

Studying the QCD Phase Structure through Higher Moments at RHIC-BES at STAR

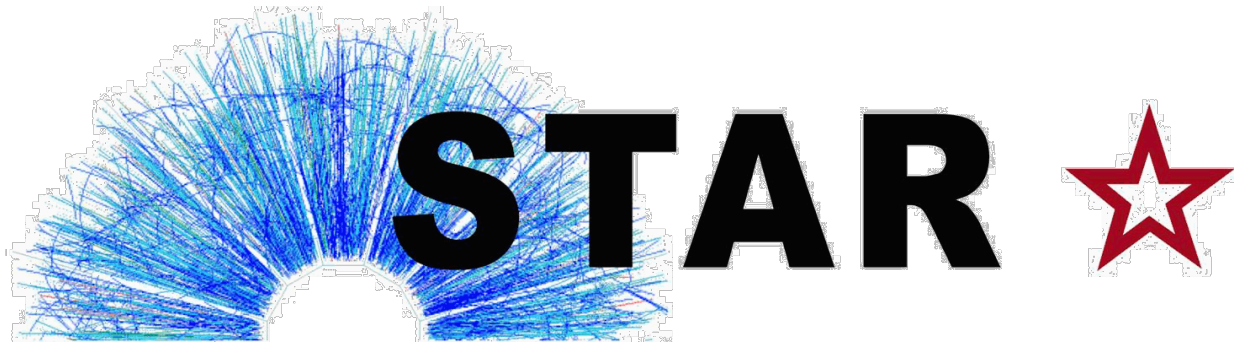
Toshihiro Nonaka University of Tsukuba

For the STAR collaboration

ICNFP2021 @Crete, Greece (Zoom)



筑波大学
University of Tsukuba



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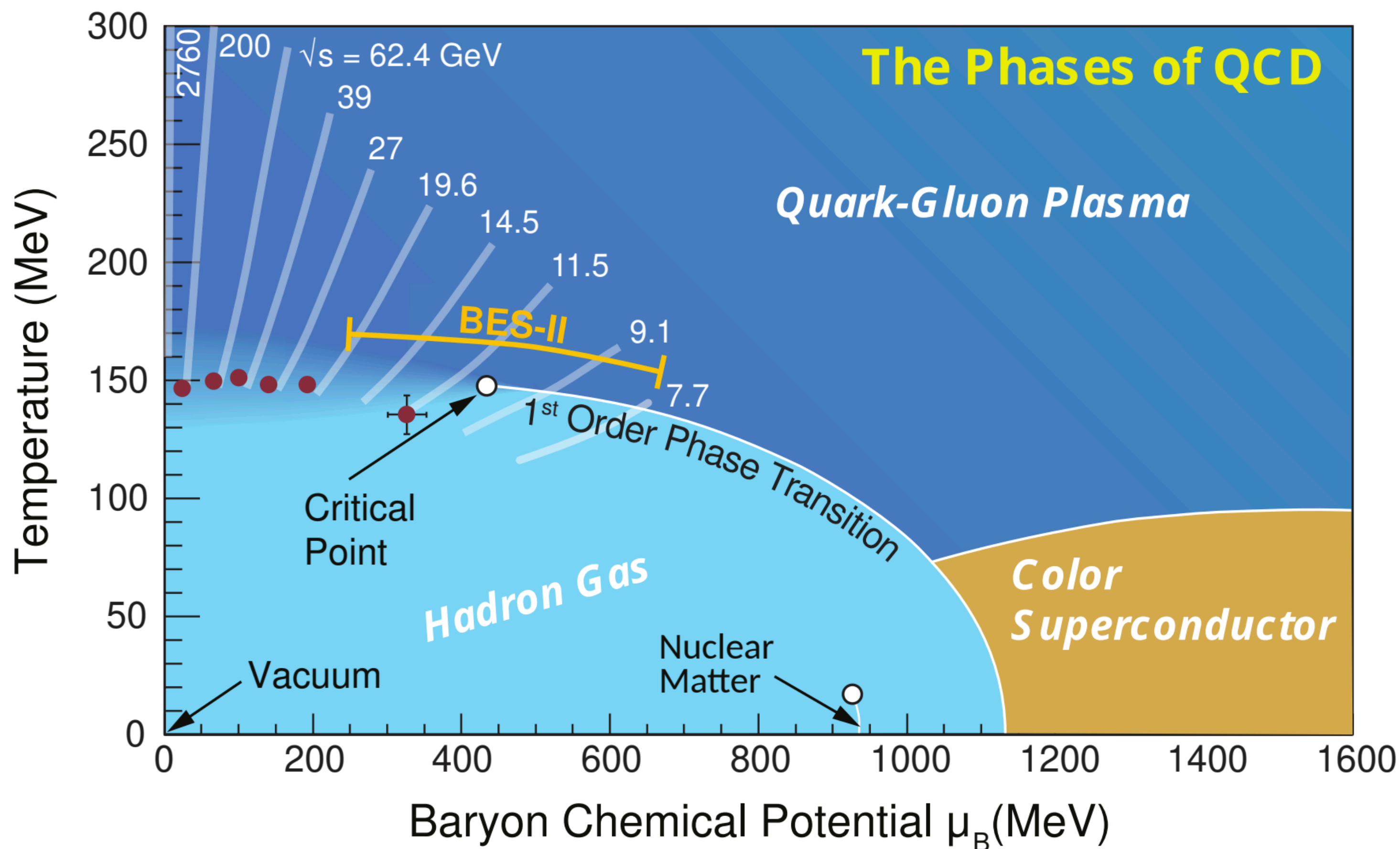
Outline

- ***Introduction***
- ***C_4/C_2 for critical point search***
- ***C_5/C_1 and C_6/C_2 for crossover search***
- ***BES-II and fixed-target programs***
- ***Experimental challenges***
- ***Summary***



QCD phase diagram

✓ QCD phase structure in wide (μ_B, T) region.

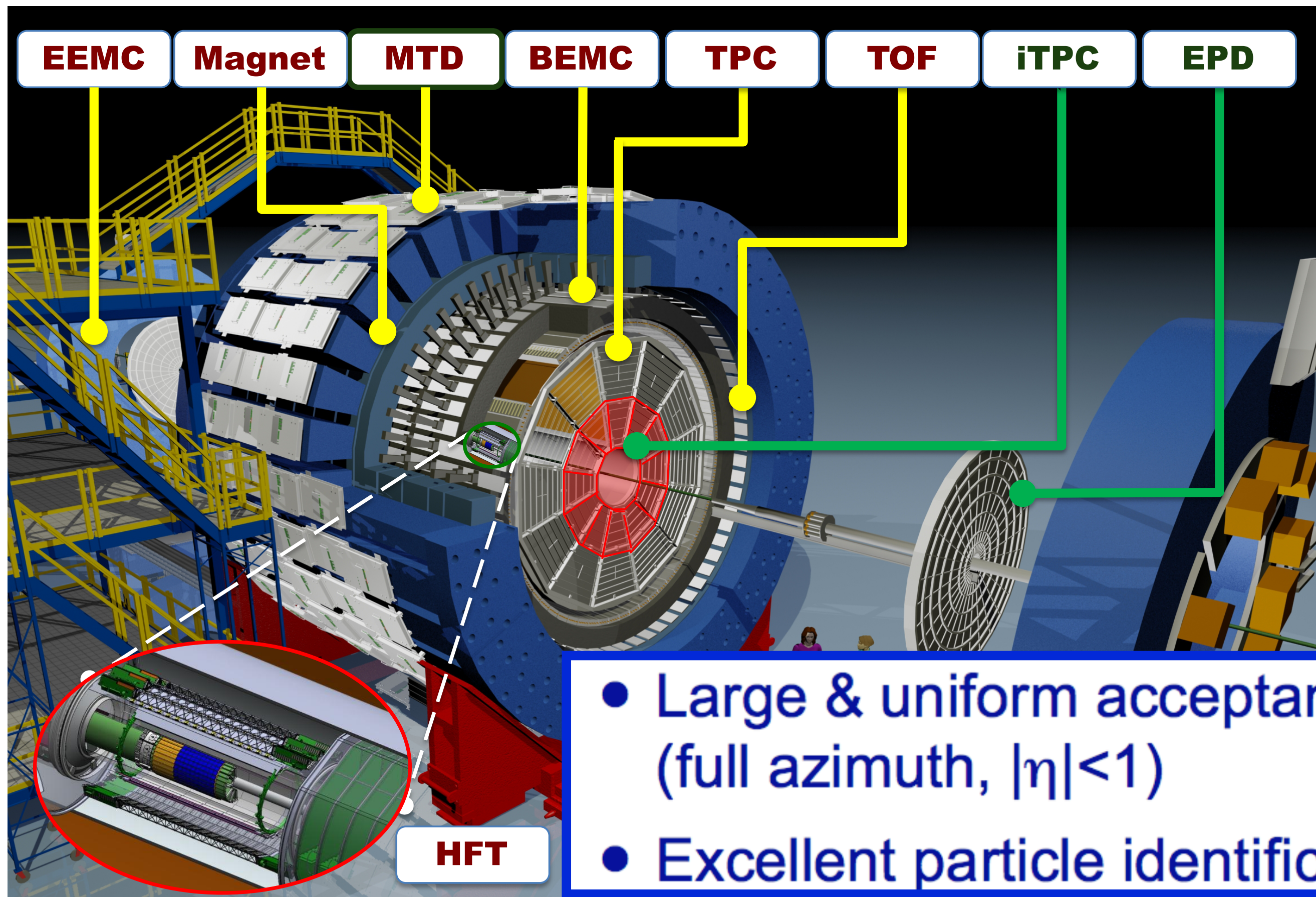


- **Crossover at $\mu_B = 0$ MeV**
Y. Aoki et al, Nature 443, 675(2006)
- **1st-order phase transition at large μ_B ?**
- **Critical point?**



Beam Energy Scan

✓ Need to investigate the QCD phase structure in wide (μ_B, T) region.



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Beam Energy Scan

✓ Need to investigate the QCD phase structure in wide (μ_B, T) region.

$\sqrt{s_{NN}}$ (GeV)	No. of events (million)	T_{ch} (MeV)	μ_B (MeV)
200	238	164.3	28
62.4	47	160.3	70
54.4	550	160.0	83
39	86	156.4	160
27	30	155.0	144
19.6	15	153.9	188
14.5	20	151.6	264
11.5	6.6	149.4	287
7.7	3	144.3	398

2010-
2017

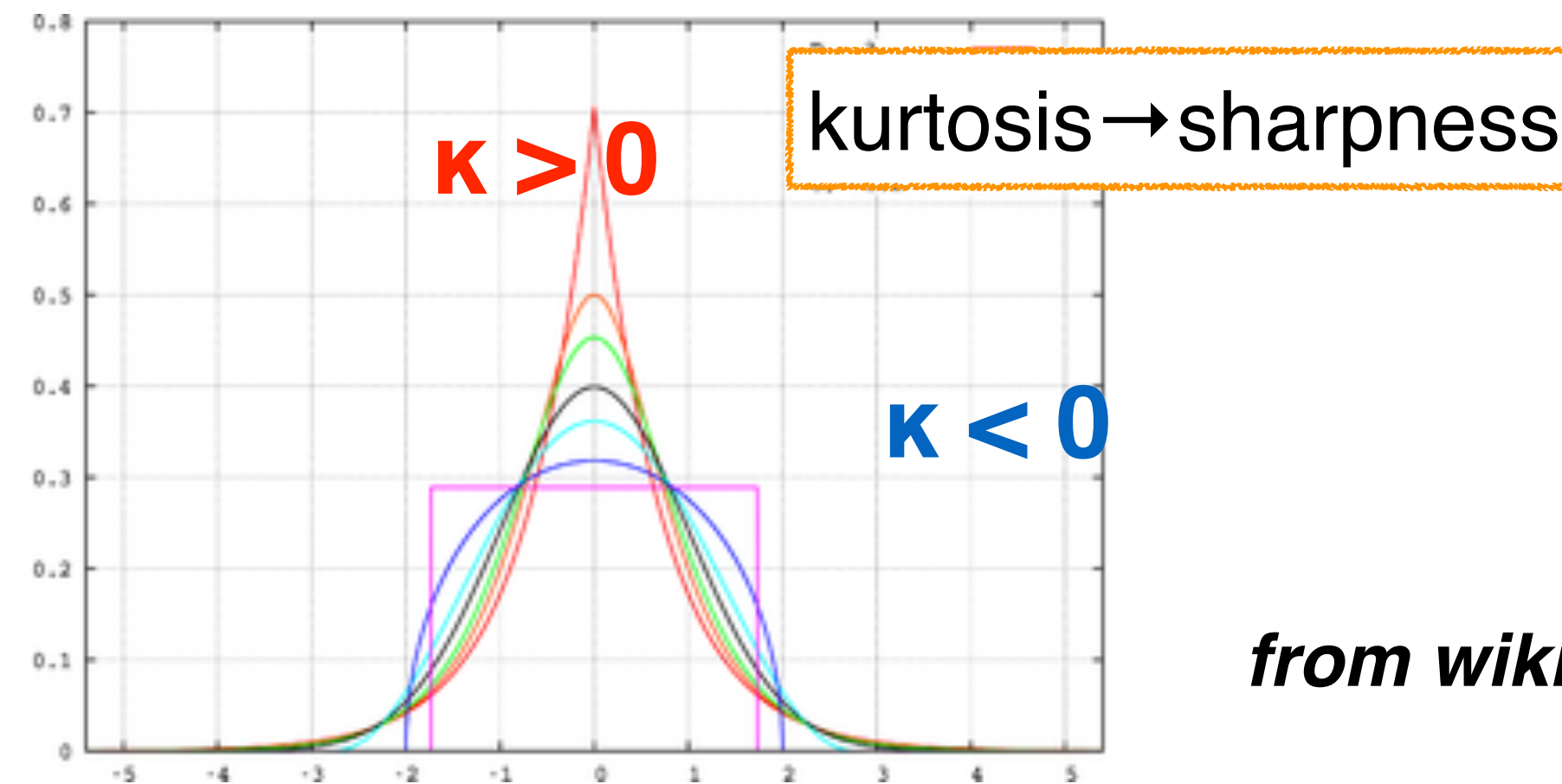
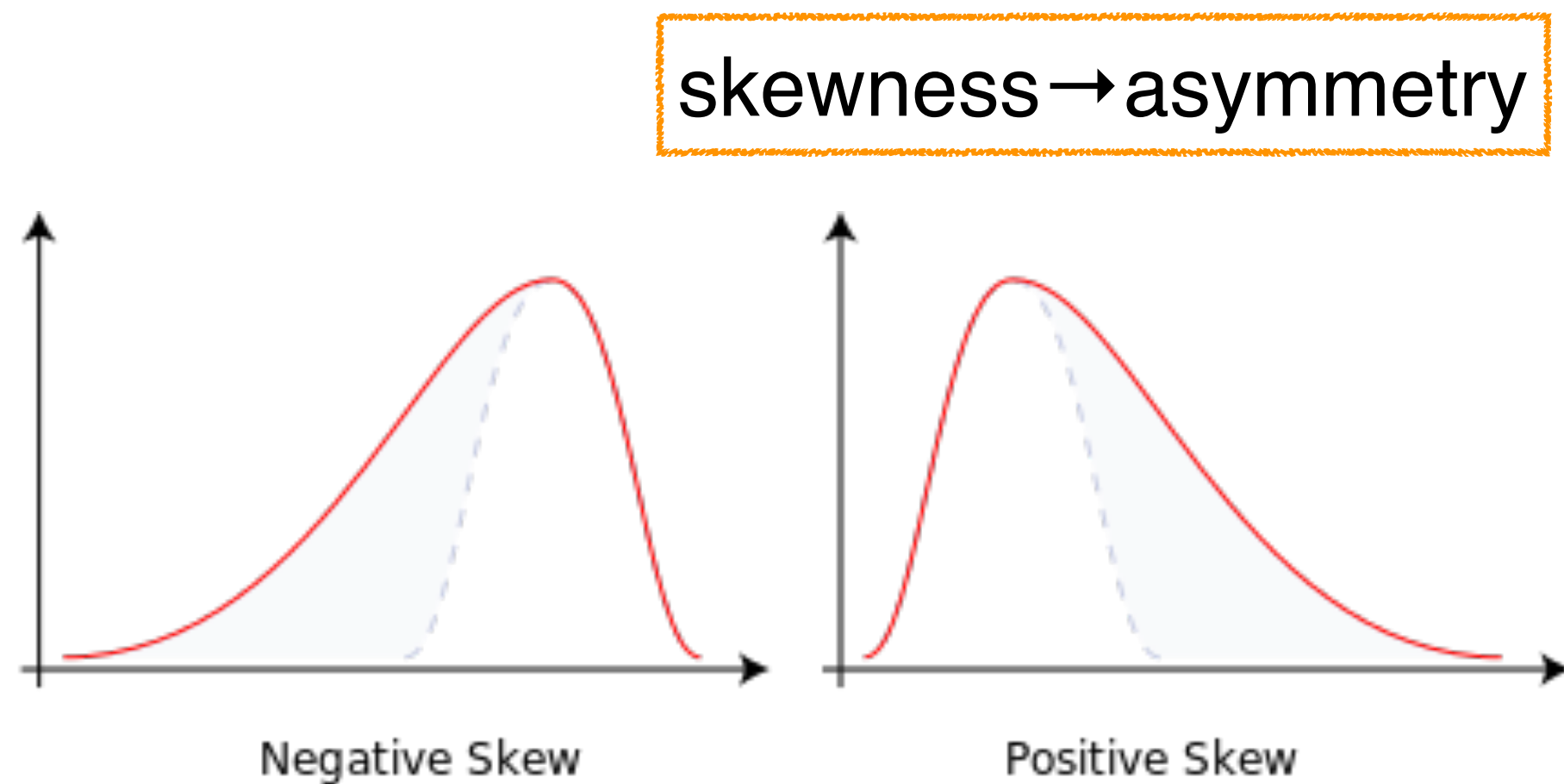
- **Crossover at $\mu_B = 0$ MeV**
Y. Aoki et al, Nature 443, 675(2006)
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- **Critical point?**



Higher-order fluctuations

◆ Moments and cumulants are mathematical measures of “shape” of a distribution which probe the fluctuation of observables.

