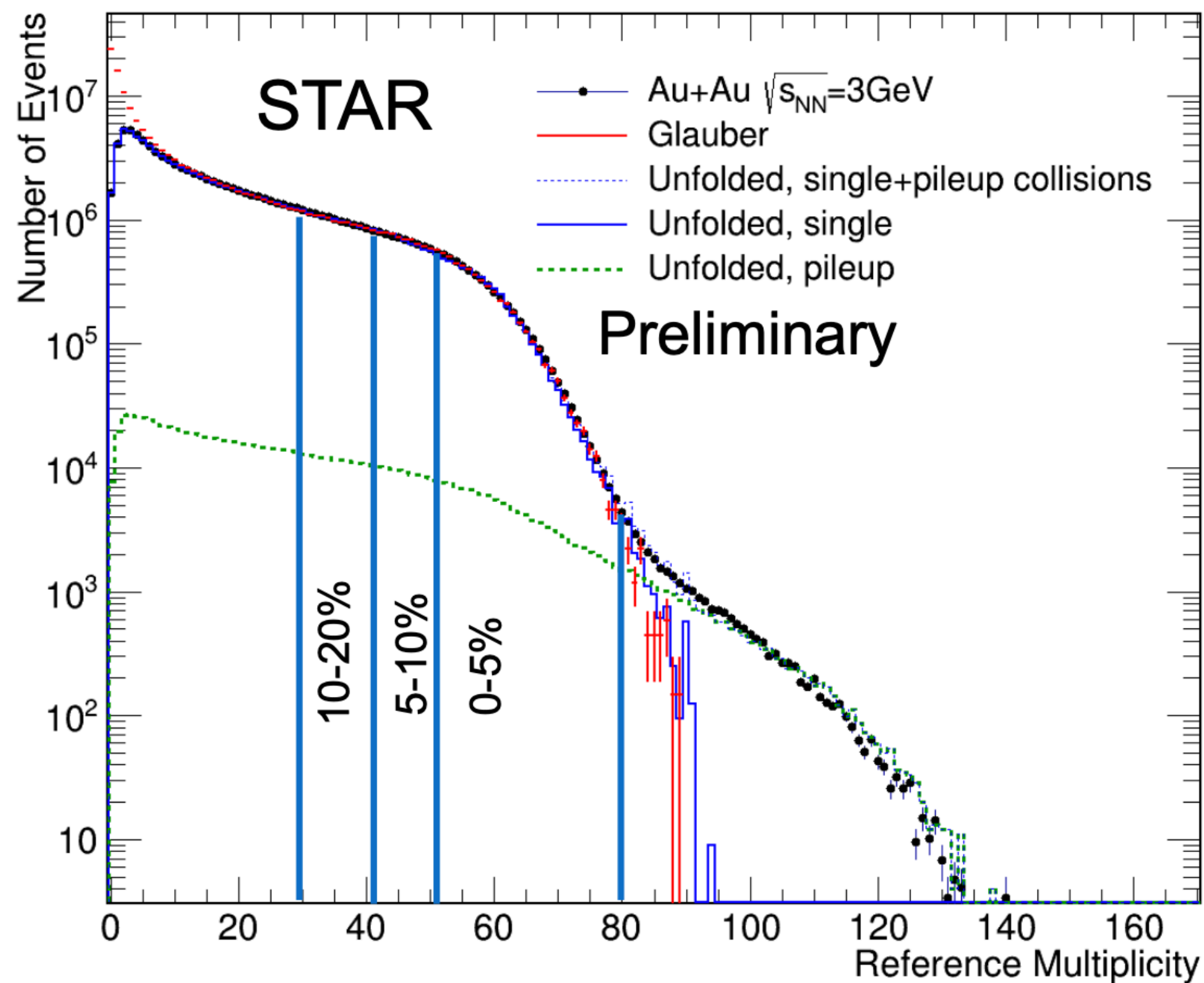


**Analysis window:**  
 $0.4 < p_T < 2.0 \text{ GeV/c}, -0.5 < y < 0$   
**Proton identification:**  
TPC when  $p < 2.0 \text{ GeV/c}$   
TPC+TOF when  $p > 2.0 \text{ GeV/c}$



# Pileup events

✓ Visible tail in multiplicity distribution due to pileup events, which is known to enhance the value of cumulants.



