Methods and Results on Conserved Charge Fluctuations from RHIC BES & FXT

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ISMD2023 @Gyöngyös, Hungary
Outline

• Introduction
• Results from BES-I and FXT
• New analysis for baryon-strangeness correlations
  • Purity correction
  • Results
• Summary
“Conjectured” QCD phase diagram

- Crossover at $\mu_B = 0$ MeV
- 1st-order phase transition at large $\mu_B$?
- Critical point?

Beam Energy Scan Phase-I (BES-I)

- Crossover at $\mu_B = 0$ MeV
- 1st-order phase transition at large $\mu_B$?
- Critical point?

<table>
<thead>
<tr>
<th>$\sqrt{s_{NN}}$ (GeV)</th>
<th>No. of events (million)</th>
<th>$T_{ch}$ (MeV)</th>
<th>$\mu_B$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>238</td>
<td>164.3</td>
<td>28</td>
</tr>
<tr>
<td>62.4</td>
<td>47</td>
<td>160.3</td>
<td>70</td>
</tr>
<tr>
<td>54.4</td>
<td>550</td>
<td>160.0</td>
<td>83</td>
</tr>
<tr>
<td>39</td>
<td>86</td>
<td>156.4</td>
<td>160</td>
</tr>
<tr>
<td>27</td>
<td>30</td>
<td>155.0</td>
<td>144</td>
</tr>
<tr>
<td>19.6</td>
<td>15</td>
<td>153.9</td>
<td>188</td>
</tr>
<tr>
<td>14.5</td>
<td>20</td>
<td>151.6</td>
<td>264</td>
</tr>
<tr>
<td>11.5</td>
<td>6.6</td>
<td>149.4</td>
<td>287</td>
</tr>
<tr>
<td>7.7</td>
<td>3</td>
<td>144.3</td>
<td>398</td>
</tr>
</tbody>
</table>

Complementary measurements at RHIC: BES-II, FXT (2019-2022)

Fluctuations of conserved charges have been measured for BES-I data.
STAR detectors

- Large & uniform acceptance (full azimuth, $|\eta|<1$)
- Excellent particle identification

FXT mode: $\mu_B=760$ MeV @ 3GeV
Cumulants of conserved charges

- Measure event-by-event distributions of net-baryon, net-charge, and net-strangeness number

\[ \Delta N_q = N_q - N_{\bar{q}}, \quad q = B, Q, S \]

1. Sensitive to the correlation length

\[
\begin{align*}
C_2 &= \langle (\delta N)^2 \rangle_c \approx \xi^2 \\
C_3 &= \langle (\delta N)^3 \rangle_c \approx \xi^{4.5} \\
C_4 &= \langle (\delta N)^4 \rangle_c \approx \xi^7
\end{align*}
\]

M. A. Stephanov, PRL102.032301(2009), PRL107.052301(2011)
M. Asakawa, S. Ejiri, and M. Kitazawa, PRL103262301(2009)

2. Comparison with susceptibilities

\[
\begin{align*}
S\sigma &= \frac{C_3}{C_2} = \frac{\chi_3}{\chi_2} \\
\kappa\sigma^2 &= \frac{C_4}{C_2} = \frac{\chi_4}{\chi_2} \\
\chi^q_n &= \frac{1}{VT^3} \times C^q_n = \frac{\partial_n p/T^4}{\partial \mu_q^n}, \quad q = B, Q, S
\end{align*}
\]

Raw net-proton multiplicity distribution

- Need to consider various experimental effects.

STAR, PRL126.092301(2021), PRC104.024902(2021)
Experimental challenges

- Detector efficiency correction
  - Binomial distribution
    - M. Kitazawa and M. Asakawa, PRC86.024904(2012), A. Bzdak and V. Koch, PRC86.044904(2012), X. Luo, PRC91.034907(2016),
  - Non-binomial distribution
    - T. Nonaka, M. Kitazawa, S. Esumi, NIMA906 10-17(2018)
    - S. Esumi, K. Nakagawa, T. Nonaka, NIMA987.164802(2021)
- Initial volume fluctuation
- Pileup events
- Particle identification
  - M. Arslandok and A. Rustamov, NIMA946.162622 (2019)
- More to be resolved…
  - Net-proton≠net-baryon, purity correction, acceptance dependence for comparison with theory, ….