- ✓ Moments: mean (M), standard deviation (σ), skewness (S) and kurtosis (κ).
- ✓ S and κ are sensitive to non-gaussian fluctuations.



from wikipedia

✓ Cumulant \Leftrightarrow Central Moment

$$\langle \delta N \rangle = N - \langle N \rangle$$

$$C_1 = M = \langle N \rangle$$

$$C_2 = \sigma^2 = \langle (\delta N)^2 \rangle$$

$$C_3 = S\sigma^3 = \langle (\delta N)^3 \rangle$$

$$C_4 = \kappa\sigma^4 = \langle (\delta N)^4 \rangle - 3 \langle (\delta N)^2 \rangle^2$$

✓ Cumulant : additivity

$$C_n(X + Y) = C_n(X) + C_n(Y)$$

➔ proportional to volume

◆ **Net baryon, net charge and net strangeness**

“Net” : positive - negative

$$\Delta N_q = N_q - N_{\bar{q}}, \quad q = B, Q, S$$

Fill in histograms
over many collisions

No. of **positively charged**
particles in one collision

No. of **negatively charged**
particles in one collision

(1) Sensitive to correlation length

$$C_2 = \langle (\delta N)^2 \rangle_c \approx \xi^2 \quad C_5 = \langle (\delta N)^5 \rangle_c \approx \xi^{9.5}$$

$$C_3 = \langle (\delta N)^3 \rangle_c \approx \xi^{4.5} \quad C_6 = \langle (\delta N)^6 \rangle_c \approx \xi^{12}$$

$$C_4 = \langle (\delta N)^4 \rangle_c \approx \xi^7$$

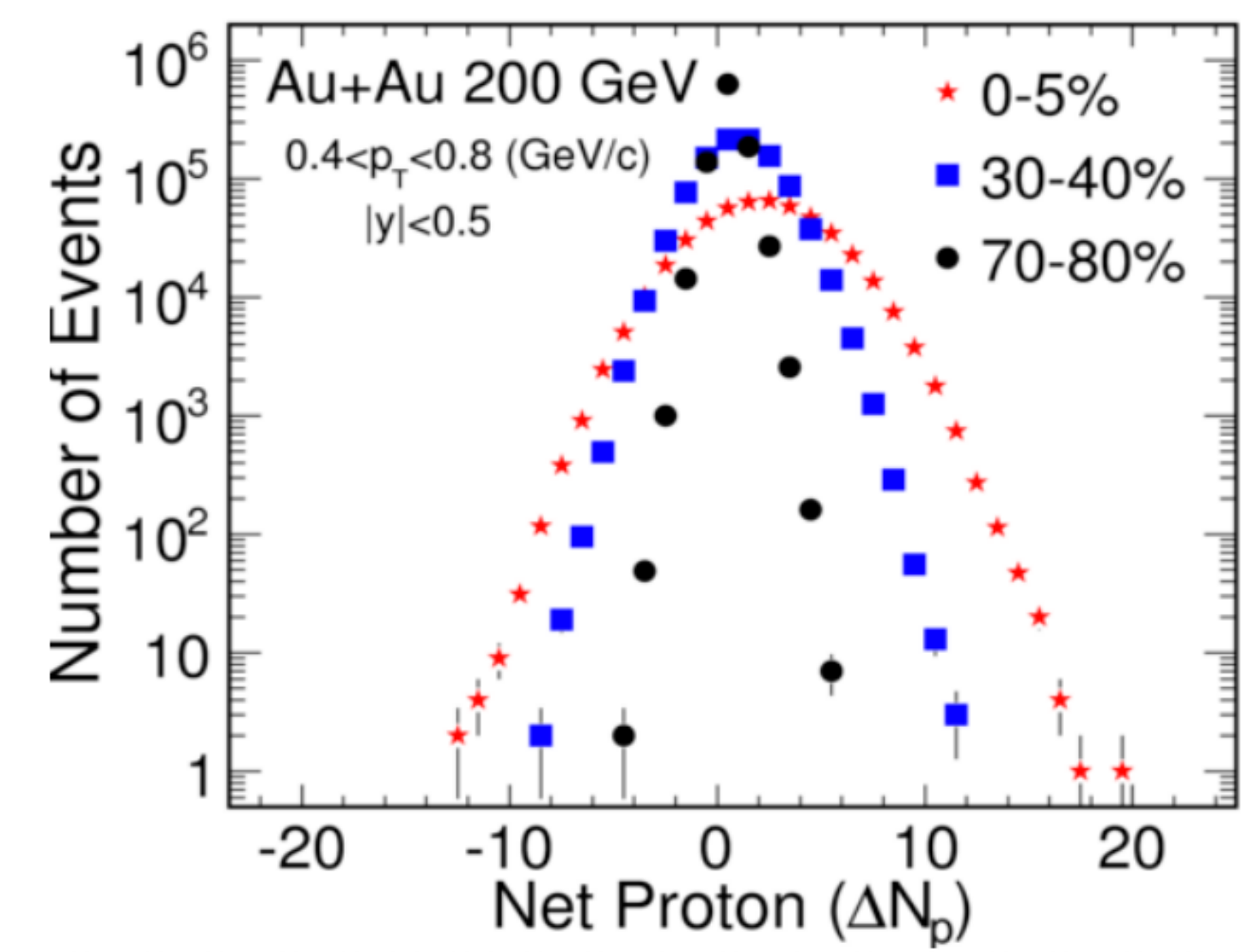
M. A. Stephanov, *Phys. Rev. Lett.* 102, 032301 (2009)
M. A. Stephanov, *Phys. Rev. Lett.* 107, 052301 (2011)
MAsakawa, S. Ejiri and M. Kitazawa, *Phys. Rev. Lett.* 103, 262301 (2009)

(2) Direct comparison with susceptibilities.

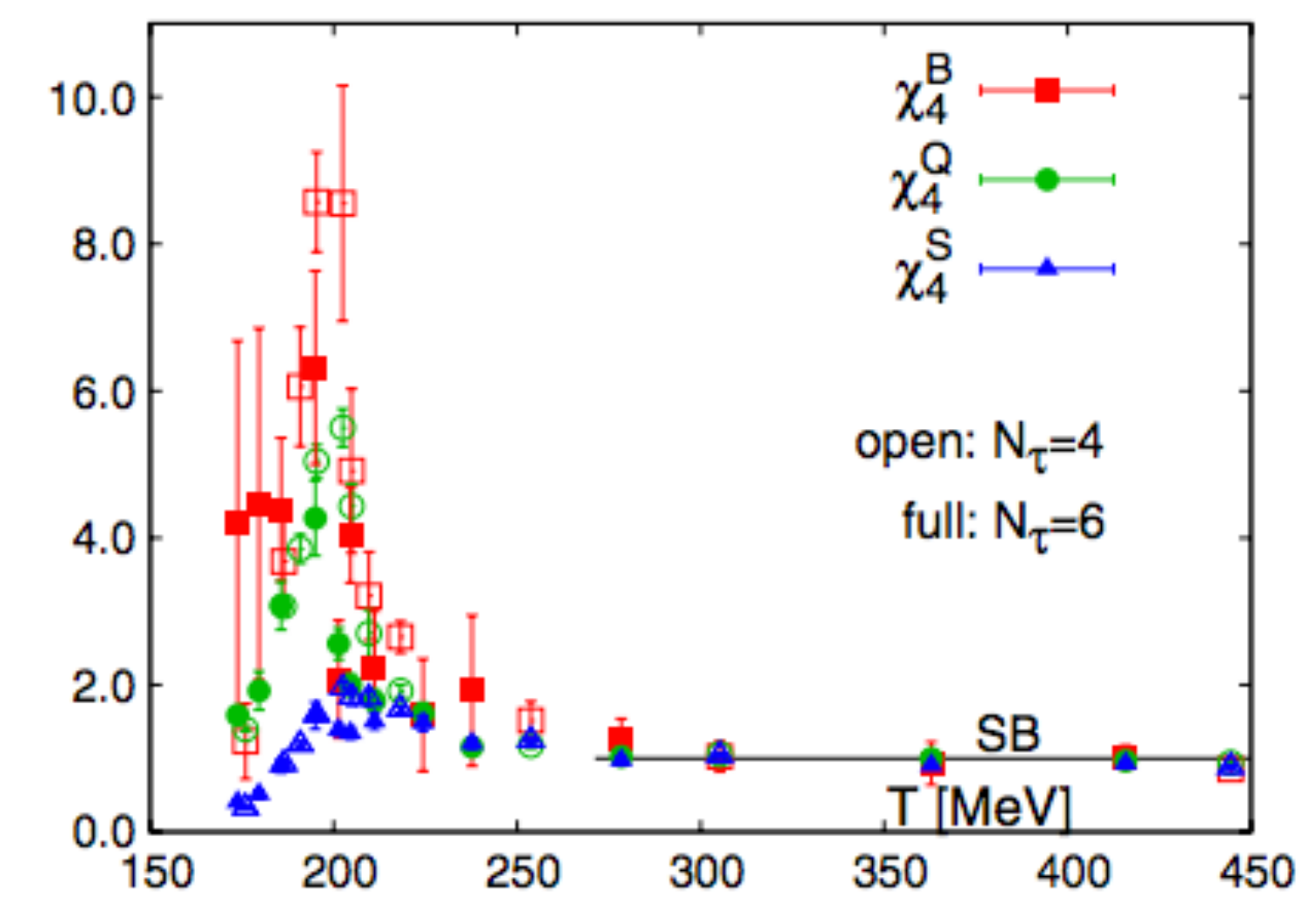
M. Cheng et al, *PRD* 79, 074505 (2009)

$$S\sigma = \frac{C_3}{C_2} = \frac{\chi_3}{\chi_2} \quad \kappa\sigma^2 = \frac{C_4}{C_2} = \frac{\chi_4}{\chi_2}$$

$$\chi_n^q = \frac{1}{VT^3} \times C_n^q = \frac{\partial^n p/T^4}{\partial \mu_q^n}, \quad q = B, Q, S$$



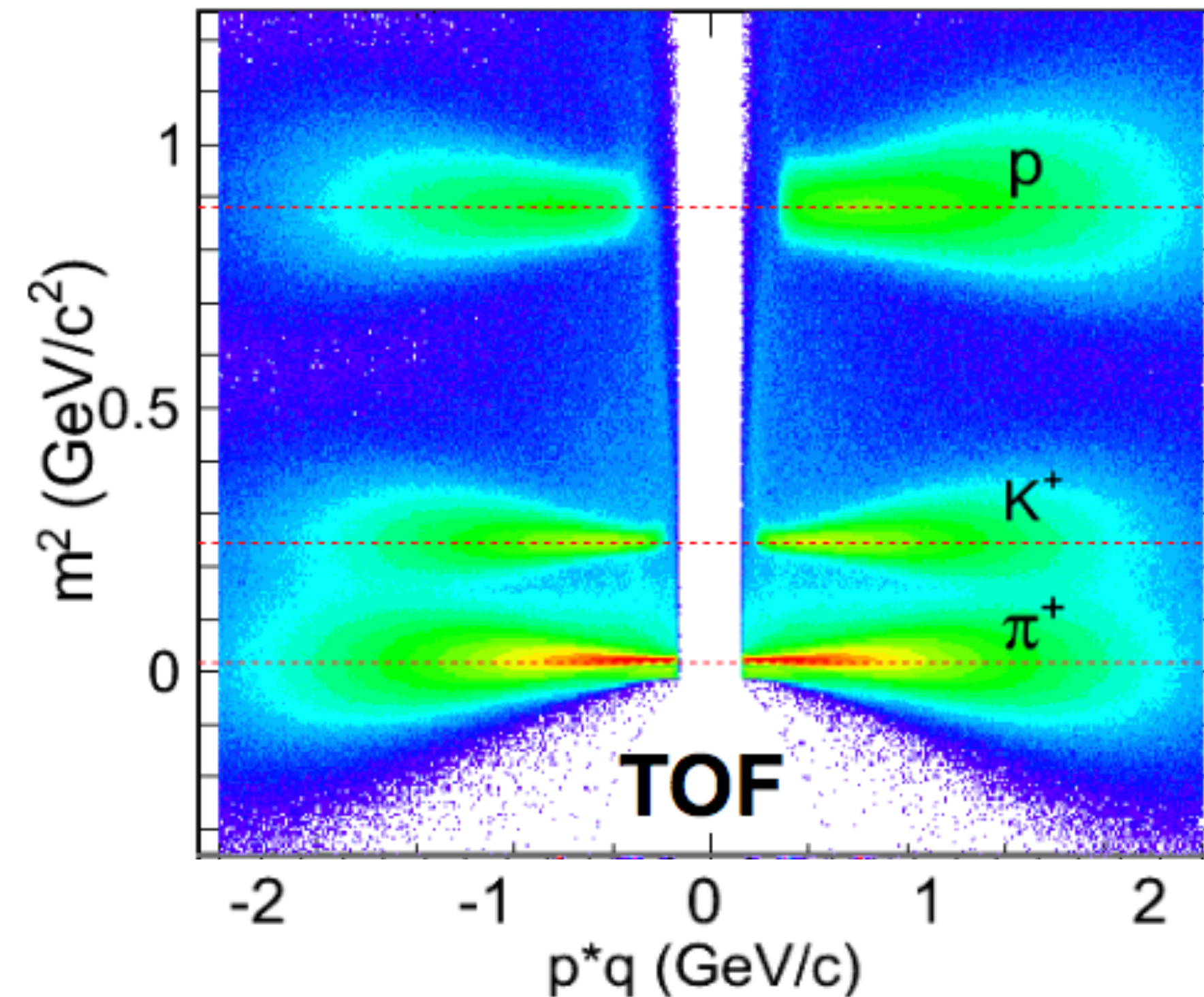
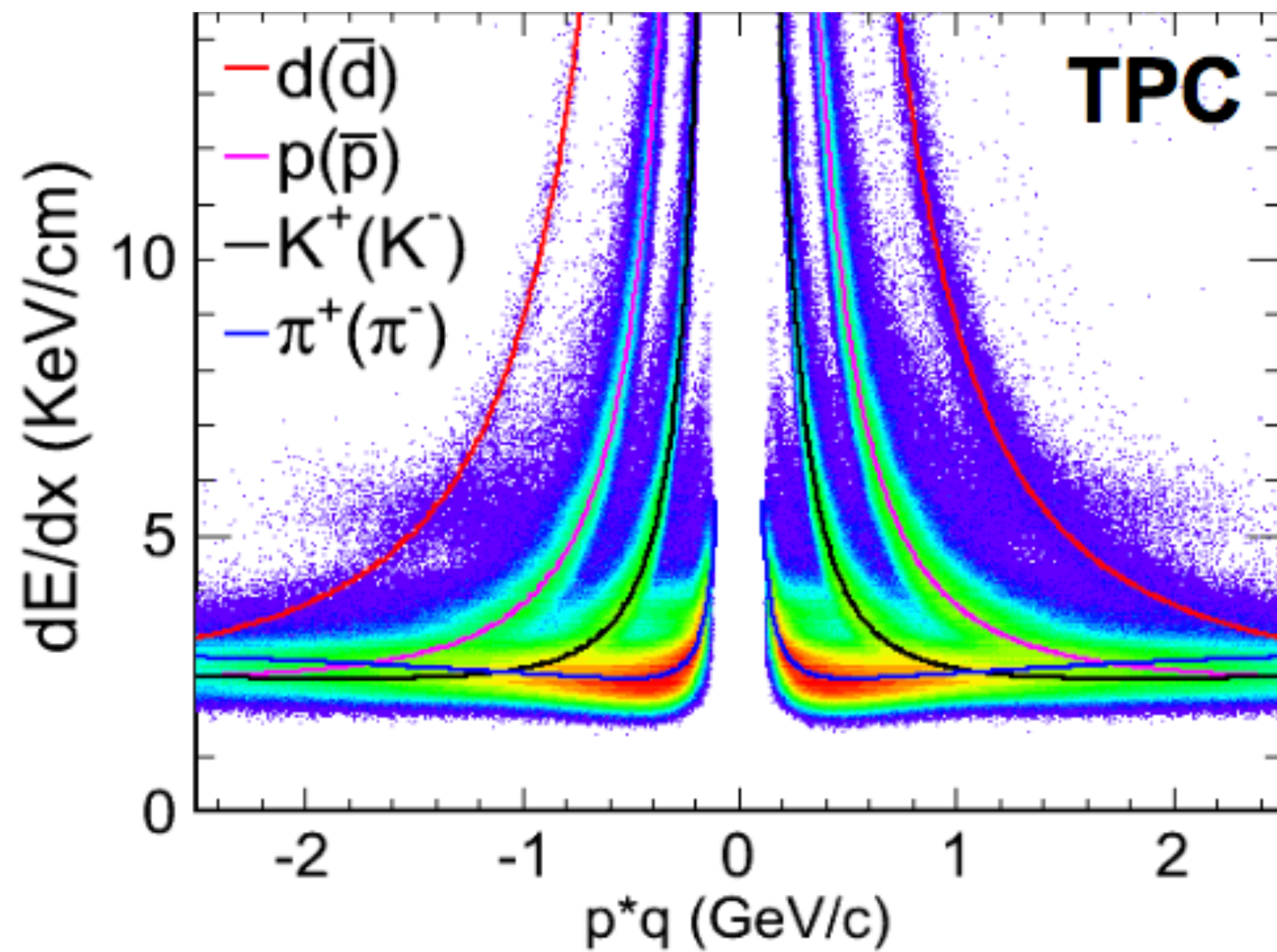
→ neutrons cannot be measured