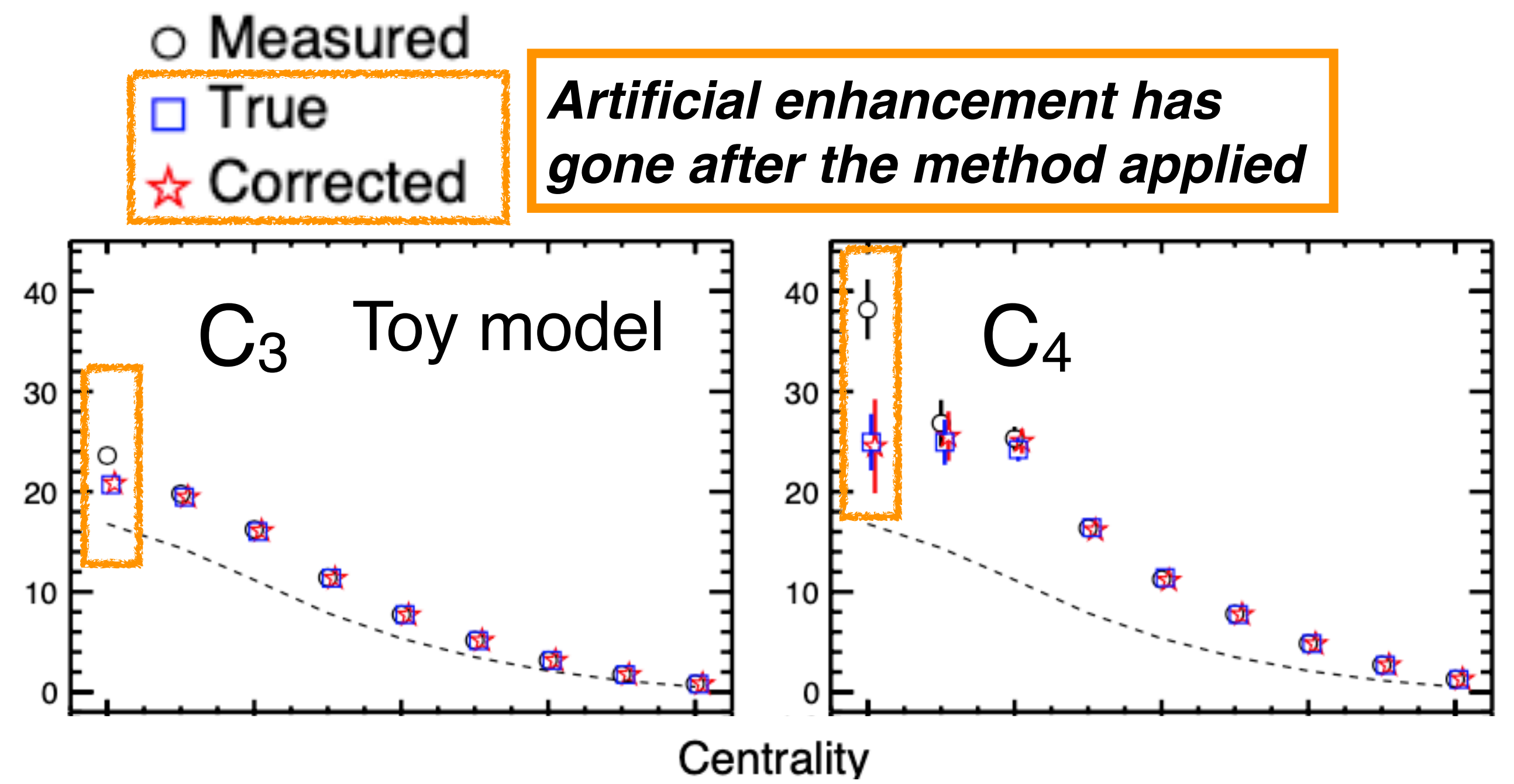
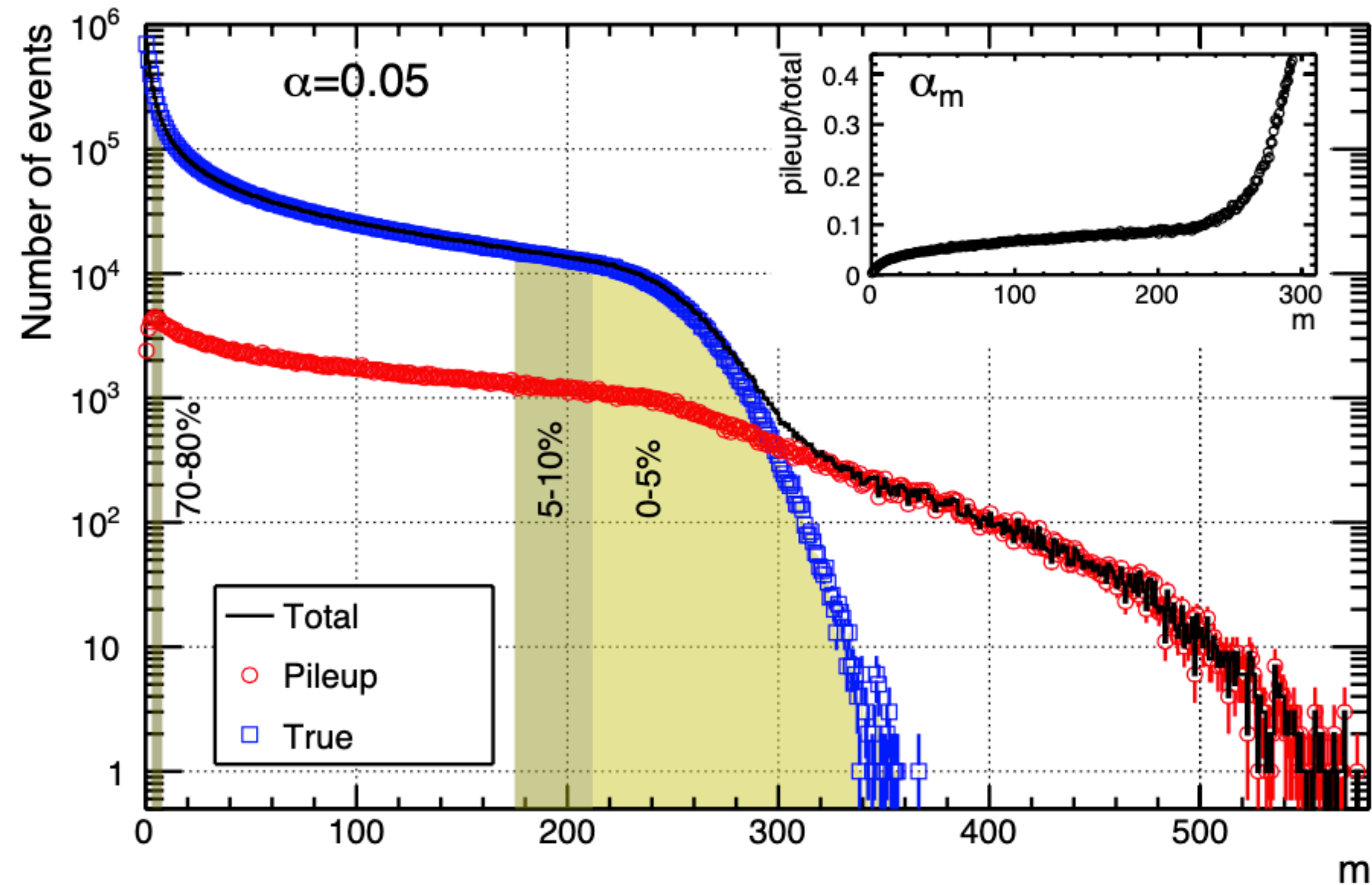
T. Nonaka, M. Kitazawa, S. Esumi, NIMA.984.164632(2020)

Y. Zhang, APS April meeting 2021



# Pileup correction

- ✓ Data-driven approach of the pileup correction is available once true and pileup multiplicity distributions are determined by simulations.
- ✓ Necessary information: True multiplicity distribution from single-collision



T. Nonaka, M. Kitazawa, S. Esumi, NIMA.984.164632(2020)  
Yu Zhang et al., arXiv: 2108.10134



# Summary

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- ✓ Non-monotonic beam energy dependence of net-p  $C_4/C_2$  has been observed in BES-I: ***Critical point at  $\sqrt{s_{NN}} \leq 7.7 \text{ GeV}$ ?***
- ✓ Negative value of net-p  $C_6/C_2$  at  $\sqrt{s_{NN}} = 200 \text{ GeV}$  central collisions: ***Crossover at  $\mu_B \sim 20 \text{ MeV}$ ?***
- ✓ Stay tuned for precise measurement from ***BES-II and FXT energies (3.0-19.6 GeV)*** to have more definitive messages.

***Thank you for your attention***

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