*Not all important studies are listed here*
Results from BES-I & FXT
Net-proton $C_4/C_2$

- Non-monotonic beam energy dependence (3.1$\sigma$) of net-p $C_4/C_2$ in Au+Au central collisions.
- Enhancement at ~7.7 GeV is not reproduced by non-critical baselines.
- Qualitatively consistent with the model prediction incorporating a critical point.
• No clear enhancement is observed for 2.4 and 3.0 GeV data from HADES and STAR.
• Negative value at 3GeV is reproduced by UrQMD, which incorporates baryon number conservation.
• The data implies that the QCD critical region could only exist at energies > 3GeV.
Net-proton $C_6/C_2$ for crossover search

- $C_6/C_2$ values are progressively negative from peripheral to central collisions at 200 GeV, which is consistent with LQCD calculations.
- Could suggest a smooth crossover transition at top RHIC energy.

*STAR, PRL127.262301(2021)*
Energy dependence of $C_5/C_1$ and $C_6/C_2$

- The $C_6/C_2$ values decrease with decreasing the collision energy, which is quite similar to the LQCD calculation.

**STAR, PRL130.082301(2023)**

- The $C_6/C_2$ values decrease with decreasing the collision energy, which is quite similar to the LQCD calculation.

**STAR preliminary: $R_{12}^B$**

Bazavov et al, PRD101.074502(2020)
New results

Notation

\( \langle X^r \rangle_c \) : \( r \)th-order cumulant of particle \( X \)
\( \langle XY \rangle_c = \langle XY \rangle - \langle X \rangle \langle Y \rangle \) : 2nd-order mix-cumulant b/w \( X \) and \( Y \)
\( \langle BS \rangle_c \) : 2nd-order mix-cumulant b/w net-baryon and net-strangeness
\( \langle S^2 \rangle_c \) : 2nd-order net-strangeness cumulant
Revisiting 2\textsuperscript{nd}-order fluctuations

• Mix-cumulants among conserved charges are suggested to be sensitive to the magnetic field as well as the temperature.
• Previous STAR measurements on baryon-strangeness correlations are far away from the theoretical guidance.

$H$-T.Ding et al, EPJA57.202(2021)

A. Bazavov et al, PRL111.082301(2013)

V. Koch et al, PRL95.182301(2005)

$C_{BS} = -3 \frac{\langle BS \rangle_c}{\langle S^2 \rangle_c}$

STAR, PRC105.29901(2022): $p, \bar{p}, K^\pm$
What is missing?

- Model studies indicate that the most of baryon-strangeness correlations are carried by hyperons.
- Measuring event-by-event fluctuations of hyperons is challenging, because of the combinatorial backgrounds and low reconstruction efficiency.

\[ \langle N_{\text{part}} \rangle \]

*UrQMD: Z. Yang et al, PRC95.014914(2017)*
Purity correction: methodology

T. Nonaka, *NIMA.1039.167171*(2022)

- $\Lambda_S$ and $\Lambda_N$ cannot be obtained directly.

\[
\Lambda_{SN} = \Lambda_S + \Lambda_N
\]

\[
\langle \Lambda_S^2 \rangle_c = \langle \Lambda_{SN}^2 \rangle_c - \langle \Lambda_S^n \rangle_c - 2\langle \Lambda_S \Lambda_N \rangle_c
\]

**Assumption**

Particle number distribution of the backgrounds under the signal peak is consistent with that in sideband.

\[
\langle \Lambda_S^2 \rangle_c = \langle \Lambda_{SN}^2 \rangle_c - \langle \Lambda_R^n \rangle_c - 2\langle \Lambda_{SN} \Lambda_{R,i} \rangle_c + 2\langle \Lambda_{R,i} \Lambda_{R,j} \rangle_c \quad (i \neq j)
\]

- If purity correction works, the efficiency/purity corrected cumulants should be consistent w.r.t various topological cuts having different efficiency/purity.
The 2\textsuperscript{nd}-order $\Lambda$ cumulant is analyzed for various topological cut conditions having different purity/significance, which increases with decreasing the purity.
Purity correction: validation in STAR data

- The 2\textsuperscript{nd}-order $\Lambda$ cumulant is analyzed for various topological cut conditions having different purity/significance, which increases with decreasing the purity.

- Purity-corrected cumulants are flat w.r.t purity, and crosses with the uncorrected cumulants at the highest purity, indicating the validity of the methodology.

- First time to address the effect of combinatorial backgrounds on event-by-event hyperon fluctuations.
Net-hyperon cumulants

- First measurement of net-$\Xi$ ($\Xi^- - \Xi^+$) fluctuations.
- $\langle \Delta \Xi^2 \rangle_c$ is systematically enhanced w.r.t the Poisson baseline in central collisions, while $\langle \Delta \Lambda^2 \rangle_c$ is consistent with the Poissonian.
Results

- The $C_{BS}$ values are significantly enhanced by including $\Lambda$ and $\Xi$ hyperons compared to the previous measurements.

- The $C_{BS}$ values are systematically larger than the Poisson baselines.

- The largest enhancement is seen in most central collisions in case including $\Xi$. 