M. Cheng et al, *PRD* 79, 074505 (2009)

Particle identification

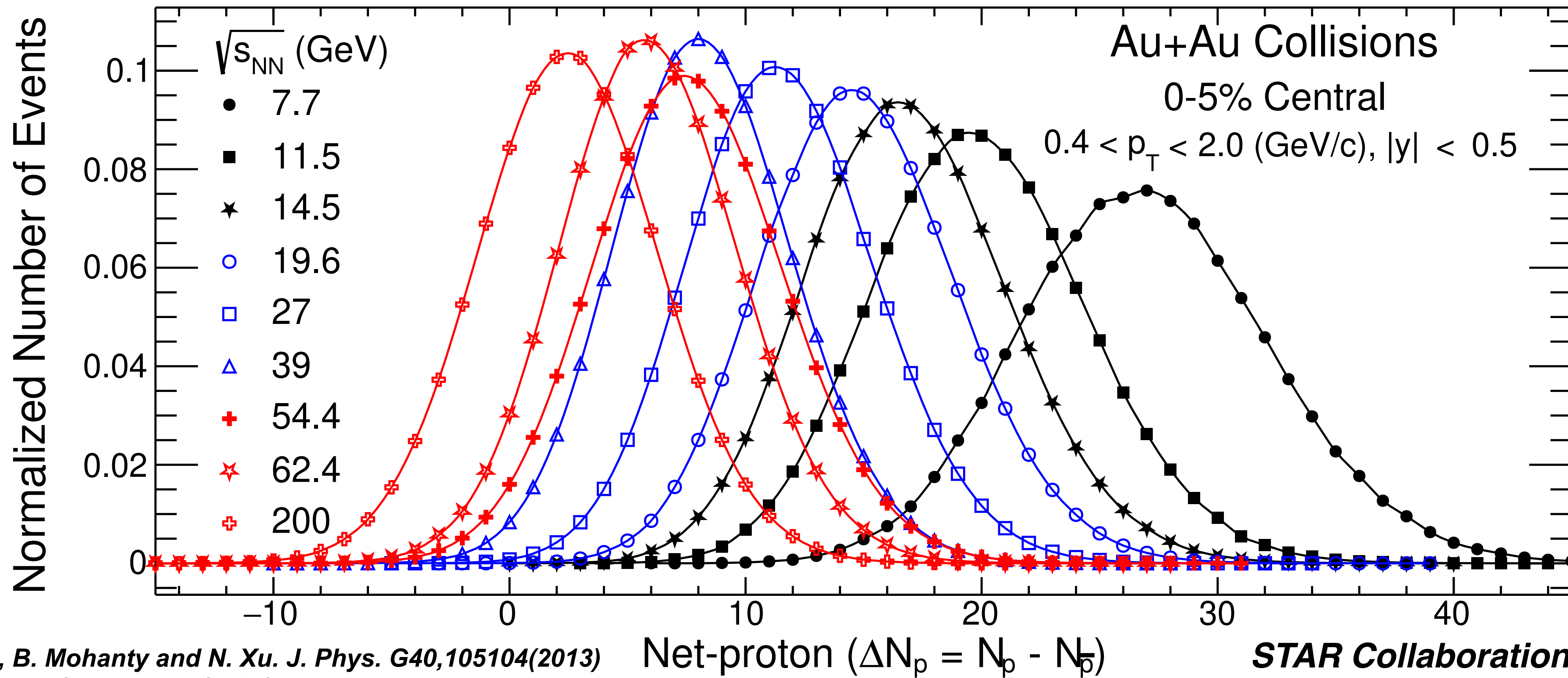
- ✓ dE/dx measured with TPC is used for proton identification at low p_T region.
- ✓ The combined PID with m^2 from TOF is used at high p_T region.





Raw net-proton distribution

- ✓ Avoid auto-correlation effects : New centrality definition
- ✓ Suppress initial volume fluctuation : Centrality bin width correction
- ✓ Detector efficiency correction : Binomial model



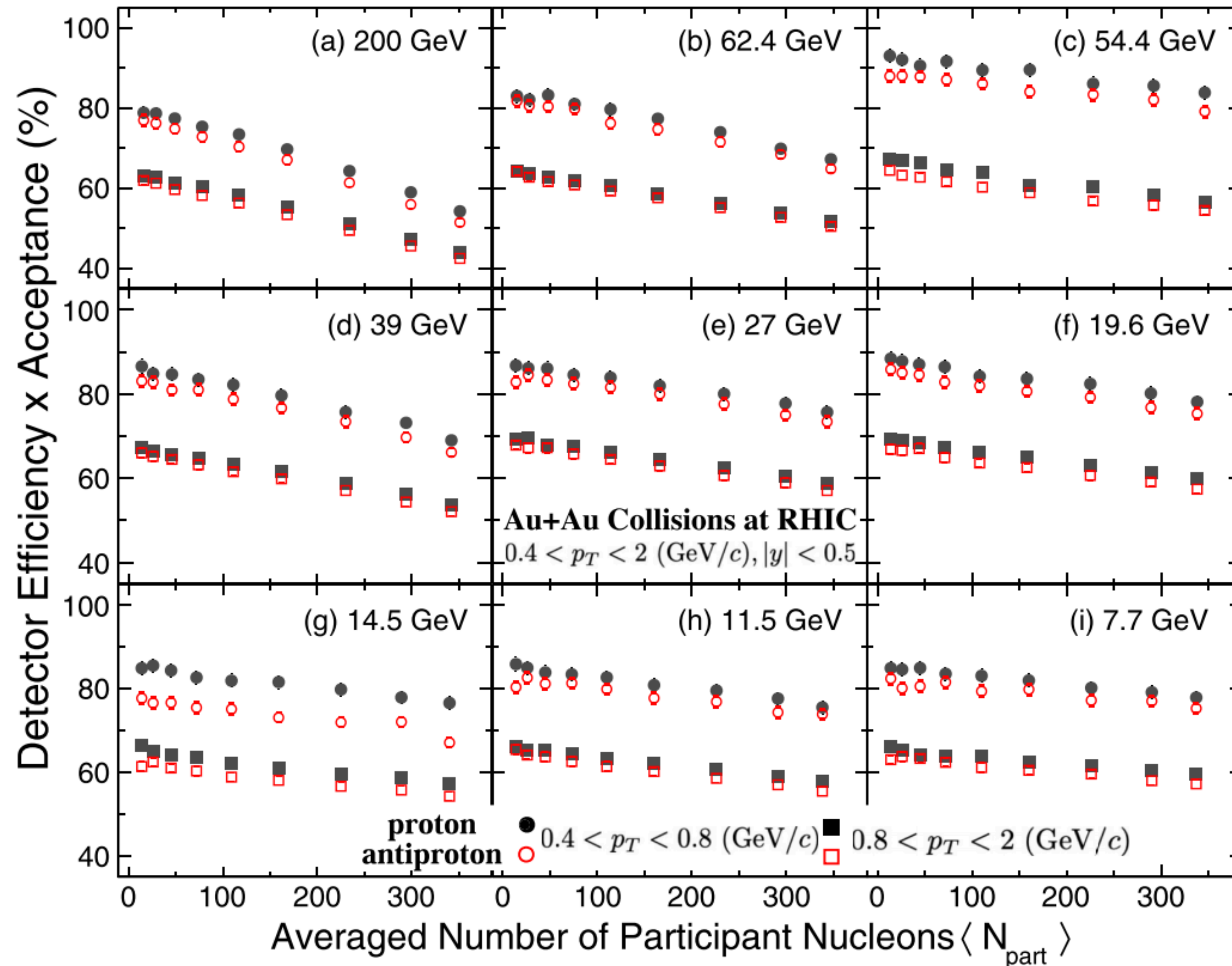
X.Luo, J. Xu, B. Mohanty and N. Xu. *J. Phys. G*40,105104(2013)
M. Kitazawa : *PRC*.86.024904(2012)
A. Bzdak and V. Koch : *PRC*.86.044904(2012), X. Luo : *PRC*.91.034907(2016)
T. Nonaka, M. Kitazawa, S. Esumi : *PRC*.95.064912(2017), *NIMA*906 10-17 (2018),
*NIMA*984(2020)164632
X. Luo, T. Nonaka : *PRC*.99.044917(2019)

STAR Collaboration,
***PRL*.126.092301(2021)**
***PRC*.104.024902(2021)**



Efficiency correction

STAR Collaboration, PRC.104.024902(2021)



- ✓ Detector efficiencies are studied in embedding simulations.
- ✓ Efficiencies are assumed to follow binomial distributions.
- ✓ Multiplicity and p_T dependence are taken into account.

M. Kitazawa : PRC.86.024904(2012)

A. Bzdak and V. Koch : PRC.86.044904(2012)

X. Luo : PRC.91.034907(2016)

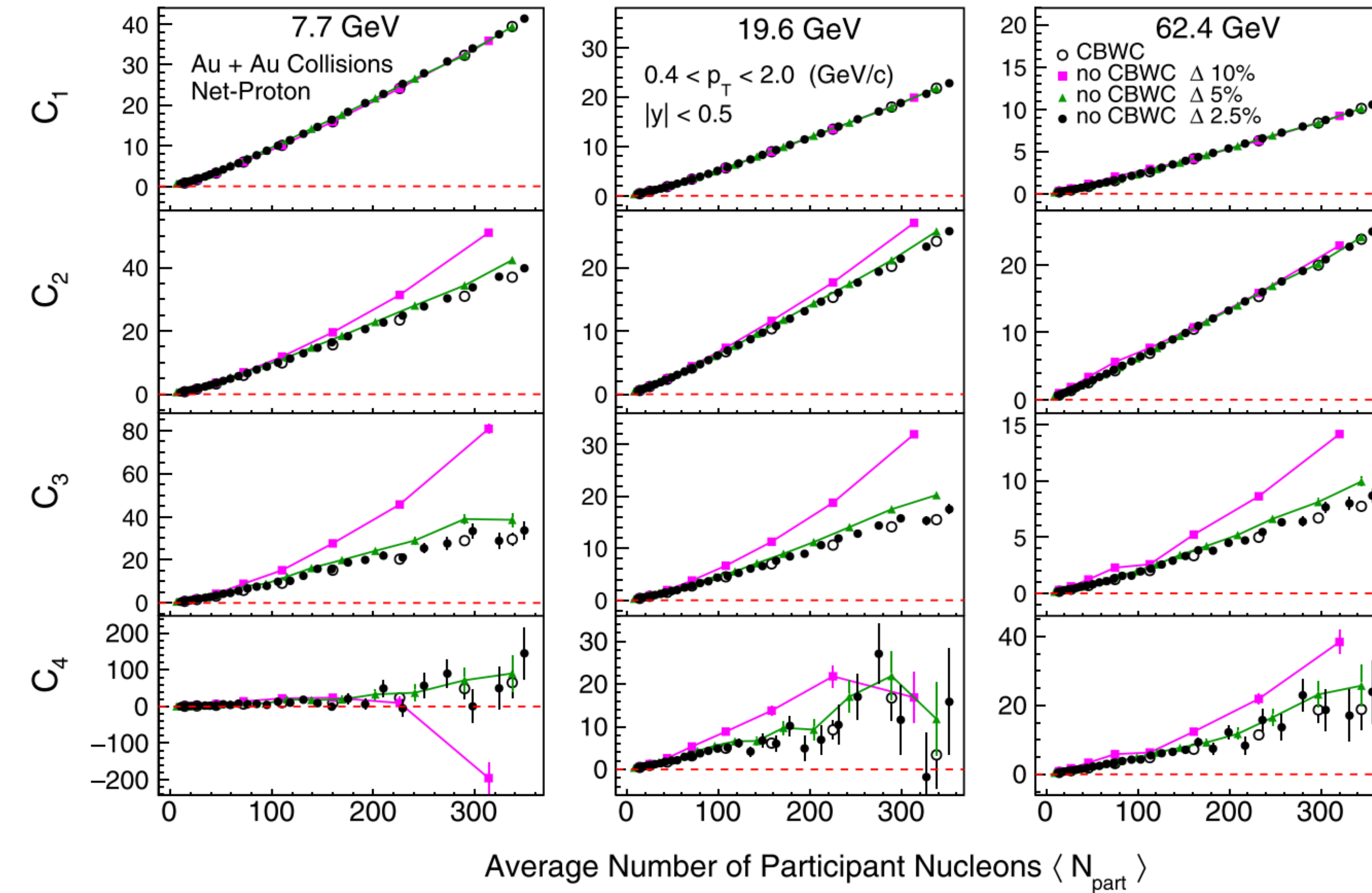
T. Nonaka, M. Kitazawa, S. Esumi : PRC.95.064912(2017)

X. Luo, T. Nonaka : PRC.99.044917(2019)



Centrality bin width correction

STAR Collaboration, PRC.104.024902(2021)

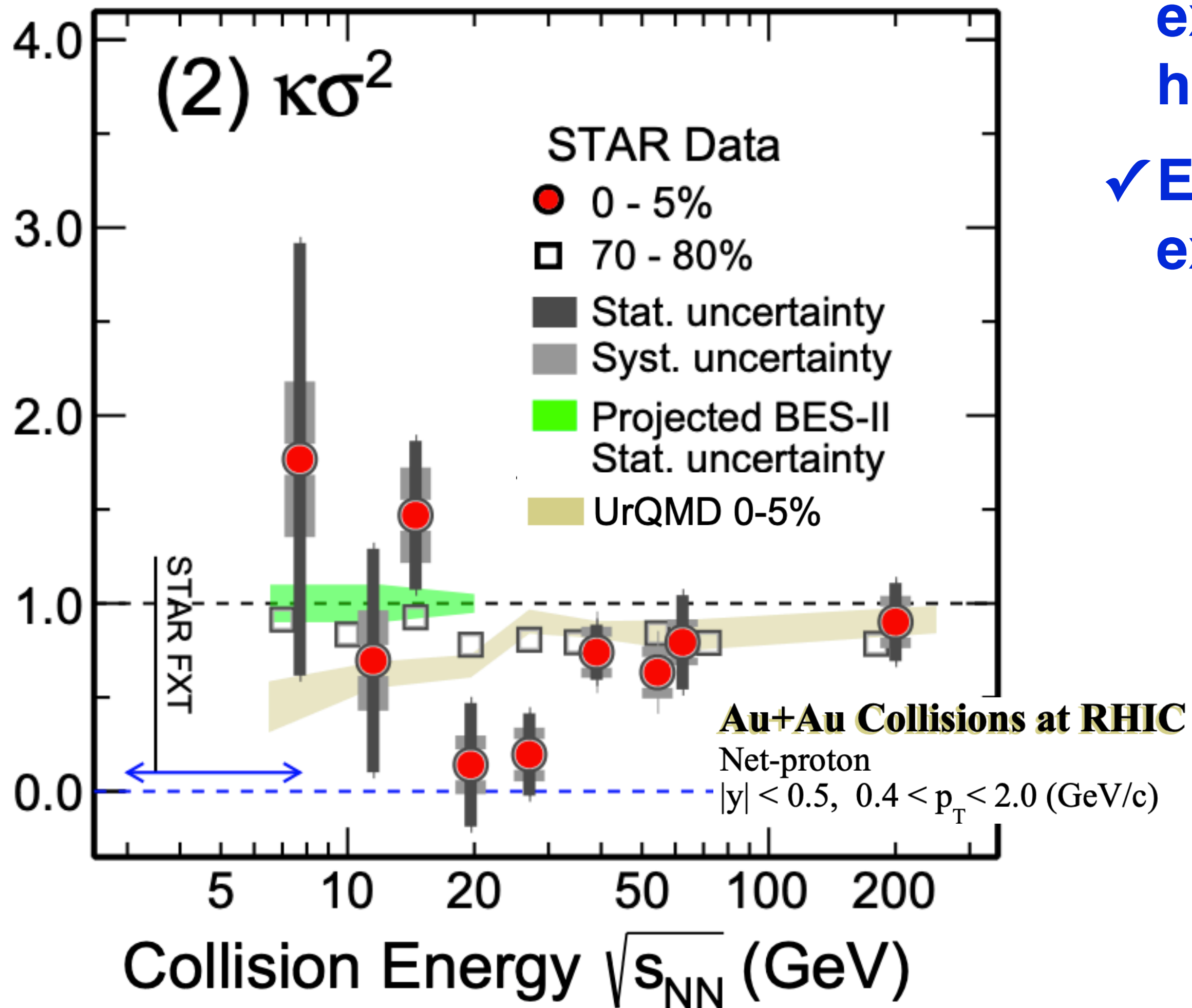


- ✓ Final state multiplicity and initial geometry are not one-to-one corresponding → volume fluctuation
- ✓ Data driven approach (CBWC) is applied.

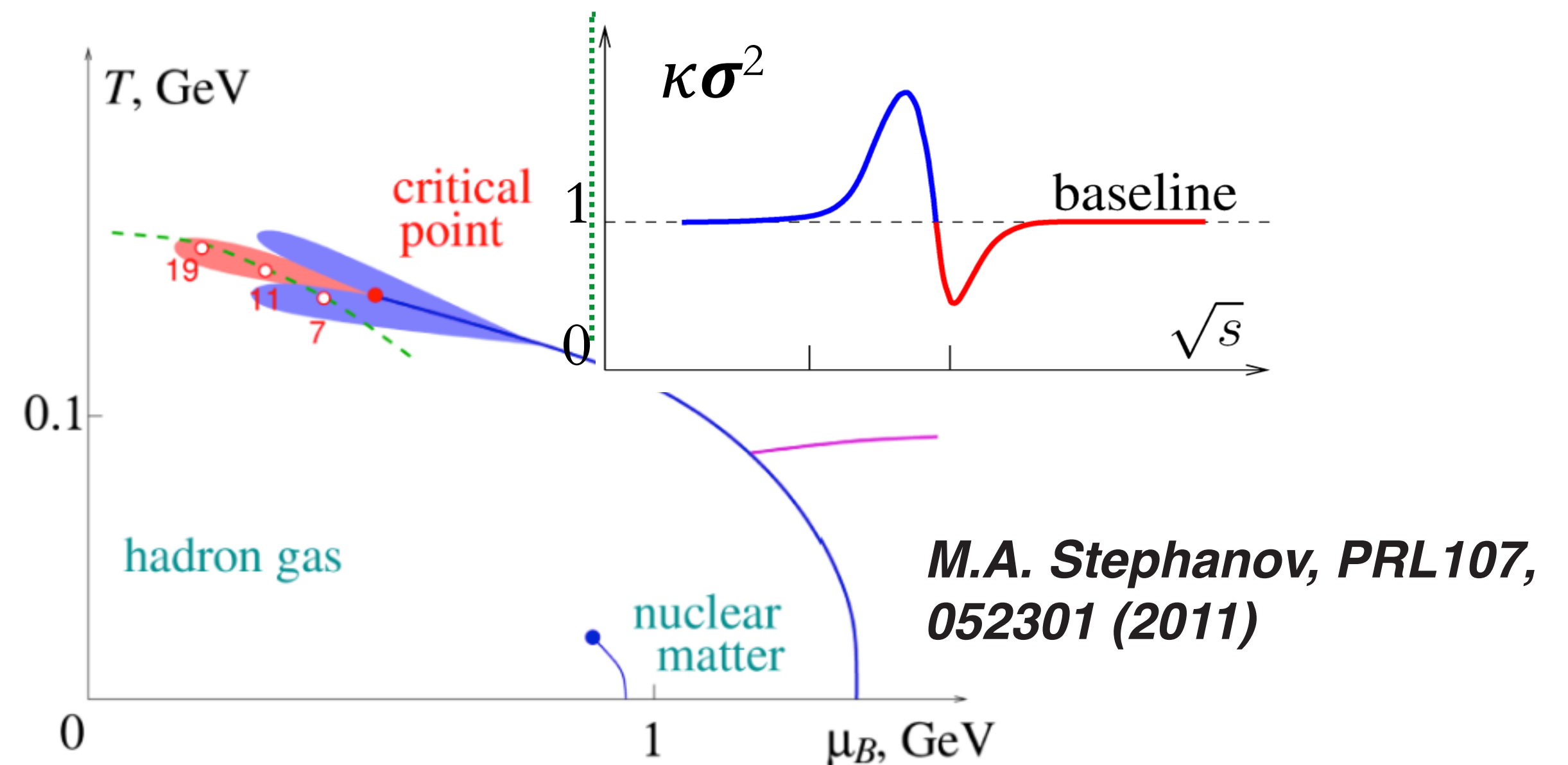


C_4/C_2 for critical point search

STAR Collaboration, PRL.126.092301(2021)

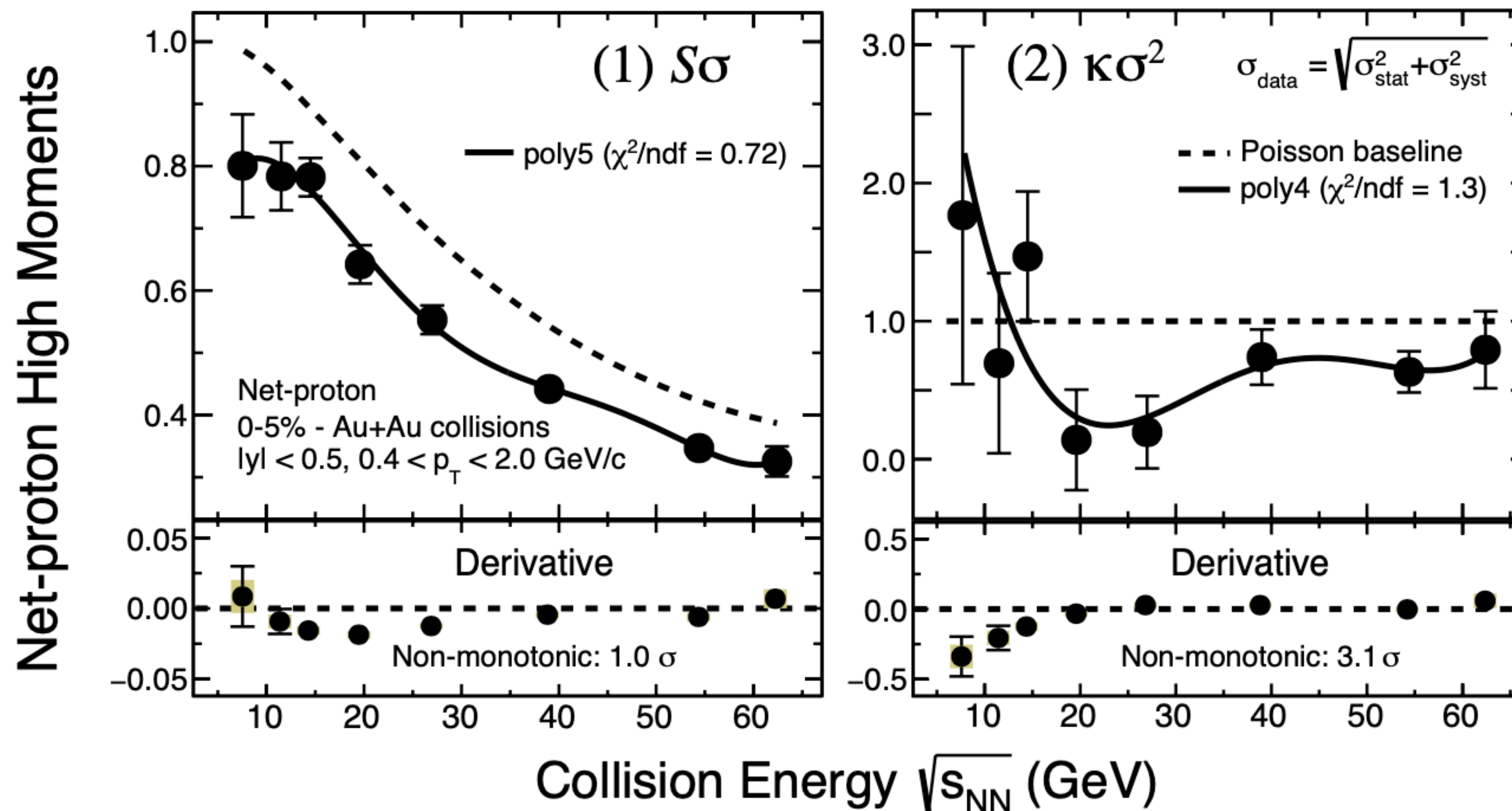


- ✓ Net-proton $\kappa\sigma^2$ (C_4/C_2) shows a non-monotonic behaviour. The trend is consistent with the expectation from theoretical calculations having a critical point.
- ✓ Enhancement at low beam energies cannot be explained by baryon number conservation.



Non-monotonicity

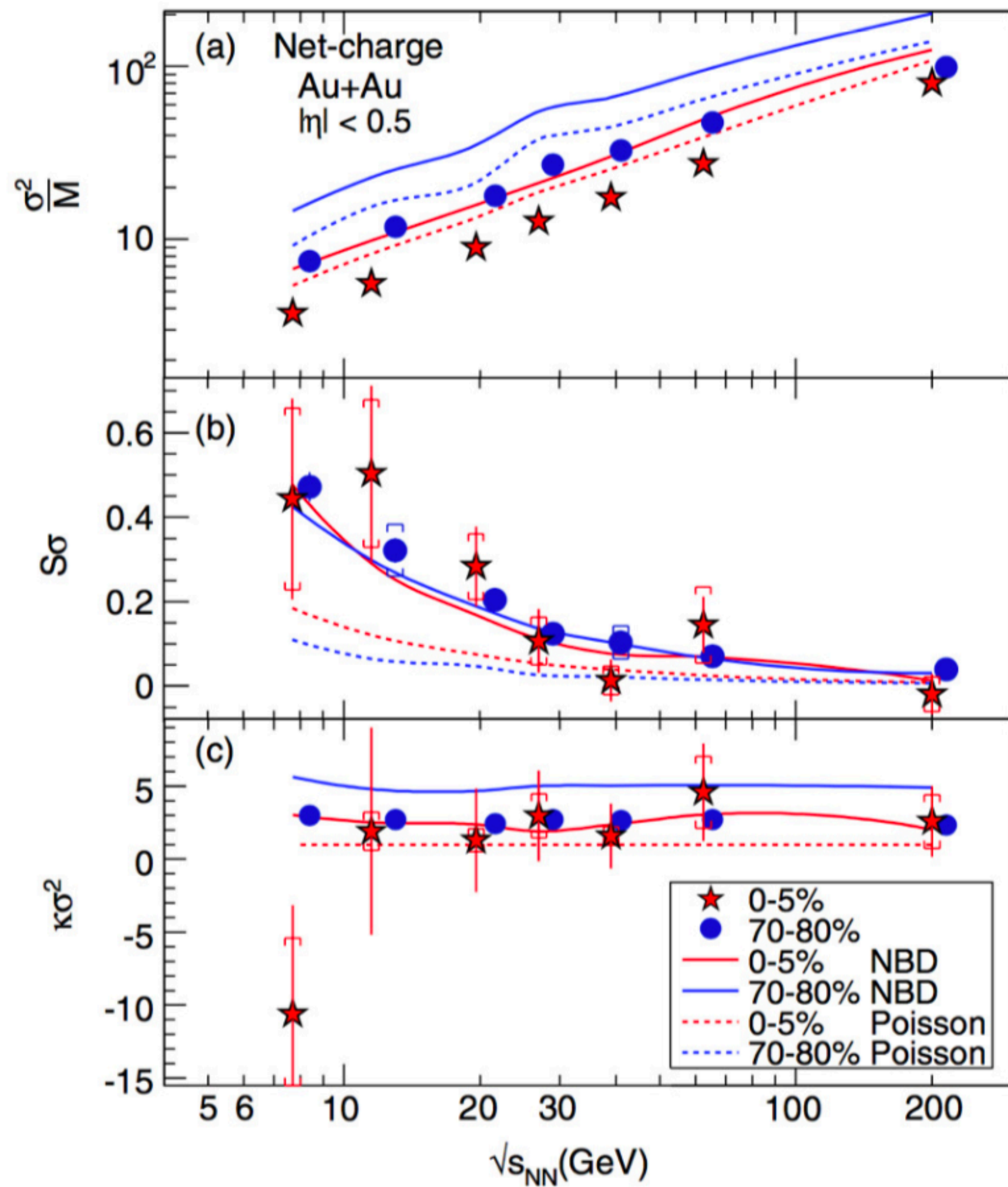
- ✓ Polynomial fits are done varying the data point within uncertainties.
- ✓ Check the probability that at least one point of derivatives at 8 energies has different sign from others → **3.1 σ significance of non-monotonicity for $\kappa\sigma^2$**



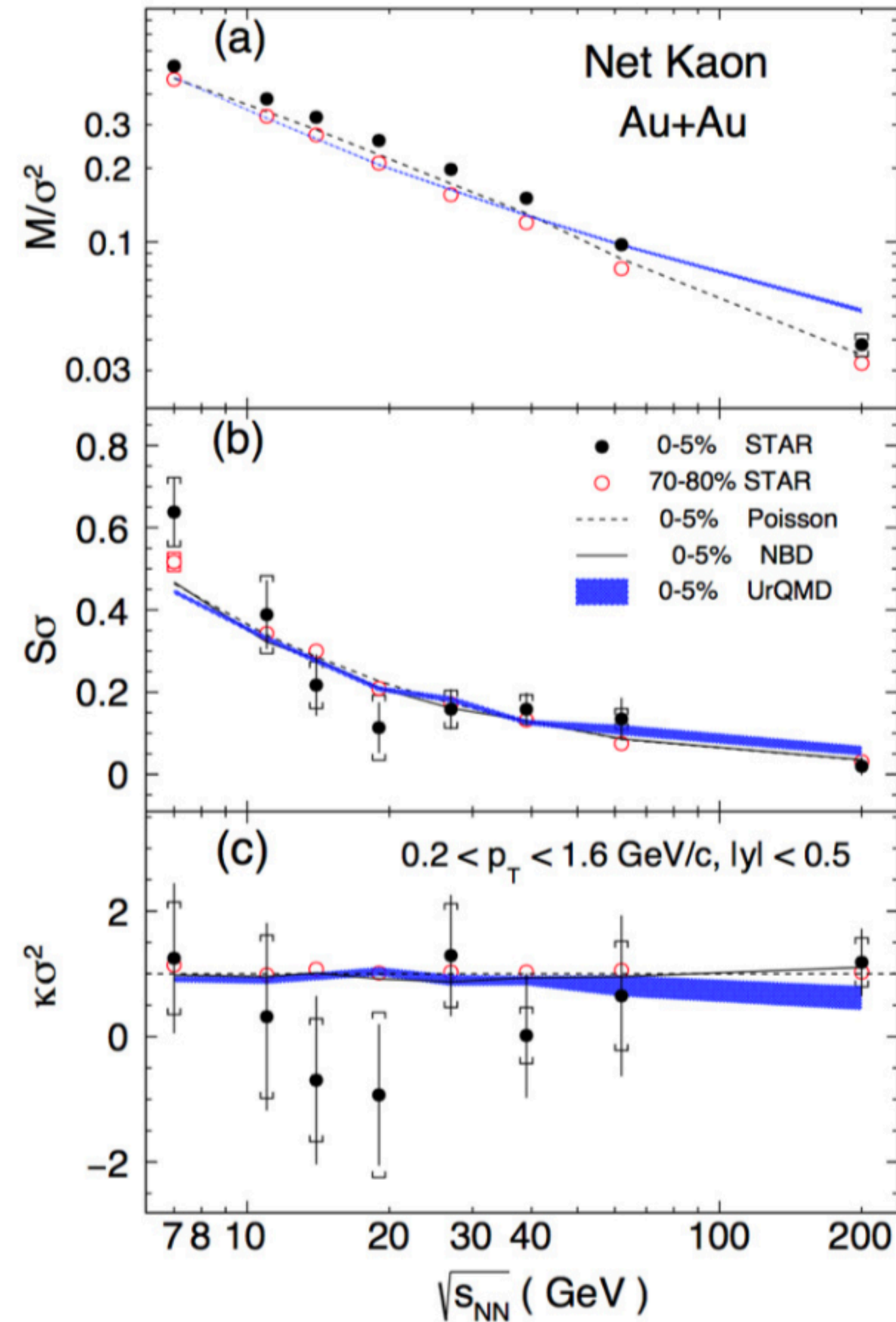


Net-charge and net-kaon

STAR: PRL 113, 092301(2014)



STAR: PLB, 785, 551(2018)



$$error(\kappa\sigma^2) \propto \frac{\sigma^2}{\epsilon^2} \frac{1}{\sqrt{N_{evts}}}$$

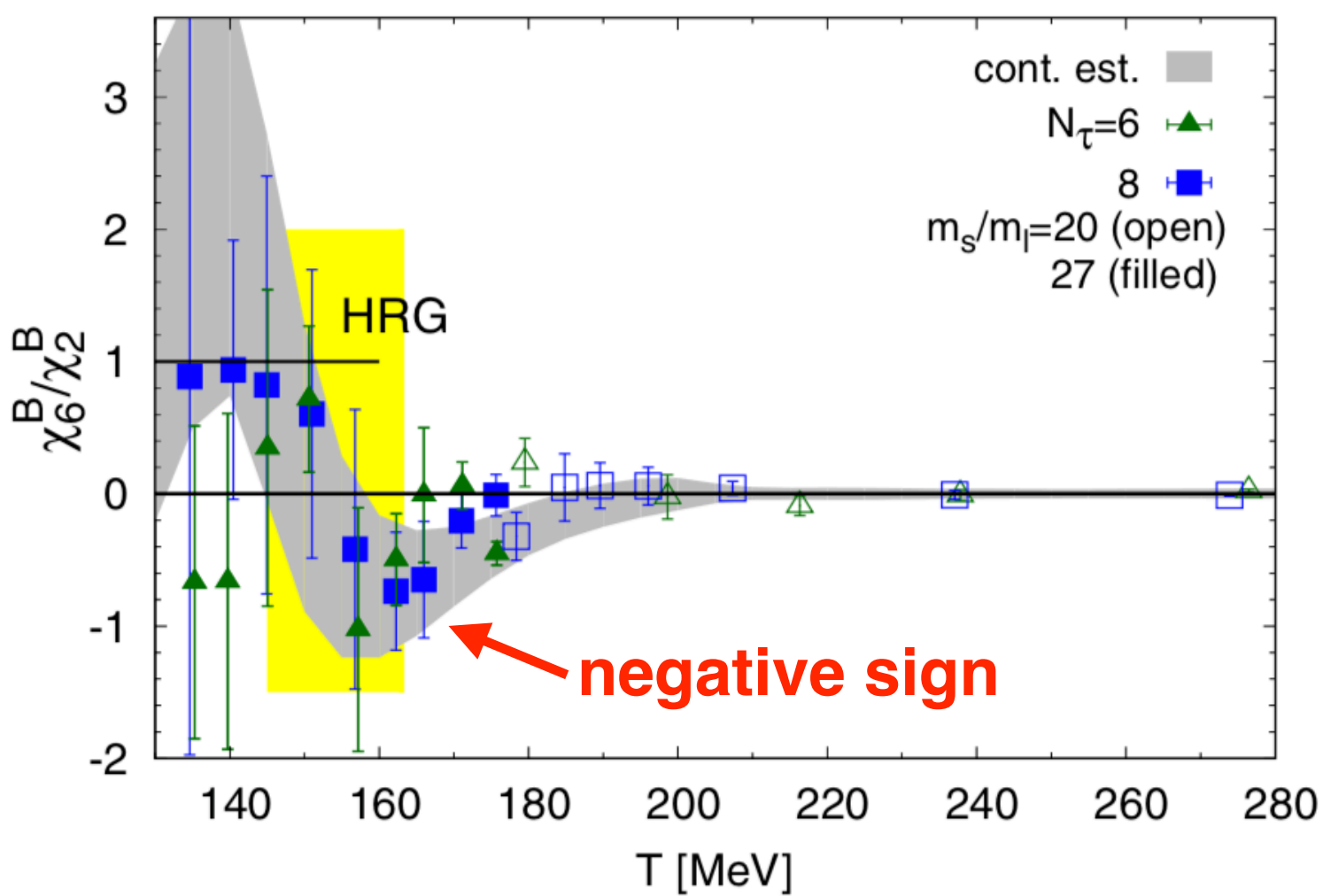
✓ Large statistical uncertainties, need more data.