![Graph showing $C_{BS}$ values comparison]
Comparison with LQCD

- Our measurements are consistent with LQCD calculations so far.
- Caution: the $C_{BS}$ value strongly depends on particle types included in the measurements.
Comparison with UrQMD

• UrQMD is analyzed by using the same particle species (+ Σ^0) as the measurements.

• UrQMD underestimates the data.
Summary

• Some interesting hints on the QCD critical point, crossover, and phase boundary have been obtained through the measurements of higher-order cumulants of net-proton distributions. Stay tuned for precise measurements for BES-II.

• Purity correction has been established to remove the effect of combinatorial backgrounds from hyperon number fluctuations.

• The values of baryon-strangeness correlations (C_{BS}) have been significantly enhanced by including Λ and Ξ hyperons.

• Theoretical inputs are needed to make physics conclusion.
Thank you for your attention
Higher-order fluctuation

• Moments and **cumulants** are mathematical measures of “shape” of a distribution, which probes fluctuations of an observable.

  **Skewness (S)** → asymmetry

  ![Negative Skew](image1) ![Positive Skew](image2)

• **Cumulant ↔ Central moment**

  \[
  \begin{aligned}
  C_1 &= \langle N \rangle , \quad C_2 = \langle (\delta N)^2 \rangle \quad \delta N = N - \langle N \rangle \\
  C_3 &= \langle (\delta N)^3 \rangle \quad C_4 = \langle (\delta N)^4 \rangle - 3 \langle (\delta N)^2 \rangle^2 \\
  C_5 &= \langle (\delta N)^5 \rangle - 10 \langle (\delta N)^2 \rangle \langle (\delta N)^3 \rangle \\
  C_6 &= \langle (\delta N)^6 \rangle + 30 \langle (\delta N)^2 \rangle^3 - 15 \langle (\delta N)^2 \rangle \langle (\delta N)^4 \rangle
  \end{aligned}
  \]

• **Cumulants have additivity**: proportional to the system volume

  \[
  C_n(X + Y) = C_n(X) + C_n(Y)
  \]
C₅ and C₆ for crossover search

- No direct experimental evidence for a smooth crossover at μₜ~0 MeV.
- C₆/C₂ < 0 is predicted as a sign of crossover transition
- High-statistics data sets at 27, 54.4, 200 GeV were analyzed.

**A. Bazavov et al, PRD.95.054504**


<table>
<thead>
<tr>
<th>Freeze-out conditions</th>
<th>(x^B_4/x^B_2)</th>
<th>(x^B_6/x^B_2)</th>
<th>(x^Q_4/x^Q_2)</th>
<th>(x^Q_6/x^Q_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRG</td>
<td>1</td>
<td>1</td>
<td>~2</td>
<td>~10</td>
</tr>
<tr>
<td>QCD:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(T^{\text{freeze}}/T_{pc} \lesssim 0.9)</td>
<td>~1</td>
<td>~1</td>
<td>~2</td>
<td>~10</td>
</tr>
<tr>
<td>QCD:</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>(T^{\text{freeze}}/T_{pc} \approx 1)</td>
<td>~0.5</td>
<td>&lt;0</td>
<td>~1</td>
<td>&lt;0</td>
</tr>
</tbody>
</table>

**Predicted scenario for this measurement**
System size dependence

- Ratios approach LQCD calculations with increasing the multiplicity, which imply that the created system approach thermalized medium at high multiplicity region.


Ho-San Ko, QM2022
Purity correction: methodology

T. Nonaka, *NIMA.1039.167171* (2022)

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Sideband cumulants

- Sidebands are divided into small windows based on the yield of the signal candidates.
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- Sidebands are divided into small windows based on the yield of the signal candidates.
- The flatness seen in 2nd-order cumulants and correlation with signal candidates w.r.t. the invariant mass imply the particle number distribution is similar for those regions.

![Cumulant plots showing Sideband cumulants for different mass regions and particle yields.](https://via.placeholder.com/150)
Correlation terms

- Most correlation terms have positive contribution on $\langle BS \rangle_c$.

$$
\langle BS \rangle_c = \langle (\Delta p + \Delta \Lambda + \Delta \Xi)(\Delta K - \Delta \Lambda - 2 \Delta \Xi) \rangle_c \\
= \langle \Delta p \Delta K \rangle_c - \langle \Delta p \Delta \Lambda \rangle_c - 2 \langle \Delta p \Delta \Xi \rangle_c + \langle \Delta \Lambda \Delta K \rangle_c - \langle \Delta \Lambda \rangle_c^2 - 3 \langle \Delta \Lambda \Delta \Xi \rangle_c + \langle \Delta \Xi \Delta K \rangle_c - 2 \langle \Delta \Xi \rangle_c^2
$$