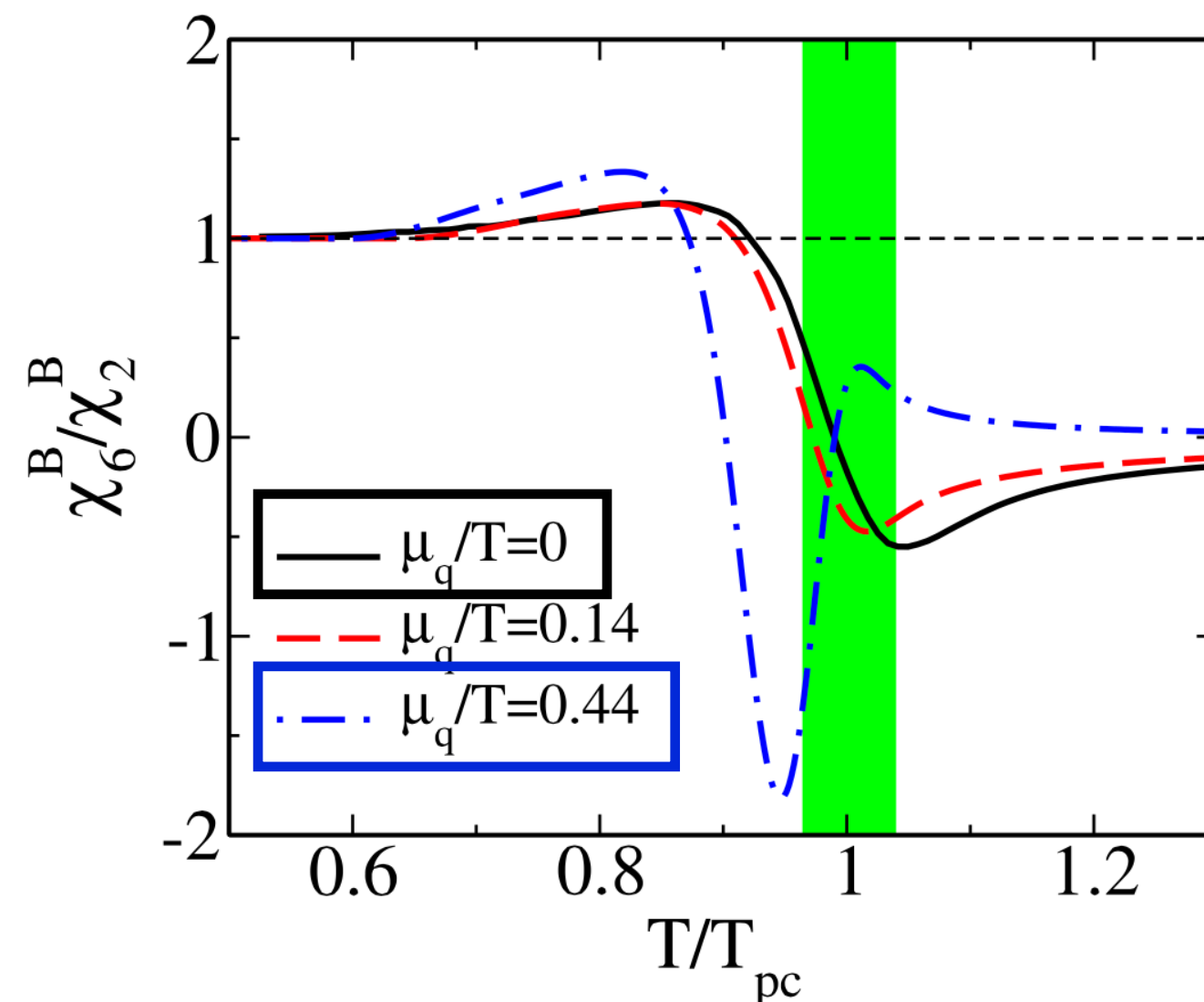
C_6/C_2 for crossover search

- ✓ There isn't yet any direct experimental evidence for the smooth crossover at $\mu_B \sim 0$.
- ✓ $C_6/C_2 < 0$ is predicted as a signature of crossover transition.
- ✓ High-statistics data sets at $\sqrt{s_{NN}} = 27, 54.4, \text{ and } 200 \text{ GeV}$ are analyzed to look for the **experimental signature of crossover transition**.

A. Bazavov et al,
PhysRevD.95.054504 : LQCD



Friman et al, Eur. Phys. J. C (2011)
71:1694 : PQM model



C.Schmidt, Prog.Theor.Phys.Suppl.186,563–566(2010)

Cheng et al, Phys. Rev. D 79, 074505 (2009)

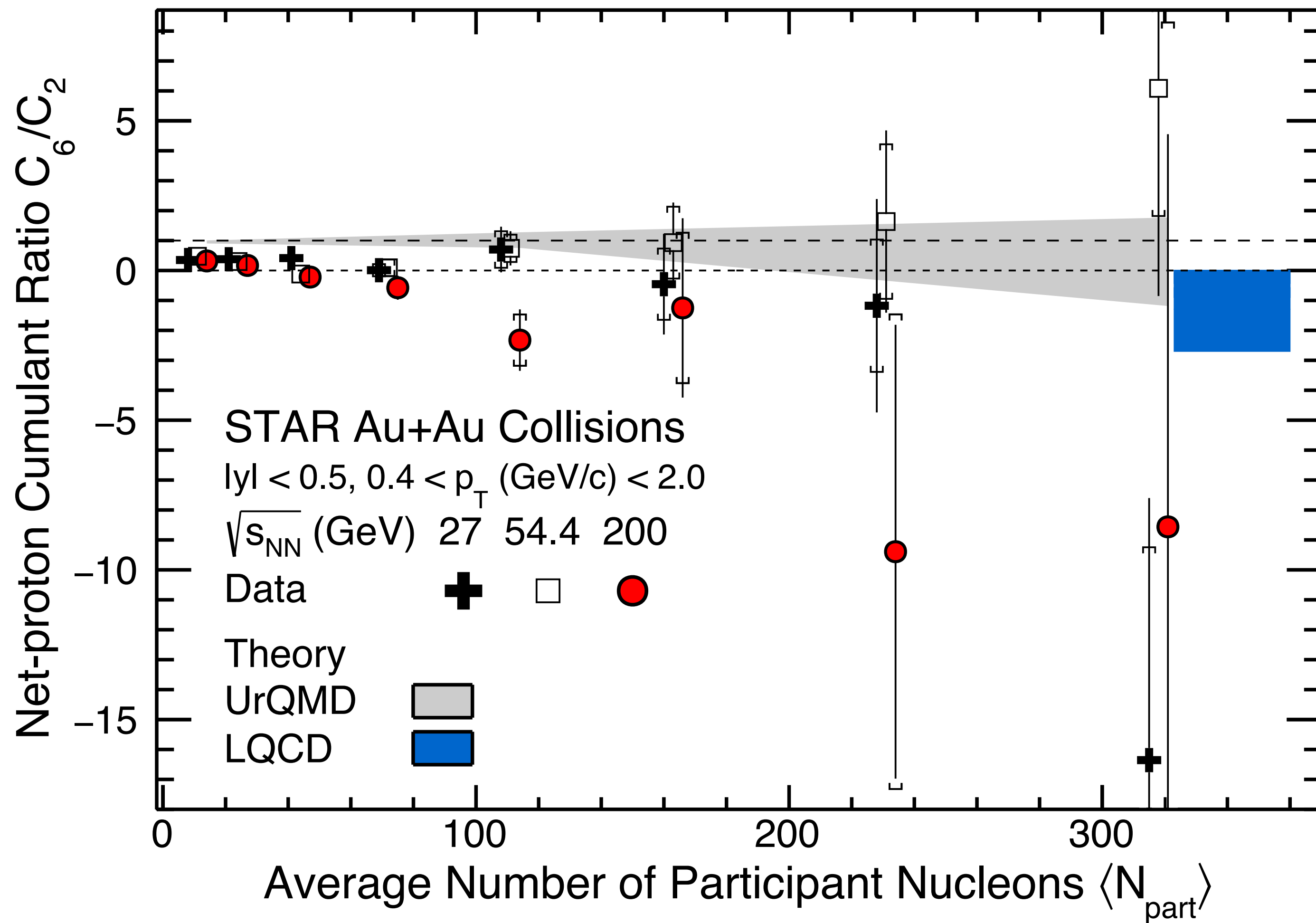
Friman et al, Eur. Phys. J. C (2011) 71:1694

Freeze-out conditions	χ_4^B / χ_2^B	χ_6^B / χ_2^B	χ_4^Q / χ_2^Q	χ_6^Q / χ_2^Q
HRG	1	1	~ 2	~ 10
QCD: $T^{\text{freeze}} / T_{pc} \lesssim 0.9$	$\gtrsim 1$	$\gtrsim 1$	~ 2	~ 10
QCD: $T^{\text{freeze}} / T_{pc} \simeq 1$	~ 0.5	< 0	~ 1	< 0

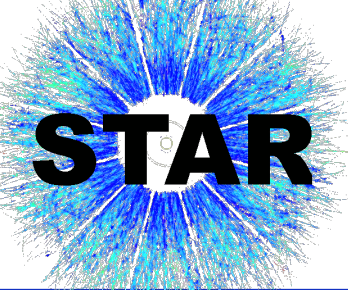
Predicted scenario for this measurement



Centrality dependence



- ✓ C_6/C_2 values are progressively negative from peripheral to central collisions at 200 GeV, which is consistent with LQCD calculations.
- ✓ Could suggest a smooth crossover transition at top RHIC energy.

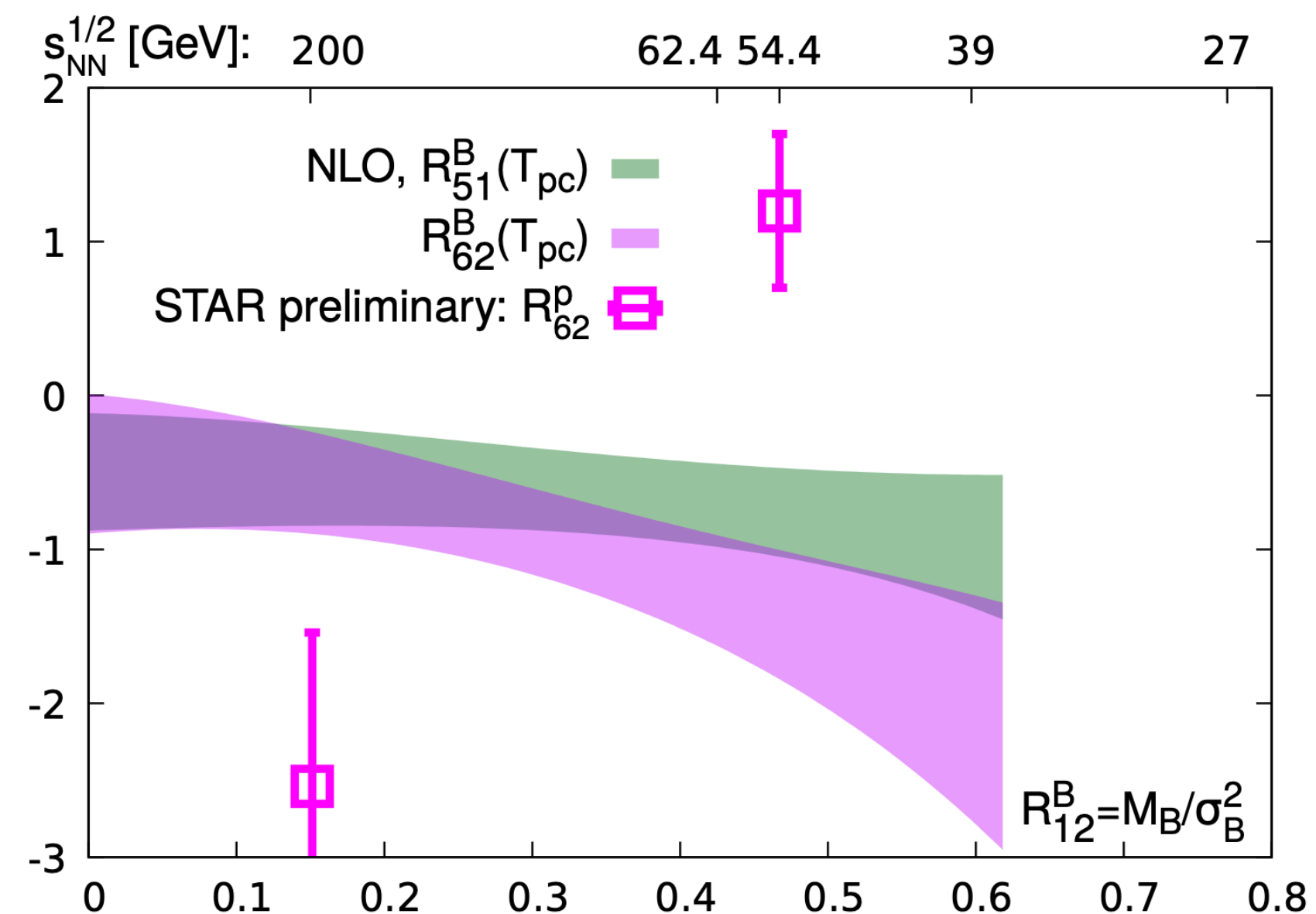
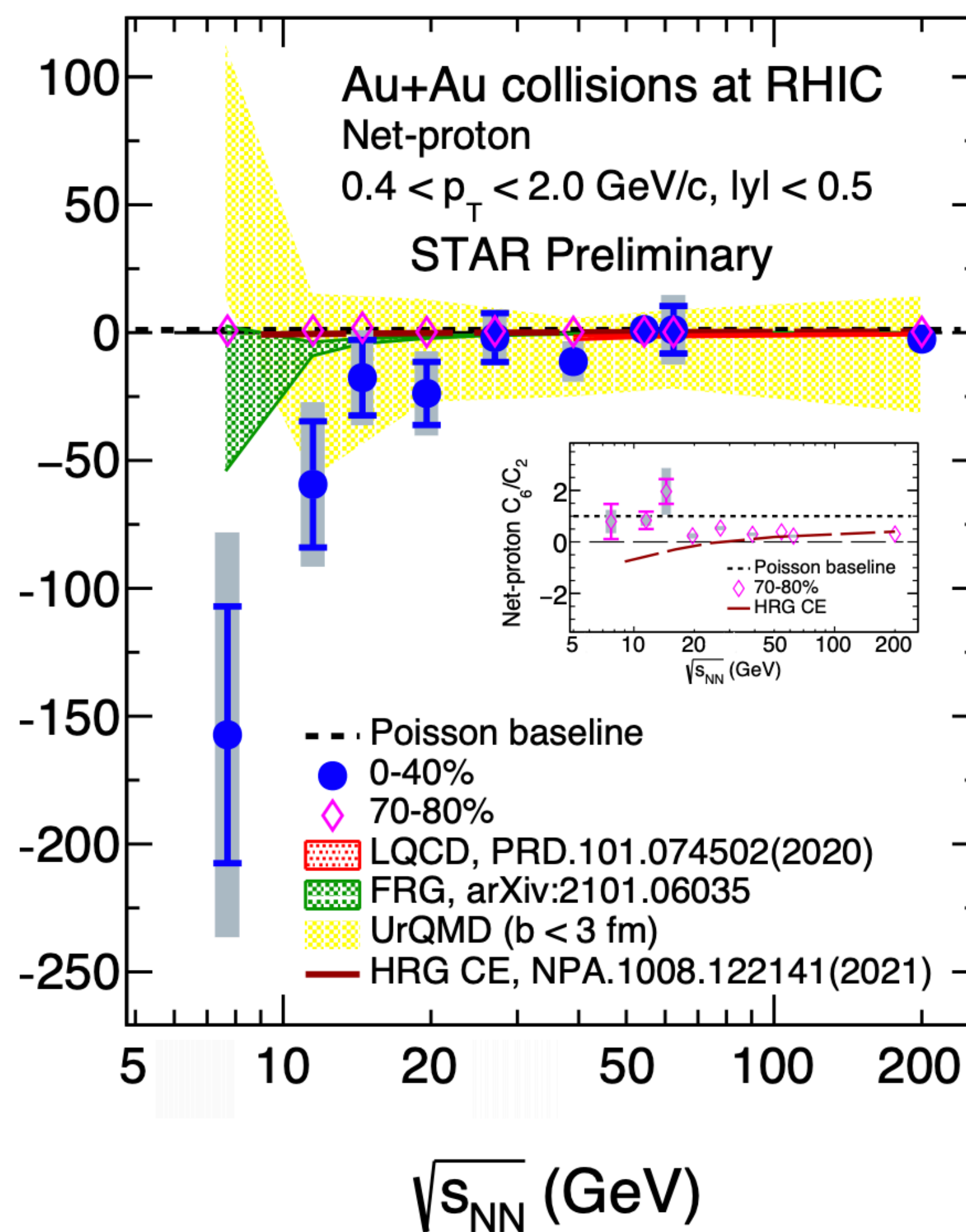
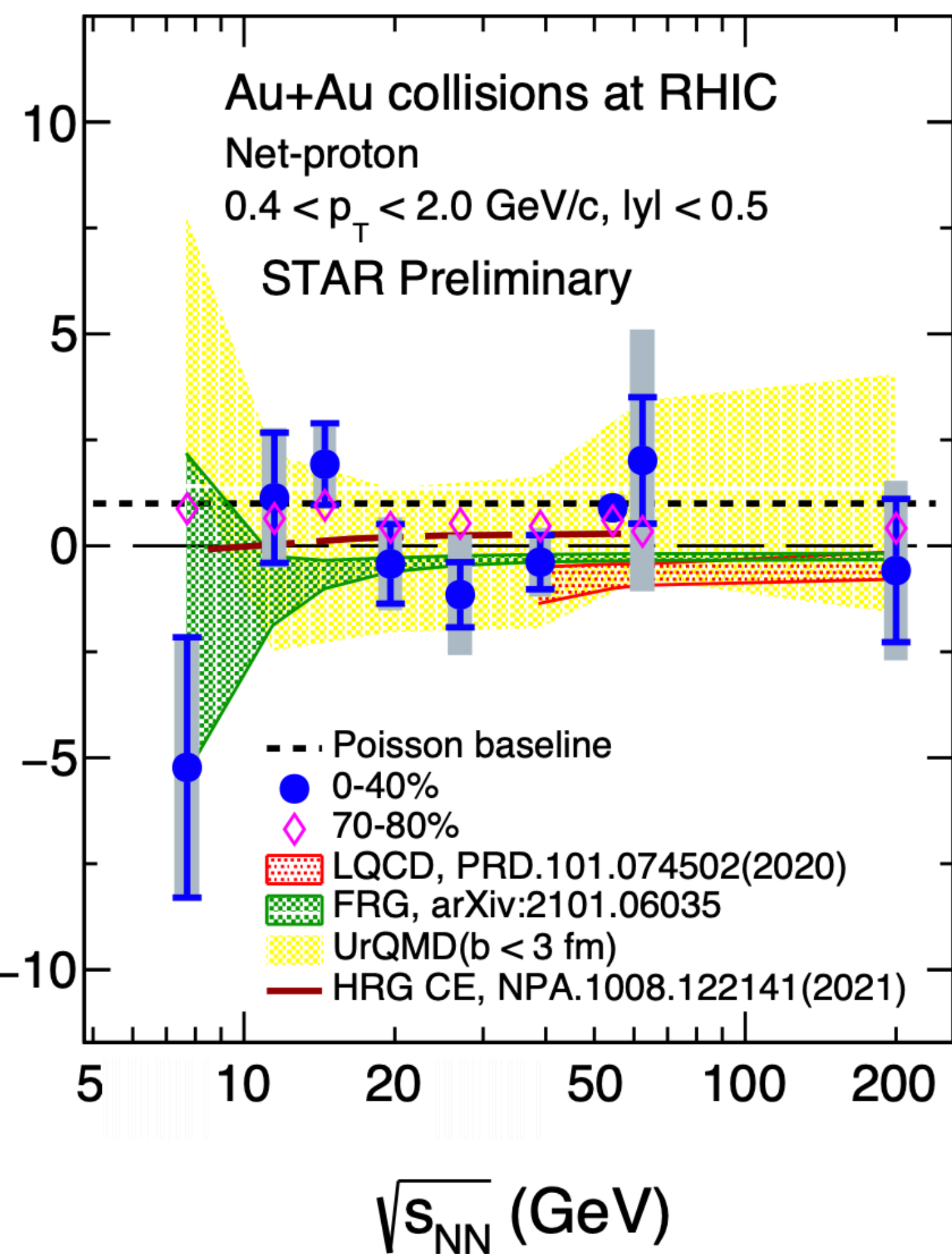


Energy dependence of C_5/C_1 and C_6/C_2

C_5/C_1

C_6/C_2

- ✓ Weak collision energy dependence observed for 0-40% centrality.
- ✓ Deviations from zero at a level of $< 2\sigma$ observed for 0-40% centrality.

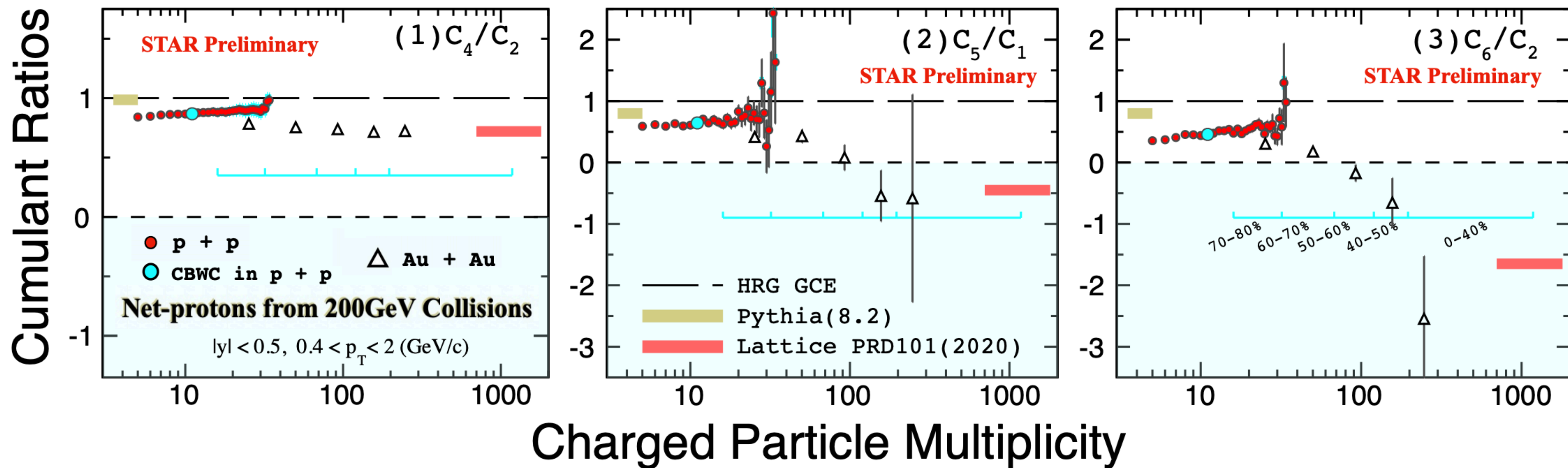


Bazavov et al., Phys.Rev.D101,074502 (2020)



Multiplicity dependence

- ✓ C_5/C_1 and C_6/C_2 are positive for p+p collisions, while negative for central Au+Au collisions.
- ✓ Lattice calculations imply chiral phase transition in the thermalized QCD matter, which is not the case in 200 GeV p+p collisions.

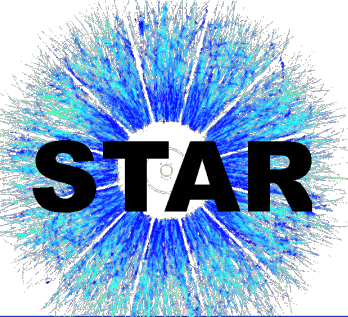


- Only statistical errors are shown for Au+Au results
- Efficiency is not corrected for x-axis

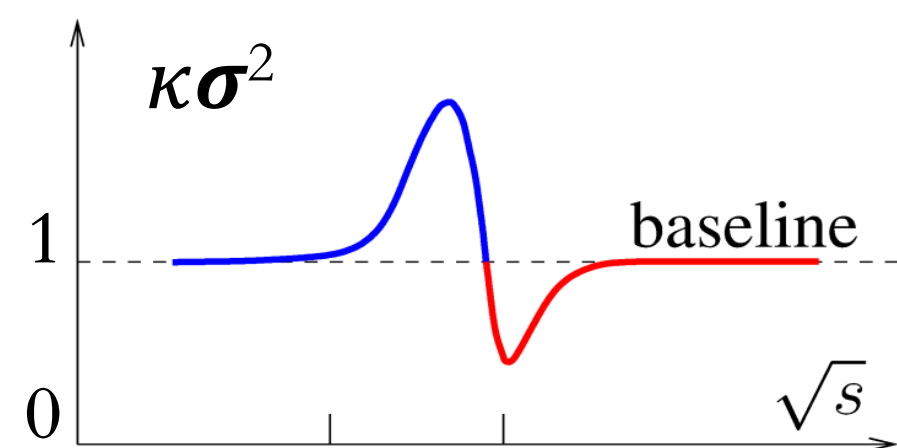
STAR Collaboration,
PRC.104.024902(2021)

LQCD : Phys. Rev. D 101, 074502 (2020)

STAR Collaboration,
Nuclear Physics A, 1005,
121882 (2021)

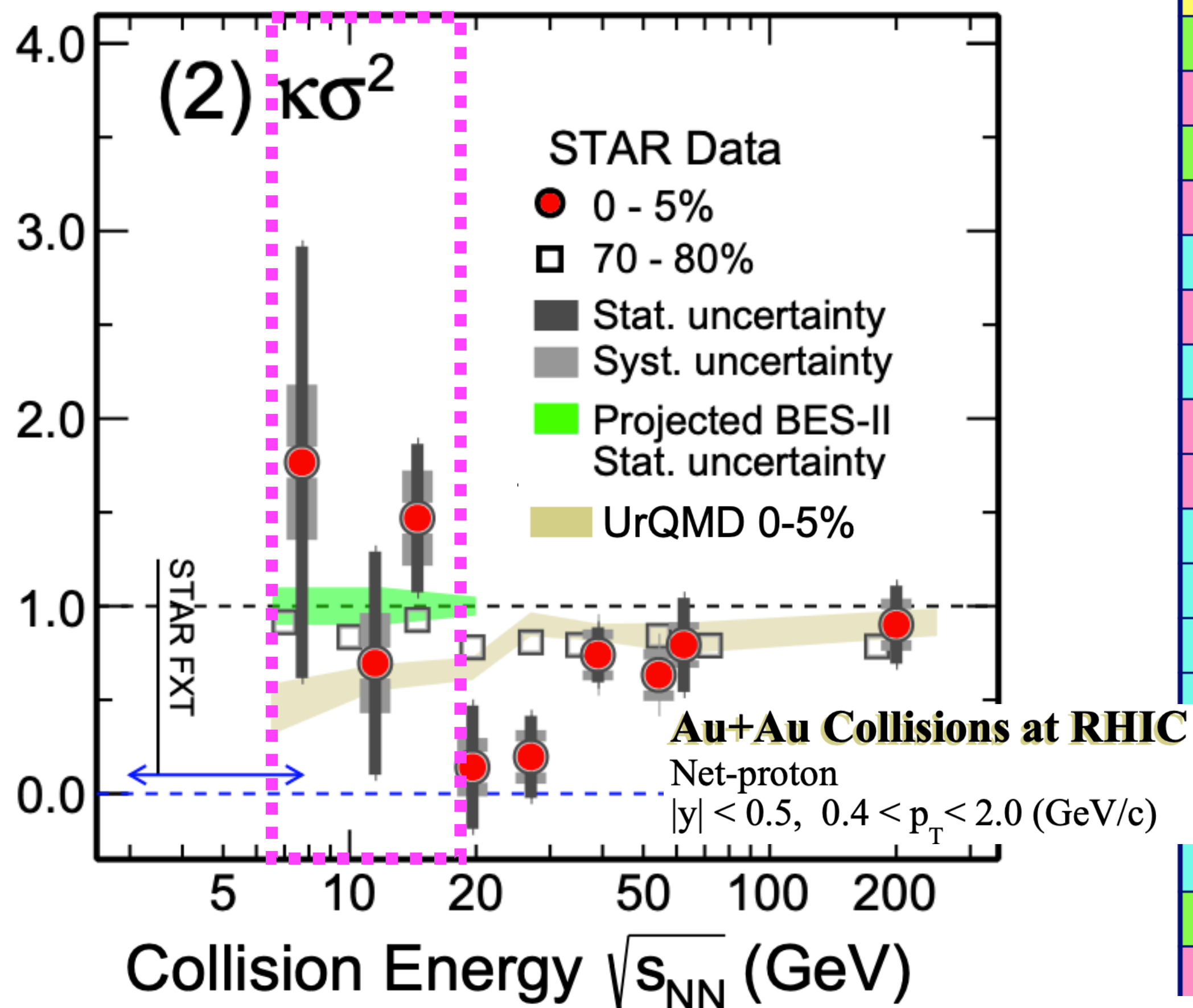


BES-II and Fixed-target programs



M.A. Stephanov,
PRL 107, 052301 (2011)

✓ 10-20 times larger statistics than BES-I have been successfully collected.

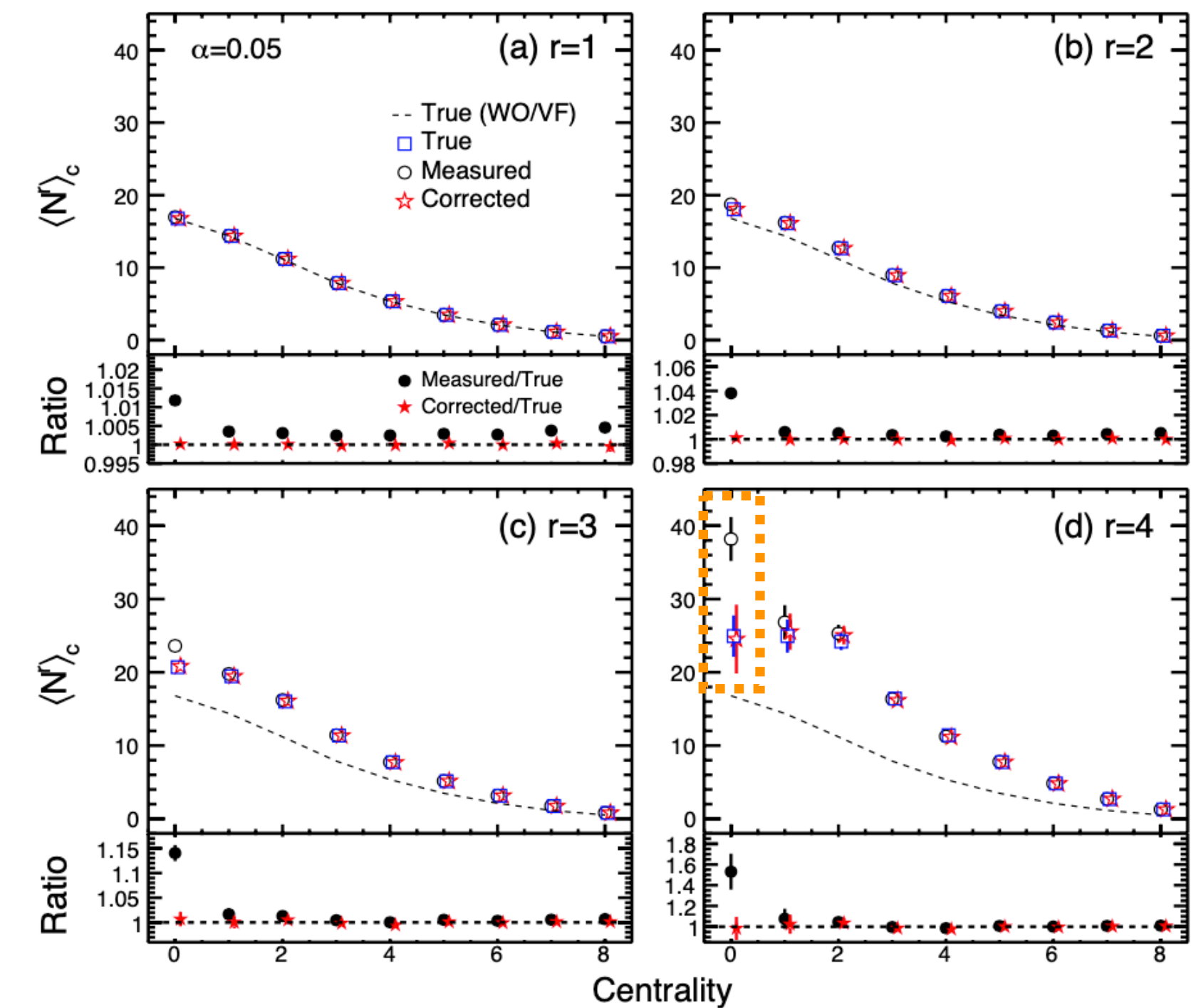
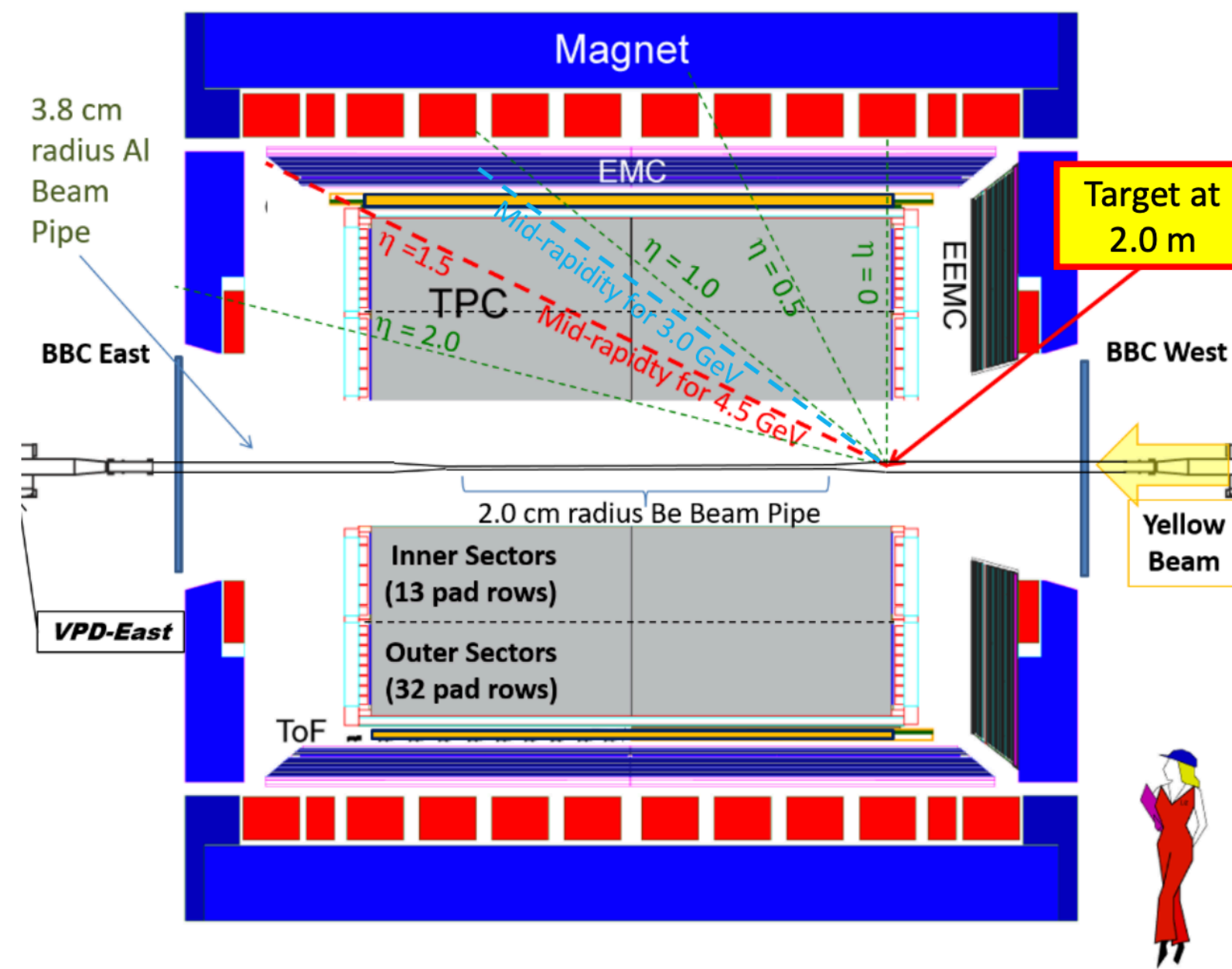
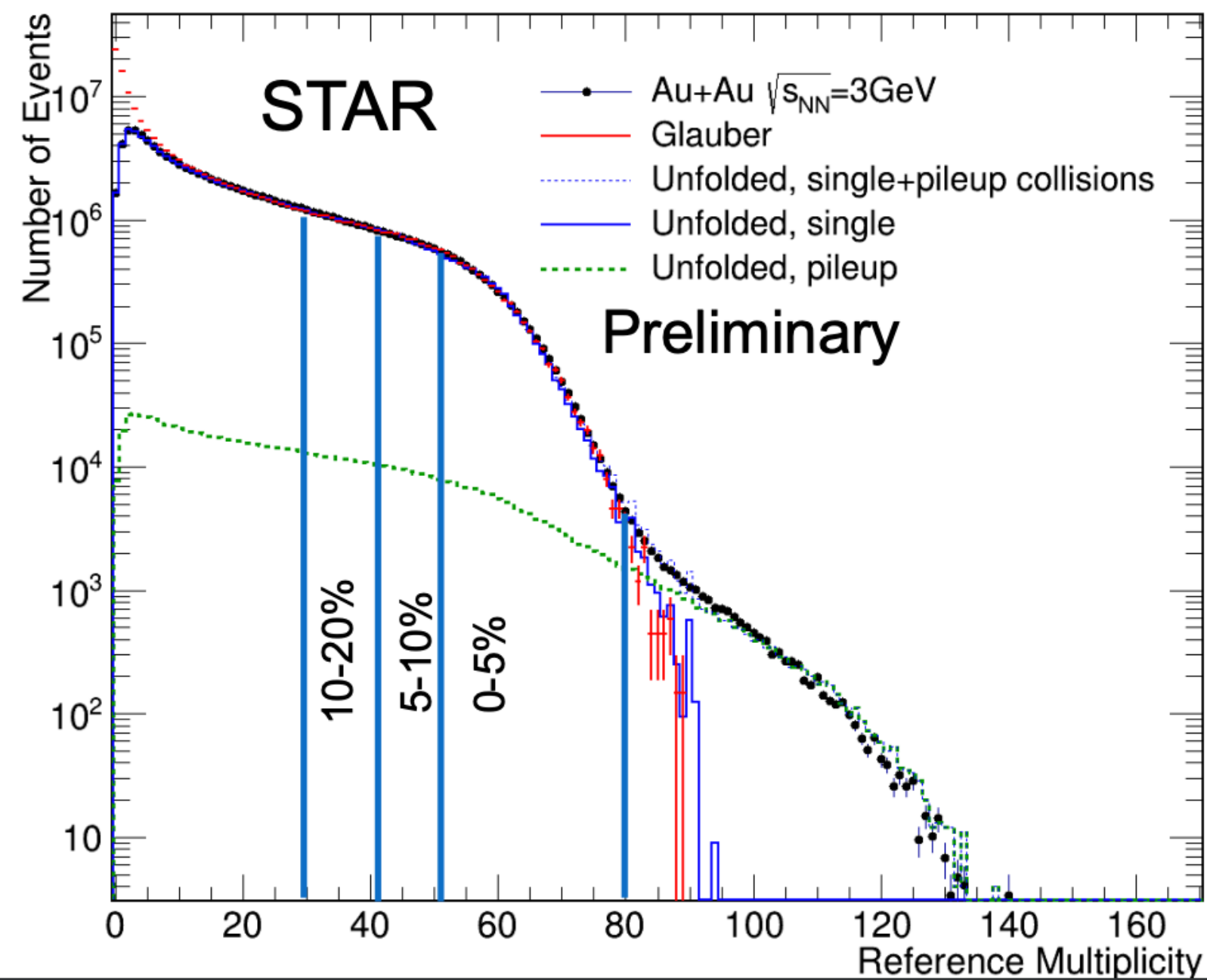


$\sqrt{s_{NN}}$ (GeV)	Beam Energy (GeV/nucleon)	Collider or Fixed Target	$y_{\text{center of mass}}$	μ_B (MeV)	Run Time (days)	No. Events Collected (Request)	Date Collected
200	100	C	0	25	2.0	138 M (140 M)	Run-19
27	13.5	C	0	156	24	555 M (700 M)	Run-18
19.6	9.8	C	0	206	36	582 M (400 M)	Run-19
17.3	8.65	C	0	230	14	256 M (250 M)	Run-21
14.6	7.3	C	0	262	60	324 M (310 M)	Run-19
13.7	100	FXT	2.69	276	0.5	52 M (50 M)	Run-21
11.5	5.75	C	0	316	54	235 M (230 M)	Run-20
11.5	70	FXT	2.51	316	0.5	50 M (50 M)	Run-21
9.2	4.59	C	0	372	102	162 M (160 M)	Run-20+20b
9.2	44.5	FXT	2.28	372	0.5	50 M (50 M)	Run-21
7.7	3.85	C	0	420	90	100 M (100 M)	Run-21
7.7	31.2	FXT	2.10	420	0.5+1.0+ scattered	50 M + 112 M + 100 M (100 M)	Run-19+20+21
7.2	26.5	FXT	2.02	443	2+Parasitic with CEC	155 M + 317 M	Run-18+20
6.2	19.5	FXT	1.87	487	1.4	118 M (100 M)	Run-20
5.2	13.5	FXT	1.68	541	1.0	103 M (100 M)	Run-20
4.5	9.8	FXT	1.52	589	0.9	108 M (100 M)	Run-20
3.9	7.3	FXT	1.37	633	1.1	117 M (100 M)	Run-20
3.5	5.75	FXT	1.25	666	0.9	116 M (100 M)	Run-20
3.2	4.59	FXT	1.13	699	2.0	200 M (200 M)	Run-19
3.0	3.85	FXT	1.05	721	4.6	259 M -> 2B(100 M -> 2B)	Run-18+21



Fixed-target program (FXT)

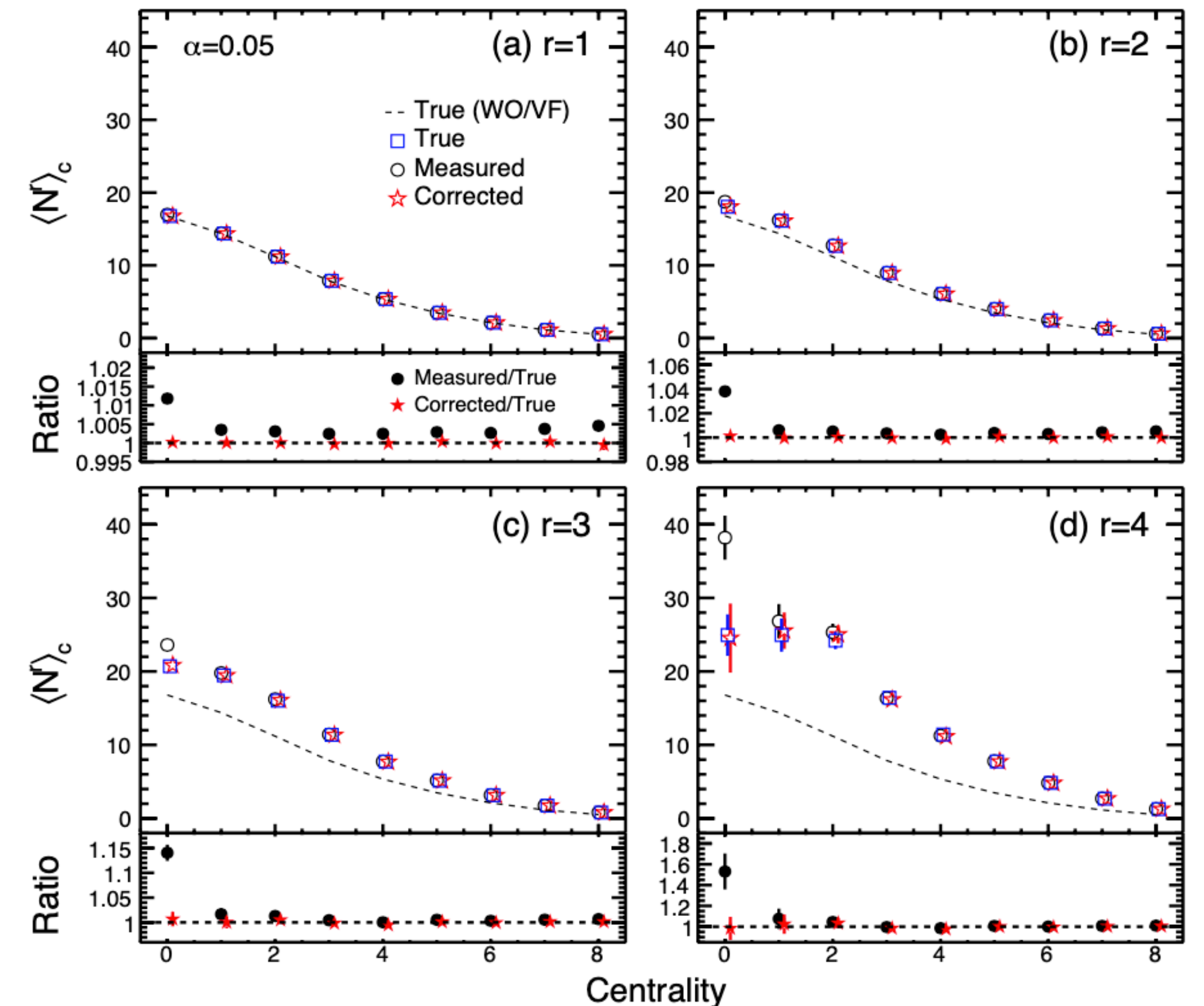
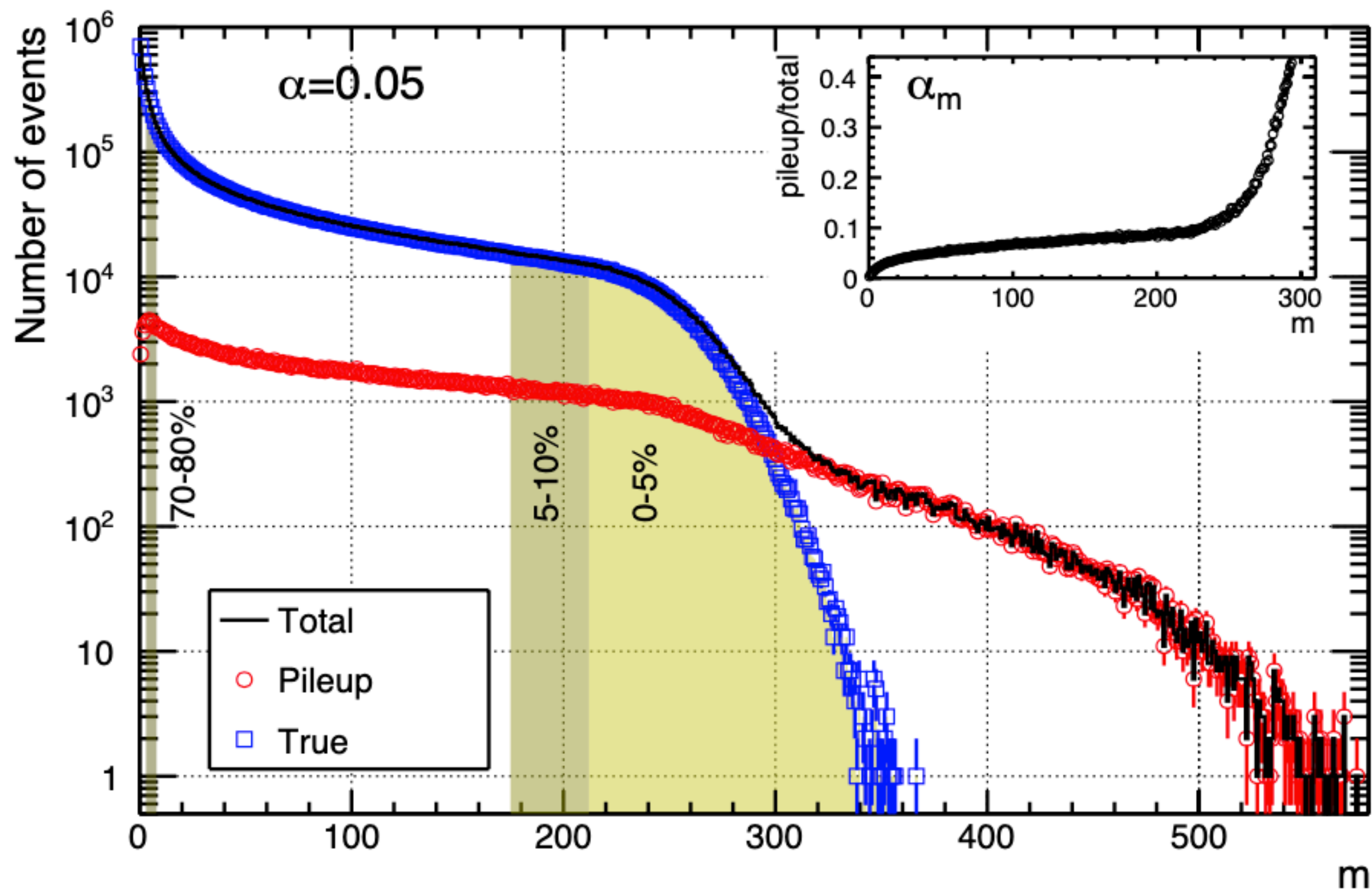
- ✓ μ_B region has been extended up to 720 MeV.
- ✓ Huge datasets were collected in FXT at 9 beam energies from 3 GeV to 7.7 GeV to confirm the peak structure of $\kappa\sigma^2$.
- ✓ Pileup is a critical issue on higher-moment analysis especially at most central collisions.
 - P. Garg et al., Phys. Rev. C 96 044908 (2017), S. Sombun et al., J. Phys. G45 025101 (2018)





FXT: pileup correction

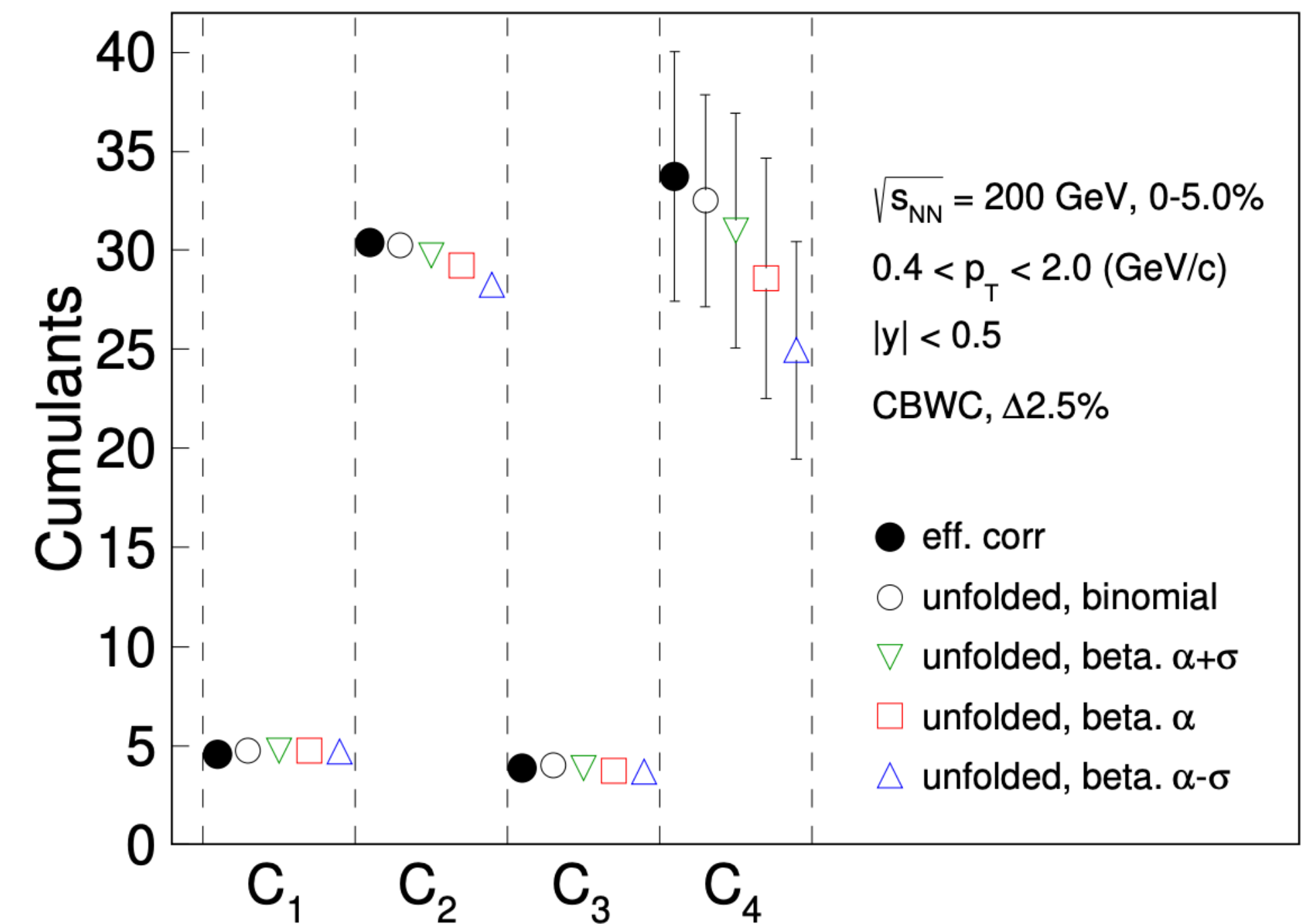
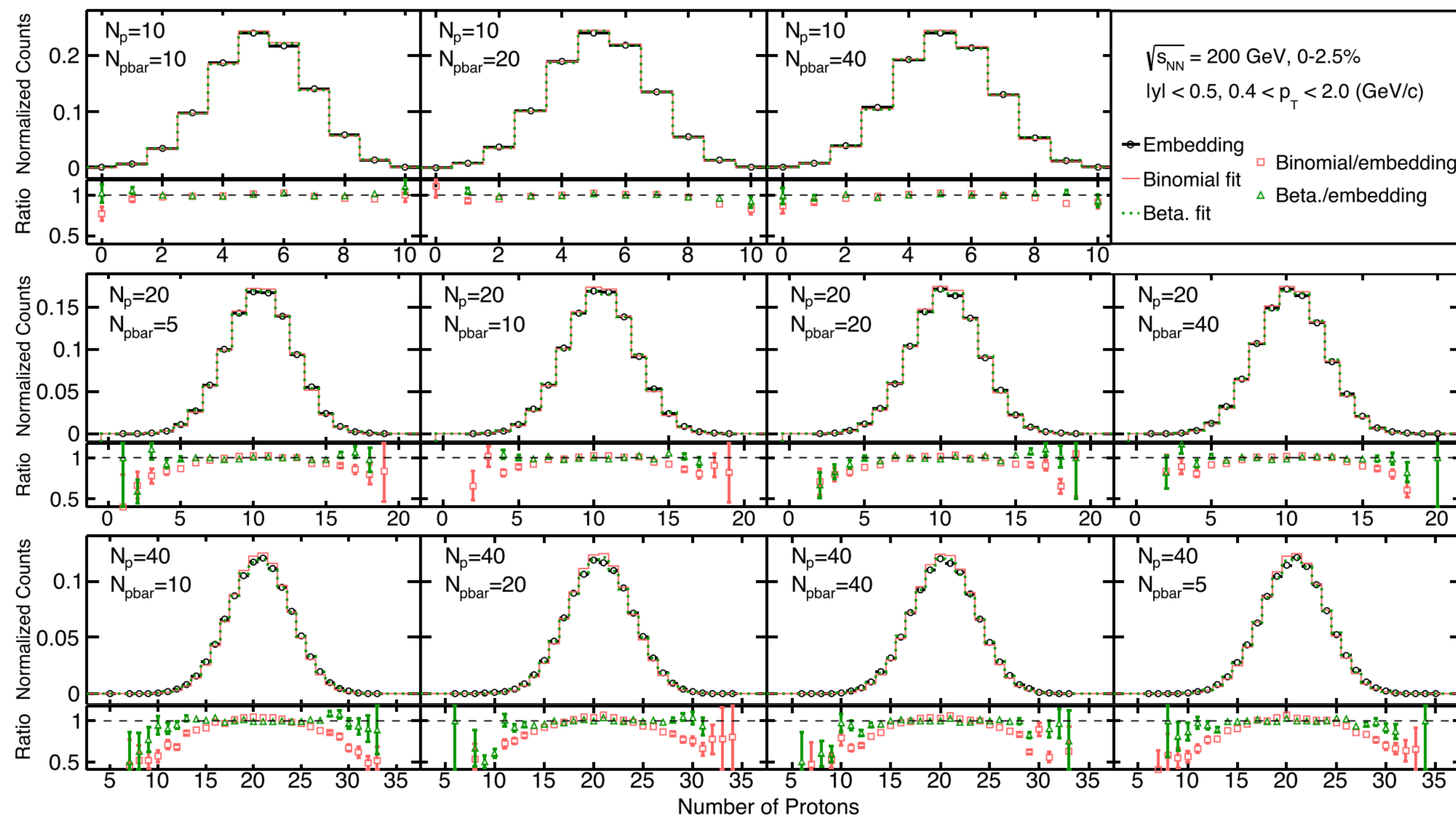
✓ Data-driven approach of the pileup correction is available once true and pileup multiplicity distributions are determined by simulations.





Challenges : Non-binomial efficiency correction

- ✓ Conventional correction method cannot be applied if response functions of detector efficiencies deviate from binomial distribution.
- ✓ Detector response functions need to be carefully studied via simulations.



STAR Collaboration, PRC.104.024902(2021)

STAR Collaboration, PRL.126.092301(2021)

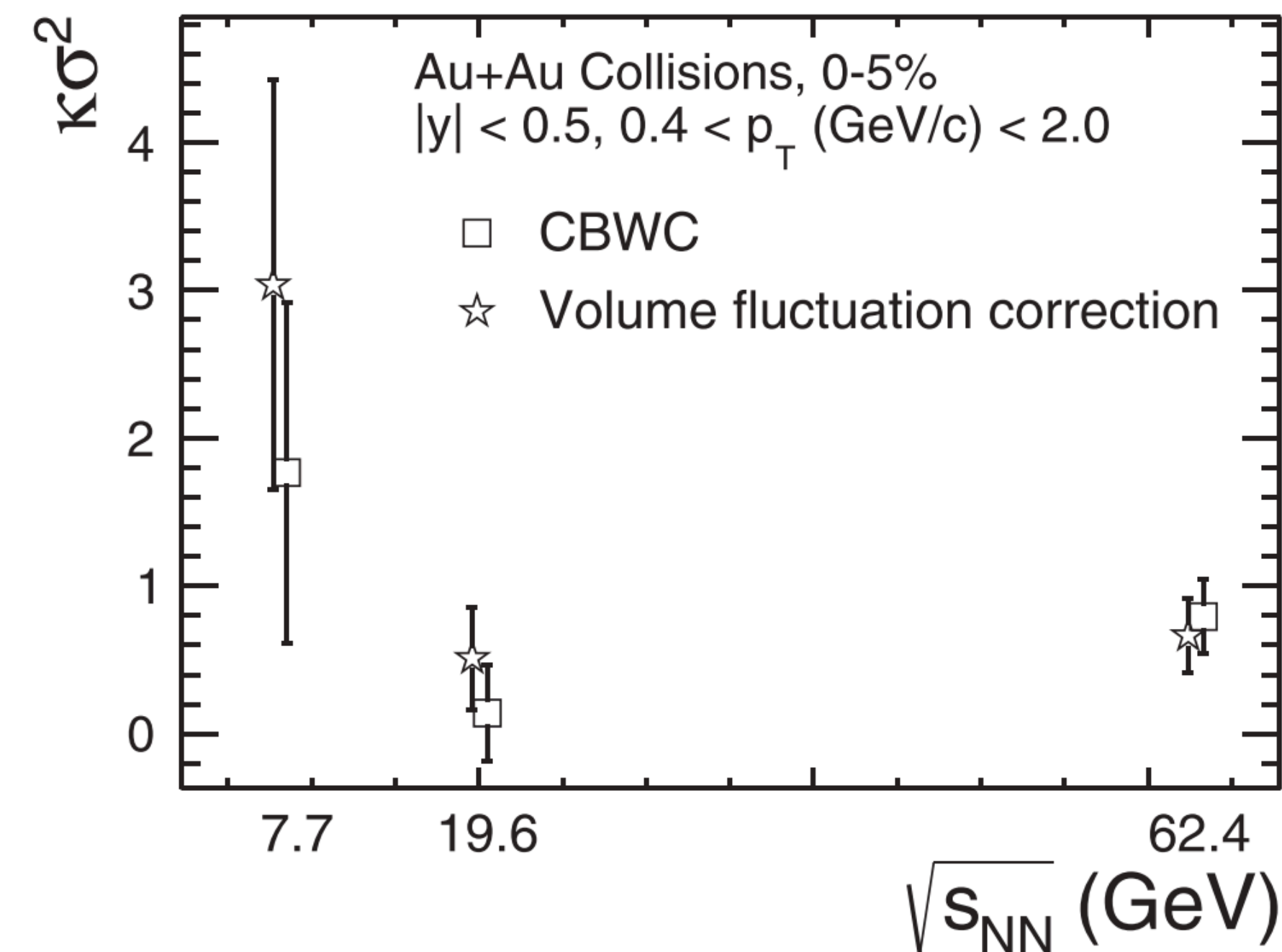
Unfolding: Esumi, Nakagawa, Nonaka, NIMA.987.164802(2021)

Moment expansion: Nonaka, Kitazawa, Esumi, NIMA906 10-17 (2018)



Challenges : Volume fluctuation

- ✓ Data driven approach (CBWC) and model dependent method (VFC) are consistent with each other in BES-I data.
- ✓ Due to less centrality resolution in lower collision energies, results will need to be carefully checked among different ways of centrality determination.



Measured cumulant	True cumulant	
$\kappa_1(\Delta N) = \langle N_W \rangle \kappa_1(\Delta n)$	$\kappa_1(\Delta N) = \langle N_W \rangle \kappa_1(\Delta n)$	
$\kappa_2(\Delta N) = \langle N_W \rangle \kappa_2(\Delta n) + \langle \Delta n \rangle^2 \kappa_2(N_W)$	$\kappa_2(\Delta N) = \langle N_W \rangle \kappa_2(\Delta n) + \langle \Delta n \rangle^2 \kappa_2(N_W)$	Additional terms appear from the event by event participant fluctuation
$\kappa_3(\Delta N) = \langle N_W \rangle \kappa_3(\Delta n) + 3 \langle \Delta n \rangle \kappa_2(\Delta n) \kappa_2(N_W) + \langle \Delta n \rangle^3 \kappa_3(N_W)$	$\kappa_3(\Delta N) = \langle N_W \rangle \kappa_3(\Delta n) + 3 \langle \Delta n \rangle \kappa_2(\Delta n) \kappa_2(N_W) + \langle \Delta n \rangle^3 \kappa_3(N_W)$	
$\kappa_4(\Delta N) = \langle N_W \rangle \kappa_4(\Delta n) + 4 \langle \Delta n \rangle \kappa_3(\Delta n) \kappa_2(N_W) + 3 \kappa_2^2(\Delta n) \kappa_2(N_W) + 6 \langle \Delta n \rangle^2 \kappa_2(\Delta n) \kappa_3(N_W) + \langle \Delta n \rangle^4 \kappa_4(N_W)$	$\kappa_4(\Delta N) = \langle N_W \rangle \kappa_4(\Delta n) + 4 \langle \Delta n \rangle \kappa_3(\Delta n) \kappa_2(N_W) + 3 \kappa_2^2(\Delta n) \kappa_2(N_W) + 6 \langle \Delta n \rangle^2 \kappa_2(\Delta n) \kappa_3(N_W) + \langle \Delta n \rangle^4 \kappa_4(N_W)$	

Δn : net-proton per N_W
 ΔN : net-proton

P. Braun-Munzinger, A. Rustamov, J. Stachel: *NPA.2017.01.011*



Summary

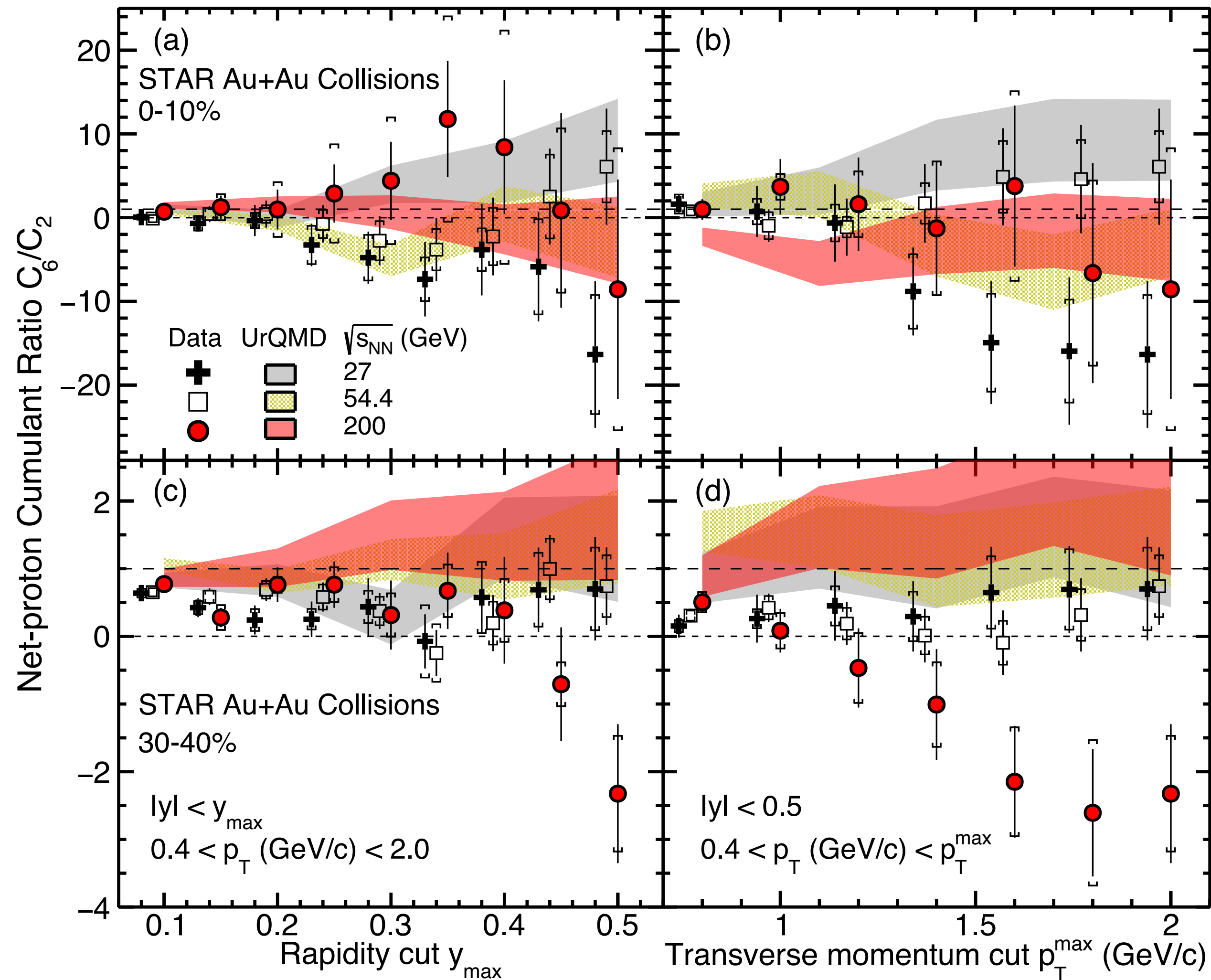
- ✓ Non-monotonic beam energy dependence of C_4/C_2 has been observed in BES-I. Stay tuned for precise measurement from BES-II and FXT energies (3.3-19.6 GeV) to have more definitive messages.
- ✓ Negative value of C_6/C_2 at $\sqrt{s_{NN}} = 200$ GeV central collisions could suggest a smooth crossover transition at RHIC top energy. This is not the case for p+p 200 GeV high multiplicity events.
- ✓ FXT 3 GeV analysis is ongoing. Pileup effects can be corrected.
- ✓ Detector efficiencies and volume fluctuations need to be carefully studied for each experiment.

Thank you for your attention

Back up



Acceptance dependence



- ✓ Consistent with zero for UrQMD
- ✓ C_6/C_2 values are negative for 200 GeV 30-40% centrality.

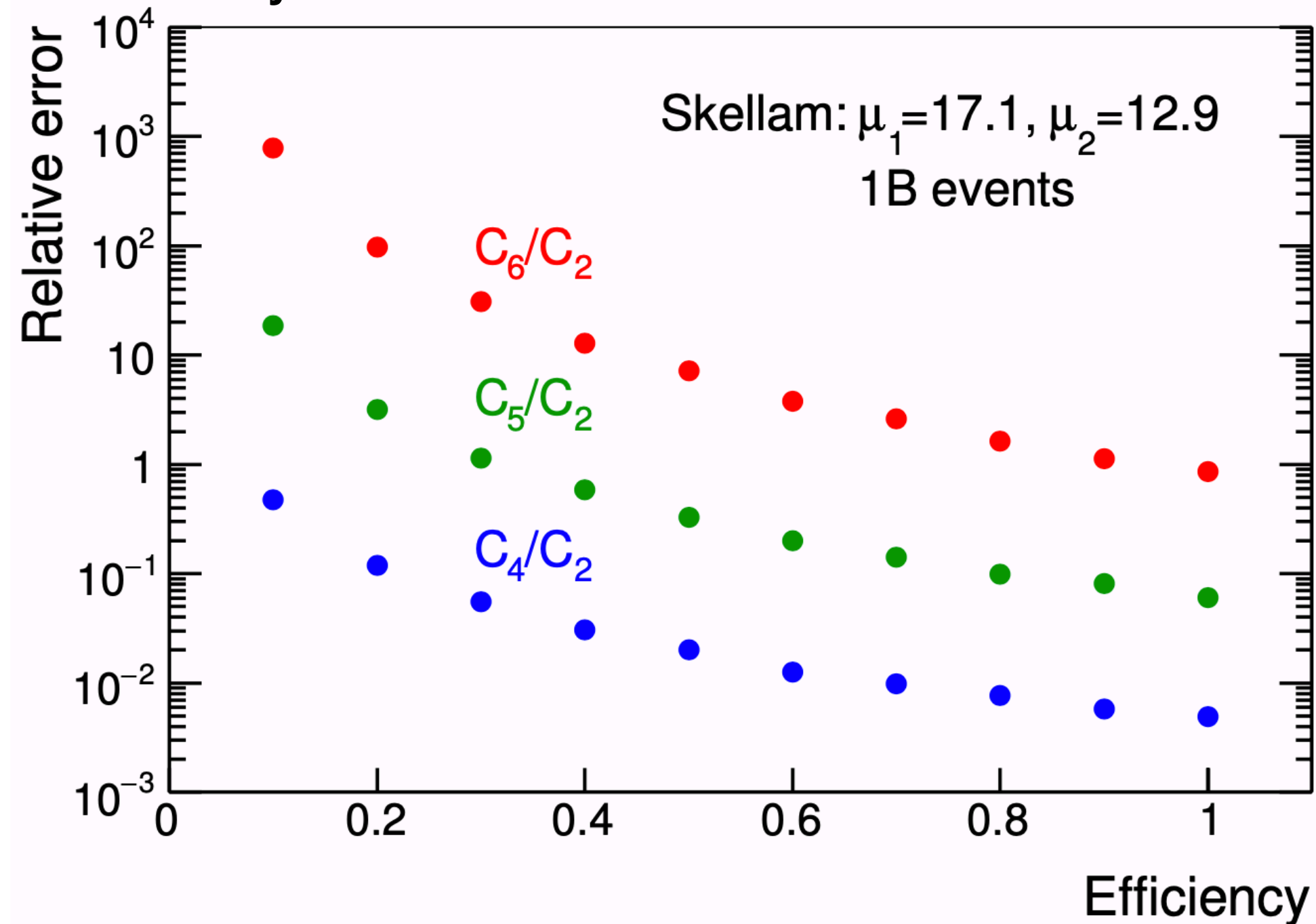


Statistical uncertainties

$$error(k\sigma^2) \propto \frac{\sigma^2}{\epsilon^2} \frac{1}{\sqrt{N_{evts}}}$$

- ✓ Larger statistical uncertainties in central than peripheral collisions
 - Broader distributions in central collisions (large σ^2)
 - Lower detector efficiencies in central collisions due to high-multiplicity.
- ✓ 100 times larger errors for C_6 than C_4 !

Toy model calculations



Toy model calculations

